Mean Reversion of Interest-Rate Term Premiums and Profits from Trading Strategies with Treasury Futures Spreads

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Financial futures contracts offer investors a wide variety of spread positions. This article studies two of the most popular types of intercommodity spread strategies on Treasury futures contracts: the note over bond (NOB) spread and the 5-year note over bond (FOB) spread. The NOB and the FOB spread strategies attempt to isolate the amount of the spread between Treasury instruments of different maturities due to the different ways their price movements respond to similar changes in yield. In addition, the spreads offer a good vehicle for taking a position on a view of the shape of the yield curve.

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During periods of economic expansion when demand for funds increases, interest rates normally rise, with short-term rates rising faster than long-term rates. When short-term rates are below long-term rates, the yield curve has a positive slope. As a cyclical peak approaches, short-term rates may rise above long-term rates and the yield curve will invert, taking on a negative slope. When a recession follows the cyclical peak, rates begin to fall, with short-term rates normally falling faster than long-term rates. Eventually, this should again lead to a positively sloped yield curve. Thus, the spread between long-term rate and short-term rate, or term premium of interest rates, changes as the economy goes through various cycles.

The cyclical trend of term premium could be used to realize a profitable trading strategy based on the futures spread on the long-term and short-term Treasury issues. Although changes in the Treasury bill over the Treasury bond spread also capture the change in the shape of the yield curve, this study focuses on the spread between long-term and intermediate-term interest rates by using the note over bond spread. In contrast to the T-bill T-bond futures spread, both note and bond futures contracts are traded on the same exchange (the Chicago Board of Trade) and have similar structures. This makes it much easier to engage in NOB spread trading as opposed to T-bill/T-bond spread trading.

The rest of the article is organized as follows. In the next section, a test for mean reversion of interest rate term premiums during a five and one half year period is presented. The second section analyzes the time-series behavior of the NOB and the FOB spreads with the equivalent spot price spreads, and finds that the futures spreads and the spot spread are significantly co-integrated. The third section tests the forecastability of the mean reversion of the futures spread, and designs a trading strategy based on the mean reverting pattern of the spread. The strategy is applied to the actual futures prices for the next 5½-year period. The average profits from the spread trades are significantly positive and the returns are, on average, much higher than the risk-free investment, whereas the risk of the trades is comparable to that of the risk-free investment. Some concluding remarks are provided in the final section.

MEAN REVERSION OF TERM PREMIUMS OF LONG-TERM OVER INTERMEDIATE-TERM INTEREST RATES

Mean reversion of interest-rate series is a common assumption for pricing debt securities, for example, Brennan and Schwartz (1982), Cox, Ingersoll, and Ross (1985), Ramaswamy and Sundaresan (1986), and has been

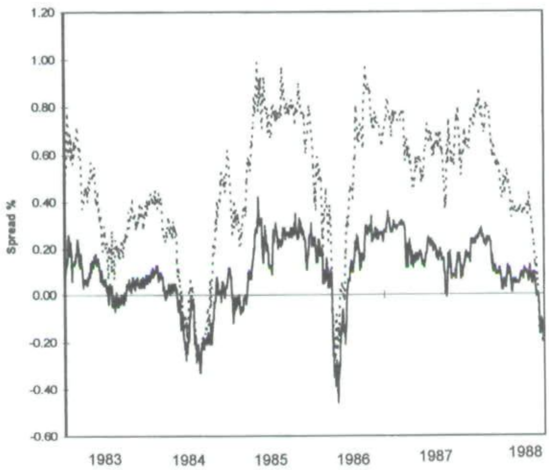

FIGURE 1

Term Premiums of Interest Rates (1/3/83–5/31/88) (Solid line, 20-year rate over 10-year rate; dotted line, 20-year rate over 5-year rate). The daily interest rates are the Treasury constant maturity rates for 5 years, 10 years, and 20 years, respectively, and are from the Federal Reserve Report H.15.

documented by Keim and Stambaugh (1986), Fama and Bliss (1987), Fama (1990), and Jorion and Mishkin (1991), among others. Mean reversion of interest-rate term premiums has been shown in Fama and French (1989) and Jorion and Mishkin (1991). The expected term premium is typically nonzero and varies between positive and negative values, in accordance with the business cycle. No previous study has tested for mean reversion of the long-term over the intermediate-term premium, which is applicable to the NOB or the FOB spreads.

Figure 1 shows the term premiums of interest rates between January 3, 1983 and May 31, 1988. In the figure, the time-varying and cyclical natures of term premiums are evident, which is consistent with long-term over short-term interest-rate premiums. The two premiums are the 20-year rate over the 10-year rate and the 20-year rate over the 5-year rate. The 20-, 10-, and 5-year rates are used as the relevant spot rates for Treasury bond, 10-year-note, and 5-year-note futures. The deliverable bonds for the bond futures are T-bonds having at least 15 years remaining until maturity or to their first call date. Those for the 10-year-note futures are T-notes with maturities between 6½ years and 10 years, and those for the 5-year note futures are T-notes with maturities greater than 4 years and 3 months and less than 5 years and 3 months.

The interest rates used in this study are the rates with the longest terms in each maturity category of note futures, e.g., the 10-year rate for

TABLE I

Autocorrelation Series of Interest-Rate Term Premiums (1/3/83–5/31/88)

			Autocorrelations (Daily Lag)									
Series	$E(x)$	$\sigma(x)$	*1*	*2*	*3*	*5*	*10*	*50*	*100*	*150*	*200*	Q^*
$r_{20} - r_{10}$	0.104	0.147	0.98	0.97	0.96	0.93	0.86	0.44	0.16	−0.05	−0.14	41,038
$r_{20} - r_5$	0.440	0.540	0.99	0.98	0.97	0.94	0.90	0.53	0.17	−0.05	−0.16	49,177

Q^* is the Ljung–Box test for the joint significance of autocorrelations and distributed χ^2 under the null hypothesis of no significance. The 95% critical value of significance is 146.5.

Correlation of the Nonoverlapping Changes in Interest-Rate Term Premiums

	T(Daily Lag)								
Series	*1*	*2*	*3*	*5*	*10*	*50*	*100*	*150*	*200*
$(r_{20} - r_{10})_{t+T} - (r_{20} - r_{10})_t$ and $(r_{20} - r_{10})_t - (r_{20} - r_{10})_{t-T}$	−0.14	−0.02	0.04	0.02	−0.04	−0.21	−0.35	−0.54	−0.59
$(r_{20} - r_5)_{t+T} - (r_{20} - r_5)$ and $(r_{20} - r_5)_t - (r_{20} - r_5)_{t-T}$	−0.03	0.03	0.05	0.02	−0.01	−0.08	−0.31	−0.55	−0.57

10-year-note futures, and the 5-year rate for 5-year-note futures. Because the actual note futures prices depend on the cheapest-to-deliver notes, a note with the lowest price dictates the futures price. Because longer-term rates are usually higher than short-term rates in a typical upward sloping yield curve environment (so that longer-term notes would be expected to determine futures prices), using the longest-term rates in each category of maturities should not pose any serious bias for comparison purposes. The interest rate applicable to bond futures is based on 20 years, because 20 years represent the maturity implicit in bond futures.[1]

Table I shows the autocorrelation series of daily interest-rate term premiums for the 20-year over the 10-year rate and the 20-year rate over the 5-year rate. During the January, 1983 and May, 1988 period, the premium of the 20-year rate over the 10-year rate averages 0.105% and the premium of the 20-year rate over the 5-year rate averages 0.450%. The autocorrelations of both term premiums are close to 1.0 at short lags,

[1]Although bond futures can have deliverable bonds with maturities up to 30 years, there usually is little slope in the yield curve after 20 years. In addition, the delivery value of bond futures are based on 20-year bonds. Recently, bond futures have become a popular method to hedge mortgage-backed securities whose durations are much closer to 20-year bonds than to 30-year bonds.

but decay across longer lags. When the daily lag approaches 150, the term premiums change sign and stay that way. If the term premium is highly autocorrelated but slowly mean reverting, the correlation of the nonoverlapping changes, $(prem_{t+T} - prem_t)$ and $(prem_t - prem_{t-T})$, is close to 0.0 for small values of T but approaches -0.5 for large values of T.[2] As reported in Table I, the correlations between nonoverlapping changes approaches -0.5 with 150-day lags. Thus, the autocorrelations of term premiums and the correlation of the nonoverlapping changes in term premiums show mean reversions occurring at around 150 days apart. Overall, these results are consistent with the burgeoning evidence on return predictability in asset markets [e.g., the survey in Fama (1991)]. The remaining question is whether the forecasting power of such mean reversion of term premiums can be utilized in trading the NOB and the FOB spreads.

THE FUTURES SPREADS AND THE SPOT SPREADS

Changes in the NOB (and the FOB) spread depend on two factors: general interest-rate movements and changes in the yield curve shape. Because the prices of longer-term issues are more sensitive to a given parallel change in yield than prices of shorter-term notes, the spread will widen if the overall interest rates increase. If interest rates decline in a parallel trend, the spread will narrow. Shape changes in the yield curve have important implications also for the behavior of the NOB spread. Changes in the NOB spread will depend on the degree to which the term premium changes. When an upward-sloping yield curve begins to flatten due to a sharp increase in intermediate-term rates relative to the long-term rate, the NOB spread can narrow if the faster rise in the short-term rate offsets the greater price sensitivity of long-term issues. The opposite will occur when a negatively sloped yield curve begins to invert and turn positive. As rates decline, the greater fall in intermediate-term rates offsets the effect of bond prices rising faster than note prices, thus widening the NOB spread. Thus, the general movement of the NOB spread is determined by the relative changes in intermediate-term and long-term rates and the price sensitivities of short-term and long-term issues.

In general, if the term premium of interest rates follows a cycle of highs and lows with a mean reversion, it may be compelling to assume

[2]As Fama and Bliss (1987) demonstrate, if the correlation pattern of nonoverlapping changes in a variable has these characteristics, the variable itself in level form is mean reverting.

TABLE II
Unit Root Tests and Cointegration Tests (6/1/88–11/26/93)

Spot NOB Spread: 20-Year Rate over 10-Year Rate
Spot FOB Spread: 20-Year Rate over 5-Year Rate
Futures NOB Spread: Bond Futures Yield over 10-Year Note Futures Yield
Futures FOB Spread: Bond Futures Yield over 5-Year Note Futures Yield

A. Unit Root Tests

	Price Levels		Differences	
	PPT	PP	PPT	PP
Spot NOB Spread	−1.840	−.450	−35.661	−37.256
Spot FOB Spread	1.129	−.243	−33.790	−34.743
Futures NOB Spread	−2.049	−1.488	−35.502	−36.436
Futures FOB Spread	−2.134	−1.571	−34.382	−35.378
95% critical value	−3.70	−3.57	−3.50	−3.27

B. Cointegration Tests
$\text{Spot Spread}_t = \delta + \gamma\ \text{Futures Spread}_t + \varepsilon_t$

	DF	*ADF(4)*	*PP*
NOB Spread	−8.242	−3.534	−6.740
FOB Spread	−10.436	−4.445	−8.238
95% critical value	−3.17	−3.37	−3.47

The PPT is the Phillips–Perron *t* statistic in an estimated model with a time trend. The PP is the corresponding statistic in the estimated model without a time trend. The DF is the Dickey–Fuller statistic for the cointegration equation between spot and futures spreads. The ADF(4) statistic is a fourth-order augmented Dickey–Fuller test for co-integration. Critical values can be found in Engle and Granger (1987) and Phillips and Ouliaris (1990).

that the NOB (and the FOB) spread should also follow a similar cycle. However, such an assumption cannot be made until an exact relationship between the spread of futures contracts with the equivalent interest-rate term premiums is found.

For the remainder of the article the futures yield spreads, rather than the futures price spreads, are referred to as the futures spread because the futures trading strategies are based on the signals of the possible reversion in futures yield spreads. Several tests on the distributional properties of the futures and the spot spreads are performed and the results are reported in Table II. The futures spreads are the differences between bond futures yields (calculated based on an 8% coupon, 20-year bond) and note futures yields (calculated based on 8% coupon, 5-year and 10-year notes). Although the quoted prices of the futures contracts are not necessarily based on 8% coupon issues, such simplification is applied for expositional purposes. Table II shows the stationarity test results for the spot and the futures data with the use of the Phillips and Perron (1988) test for unit roots. If the Phillips and Perron t statistic is below the critical value, the null hypothesis of the presence of a unit root cannot be rejected. The results show the significance of unit roots and the nonstationarity of the yield spread, but not of the first differences of the yield spread series. Because any relationship between two nonstationary series (each of which becomes stationary on first differencing) can be spurious, tests of co-integration between the two series are conducted also.

Table II reports the results of co-integration tests between the spot and the futures spreads. Based on the tests of the residuals of the co-integrating equation, it is apparent that the spot and the futures spreads are significantly co-integrated. If the spot and the futures spreads are co-integrated, then the mean reverting pattern is also expected from the futures spread. With this finding, filter rules based on mean reversion after 150 days are introduced to the NOB and the FOB spreads for the subsequent 5½-year period.

SPREAD TRADING STRATEGIES AND THE SIMULATED TRADING RESULTS

Data and the Preliminary Test for the Forecastability of Mean Reversion

Daily data on the CBOT Treasury bond, 10-year Treasury note, and 5-year Treasury note futures contracts and equivalent spot rates are from Commodity Services Inc. for an approximately 5½-year period from June 1, 1988 to November 26, 1993. All three contracts mature in March, June, September, and December. Only the most recent contracts are considered and they are rolled over to the next contracts on the expiration months. As a result, the nearest contract series provides 1391 observations of the daily settlement prices of the bond and the note futures.

On each day, futures yields for 20-, 10-, and 5-year Treasuries are calculated from futures settlement prices. The yield spread (20 over 10 or 20 over 5) from futures contracts is computed and compared to the 150-day moving average of the previous 150 days. The 150-day lag is chosen because of the expected mean reversion of the spread after 150 days. If the direction of the change in the spread changes after 150 days, a futures position can be taken to take advantage of the expected direction of the spread. Profits are expected if the anticipated convergence/

divergence of the two futures prices occurs. For both the NOB and the FOB spreads, the following filter rules are possible to initiate a position.

$$\text{Long if } SD_t < MA_t - \alpha\, MA_t$$
$$\text{Short if } SD_t > MA_t + \alpha\, MA_t$$

$$\text{Long if } SD_t < MA_t - \beta\, STD_t$$
$$\text{Short if } SD_t > MA_t + \beta\, STD_t$$

where SD_t is the spread, MA_t is the moving average with 150-day lag, and STD_t is the moving standard deviation with 150-day lag, on day t. In the first case, if the term spread is lower than its moving average, it is expected to rise soon based on the 150-day cycle. To take advantage of the rising term spread, bond futures are sold and note futures are bought (i.e., long NOB or FOB position). The constants, α and β, are the tolerance levels on the deviation from the moving average, where the former is in proportion to the moving average and the latter in proportion to the standard deviation. As the tolerance level increases, fewer positions will be taken overall, but whether profits will increase remains to be seen. To make the trading strategies selective, a double-filter rule is applied such that both 150-day moving averages and standard deviations are used to create a filter.

$$\text{Long if } SD_t < MA_t - \alpha\, MA_t \quad \text{and} \quad SD_t < MA_t - \beta\, STD_t \tag{1a}$$

$$\text{Short if } SD_t > MA_t + \alpha\, MA_t \quad \text{and} \quad SD_t > MA_t + \beta\, STD_t \tag{1b}$$

A preliminary test requires an analysis of whether the above filters can provide forecasts of the movement of futures term spreads that are different from what might have been expected from an efficient market. An appropriate statistical procedure is a market-timing test based on the hypergeometric distribution that was developed by Merton (1981) and Henriksson and Merton (1981). The Henriksson–Merton test is implemented here to assess the reliability of the filter's signal of a reversion in the direction of changes in the term spread. Table III provides the Henriksson–Merton test statistics on the forecastability of term spread movements with different number of days into the future subsequent to a reversion signal.

Once a signal is made, the signal is correct if the spread change from the day of the signal conforms to the mean reverting pattern forecasted by the filter. The p stat in Table III shows the significance of such forecastability against a random prediction for 10 different tolerance levels. For the NOB spread, tighter filters (lower tolerance levels) result in more

TABLE III

Test Statistics of Forecastability of Spread Changes by Filter Rules (6/1/88–11/26/93)

	α	β	*p Stat* after (waiting period in days)* 10	20	30	50	70	100	120	150
NOB										
	0.1	0.2	1.20*	1.26*	1.31*	1.34*	1.45*	1.47*	1.41*	1.20*
	0.2	0.4	1.15*	1.26*	1.32*	1.42*	1.54*	1.59*	1.49*	1.30*
	0.3	0.6	1.21*	1.27*	1.39*	1.53*	1.64*	1.70*	1.56*	1.40*
	0.4	0.8	1.26*	1.35*	1.48*	1.62*	1.77*	1.82*	1.66*	1.51*
	0.5	1.0	1.22*	1.38*	1.61*	1.69*	1.88*	1.93*	1.78*	1.68*
	0.6	1.2	1.17	1.29*	1.60*	1.69*	1.93*	1.95*	1.90*	1.79*
	0.7	1.4	1.11	1.29*	1.61*	1.67*	1.93*	1.94*	1.92*	1.83*
	0.8	1.6	1.10	1.42*	1.66*	1.74*	2.00*	1.98*	2.00*	1.81*
	0.9	1.8	1.40	1.50*	1.79*	1.96*	2.00*	1.94*	2.00*	1.88*
	1.0	2.0	1.17	1.20	2.00*	2.00*	2.00*	2.00*	2.00*	1.50*
FOB										
	0.1	0.2	1.16*	1.20*	1.24*	1.38*	1.47*	1.50*	1.50*	1.39*
	0.2	0.4	1.16*	1.17*	1.23*	1.35*	1.51*	1.55*	1.52*	1.42*
	0.3	0.6	1.15*	1.11*	1.17*	1.30*	1.50*	1.59*	1.51*	1.46*
	0.4	0.8	1.10	1.07	1.16*	1.29*	1.51*	1.63*	1.54*	1.49*
	0.5	1.0	1.04	0.99	1.14	1.32*	1.52*	1.61*	1.59*	1.54*
	0.6	1.2	0.95	0.93	1.08	1.27*	1.56*	1.58*	1.63*	1.64*
	0.7	1.4	1.02	0.88	0.95	1.14	1.54*	1.53*	1.59*	1.72*
	0.8	1.6	0.92	0.89	0.84	0.91	1.50*	1.44*	1.50*	1.68*
	0.9	1.8	0.89	0.88	0.79	0.81	1.49*	1.42*	1.48*	1.69*
	1.0	2.0	0.86	0.80	0.97	0.68	1.41*	1.38*	1.39*	1.65*

[a] *p*-stat $= n_1/N_1 + n_2/N_2$ where n_1 = number of correct signals by the filter rule given an increase in the spread, N_1 = number of cases where the spread increases, n_2 = number of incorrect signals given a decrease in the spread, n_3 = number of correct signals given a decrease in the spread, and N_2 = number of cases where the spread declines. The critical value of *p*-stat depends on the total number of signals, which is different for each filter and each waiting period. Specifically, the confidence level on which to base the significance test is calculated as in McIntosh and Dorfman (1992)

$$c = 1 - \sum_{x=n_1}^{\min(N_1, n)} \binom{N_1}{x}\binom{N_2}{n-x} \Big/ \binom{N}{n}, \quad \text{where } N = N_1 + N_2, \text{ and } n = n_1 + n_2$$

*Rejects the null hypothesis that the filter rule does not forecast the change of term spread at the 99% confidence level.

significant forecastability, or more frequent accurate prediction, of changes in the spread after 10 days from the signal date. With 20 days or longer after the signal is made, reversion of the spread is significantly forecasted at all tolerance levels.[3] For the FOB spread, the reversion takes longer and is forecasted more accurately by the filter rule with a longer waiting period. Similar to the NOB spread, the FOB spread reverses its

[3] At $\alpha = 1$ and $\beta = 2$, the reversion signal is not significantly forecasted for the 20-day period. This result may be a consequence of the small sample size for the high tolerance level signal. As is shown in Table IV(a), only 16 signals are made during the testing period.

course faster with lower tolerance levels (tighter filters). At all tolerance levels, the FOB spread's reversion is significantly forecasted 70 days after a reversion signal is made. Thus, a simple trading strategy can be based on staying with a spread position at least 20 (70) days once a possible reversion signal is made for the NOB (FOB) spread. Alternatively, a more refined trading strategy is to stay in a given position until a predetermined level of the change in the spread is realized. This strategy is discussed in the next section.

Simulation Design

Buying or selling intercommodity spreads between Treasury issues is generally done on a 1:1 basis, if the trader expects that changes in yields will be approximately equal for all maturities, or if he intends to enter and exit the market quickly. If the trader's goal is to profit from changes in the shape of the yield curve, it is often more prudent to weight the spread so as to hold the relative price sensitivity of the two instruments constant [Sehap (1992)]. This can be accomplished by setting up an equivalency ratio, which takes into account the duration of the cheapest-to-deliver issues. The equivalency ratio is calculated as follows:

$$\begin{array}{c}\text{Equivalency Ratio}\\ \{\text{Notes per Bond}\}\end{array} = \frac{BPV_{\text{CTD Bond}}}{BPV_{\text{CTD Note}}} \times \frac{CF_{\text{CTD Note}}}{CF_{\text{CTD Bond}}} \tag{2}$$

where BPV is the basis point value, or the price change of a debt instrument given a one-basis-point (0.01 percent) change in the yield of that instrument (i.e., discretized duration), CTD is the cheapest-to-deliver issue, and CF is the conversion factor for each contract. By trading the equivalency ratio of note futures against bond futures, the investor can isolate the spread from all but changes in curve shape. Thus, the equivalency ratio is the hedge ratio that immunizes the spread position against any change in the overall interest rates.

As in a common calculation of profit from spread positions, a conversion factor of 1 is assumed and the accrued coupons are ignored for the simulation. The equivalency ratio is calculated as the ratio of durations of bonds and notes. In computing the durations of bonds and notes, 8% coupon bonds with 20 years to maturity and 8% coupon notes with 10 years and 5 years to maturity are assumed. Although durations change with the underlying interest rates and thus require changing the ratio daily, a constant interest rate of 8% is applied in all cases. For example, on January 4, 1989 (which is 150 days after the first date in the database), the ratio of bonds over 10-year notes is 1.47 and the ratio of bonds over 5-year notes is 2.38. If a long NOB (FOB) trade is initiated on this date, 147 (238) 10-year-note futures (5-year-note futures) are bought and 100 bond futures are sold. Once the equivalency ratio is calculated on the trade date, the ratio remains constant until the position is closed. Using a constant equivalency ratio for each trade ensures that transactions costs remain reasonable during volatile periods.

Once a trade is initiated, the position needs to be closed after the level of term spread supposedly reverses its course. A simple rule is used to close each position when the difference between the spread and its 150-day moving average changes sign:

$$\text{close long position if } SD_t \geq MA_t \tag{2a}$$

$$\text{close short position if } SD_t \leq MA_t \tag{2b}$$

For example, once a long (short) position is taken, it is held until the spread reaches or exceeds (falls below) the 150-day moving average, and no new one is taken until the next significant deviation occurs.

The trading simulations that follow are based on the following assumptions:

1. Each simulated position is opened on the next day after the trading signal is made. Simulated NOB positions are opened and closed at every day's settlement prices.
2. One contract for each futures is bought or sold on a \$100,000 face value. To have both a meaningful equivalency ratio and investment amounts, 10 bond futures contracts and the equivalent note futures are traded. If the equivalency ratio is 1.53, for instance, 15 note futures are bought/sold for 10 bond futures sold/bought.
3. The rate of return computations assume that an investor posts \$600,000, or 15%, of the initial dollar value of his position for 40 futures contracts as margin. In all cases, the maximum number of bond and note futures contracts required does not exceed 40. Thus, \$600,000 is set aside for initial margin, enough for 40 contracts, because a fixed amount for initial investment is required to compare different trades. The margin requirement of 15% is a much higher margin requirement for a spread position than usual.[4] This relatively conservative strategy was chosen to avoid margin calls.

[4]The margin required for a spread between a 10-year T-note and T-bond futures is \$500 per position, compared with \$2000 for bonds and \$1000 for notes for an outright position. With the spread position, the probability of the position falling below the minimum margin requirement is less than an outright position.

4. Normally, large account holders would satisfy 70–90% of their margin requirements by posting Treasury bills. The interest income they would earn is significant in some trades that last several months. On each trade, \$420,000, or 70% of \$600,000, earns the 3-month Treasury-bill rates prevailing during the trading period.
5. Transactions costs are assumed to be \$100 per round trip, per contract. Based on a survey, discount brokers typically charge, for a round-trip trade, retail customers as little as \$29.75 to \$50 for an outright position in T-note futures and T-bond futures, and \$40 to \$60 for a NOB spread position. Also, the bid-ask spread is taken into account by deducting the amount for one tick size (\$31.25) from the profit in the transaction of each contract. The same transactions costs are considered when the futures contracts are rolled over to the next contracts on the first day of the expiration months.

Simulation Results

Table IV(A) presents the results of the trading simulations over the 5½-year period from June, 1988 to November, 1993. All reported results are net of transaction costs. The simulation results are based on 12 different tolerance levels with respect to the moving average (α) and standard deviation (β). The total profit is the sum of all the profits from all possible trades with a \$600,000 initial investment in each trade. The average profit is the average of profits earned by all the trades. The average daily profit is the "weighted" average of daily profits from each trade, which are the profits from each trade divided by the number of days in the trade. As shown below in (3*a*), this average daily profit assigns more weight on daily profits earned on longer periods, and it is the same as total profits divided by the total number of days in all the trades.[5]

$$avg.\ profit = \sum_{i=1}^{N} profit(i)/N = total\ profits/N,$$

$$avg.\ daily\ profit = \frac{\sum_{i=1}^{N} day(i) \cdot profit(i)/day(i)}{\sum_{i=1}^{N} day(i)} = \frac{total\ profits}{\sum_{i=1}^{N} days(i)}, \quad (3a)$$

[5]This way of weighing the number of days in each trade to the daily profit is designed to avoid a possible bias arising from the different number of days in each trade. For instance, the unweighted average daily profit can be still positive even if total profits from all the trades are negative. If most of the trades with negative profits are from staying in a position much longer than the trades with a positive profit, the unweighted average daily profit will be positive, but the correct average daily profit should be negative because the total profits are negative.

TABLE IV(a)

Trading Profits from the Moving Average NOB and FOB Spreads Strategies (6/1/88–11/26/93)

α	β	*Total Profit* ($)	*Average Profit per Trade* ($)	*Avg. Daily Profit* ($)	*Long Trades*	*Short Trades*	*Avg. No. of Days*	*Profitable Trades* (%)	σ_P	σ_{DP}
					NOB					
0.1	0.2	3378070.50	4975.07	151.61	215	464	33	75	6281.65	257.91
0.2	0.4	3196441.00	6874.07	184.24	129	336	37	88	5911.16	229.72
0.3	0.6	2892482.50	8057.05	202.71	94	265	40	94	5467.76	213.07
0.4	0.8	2191578.50	8766.31	211.64	76	174	41	98	5248.34	203.66
0.5	1.0	1519306.50	9264.06	205.92	63	101	45	99	4443.42	139.68
0.6	1.2	1084022.50	9854.75	195.00	42	68	51	100	3519.91	108.82
0.7	1.4	909731.00	10108.12	189.17	34	56	53	100	3284.29	91.35
0.8	1.6	753569.19	10921.29	197.22	24	45	55	100	2885.64	85.40
0.9	1.8	463860.63	11044.30	205.25	15	27	54	100	2518.07	85.04
1.0	2.0	149822.27	9363.89	247.64	1	15	38	100	2149.90	133.09
1.1	2.2	32572.71	8143.18	452.40	0	4	18	100	947.18	61.43
1.2	2.4	8932.60	8932.60	425.36	0	1	21	100	0.00	0.00
					FOB					
0.1	0.2	5879423.50	7645.54	170.18	239	530	45	76	15474.54	459.39
0.2	0.4	5358384.00	9568.54	188.48	153	407	51	80	15115.40	397.04
0.3	0.6	4579079.50	11007.40	199.99	104	312	55	84	13963.58	358.84
0.4	0.8	3404394.75	11501.33	191.79	84	212	60	85	13141.63	327.38
0.5	1.0	2713547.50	12278.50	196.72	69	152	62	86	13135.53	331.41
0.6	1.2	1999148.63	11555.77	177.36	62	111	65	84	13782.57	337.15
0.7	1.4	1431367.75	10524.76	152.05	48	88	69	79	14674.42	349.65
0.8	1.6	1093415.00	10513.61	145.44	30	74	72	78	14152.96	343.74
0.9	1.8	1034992.69	11499.92	157.70	25	65	73	78	13779.73	354.00
1.0	2.0	849185.19	11960.35	164.38	15	56	73	76	13727.70	376.95
1.1	2.2	636369.31	12987.13	190.59	3	46	68	71	15048.16	457.46
1.2	2.4	473019.84	12447.89	182.14	0	38	68	66	15973.81	475.08

where *profit*(i) and *days*(i) are the profit and the number of days in trade i, respectively, and N is the total number of trades in each simulation. Similarly, the standard deviation of profits (σ_P) and of daily profits (σ_{DP}) are calculated as:

$$\sigma_P = \left(\sum_{i=1}^{N} (\mathit{profit}(i) - \mathit{avg.\ profit})^2/N\right)^{1/2},$$

$$\sigma_{DP} = \left(\sum_{i=1}^{N} \mathit{days}(i)\left(\frac{\mathit{profit}(i)}{\mathit{days}(i)} - \mathit{avg.\ daily\ profit}\right)^2 \Bigg/ \sum_{i=1}^{N} \mathit{days}(i)\right)^{1/2} \tag{3b}$$

As the tolerance level increases or the filters are widened, fewer trades are initiated. However, this tends to increase the average daily profit per trade and reduces the standard deviation of daily profits, especially for the NOB trades. The average number of days to remain in a given spread position is between 1 month and 3 months (based on trading days). As indirectly predicted in the previous section, the FOB trades, on average, last longer than the NOB trades. For both NOB and FOB trades, more short positions than long positions are initiated, indicating that term spreads are increasing more frequently (so that more short trades are signalled). In fact, the simulation period misses the late 1993 and 1994 period during which the yield curve flattened substantially. Also, during the simulation period, the NOB trades are made less often than the FOB trades, but most of the NOB trades are profitable (greater than zero), as shown in the percentage of profitable trades.

Although most of the spread trades are profitable, a more relevant issue is their performance relative to a risk-free investment strategy. Rate of returns and daily rates of returns from the spread trades are calculated, and their summary statistics are shown in Table IV(b). The rates of returns are based on \$600,000 initial investment, and the average daily returns and standard deviation of daily returns are calculated by assigning the number of days in each trade as weights, as in the above example. The risk-free rate of return is the return from investing \$600,000 in 3-month Treasury bills (at the rates prevailing during the period of each trade) for the same number of days in each trade. The NOB trades perform remarkably well against the T-bills, whereas the FOB trades outperform the T-bills only slightly. For all tolerance levels, the average (daily) returns of the spread trades are greater than the equivalent returns from the T-bills. The risks of the spread positions shown as σ_R (for the return for each trade period) and σ_{DR} (for daily returns) are somewhat higher than that of the T-bills. The NOB trades have relatively lower risk than the FOB trades, and the risk from the NOB trades tends to decline with greater tolerance levels (although this decline is due in part to a decrease in the number of trades).

Figures 2(a) and 2(b) graphically illustrate the average daily excess returns from the NOB and FOB trades over the risk-free returns with different tolerance levels for the filter. Higher tolerance levels (wider filters) than shown in the graphs are not implemented because the number of trades decline drastically with higher tolerance levels, especially for the NOB spread. In all cases for the NOB and most for the FOB, excess returns are positive. Increasing the tolerance level of moving average, α, raises the excess returns, although not uniformly. Increasing the tolerance level for β, however, does not raise the excess returns, especially with the

TABLE IV(b)

Average Returns from the Moving Average NOB and FOB Spreads Strategies (6/1/88–11/26/93)

α	β	R (%)	R_F	σ_R	σ_F	*Positive Excess Return* (%)	DR	DR_F	σ_{DR}	σ_{DF}
						NOB				
0.1	0.2	0.829	0.512	1.047	0.505	61	0.025	0.017	0.043	0.007
0.2	0.4	1.146	0.612	0.985	0.513	70	0.031	0.017	0.038	0.007
0.3	0.6	1.343	0.673	0.911	0.511	75	0.034	0.018	0.036	0.007
0.4	0.8	1.461	0.785	0.875	0.538	79	0.035	0.019	0.034	0.006
0.5	1.0	1.544	0.975	0.741	0.548	80	0.034	0.022	0.023	0.004
0.6	1.2	1.642	1.137	0.587	0.540	79	0.033	0.023	0.018	0.002
0.7	1.4	1.685	1.215	0.547	0.532	80	0.032	0.023	0.015	0.003
0.8	1.6	1.820	1.244	0.481	0.523	86	0.033	0.023	0.014	0.002
0.9	1.8	1.841	1.216	0.420	0.528	93	0.034	0.023	0.014	0.002
1.0	2.0	1.561	0.801	0.358	0.495	94	0.041	0.022	0.022	0.002
1.1	2.2	1.357	0.388	0.158	0.066	100	0.075	0.022	0.010	0.001
1.2	2.4	1.489	0.456	0.000	0.000	100	0.071	0.023	0.000	0.000
						FOB				
0.1	0.2	1.274	0.719	2.579	0.666	61	0.028	0.016	0.077	0.006
0.2	0.4	1.595	0.836	2.519	0.649	63	0.031	0.016	0.066	0.006
0.3	0.6	1.835	0.949	2.327	0.643	64	0.033	0.017	0.060	0.006
0.4	0.8	1.917	1.099	2.190	0.619	64	0.032	0.018	0.055	0.005
0.5	1.0	2.046	1.236	2.189	0.623	62	0.033	0.020	0.055	0.004
0.6	1.2	1.926	1.377	2.297	0.620	59	0.030	0.022	0.056	0.003
0.7	1.4	1.754	1.497	2.446	0.643	56	0.025	0.022	0.058	0.002
0.8	1.6	1.752	1.557	2.359	0.652	56	0.024	0.022	0.057	0.002
0.9	1.8	1.917	1.575	2.297	0.649	57	0.026	0.022	0.059	0.002
1.0	2.0	1.993	1.551	2.288	0.672	54	0.027	0.022	0.063	0.002
1.1	2.2	2.165	1.406	2.508	0.761	51	0.032	0.021	0.076	0.002
1.2	2.4	2.075	1.389	2.662	0.751	47	0.030	0.020	0.079	0.001

R and R_F are the average return per trade from the spread trades and risk-free investment, respectively. σ_R and σ_F are the standard deviation of the returns from the spread trades and risk-free investment, respectively. DR, DR_F, σ_{DR}, and σ_{DF} are the daily average returns and standard deviations of returns from these two types of investments.

FOB spreads. These figures clearly illustrate the appropriate filters a potential investor should adopt to maximize profits from spread trading.

Analysis of the performance of the spread trades' returns relative to risk-free investment is difficult to implement, because each trade involves a position taken for a nonstandard time frame. To account for the heterogeneity in the length of time in the spread positions, the average and standard deviation of daily returns is weighed by the number of days for each trade. In Figures 3(a) and 3(b) some graphical illustrations are provided of the comparable performance of the (NOB and FOB) spread trades relative to the risk-free investments (T-bills) as well as naked positions in note or bond futures. Each figure is divided into four separate

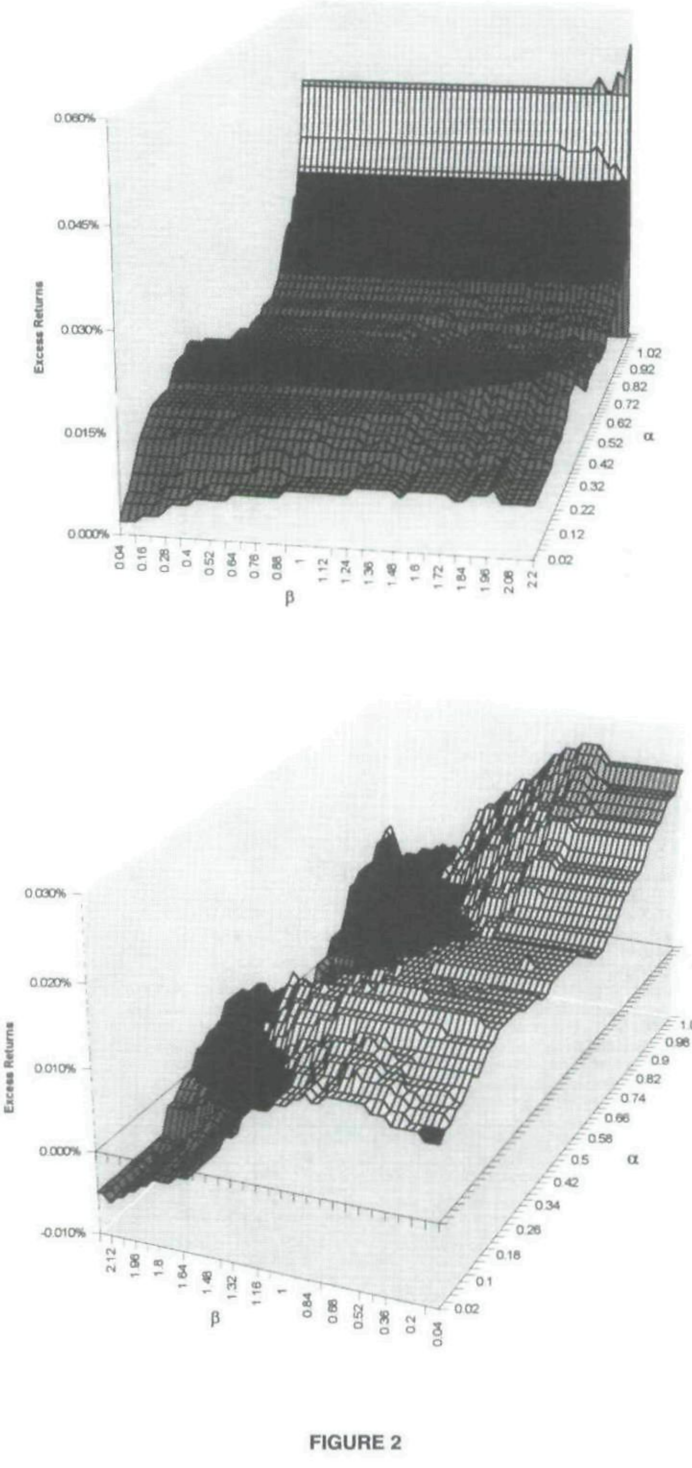

FIGURE 2

(a) Average daily excess returns from the NOB spread trading. (b) Average daily excess returns from FOB spread trading.

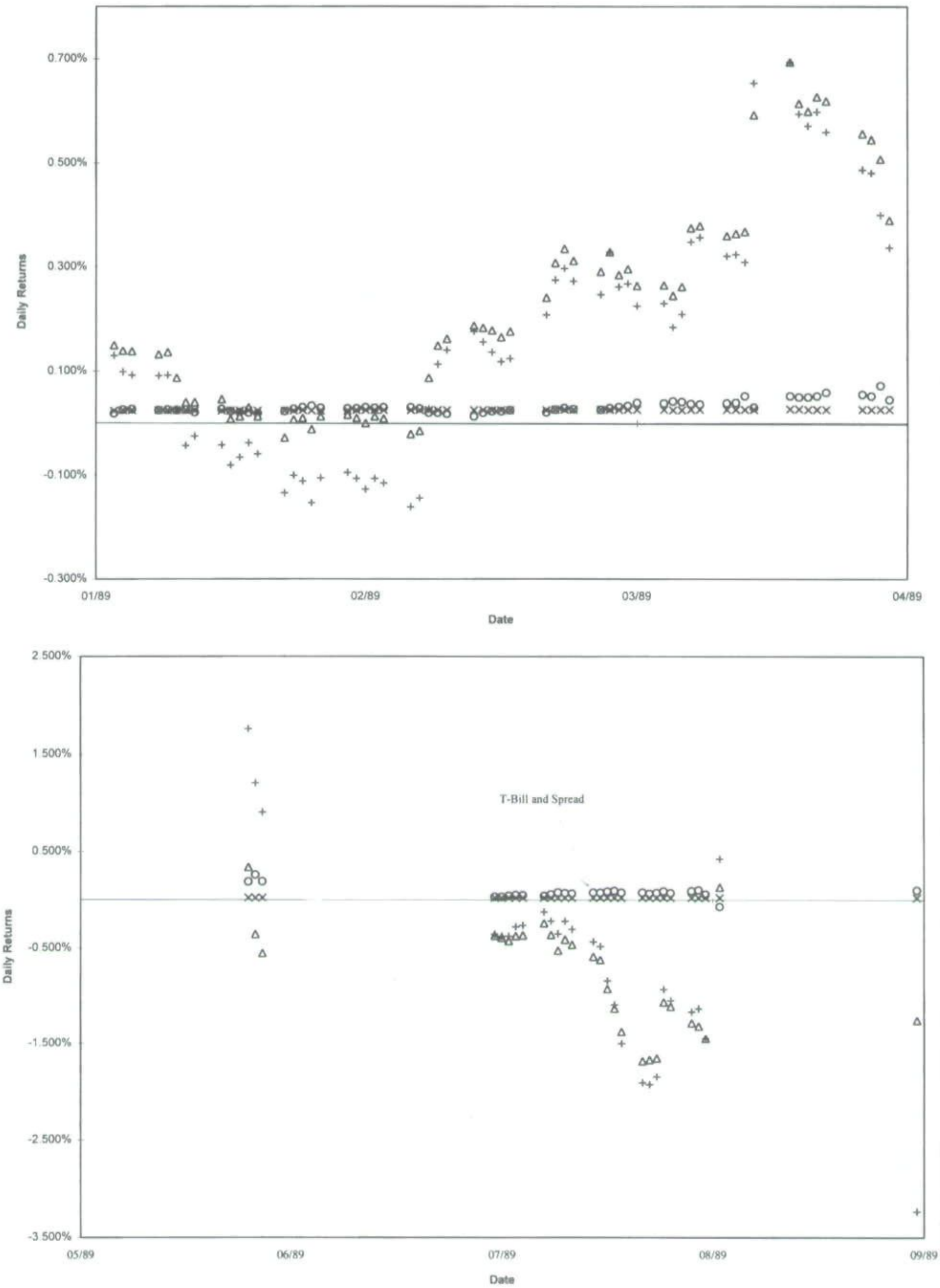

FIGURE 3

(a) Daily returns from NOB spread trading, risk-free investments, and naked positions in note and bond futures.

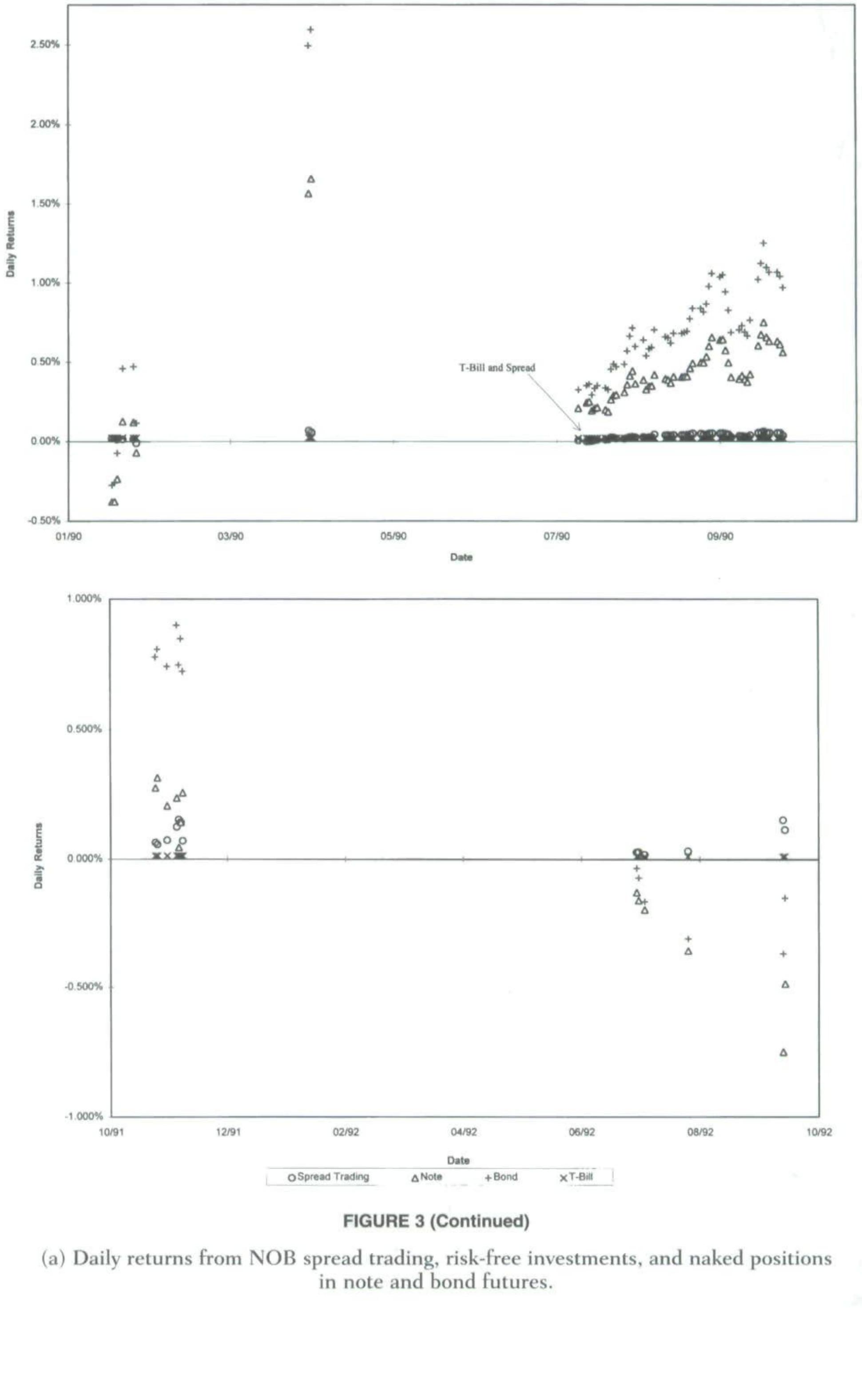

FIGURE 3 (Continued)

(a) Daily returns from NOB spread trading, risk-free investments, and naked positions in note and bond futures.

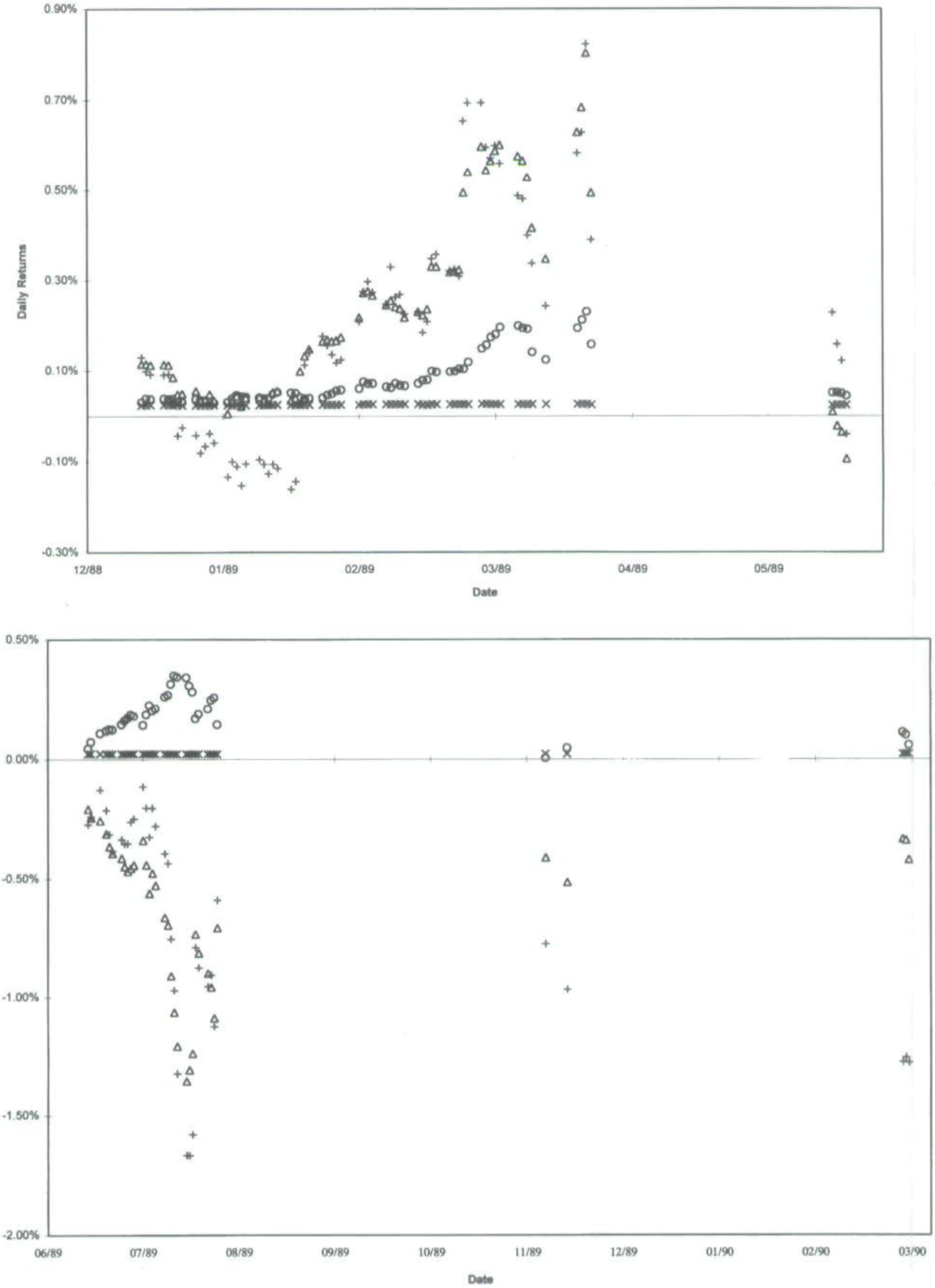

FIGURE 3 (Continued)

(b) Daily returns from FOB spread trading, risk-free investment, and naked positions in note and bond futures.

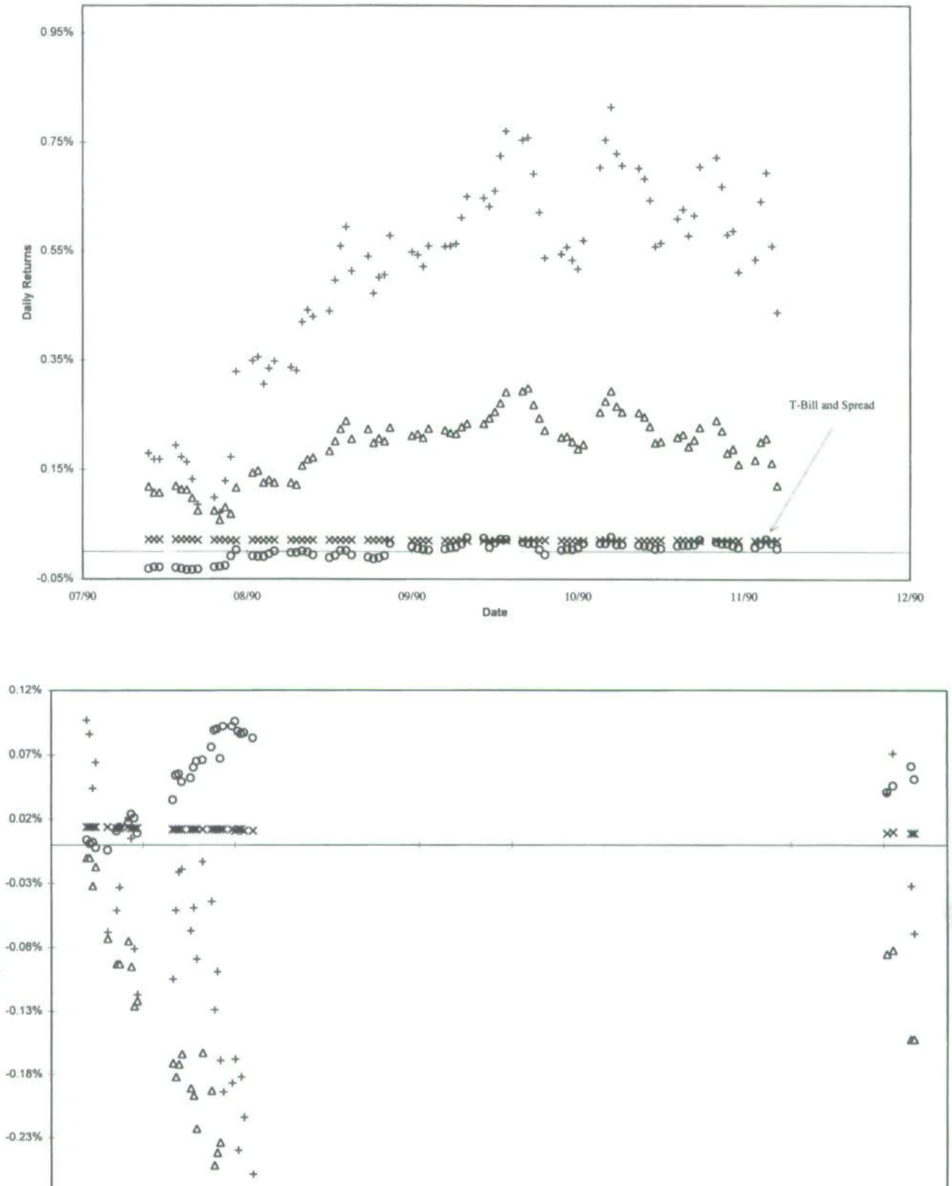

FIGURE 3 (Continued)

(b) Daily returns from FOB spread trading, risk-free investment, and naked positions in note and bond futures.

periods during which spread tradings signals occur so that clear contrasts among the different investments are shown. Daily returns in Figure 3 are calculated as the return from each trade divided by the number of days for each trade. The tolerance level for the spread positions is arbitrarily set at $\alpha = 0.5$ and $\beta = 1.0$. The returns from naked positions are calculated based on a \$600,000 initial investment, and 40 contracts are bought (the return from short positions is exactly the reverse). Both NOB and FOB trades display much lower volatility in their return patterns than the returns from the naked positions. Combined with the positive excess return from the previous table, the spread trades show handsome profits with comparable risk to the proxy for the risk-free investment.

SUMMARY

The simulations show that spread trading is generally a profitable trading strategy for the past 5½-year period. As the interest-rate term premiums have shown mean reversion in the preceding 5½-year period, the NOB and the FOB yield spreads exhibit a certain mean reversion. Because the spread position used in the simulation is not immunized against interest rate risk and the spread does not always move in the anticipated direction, the profit from a spread position is accompanied by risk. Nevertheless, the risk is comparable to that of Treasury bills.

The results are of obvious interest to speculators. The trading rules described in this article are deliberately naive to demonstrate that the futures spread exhibits a mean reversion. This would be less clear if more complicated trading tactics are simulated. For instance, the term premiums could be estimated with a time-varying heteroskedastic model with a mean reversion factor to produce a more accurate forecast of the spread. Dynamic hedging of bonds and notes, contrary to the stationary hedging used here, can eliminate the interest-rate risk completely from the spread position and make the profit from the spread depend entirely on the change of the interest-rate term premiums. However, the costs associated with the dynamic hedging may prevent an arbitrage profit even when an obvious trend in the spread is about to emerge.

BIBLIOGRAPHY

Brennan, M. J., and Schwartz, E. S. (September, 1982): "An Equilibrium Model of Bond Pricing and a Test of Market Efficiency," *Journal of Financial and Quantitative Analysis*, 17:301–329.

Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985): "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53:385–407.

Engle, R. F., and Granger, C. W. J. (1987): "Cointegration and Error Correction: Representation, Estimation and Testing," *Econometrica*, 55:251–276.

Fama, E. F. (1990): "Term-Structure Forecasts of Interest Rates, Inflation, and Real Returns," *Journal of Money Economics*, 25:59–76.

Fama, E. F. (December, 1991): "Efficient Markets II," *Journal of Finance*, 1575–1617.

Fama, E. F., and Bliss, R. R. (September, 1987): "The Information in Long-Maturity Forward Rates," *The American Economic Review*, 680–692.

Fama, E. F., and French, K. R. (1989): "Business Conditions and Expected Returns on Stocks and Bonds," *Journal of Financial Economics*, 25:23–49.

Henriksson, R., and Merton, R. C. (1981): "On Market Timing and Investment Performance. II. Statistical Procedure for Evaluating Forecasting Skills," *Journal of Business*, 54:513–533.

Jorion, P., and Mishkin, F. (1991): "A Multicountry Comparison of Term-Structure Forecasts at Long Horizons," *Journal of Financial Economics*, 29:59–80.

Keim, D. B., and Stambaugh, R. F. (1985): "Predicting Returns in the Stock and Bond Markets," *Journal of Financial Economics*, 17:357–390.

McIntosh, C. S., and Dorfman, J. H. (1992): "Qualitative Forecast Evaluation: A Comparison of Two Performance Measures," *American Journal of Agricultural Economics*, 74:207–214.

Merton, R. C., (1981): "On Market Timing and Investment Performance. I. An Equilibrium Theory of Value for Market Forecasts," *Journal of Business*, 54:363–406.

Phillips, P. C. B., and Ouliaris, S. (1990): "Asymptotic Properties of Residual Based Tests for Cointegration," *Econometrica*, 58:165–193.

Phillips, P. C. B. and Perron, P. (1988): "Testing for a Unit Root in Times Series Regression," *Biometrika*, 75:335–346.

Ramaswamy, K., and Sundaresan, S. M. (1986): "The Valuation of Floating-Rate Instruments," *Journal of Financial Economics*, 17:251–272.

Sehap, K. (July, 1992): "Trading on a Curve: Spreading NOB Volatility," *Futures*, 42–43.