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# THE VALUE OF INFORMATION IN THE PRESENCE OF FUTURES MARKETS

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## **INTRODUCTION**

The important role of information in various economic models is well known. Recent literature includes a tremendous amount of work demonstrating the crucial role that information plays in decision making under uncertainty. In a real world situation, decision makers operating under uncertainty use signals (which are correlated with the uncertain state of the economy) as a source of information. The interpretation of such signals (through an "information system") is important, since it determines the updating of their beliefs regarding the random state of nature (for example, price) and, hence, the actions to be taken.

Comparing statistical experiments, Blackwell (1953) characterized the notion of a "more informative" information system in various ways. To derive his result, Blackwell assumed that, after a signal is revealed, the feasible set of actions that each economic agent faces does not change. However, this is not the case in most economic frameworks:

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once a signal is observed by the decision makers, they update their probability distributions and, due to the resulting changes in prices, the feasible sets of actions vary. The economic implication of this observation (which results in the failure of Blackwell's theorem in a wide range of economic circumstances) is that more information may be disadvantageous. This article demonstrates the possibility of "disadvantageous information" in an economy with risk-averse economic agents who face random exchange rates, but can hedge their exchange risk by using currency futures/forward markets.

The question posed here is the following: Are the risk-averse economic agents better off when they can derive more information from each signal observed? In other words, is "more information" always beneficial? As shown later, this is not always the case. Similar phenomena have been shown in the literature, although in frameworks completely different from this study's [See for example Hirschleifer (1971), Chen (1995)].

In the next section this study's model is introduced with signals and information systems. In the following section an economy with competitive risk-averse firms engaged in international trade under random exchange rates is considered. Producers make their production decisions after observing certain signals correlated to the (random) exchange rate (e.g., trade deficit, interest rates). It is shown that, in the absence of currency forward/futures markets, exporters prefer more information (derived from each signal). In the fourth section, it is shown that under some circumstances more information can be disadvantageous to firms that hedge their foreign exchange risk by selling (buying) forward part of their foreign currency proceeds. This phenomenon holds regardless of whether the currency futures market is *unbiased* or *biased*.

## THE MODEL

Assume that a decision maker operates in the following environment: The payoff depends upon an action that should be chosen prior to the revelation of the random state of nature (e.g., price or exchange rate). Before taking action (e.g., making a decision about production level, hedging, etc.) some signal correlated to the state of nature is observed (for example, the consumer price index, monetary policy, interest rate, etc.). As a result, the decision maker can update prior beliefs about the distribution of the random state of nature. Formally, let  $S = \{s_1, \dots, s_n\}$  be the set of states of nature, and  $\pi = (\pi_1, \dots, \pi_n)$  be a prior probability distribution over  $S$ . The decision maker is assumed to be an expected

utility maximizer whose von-Neumann Morgenstern utility function  $U$  is defined over payoffs  $\xi(a, s_i)$  where  $a \in B$ ,  $B$  is the set of feasible actions. Before taking an action, the decision maker observes a signal  $y$  which is correlated to the state of nature. The possible set of signals is denoted by  $Y = \{y_1, \dots, y_m\}$ . For simplicity  $m = n$ ,  $u(a, s_i) = U(\xi(a, s_i))$ .

An information system  $P$  is an  $n \times n$  row stochastic matrix specifying for each state of nature a probability vector over the set of signals. The  $(i, j)$  element of  $P$  is denoted by  $P_{ij}$ , which stands for the conditional probability that if the state is  $s_i$ , then the signal  $y_j$  will be observed. The hypothetical decision maker does not observe the true (i.e., prevailing) state of nature but rather observes signals which are generated by those states. Upon receiving a signal, the a priori probability vector (i.e., the decision maker's beliefs) is updated, using Bayes rule, and then actions are chosen in a way that maximizes expected utility.

Let  $P$  and  $Q$  be two information systems.  $P$  is more informative than  $Q$ , denoted by  $P \succeq Q$ , if there exists an  $n \times n$  row stochastic matrix  $R$  such that  $Q = PR$ . The motivation for this definition is that multiplying by  $R$  adds some noise (through a process of randomization) to the information contained in  $P$ .

This definition is demonstrated via a simple case. Consider the two extreme information systems. The full information system

$$I_n = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & & & \\ \vdots & & & \\ 0 & \dots & \dots & 1 \end{bmatrix}$$

which fully reveals the prevailing state of nature and

$$E_n = \begin{bmatrix} \frac{1}{n} & \dots & \frac{1}{n} \\ \vdots & & \\ \vdots & & \\ \frac{1}{n} & \dots & \frac{1}{n} \end{bmatrix}$$

the null information system, which does not affect the prior distribution. That is, a decision maker using  $I_n$  will operate under certainty after observing any signal, whereas a decision maker using  $E_n$  will not alter his/her beliefs after observing a signal. It is assumed that  $I_n$  is "more informative" than  $E_n$ , that is,  $E_n = I_n R$  with  $R = E_n$ .

Denote by  $q_j$  the probability that the signal,  $y_j$ , would be received. In fact,  $q_{y_j} = \sum_{i=1}^n \pi_i P_{ij}$ . Using Bayes rule,  $\pi_i(y_j) = \pi_i P_{ij} / q_{y_j}$ , the updated

(posterior) probability that the state is  $s_i$  given the signal,  $y_j$  (under the information system  $P$ ).

Define  $V(P, \pi, U)$  to be the maximal expected utility that can be derived under the information system  $P$ . It is given by:

$$V(P, \pi, U) = \sum_{y_j \in Y} q_{y_j} \max_{a \in B} \sum_{i=1}^n \pi_i(y_j) u(a, s_i)$$

**Theorem (Blackwell).**  $P \succeq Q$  if and only if  $V(P, \pi, U) \geq V(Q, \pi, U)$  for all  $\pi, U$ .

Blackwell's theorem states that an information system,  $P$ , is "more informative" than an information system,  $Q$ , if and only if every expected utility decision maker prefers (weakly) using  $P$  to using  $Q$ .

It is noteworthy that the importance of Blackwell's theorem stems from the fact that it relates a purely statistical concept, with no a priori economic context, to a very meaningful economic concept. In other words, there are many definitions of informativeness, but none of these notions is equivalent to the well-accepted criterion of expected utility maximization.

The following example is used to demonstrate this result. Let  $n = 2$  with two feasible actions:  $B = \{a_1, a_2\}$ .  $P = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , the full information systems; and  $Q = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ , the null information system. Let  $\pi = (\frac{2}{3}, \frac{1}{3})$  and  $U$  is any utility function.

$$\begin{aligned} V(I_2, \pi, U) &= \frac{2}{3} \max_B u(a, s_1) + \frac{1}{3} \max_B u(a, s_2) \\ V(Q, \pi, U) &= \frac{1}{2} \max_B \left[ \frac{2}{3} u(a, s_1) + \frac{1}{3} u(a, s_2) \right] \\ &\quad + \frac{1}{2} \max_B \left[ \frac{2}{3} u(a, s_1) + \frac{1}{3} u(a, s_2) \right] \\ &= \max_B \left[ \frac{2}{3} u(a, s_1) + \frac{1}{3} u(a, s_2) \right] \leq V(I_2, \pi, U) \end{aligned}$$

Consider the following extension of the model described above. Assume that the set  $B$  of feasible actions does not remain the same regardless which information system is available and which signal is received. Instead, assume that to every signal,  $y$ , there corresponds a feasible set,  $B(P, y)$ . The notation,  $B(P, y)$ , emphasizes the dependence on both elements: the information system and the specific signal.

Within this extended model, the value function of information should be adjusted as follows:

$$V^*(P, \pi, U) = \sum_{y_j} q_{y_j} \max_{a \in B(P, y_j)} \sum_{i=1}^n \pi_i(y_j) u(a, s_i) \quad (1)$$

In such a generalized framework, the question whether Blackwell's theorem still holds seems natural. Put differently, given a model with signal-dependent feasible set, is it always the case that more information is advantageous? It is not difficult to show that for any two information systems with  $P \succ Q$  there exist two families of feasible sets  $\{B(P, y_i)\}_{i=1}^n$ ,  $\{B(Q, y_i)\}_{i=1}^n$ , a state dependent utility function,  $U$ , and an a priori probability vector,  $\pi$ , such that:  $V^*(P, \pi, U) < V^*(Q, \pi, U)$ . Surprisingly, the following example can be shown:

*Example.* As before, there are two information systems:  $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , the full information system; and  $Q = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ , the null information system.  $\pi = (\frac{1}{2}, \frac{1}{2})$ . Let:

$$B(I_2, y_1) = B(E_2, y_1) = \{a_1\}$$

$$B(I_2, y_2) = B(E_2, y_2) = \{a_2\}$$

Choose a utility function which satisfies

$$u(a_1, s_2) = u(a_2, s_1) = 3$$

and elsewhere it is zero. It is easy to verify that

$$V^*(P, \pi, U) = 0 \quad \text{and} \quad V^*(Q, \pi, U) = 1.5$$

This example demonstrates that allowing the feasible set to vary with signals results in the collapse of Blackwell's theorem even if  $B(P, y_i) = B(Q, y_i)$   $i = 1, 2$ . As shown later, such circumstances of varying feasible sets are not rare in economic models. On the other hand, since in many economic models with uncertainty and signals, signal-dependent opportunity sets arise naturally, it would be useful to find some criterion that guarantees that more information is preferable. Such criterion was found in Sulganik and Zilcha (1994). It is restated here for the reader's convenience.

**Theorem 1 [from Sulganik and Zilcha (1994)].** Let  $P, Q$  be two information systems.  $\{B(P, y_i)\}_{i=1}^n$  and  $\{B(Q, y_i)\}_{i=1}^n$  are the associated families of feasible sets. Assume that  $P \succ Q$ , i.e., there exists a stochastic matrix  $R$

such that  $PR = Q$ . If the following condition holds for all  $k, j$ :

$$R_{kj} > 0 \rightarrow B(P, y_k) \supseteq B(Q, y_j) \quad (2)$$

Then, for all  $U$  and  $\pi$ ,  $V(P, \pi, U) \geq V(Q, \pi, U)$ .

The intuition behind this result is: If  $Q$  is less informative than  $P$ , then one can think of  $Q$  as being generated by adding noise to  $P$ . Thus, after observing a signal,  $y_k$  under  $P$ , there is another randomization and signal,  $y_j$ , is observed with probability,  $R_{kj}$ . Say that  $y_j$  (under  $Q$ ) follows  $y_k$  (under  $P$ ) if  $R_{kj} > 0$ , i.e.,  $y_j$  is sometimes chosen by the noise process following  $y_k$ . If fewer actions are available under  $Q$  at all signals following  $y_k$  (than under  $P$  at  $y_k$ ), then  $P$  is more valuable than  $Q$ . Note, however, that this is sufficient condition only.

Consider again the above example. In this case,  $R = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ , i.e.,  $R_{kj} > 0$  for  $k, j = 1, 2$ ; but  $B(I_2, y_k)$  does not include  $B(E_2, y_j)$ , i.e., condition eq. (2) is violated.

## EXPORTING FIRM: PRICE UNCERTAINTY WITHOUT FUTURES MARKETS

In recent years, commodity price volatility has been high and exporting/importing firms face significant exchange rate fluctuations in the major industrial currencies. For those risk-averse economic agents who have future commitments in foreign currency, the higher uncertainty about their profits results in adverse effects upon their economic activities, such as the production level [see Sandmo (1971)]. As a result, financial risk-sharing markets, such as commodity futures and foreign currency forward markets, are utilized. The economic impacts of such hedging tools on risk-averse agents have been studied extensively in the literature; for example, Danthine (1978), Holthausen (1979), Kawai (1981), and Kawai and Zilcha (1986). It has been demonstrated in various frameworks that introducing forward/futures markets increases the production level and enhances international trade under some mild conditions about the forward/futures markets.

A specific model where the uncertainty is due to a random exchange rate is presented now. Consider an exporting firm selling its product on the international market. The firm is competitive and risk averse facing a fixed price,  $P^*$ , abroad, but the exchange rate,  $\tilde{e}$ , is a random variable. Production takes place at the home country and let  $C(x)$  be the cost function (in the domestic currency). Assume that  $C'(x) > 0$  and  $C''(x) \geq 0$ . Let  $U(\xi)$  be the von-Neumann Morgenstern utility

function defined on profits denominated in the local currency. Assume that  $U' > 0$  and  $U'' < 0$ . The random exchange rate,  $\tilde{e}$ , is assumed to have  $n$  values,  $\{e_1, \dots, e_n\}$ , and, without loss of generality,  $P^* = 1$ .

Consider now the following order of events. At date, 0, the firm observes a signal,  $y$ , correlated with the state of nature (it is observed by all agents in this economy) and at that time a decision regarding the production level must be made. At date  $t = 1$ , the firm exports its output and the exchange rate,  $\tilde{e}$ , is observed. The firm, and other agents, use at  $t = 0$  a certain information system to revise their prior expectations about the probabilities of states of nature once the signal becomes known. Clearly, the choice of information system will have an impact on the production decisions made at date 0.

For each signal  $y_j$  and information system,  $P$ , the posterior probability of state,  $i$  (given  $y_j$ ) is given by  $\pi_i(P, y_j)$ . A signal,  $y_j$ , is called an *excluding signal* if for some state  $i$   $\pi_i(P, y_j) = 0$ , while for the given prior  $\pi$   $\pi_i > 0$ , and as a result the feasible set of actions changes. Given  $y_j$  and  $P$  the feasible set of production levels is:  $B(P; y_j) = \{x \geq 0 | e_i x - C(x) \geq -C^* \text{ for each state } i \text{ with } \pi_i(P, y_j) > 0, i = 1, 2, \dots, n\}$ .

For some exogenously given nonnegative constant  $C^*$ . Thus, losses are not allowed above some constant  $C^*$  in any state of nature which can occur with positive probability. Since some signals may be excluding signals, the feasible set is signal-dependent. It is possible to define  $B(P, y_i)$  as signal-independent without any significant change in the optimization by requiring that  $e_i x - C(x) \geq -C^*$  for *all* states  $i$ . However, this presentation facilitates a generalization of this case to a more general framework. Now, given  $\pi$  and  $U$  (to be taken fixed in the sequel), the value of the information system,  $P$ , under the above feasible sets of actions, is given by:

$$V(P) = \sum_j q_{y_j} \left[ \max_{x \in B(P, y_j)} \sum_i \pi_i(P, y_j) U(e_i x - C(x)) \right]$$

It is shown now that in the absence of a risk-sharing mechanism (such as currency futures markets) the firm is better off with the more informative information system.

**Theorem 2.** *In the absence of risk-sharing markets, if  $P \succ Q$ , then  $V(P) \geq V(Q)$ .*

**Proof.** To prove this theorem, Theorem 1 is applied. Assume that  $P \succ Q$ . Hence, by definition there exists a stochastic matrix,  $R$ , such that  $PR = Q$ . Using the result proven therein, if it is shown that for all  $k, j$  [ $R_{kj} > 0 \rightarrow B(P, y_k) \supseteq B(Q, y_j)$ ] holds, then  $V(P) \geq V(Q)$ .

Consider first a nonexcluding signal,  $y_j$  (hence the elements of  $\pi(Q, y_j)$  are all not zero). In this case,

$$R_{kj} > 0 \longrightarrow B(P, y_k) \supseteq B(Q, y_j)$$

since it may be possible that the probability vector of  $\pi(P, y_k)$  has zeroes. Now, consider an excluding signal,  $y_j$ . In this case,  $R_{kj} > 0$  implies that the number of zeroes of  $\pi(P, y_k)$  are at least as much as that in  $\pi(Q, y_j)$ . But this implies, by definition, that  $B(P, y_k) \supseteq B(Q, y_j)$ ; hence, condition eq. (2) holds. Thus,  $V(P) \geq V(Q)$ .

## FUTURES MARKETS AND HEDGING

Now consider the above framework when currency futures markets exist. The futures exchange rate is denoted by  $e_f$ . In this case the order of events is as follows:

*Date 0:* An information system,  $Q$ , used economy-wide, has to be chosen. Given a signal,  $y$  (observed by all agents) and  $e_f$ , the firm determines its production level,  $X$ , and its futures contracting (sale or purchase) of foreign currency,  $Z$ . This forward contract is to be executed when production is completed at date 1 (where the spot market for foreign exchange operates and  $\tilde{e}$  is realized).

*Date 1:* The exchange rate,  $\tilde{e}$ , is revealed. Hence, a currency spot market operates. The firm exports its product and receives foreign currency (which is used to cover part of its futures contract obligations). Note that since all transactions take place at date 1, there is no difference between forward and futures markets [see Cox, Ingersoll, and Ross (1981)].

For a given information system and a signal,  $y$ , the posterior probability distribution over states of nature is known to all agents. It is well founded in the literature on currency forward (futures) markets that the assumption of unbiasedness is a reasonable one [see Chowdhury (1991), Maberly (1985)]; namely, that the forward exchange rate,  $e_f$ , is an unbiased estimator of the currency spot price,  $\tilde{e}$ . Thus, it is assumed throughout the following example that

$$e_f(Q, y) = E[\tilde{e}|Q, y] \quad (3)$$

for all  $y$ .

Now, write the feasible set for the firm in this framework for a given information system,  $Q$ , and a signal,  $y$ . Note that due to the choice of the international price,  $P^* = 1$ , the firm receives  $\tilde{e}$  for each unit it sells on this competitive market. The firm's profits in state  $i$  are denoted by

$\xi_i$ , i.e.,:

$$\xi_i(X, Z, Q, y_j) = e_i(X - Z) + e_f(Q, y_j)Z - C(X)$$

The signal-dependent feasible sets in this case are

$$B(Q, y_j) = \{(X, Z) | X \geq 0, \text{ and for each } i, \xi_i(X, Z, Q, y_j) \geq -C^* \\ \text{whenever } \text{Prob}\{\tilde{e} = e_i | Q, y_j\} > 0\}$$

The value function is:

$$V(Q) = \sum_j q_{y_j} \left[ \max_{(X, Z) \in B(Q, y_j)} \sum_i \pi_i(Q, y_j) U(\xi_i(X, Z, Q, y_j)) \right]$$

Now, consider Theorem 2 when currency forward/futures markets are available. It is hypothesized that in this case it is possible that the firm will be worse off when more information is derived from each signal.

**Theorem 3.** *In the presence of currency forward/futures market it is possible that  $P \succ Q$  and  $V(P) < V(Q)$ .*

The conclusion from this theorem is that revealing more information to the market does not always guarantee improving the situation of those risk-averse agents in the market. In fact, more information results in two effects which do not always work in the same direction. On the one hand more information results in having a "better" distribution function of the random price used in the process of maximizing expected profits. On the other hand, it affects the demand for futures contracts, and (in some cases) changes the futures price. In certain cases this change in the futures price can reduce the welfare of the risk-averse agents, ultimately outweighing the benefit derived from the "better" decision emanating from the more precise distribution. Although no complete characterization of these cases exists in the literature, one can use condition eq. (2) to identify cases where such phenomena occur.

*Proof.* One example where  $P \succ Q$  but  $V(P) < V(Q)$  proves this theorem. The following simple example demonstrates this claim for  $n = 3$ ,  $P = I_3$  (the full information case) and  $Q = E_3$  (the no information case).

Consider an exporting, competitive firm with a cost function,  $C(x) = \frac{1}{2}\alpha x^2$ , where  $0 < \alpha < 1$ , and a von-Neumann Morgenstern utility function,  $U(\theta) = \ln\theta$ . Assume also that the currency forward market is unbiased, i.e.,

$$e_f(P, y) = E[\tilde{e} | P, y] \quad \text{and} \quad e_f(Q, y) = E[\tilde{e} | Q, y]$$

for all  $y$ .

Let  $\tilde{e}$  assume one of the values  $\{1, 2, 3\}$  with a prior probability distribution  $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ . Given this prior probability distribution, one obtains  $q_{y_j} = \frac{1}{3}$  for  $j = 1, 2, 3$  for either  $P$  or  $Q$ . Due to the unbiasedness of the futures market the optimal hedge that the firm chooses,  $Z^*(P, y)$  for each given signal, will result in a nonrandom profit. This implies that whenever the posterior distribution of  $\tilde{e}$  is nondegenerate, the firm will sell forward all its foreign currency proceeds, hence, its profit will be risk-free [see Feder, Just, and Schmitz (1980), or Kawai and Zilcha (1986)]. Also, using what is known as the "separation theorem," the firm's optimal output will be *independent* of the probability distribution of the exchange rate and its attitude toward risk [see Danthine (1978) and the above two references]. Moreover, its optimal production (in the case where the information system is  $P$ ) is given by:

$$MC(X^*(P, y)) = e_f(P, y) \quad (4)$$

for all  $y$ ; and the optimal  $(X^*(P, y), Z^*(P, y))$  yields maximum profits:

$$\xi_j(P, y) = e_f(P, y)X^*(P, y) - C(X^*(P, y))$$

for all  $y$ .

Using the above specific form of the cost function, one can compute the optimal production levels under  $P$  or  $Q$ , for each signal,  $y_j$ . First, let the prevailing information system be  $P = I_3$ . Then,

$$e_f(P, y_1) = 1, e_f(P, y_2) = 2, e_f(P, y_3) = 3$$

Hence, by eq. (3) it is found that,

$$X^*(P, y_1) = 1/\alpha, X^*(P, y_2) = 2/\alpha, X^*(P, y_3) = 3/\alpha \quad (5)$$

Thus:

$$\begin{aligned} \xi_1(P, y_1) &= \frac{1}{2\alpha} \\ \xi_2(P, y_2) &= \frac{2}{\alpha} \\ \xi_3(P, y_3) &= \frac{9}{2\alpha} \end{aligned} \quad (6)$$

$$V(P) = \frac{1}{3} \ln\left(\frac{1}{2\alpha}\right) + \frac{1}{3} \ln\left(\frac{2}{\alpha}\right) + \frac{1}{3} \ln\left(\frac{9}{2\alpha}\right)$$

Consider now the case where  $Q = E_3$  prevails. The forward exchange rate in this no-information signals case is:

$$e_f(Q, y_j) = \frac{1}{3}1 + \frac{1}{3}2 + \frac{1}{3}3 = 2$$

for all  $y_j$

By the Full-Hedging theorem [see Kawai and Zilcha (1986)],  $Z^*(Q, y_j) = X^*(Q, y_j)$  for all  $j$ . Hence:  $X^*(Q, y_j) = 2/\alpha$  for all  $j$ ; and  $Z^*(Q, y_j) = 2/\alpha$  for all  $j$ .

Hence,  $\xi_j(Q, y_j) = \frac{4}{\alpha} - \frac{2}{\alpha} = \frac{2}{\alpha}$  for all  $j$ ; and, therefore,  $V(Q) = \ln(\frac{2}{\alpha})$ .

Since, for  $0 < \alpha < 1$ , one obtains

$$V(Q) = \ln \frac{2}{\alpha} > \frac{1}{3} \ln \left( \frac{9}{2\alpha^3} \right) = V(P)$$

this proves the claim.

To understand this result, an illustration is given of how the opportunity sets are related. Assume  $C^* = 0$  here. For the case of no-informational signals, one obtains for all  $y_j$ ,

$$\begin{aligned} B(Q, y_j) &= B^*(Q) \\ &= \left\{ (X, Z) \mid X \geq 0 \text{ and } e_i(X - Z) + 2Z - \frac{1}{2}\alpha X^2 \geq 0 \text{ for } i = 1, 2, 3 \right\} \\ &= \left\{ (X, Z) \mid X \geq 0, Z \leq X \text{ and } X + Z - \frac{1}{2}\alpha X^2 \geq 0 \right\} \\ &\cup \left\{ (X, Z) \mid X \geq 0, Z > X \text{ and } 3X - Z - \frac{1}{2}\alpha X^2 \geq 0 \right\} \end{aligned}$$

Now

$$\begin{aligned} B(P, y_1) &= \left\{ (X, Z) \mid X \geq 0 \text{ and } X - \frac{1}{2}\alpha X^2 \geq 0 \right\} \\ B(P, y_2) &= \left\{ (X, Z) \mid X \geq 0 \text{ and } 2X - \frac{1}{2}\alpha X^2 \geq 0 \right\} \\ B(P, y_3) &= \left\{ (X, Z) \mid X \geq 0 \text{ and } 3X - \frac{1}{2}\alpha X^2 \geq 0 \right\} \end{aligned}$$

Thus, it is found here that  $B(P, y_1) \subset B^*(Q)$  while  $B(P, y_3) \supset B^*(Q)$ . For  $Z = X$  it is found also that  $B(P, y_2) = B^*(Q)$ . This type of relationship between the feasible sets makes it possible to achieve  $V(Q) > V(P)$ .

In this example, due to the unbiased futures market, the firm "reacts" to the market price,  $e_f$ , and its actions are independent of its beliefs about the distribution of the exchange rate or its attitude toward risk. It is shown that such a phenomenon occurs in the biased case as well, where each economic agent's optimal hedging behavior takes into account its own beliefs and preferences. Moreover, in contrast to the logarithmic utility where absolute risk aversion is decreasing, a quadratic

utility function is chosen; hence, the absolute risk aversion is increasing. The result remains unchanged.

Let the utility function be  $U(\xi) = a\xi - \frac{1}{2}b\xi^2$  for some positive constants,  $a$  and  $b$  ( $a$  is "much larger" than  $b$ ). All parameters are the same as in the above example, except the currency futures price. Now, consider an unbiased market. The futures price is given by:

$$e_f(N, y) = (1 - \theta)E[\tilde{e}|N, y]$$

for some  $\theta \geq 0$ .

Thus, the participants pay some (risk) premium. When  $\theta > 0$  the firm sells part of its foreign proceeds, i.e.,  $Z^* < X^*$ . From the optimality conditions (i.e., when the firm chooses  $X, Z$  such that it maximizes expected utility) the separation property holds. Hence, the optimal output is given by

$$C'(X^*) = (1 - \theta)E\tilde{e}$$

However, the optimal hedge  $Z^*$  is determined by the equation [see Feder, Just, and Schmitz (1980), Kawai and Zilcha (1986)],

$$E[e_f - \tilde{e}]U'(\xi) = 0 \quad (7)$$

Let  $P = I_3$ . In this case signals provide full information, i.e., removing all uncertainty regarding the profits; hence,  $Z^*(P, y_j) = 0$  for  $j = 1, 2, 3$  for  $\theta \geq 0$ . Thus,

$$\xi_j(P, y_j) = e_j X^*(P, y_j) - \frac{1}{2}\alpha X^*(P, y_j)^2, j = 1, 2, 3$$

where  $X^*(P, y_j) = \frac{(1-\theta)e_j}{\alpha}$  for  $j = 1, 2, 3$ .

Consider now the information system,  $Q = E_3$ . In this case,

$$X^*(Q, y_j) = \frac{2(1 - \theta)}{\alpha}$$

for  $j = 1, 2, 3$ , and the profits are:

$$\xi_j(Q, y_j) = \frac{2(1 - \theta)e_j}{\alpha} + Z^*(Q, y_j)[2(1 - \theta) - e_j] - \frac{2(1 - \theta)^2}{\alpha}$$

Using all these expressions in eq. (6) the following formula is derived for optimal hedging:

$$\begin{aligned} Z^*(Q, y_j) \\ = \frac{1}{b + b\theta^2} \left[ \frac{3b(1 - \theta)}{\alpha} (1 - 4\theta) - 3a\theta - \frac{b(1 - \theta)}{\alpha} (1 + 6\theta(1 - \theta)) \right] \end{aligned}$$

Note that this expression for optimal hedging depends also upon the ex post distribution and the utility function. Moreover, in the unbiasedness case,  $\theta = 0$ , it reduces to  $Z^*(Q, y_j) = \frac{2}{\alpha}$  for  $j = 1, 2, 3$ .

The expected utility of the firm given this quadratic utility is denoted by  $\hat{V}(P, \theta)$  and  $\hat{V}(Q, \theta)$  and the above expressions for the output and hedging under  $P$  and  $Q$ , where  $\theta \geq 0$ . It is shown first that  $\hat{V}(P, 0) < \hat{V}(Q, 0)$  when  $\alpha$  is close to 1. For  $\theta = 0$ ,  $\xi_j(P, y_j)$  is given by eq. (7); hence, one obtains:

$$\hat{V}(P, 0) = \frac{7a}{3\alpha} - \frac{49b}{12\alpha^2}$$

For  $\theta = 0$  one obtains  $\xi_j(Q, y_j) = \frac{2}{\alpha}$ ,  $j = 1, 2, 3$ . Hence,

$$\hat{V}(Q, 0) = \frac{2a}{\alpha} - \frac{2b}{\alpha^2}$$

When  $b = \frac{1}{3}a$  and  $\alpha < 1$  but close to 1, it is guaranteed that  $\hat{V}(P, 0) < \hat{V}(Q, 0)$ . Now, using continuity arguments, it can be shown that for some  $0 < \bar{\theta} < 1$ , one obtains:

$$\hat{V}(P, \theta) < \hat{V}(Q, \theta)$$

for all  $\theta \in [0, \bar{\theta}]$ . Thus, more information can be disadvantageous in the biased market case, specifically, when the risk premium is positive.

## DISCUSSION

The role that information plays in determining futures prices, hedging, and speculation is significant. Economic agents acquire information about random economic parameters prior to their decisions about production and the level of engagement in futures trading. Certain economic signals are used in the process of evaluating the distribution of the relevant random variable (in this case, the exchange rate). Decision makers invest efforts and resources in acquiring "better interpretation" of signals (such as trade deficit, interest rates, money supply, etc.). This study shows that in some cases more information derived from each signal may result in an equilibrium which is less beneficial. Such an adverse effect can occur due to the widespread observation of this signal: too many participants in the market adjust their beliefs regarding the probability distribution (of the exchange rate or the random price). The subsequent changes in their actions in the market (due to the reevaluation process) may change the futures price in a way that is disadvantageous. The value of information depends on the number of

participants in the market who obtain this information and their weight in this market.

This phenomenon, where the introduction of risk-sharing markets hurts risk-averse agents even though signals convey more information, holds under more general circumstances and in a wide range of competitive economies. It is highly likely that generalizing even further to consider cases with a diversity of information systems and a diversity of prior distributions would increase the likelihood of such phenomena.

In the absence of risk-sharing markets, i.e., without futures/forward markets, more information is advantageous. However, when futures/forward markets are used, one should bear in mind that the value of information may not always be positive.

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