Put–Call Parities and the Value of Early Exercise for Put Options on a Performance Index

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INTRODUCTION

In an interesting study Zivney (1991) demonstrates that deviations from the European put–call parity are caused by the possibility of early exercise. He investigates this possibility for options on the S&P 100 index. Because options on the S&P 100 index are not corrected for dividend payments, both call and put options may contain a premium for early exercise, thereby causing deviations from the European put–call parity. The deviation from the put–call parity is very difficult to split into two parts. This is not the case if options on a performance index are considered. In a performance index, it is assumed that dividends are reinvested in the index, which makes the index similar to a nondividend paying stock. As Merton (1973) has shown, rational investors will exercise call options early only just before an ex dividend date, so

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they will not exercise call options on a performance index before their expiration date. Deviations of the put–call parity for options on a performance index can therefore only be caused by the possibility of early exercise of put options.

This study applies Zivney's concept to options on a performance index. Since May 1992, options on the DAX index have been trading on the Amsterdam Stock Exchange (ASE).[1] The DAX index is an example of a performance index [see, e.g., Grünbichler and Callahan (1994)]. On ex-dividend days, the DAX is calculated as if the dividends are reinvested in the index. Without these reinvestments there would be no difference between the DAX and a value-weighted index like the S&P 100. The fact that the DAX is a performance index allows one to investigate the value of early exercise for American put options. Knowledge about this premium is helpful in developing an option formula for American put options.

In the following section, put–call parities are discussed and a new parity relation for American options on an index with a known dividend yield is derived. The next section indexes the data and the regression analysis, which is used to investigate the premium for early exercise. The results are then presented and discussed.

PUT–CALL PARITIES AND THE VALUE OF EARLY EXERCISE

Depending upon the assumptions that are made about the possibility of early exercise and about dividend payments, different put–call parities—barring arbitrage possibilities—can be derived. Assuming, for instance, that early exercise is not possible and that a continuous and known dividend is paid on an index, the following European put–call parity can be derived [Chance, (1987)]:

$$\hat{C}_t = P_t + I_t e^{-q(T-t)} - \mathrm{X}e^{-r(T-t)} \tag{1}$$

In which:

P_t = the market value of the put option at time t,

I_t = the market value of the index at time t,

[1]These securities are listed as index *warrants*. However, they are, in fact, index options, except that they are traded on the stock exchange, instead of the options exchange. Because this involves only institutional differences which are not important for valuation purposes, this study refers to these index warrants as index options.

$q =$ the known annualized continuous dividend yield of the index,

$X =$ the exercise price of the option,

$T =$ the expiration date of the option,

$r =$ the known annualized continuous interest rate until the expiration date $(T - t)$, and

$\hat{C}_t =$ the value of the call option according to the parity relation.

If the assumption is made that early exercise is possible, then it is no longer possible to derive the put–call parity in the form of an equality. Instead, it is only possible to give a lower and an upper bound for the value of the call option in terms of the other variables. If the assumption that the index has a known dividend yield is still made and if, in addition, it is assumed that the interest rate is known and equal for all maturities until T, then the lower and upper bounds for the value of the call options become:

$$\hat{C}_t \geq P_t + I_t e^{-q(T-t)} - X \tag{2}$$

$$\hat{C}_t \leq P_t + I_t - \mathrm{X}\mathrm{e}^{-r(T-t)} \tag{3}$$

The derivation of these bounds is presented in the appendix.[2]

When no dividends are paid on the index, i.e., $q = 0$, it is clear that the upper bound for the American call option, eq. (3), is the same as the value of the call option according to the European put–call parity, eq. (1). The difference between the upper and lower bound for the American call option can then be interpreted as the maximum premium that will be paid on the put option, since the value of an American call option on a nondividend paying index will be equal to the value of its European counterpart.

Although the maximum premium for the possibility of early exercise is given by the difference between the upper and the lower bound, the actual premium for early exercise paid by investors is equal to the difference between the actual value of the call option and the upper-bound in (3). Zivney investigates the premium for early exercise in a similar way, but since he uses options on the S&P 100, which is a dividend paying index, he has to make the additional assumption that the in-the-money option will have a larger premium for early exercise

[2]Still other put–call parities can be derived if it is assumed, for instance, that discrete dividend payments are made. The parities that result in this case can be found in Jarrow & Rudd (1983).

than the out-of-the-money option. Since options on a nondividend paying index are used here, that assumption is not necessary. Moreover, the difference between the value of the call option according to the European put–call parity and the actual value of the call option will be solely determined by the premium for early exercise of the American put option. Note that this difference is also equal to the difference between the value of the American put option and the value of this put option according to the European put–call parity.

The investor has an option to exercise early. Zivney investigates whether the premium for early exercise depends on (1) the moneyness, (2) the interest rate, and (3) the time to maturity. Since this study calculates only the value of early exercise for put options, discussion is limited to explaining his hypotheses for put options. Because the probability of early exercise of an American put option is larger when the option is deeper in-the-money [Merton (1973)] it may be expected that the premium for early exercise increases when the value of the index decreases for any given exercise price. If the risk-free interest rate increases, then the present value of the exercise value decreases [see, e.g., Geske and Johnson (1984)]. This makes the possibility of early exercise more attractive. The risk-free interest rate, therefore, is expected to have a positive effect on the premium for early exercise. Finally, the time to expiration is also expected to have a positive effect on the early exercise premium. This is easy to see when one considers two American put options which differ only with respect to their expiration dates, T_1 and T_2, where $T_2 > T_1$. Since the option with the longer maturity $(T_2 - t)$ has the same possibilities as the option with the shorter maturity $(T_1 - t)$ plus all the options associated with the longer maturity $(T_2 - T_1)$, its value can never be smaller than the value of the option with the shorter maturity. In his study Zivney finds the positive signs that were, a priori, expected. In addition to the factors investigated by Zivney, it is also possible to study the effect of the volatility. This effect is expected to be positive, since a higher volatility implies that the probability for the put option to get in-the-money increases, thereby increasing the value of early exercise.

DATA DESCRIPTION AND METHODOLOGY

Closing prices for options on the DAX index and traded on the ASE are used for the period of May 12, 1992, when they were first traded, until October 7, 1993. These closing prices are taken from Datastream. The expiration date for the options series is April 15, 1994, and the exercise price is 1725 German Marks. Since only those dates for which both the call and the put options were traded are used, this results in a total of 175 observations. Closing values for the DAX index, closing values for the German Mark/Dutch Guilder exchange rate and estimates of the risk-free interest rate are also taken from Datastream. For the risk-free interest rate, the yield on German treasury bills with a maturity closest to the maturity of the options is used. Data for the exchange rate are needed because the options are traded in Dutch Guilders, while the DAX index and the exercise price are listed in German Marks. Wei (1992) shows that the value of the option traded in Dutch Guilders will be equal to the value of a similar option traded in German Marks, multiplied by the current exchange rate. Therefore, the put–call parity used here is given by:

$$\hat{P}_t = C_t - S_t I_t + S_t X e^{-r(T-t)} \tag{4}$$

where S_t is the exchange rate at time t defined as the Guilder value of one German Mark.

The behavior of the premium for early exercise with respect to the variables that were discussed in the preceding section are investigated by means of the following multiple regression equation:

$$D_t = \beta_0 + \beta_1 M_t + \beta_2 \sigma_{\mathrm{impl},t-1} + \beta_3 r_t + \beta_4 (T - t) + \beta_5 D_{t-1} + \varepsilon_t \tag{5}$$

The dependent variable D_t is defined as:

$$D_t = P_t - \hat{P}_t \tag{6}$$

As discussed the premium for early exercise is equal to $\hat{C}_t - C_t$, the difference between the value of the call option according to the parity model and the actual value of the call option, which is also equal to the difference between the actual put value and the put value according to the European put–call parity: $\hat{C}_t - C_t = P_t - \hat{P}_t$.

The regressor, M_t, measures the amount by which the put option is in-the-money:

$$M_t = S_t X - S_t I_t \tag{7}$$

As discussed, this variable is expected to have a positive effect on D_t. The volatility is measured by the implied standard deviation of the call option, $\sigma_{\mathrm{impl},t-1}$, according to the Black and Scholes (1973) formula. Since Black and Scholes assume that early exercise is not possible and since an American call option on the DAX behaves like its European counterpart, it seems appropriate to use the implied standard deviation of the call option rather than the put option. The implied standard deviation at time, $t - 1$, rather than time, t, is used, since σ_t is a function of the dependent variable and will probably not be uncorrelated with the error term, ϵ_t. Also, the interest rate, r_t, and the time to maturity, $(T - t)$, are expected to have a positive effect on the premium and thus on D_t. To correct for autocorrelation problems, D_{t-1} is also used as a regressor. Finally, to take into account the effect of possible heteroskedasticity, White standard errors for the regression estimates are reported.

RESULTS AND DISCUSSION

Figure 1 includes the lower and upper bounds of the put–call parity [eqs. (2) and (3)] as well as the actual call prices.

This figure shows that the call price is almost always between the upper and lower bound. The call price is above the upper bound on only two days. These observations are omitted in the analysis that follows.

Summary statistics for the variables, the premium and the premium as a fraction of the value of the put option according to the European

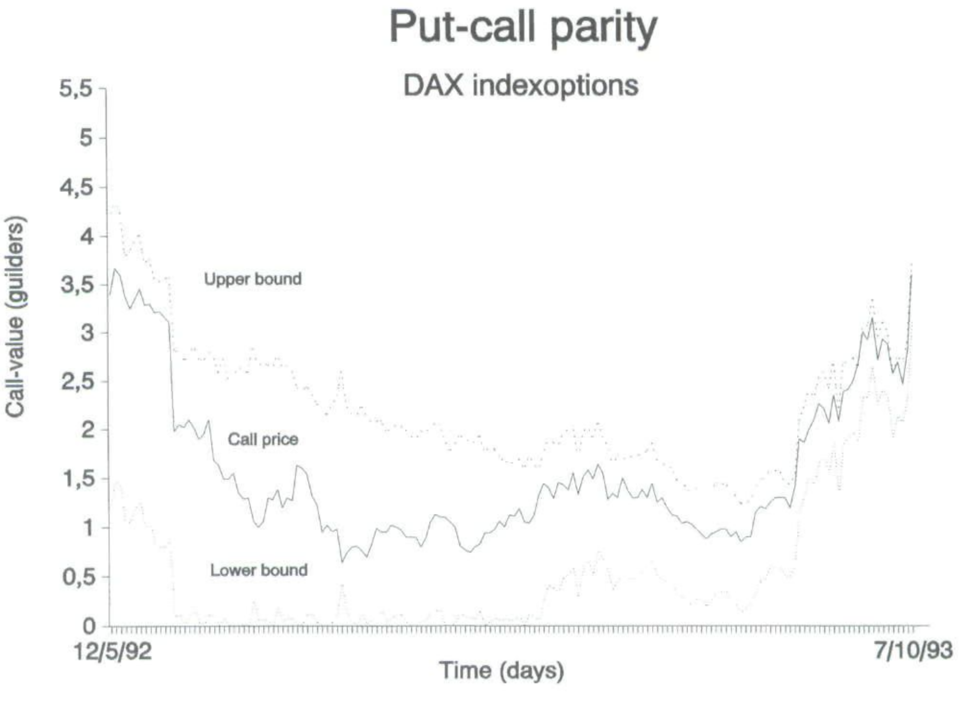

FIGURE 1
Upper and lower bounds of the put–call parity for options on the DAX index for May 12, 1992, to October 7, 1993.

put–call parity are included in Table I. Table I shows the premium for early exercise is always positive with an average of 0.616 Dutch guilders, while the average premium as a percentage of the put price is 51.3%.

Estimation of eq. (5) yields the following results (t-values in parentheses):

$$\underset{[-3.59]}{D_t = -0.62} + \underset{[5.20]}{0.06M_t} + \underset{[2.97]}{1.04\sigma_{\text{impl},t-1}} + \underset{[2.89]}{9.75r_t} - \underset{[-1.06]}{0.08(T-t)} + \underset{[11.27]}{0.66D_{t-1}} + \varepsilon_t \tag{8}$$

with $\hat{R}^2 = 0.8887$.

A Lagrange–Multiplier test for first-order autocorrelation has a value of 1.168. Under the null-hypothesis that there is no autocorrelation, this test-statistic has a $\chi^2_{(1)}$-distribution, implying that this null-hypothesis cannot be rejected at any reasonable level of significance. The value of $\hat{R}^2$ shows that the selected variables do a good job in explaining the value of early exercise. As expected, the signs for M_t, $\sigma_{\text{impl},t-1}$ and r_t are all positive, indicating that the early exercise premium indeed behaves like an option with respect to these variables. The size of the coefficient for M_t shows that this variable is not only statistically but also economically significant. If the value of the index changes with, e.g., one point, the premium changes (ceteris paribus) by 0.06 Dutch guilders, which is significant, given that the average premium is equal to 0.62 Dutch guilders. Changes of one point in I_t are not uncommon. The coefficient for $\sigma_{\text{impl},t-1}$ is also economically significant, since each percentage point change in the volatility translates (ceteris paribus) to a change of the premium of 1.04 Dutch guilders. Changes of one percentage point in the volatility are not unlikely, given the standard deviation of the volatility of 0.73% (see Table I). The coefficient of r_t

TABLE I

Summary Statistics for the Early Exercise Premium

			Quantiles				
	Average	*Std. Dev.*	*Smallest*	*5%*	*50%*	*95%*	*Largest*
$D_t = P_t - \hat{P}_t$ (in guilders)	0.616	0.171	0.067	0.133	0.535	1.404	1.964
$(P_t - \hat{P}_t)/P_t$	51.3%	18.44%	16.7%	25.5%	43.5%	104%	157%
$M_t = S_t^*(X - I_t)$ (in guilders)	87.3	149	−294	−198	107	287	343
$\sigma_{\text{impl},t-1}$	17.5%	0.73%	13.7%	14.6%	16.3%	25.1%	29.3%
r_t	6.3%	0.40%	5.9%	5.9%	7.1%	8.9%	8.9%
$T - t$ (in years)	1.2	0.42	0.51	0.61	1.26	1.85	1.89

Note: Standard deviations are corrected for autocorrelation where necessary.

is positive and is both statistically and economically significant. The positive value of D_{t-1} shows the positive autocorrelation, implying that a high premium at time t will probably be followed by a high premium at time, $t + 1$. Finally, the negative sign of the time to maturity is contrary to what might be expected. For this variable, however, there is a lack of both statistical and economical significance.

SUMMARY AND CONCLUSIONS

This study calculates the value of early exercise of a put option from deviations of the European put–call parity, using put options on a performance index. As hypothesized, the premium for early exercise is significantly and positively correlated with the moneyness, the volatility of the index and with the interest rate. Contrary to our expectations, a negative relationship with the time to maturity is found. However this relationship is not significant. The results of this study may be particularly useful for the development of a correct model for the pricing of American put options.

APPENDIX: DERIVATION OF THE AMERICAN PUT–CALL PARITY

Both eq. (2) and (3) can be derived by constructing a portfolio which must always have a positive value to exclude arbitrage possibilities. For the derivation of the lower bound, eq. (2), a portfolio needs to be constructed by taking the following positions:

1. A short position in the underlying index of $I_t e^{-q(T-t)}$;
2. A short position in a put option written on the index;
3. A long position in a call option written on the index with the same exercise price (X) and the same expiration date (T) as the put option;
4. A long position in a risk-free bond with value X.

It is assumed that if dividends have to be paid because of the short position in the index, additional units of the index are sold short by the amount of the dividend obligation. Exercise is possible at the expiration date, T, or at a time, τ, for which: $t < \tau < T$. For ease of exposition, only the possibility of early exercise of the written put option, is considered since early exercise of the call option can never reduce value for a rational investor. Thus, early exercise takes place when the put option is in-the-money, i.e., $X \geq I_\tau$.

TABLE AI
The Pay-Off for the Investor When Exercised

Value at Time t		*Exercise at* τ	*Exercise at* T	
		$X \geq I_\tau$	$X \geq I_T$	$X < I_T$
Short index	$-I_t e^{-q(T-t)}$	$-I_\tau e^{-q(T-\tau)}$	$-I_T$	$-I_T$
Short put	$-P_t$	$I_\tau - X$	$I_T - X$	0
Long call	C_t	0	0	$I_T - X$
Long bond	X	$Xe^{r(\tau-t)}$	$Xe^{r(T-t)}$	$Xe^{r(T-t)}$
	Total Pay off:	$I_\tau(1 - e^{-q(T-\tau)}) + X(e^{r(\tau-t)} - 1) \geq 0$	$X(e^{r(T-t)} - 1) \geq 0$	$X(e^{r(T-t)} - 1) \geq 0$

The pay-off for the investor in case of early exercise is given in Table A.I. Since the value at exercise is positive at any moment, the value of the portfolio must also be positive at time t, resulting in eq. (2).

For the derivation of eq. (3) a slightly different portfolio is needed, consisting of:

1. A long position in the index of I_t;
2. A long position in a put option written on the index;
3. A short position in a call option written on the index with the same exercise price, X, and the same expiration data, T, as the put option;
4. A short position in a risk-free bond with value, $Xe^{-r(T-t)}$.

If it is assumed that the dividends which are received on the index are continuously reinvested in the index, eq. (3) can be derived in a similar way as eq. (2).

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