Conditional Heteroskedasticity, Asymmetry, and Option Pricing

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INTRODUCTION

Many studies have shown that the empirical distribution of commodity futures price changes is not normal but leptokurtic [Hall et al. (1989); Hudson et al. (1987)]. That is, there are more observations around the mean and more extreme values than with a normal distribution. Skewness and serial dependence of successive price changes are also well documented [Taylor (1985); Yang and Brorsen (1993)]. However, ignoring non-normality and stochastic volatility likely leads to biased estimates of option premia. Therefore, a correct option pricing model should model not only the stochastic volatility but also the conditional non-normality. The Black-Scholes option pricing model (OPM) based on constant volatility yields systematically biased estimated of deep in-the-money and deep out-of-the-money options [Black (1975); Johnson and Shanno (1987); Hull and White (1987)]. These inaccuracies may be due to inappropriate assumptions about the futures price distribution. Eales

Authors wish to thank Robert Myers for helpful comments.

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The Journal of Futures Markets, Vol. 15, No. 8, 901–928 (1995)

CCC 0270-7314/95/080901-28

and Hauser (1990) argue that non-normality has not been considered extensively in commodity option pricing because no clearly superior distributional specification has been found.

Time-varying variance models can explain nonlinear dependence and leptokurtosis. The generalized autoregressive conditional heteroskedasticity (GARCH) process of Bollerslev (1986) was developed as an effective way of modeling the dynamics of volatility. Bollerslev (1987) suggested the GARCH (1, 1)-t model. The GARCH(1, 1)-t process fits most empirical data better than the GARCH(1, 1)-normal or a mixed diffusion-jump process [Yang (1989)]. Myers and Hanson (1993) also provided evidence that the GARCH OPM with a student distribution performs better than Black's OPM in predicting soybean option premia. Engle and Mustafa (1992) estimated GARCH parameters from option premiums and found that GARCH OPM outperforms the Black model with implied volatility.

The GARCH process does not model skewness. An exponential GARCH (EGARCH) model that captures skewness was suggested by Nelson (1991). The EGARCH model considers skewness by allowing the ARCH process to be asymmetric. Asymmetry in the dynamics of the mean return has not been considered in past research. This study introduces an asymmetric GARCH model that captures skewness in the mean equation and determines the most descriptive model of daily Kansas City wheat futures price distributions among several models that consider non-normality and conditional heteroskedasticity. The study also seeks to determine whether time-varying stochastic volatility and conditional non-normality can explain the biases in Black's option pricing model. The GARCH-t, the asymmetric GARCH-t, and the asymmetric EGARCH-t processes are estimated and the most likely model is selected.

Previous option pricing models allowing stochastic volatility [e.g., Hull and White (1987)] other than Myers and Hanson have not explicitly considered a conditional non-normal distribution. This article considers non-normality as well as stochastic volatility and determines their effects on option pricing. This study extends Myers and Hanson in several ways. Significance tests are conducted on the performances of Black's OPM and GARCH OPM both in terms of forecast error and trading returns. The simulated biases between the two OPM's are analyzed in terms of time to maturity effects and futures-exercise price ratio effects.

The GARCH and EGARCH processes with and without asymmetry are estimated by maximum likelihood. The preferred models are selected using likelihood ratio tests or the Schwarz model selection criterion. A

Monte Carlo simulation is used to determine the systematic mispricing error in Black's OPM when the GARCH process is the true model. The ability of Black OPM and the GARCH OPM to predict actual wheat option premiums of the Kansas City Board of Trade is determined. Finally, the trading returns form both OPMs are calculated and compared.

ECONOMETRIC MODELS

Many competing statistical distributions have been proposed to model the departures from normality: a symmetric stable Paretian distribution (Mandelbrot (1963); Fama (1965)], student t-distribution [Blattberg and Gonedes (1974)], a mixture of normal distributions [Kon (1984)], and a mixed diffusion-jump process [Akgiray and Booth (1988)]. However, since these models assume the independence of successive asset returns, they are inconsistent with empirical data that is known to be linearly or nonlinearly dependent. Further, these models are focused on capturing leptokurtosis. Jorion (1988) found that combining a jump process with a simple ARCH process provides a significantly better fit of the distribution of weekly exchange rates than either process alone. The mixed jump-diffusion process also models skewness. However, combining the GARCH(1, 1)-t process with a jump process is not always significantly better than the GARCH-t process alone [Brorsen and Yang (1994)]. Thus, a GARCH(1, 1)-t process is used as the benchmark model, and other alternative models are compared with it.

While the GARCH model elegantly captures the volatility clustering in asset returns, it ignores the possible asymmetric response of variance to positive and negative residuals and restricts the parameters in the variance equation to be non-negative. Nelson (1991) suggested an Exponential GARCH (EGARCH) model that overcomes these objections. LeBaron (1989) reported that the EGARCH model explains skewness better in the distribution of weekly and monthly stock indices than the GARCH model. However, Nelson's EGARCH model considers asymmetry only in the variance equation.

The GARCH under a student-t distribution and the EGARCH under a student-t distribution are considered here. Each model is estimated with and without asymmetry in the mean equation. The GARCH process can model well-documented market anomalies such as day-of-the-week [Chiang and Tapley (1983); Junkus (1986)], seasonality [Anderson (1985); Kenyon et al. (1987)] in mean and variance, and maturity [Milonas (1986)] in the variance equation. In the GARCH process, the futures price changes, R_t, can be expressed as a stochastic process:

$$R_t = f(I_{t-1}; \theta) + \varepsilon_t \tag{1}$$

where $f(I_{t-1}; \theta)$ denotes a function of I_{t-1} (the information set at time $t - 1$) and the parameter vector θ, and ε_t has a discrete time stochastic process

$$\varepsilon_t = \begin{cases} z_t h_t & \text{in the GARCH-normal model and} \\ ((v - 2)/v)^{1/2} \omega_t h_t & \text{in the GARCH-t model.} \end{cases} \tag{2}$$

where z_t is i.i.d. normal with $E(z_t) = 0$ and variance $\text{Var}(z_t) = 1$ and ω_t is i.i.d. student with degrees of freedom v, $E(\omega_t) = 0$ and $\text{Var}(\omega_t) = v/(v - 2)$. Therefore, h_t^2 is the time varying variance of ε_t. The GARCH(p, q) model expresses h_t^2 as a linear function of past variance and past squared values of the process,

$$h_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}^2 \tag{3}$$

Equation (1) is the mean equation and eq. (3) is the variance equation. Since h_t^2 is the conditional variance of ε_t, it must be non-negative. The GARCH model ensures non-negativity by making h_t^2 a linear combination of positive random variables. The EGARCH model ensures that h_t^2 remains non-negative by replacing h_t^2 with $\ln(h_t^2)$ in eq. (3). For example, the conditional variance of EGARCH(1,0)-t with one lag on the conditional variance is

$$\ln(h_t^2) = \alpha_0 + \beta \ln(h_{t-1}^2) + \eta(\varepsilon_{t-1}/h_{t-1}) + \varphi(|\varepsilon_{t-1}/h_{t-1}| - (2/\pi)^{1/2}) \tag{4}$$

Over the range $0 < \varepsilon_{t-1}/h_{t-1} < \infty$, $\ln(h_t^2)$ is linear in $\varepsilon_{t-1}/h_{t-1}$ with slope $\eta + \varphi$, and over the range $-\infty < \varepsilon_{t-1}/h_{t-1} < 0$, $\ln(h_t^2)$ is linear in $\varepsilon_{t-1}/h_{t-1}$ with slope $\eta - \varphi$. If $\eta = 0 \ln(h_t^2)$ responds symmetrically to $\varepsilon_{t-1}/h_{t-1}$, but if $\eta \neq 0$, $\ln(h_t^2)$ responds asymmetrically.

Asymmetric price transmission models have been used extensively in research on farm-retail price transmission [e.g., Kinnucan and Forker (1987); Boyd and Brorsen (1988)]. Restrictions on short selling, asymmetries in information, preferences of investors, and market psychology might cause differing responses to past price rising or falling. The mean equation of the asymmetric GARCH model is a special case of the Threshold Autoregressive model of Tong and Lim (1980).

The model with asymmetry in the mean equation is obtained by segmenting lagged price changes into one set for rising changes and another set for falling changes. The logarithmic changes in returns, R_t, are segmented as,

$$RP_t = \begin{cases} R_t, & R_t \geq 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$RN_t = \begin{cases} R_t, & R_t < 0 \\ 0, & \text{otherwise,} \end{cases}$$

where R_t is the logarithmic difference of daily returns at time t. In the asymmetric GARCH process, the mean and the variance equations to be estimated are,

$$\begin{aligned} R_t = a_0 &+ \sum_{i=1}^{m} \delta_i RP_{t-i} + \sum_{i=1}^{m} \omega_i RN_{t-i} + a_1 D_{\text{MON}} + a_2 D_{\text{TUE}} \\ &+ a_3 D_{\text{WED}} + a_4 D_{\text{THU}} + a_5\ \text{SIN}(2\pi K/252) + a_6\ \text{COS}(2\pi K/252) \\ &+ a_7\ \text{SIN}(2\pi K/126) + a_8\ \text{COS}(2\pi K/126) + \varepsilon_t \end{aligned} \tag{5}$$

$$\begin{aligned} h_t^2 = \alpha_0 &+ \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}^2 + b_1 D_{\text{MON}} + b_2 D_{\text{TUE}} \\ &+ b_3 D_{\text{WED}} + b_4 D_{\text{THU}} + b_5\ \text{SIN}(2\pi K/252) + b_6\ \text{COS}(2\pi K/252) \\ &+ b_7\ \text{SIN}(2\pi K/126) + b_8\ \text{COS}(2\pi K/126) + b_9 \text{TTM} \end{aligned} \tag{6}$$

where δ_i and ω_i represent the net effect of the ith positive and negative changes of R_t, respectively, and m is the length of lags. The length of lags in the mean equation is identified with the Schwarz criterion.[1] D denotes dummy variables for each day of the week; $D_{\text{MON}} = 1$ if Monday and 0 otherwise, $D_{\text{TUE}} = 1$ if Tuesday and 0 otherwise, $D_{\text{WED}} = 1$ if Wednesday and 0 otherwise, $D_{\text{THU}} = 1$ if Thursday and 0 otherwise. SIN and COS represent the sine and cosine functions, respectively, and π is approximated as 3.14. K in the sine and cosine functions is the number of trading days after January 1 of the particular year. Denominators in the sine and cosine functions are the specified cycle length in trading days, so 252 indicates a one-year cycle and 125 a half-year cycle. TTM is the time to maturity measured in the number of trading days prior to maturity. The variance equation for the EGARCH model is obtained by adding day-of-the-week, seasonality, and time to maturity variables into eq. (4). Maximum likelihood estimates of

[1] Schwarz's SC criterion is obtained by $\text{SC(m)} = \ln(\text{SSE}_{\text{m}}) + \text{Q}_m * \ln(T)/T$, where m is the length of lags SSE_m is the sum of squared residuals, Q_m is the number of parameters, and T is the number of observations. The value of m that minimizes SC is selected as the length of lags in the model.

alternative models are obtained using the statistical software package GAUSS (Aptech Systems Inc.).[2]

The asymmetry hypothesis is tested in two ways. In the first test, the total impact of past price increases is assumed to be same as that of past price decreases:

$$H_0 : \sum_{i=1}^{m} \delta_i = \sum_{i=1}^{m} \omega_i$$
$$H_A : H_0 \text{ is not true} \tag{7}$$

In the second test, the speed of adjustment to price increases and to price decreases are assumed to be the same [Boyd and Brorsen (1988)].

$$H_0 : \delta_i = \omega_i \,, \text{ for all } i$$
$$H_A : H_0 \text{ is not true} \tag{8}$$

For the hypothesis tests of eqs. (7) and (8), Wald-F statistics are used. The Wald test is not invariant in nonlinear models, but it is still asymptotically valid [Dagenais and Dufour (1991)]. The estimated t-ratios of the parameters of the skewness term (η) in eq. (4) provides a test of asymmetry in the variance equation of the EGARCH model.

SAMPLE DATA

The alternative statistical models are estimated with the first differences of the natural logarithms[3] of the daily futures closing prices of wheat at

[2]For the GARCH model, since the GARCH terms (α and β in the variance equation) are restricted to be non-negative, inequality restrictions are imposed by taking the exponential of the parameters. The degrees of freedom of the student distribution is restricted to be greater than three since the likelihood function is not defined for degrees of freedom less than or equal to three. The variance of the first 15 observations of price changes is used as an initial variance. The initial algorithm is Polak–Ribiere-type Conjugate Gradient method and the initial step size is one. After a few iterations, the algorithm is switched to the Davidson–Fletcher–Powell method and the Brent step-size method is used. The final estimates are obtained with the Newton method to that the Hessian is used to estimate the information matrix. All derivatives are calculated numerically. The estimation took about 3 to 15 hours on a 486DX2/66MHZ computer. Estimation time depends heavily on the starting values.

[3]Augmented Dickey–Fuller unit-root tests are conducted for futures prices, for the first differences of the futures prices, and for the log differences of the futures prices. The test statistics are –1.35, –6.43, and –6.48, respectively. The asymptotic critical value is –2.57. Therefore, the null hypothesis that the time series has a unit root is not rejected for the futures prices, but rejected for the first differences and the log differences. Both first differences and log differences remove nonstationarity from the data, but Fama (1965) provides reasons for using log differences which are percentage changes. First, the log difference is the return, with continuous compounding, from holding the asset. Second, while the variability of simple price changes increases as the price level increases, log differences neutralize price level effects.

the Kansas City Board of Trade. The first differences of the natural logarithm are rescaled by multiplying them by 100. The data are from Jan. 1982 to Sep. 1990. The data are created using Continuous Contractor from Technical Tools. Kansas City wheat futures contracts are traded based on five maturities: March, May, July, September, and December. The price series used is a continuous combination of the five contracts. The rollover date is the 21st day of the month prior to delivery.[4] Log differences are taken before splicing so that no outlier is created at the rollover date.[5] July option premia, for example, are only predicted for April 21 to June 20, the dates the July futures contract is used to estimate the GARCH model. The daily option premia over the simulation period are collected from Kansas City Grain Market Review.

Using a continuous series of nearby log changes implicitly imposes the restriction that the GARCH parameters are the same across contract months. One alternative would be to make a continuous series of July contracts, for example. The drawback of the alternative approach is that distant futures are often very thin. Closing prices reported will often be from a transaction earlier in the day or may be a bid or ask quote. The problem of thin markets are especially acute in Kansas City wheat futures. Therefore, the GARCH model is estimated and option premia are predicted only with near-to-maturity data.

The Kansas City wheat futures prices and log differences are graphed in Figure 1. The log differences show that futures prices have tranquil and volatile periods. The most volatile period, spring 1986, was when the Soviet nuclear reactor at Chernobyl exploded. Volatile periods in the summers of 1983 and 1988 were due to droughts in the Midwest when wheat prices rallied due to lower expected yields of corn and soybeans. Table I shows summary statistics of log differences of wheat futures prices at the Kansas City Board of Trade. The departures from normality are apparent from the high kurtosis and skewness. The Kolmogorov–Smirnov test rejects normality. Figure 2 clearly shows that the Kansas City wheat futures price changes have thicker tails and a sharper peak at the center than a normal distribution with the same standard deviation.

[4]The 21st is selected because options expire on the Friday 10 days before the first notice day. The first notice day is the last business day of the month.

[5]Myers and Hanson used an alternative approach. They explained (p. 125) that "a dummy variable was introduced into the GARCH model to account for the effect of switching contracts at annual intervals." Such an approach requires including a dummy variable in the variance equation the day of the rollover and the day after the rollover.

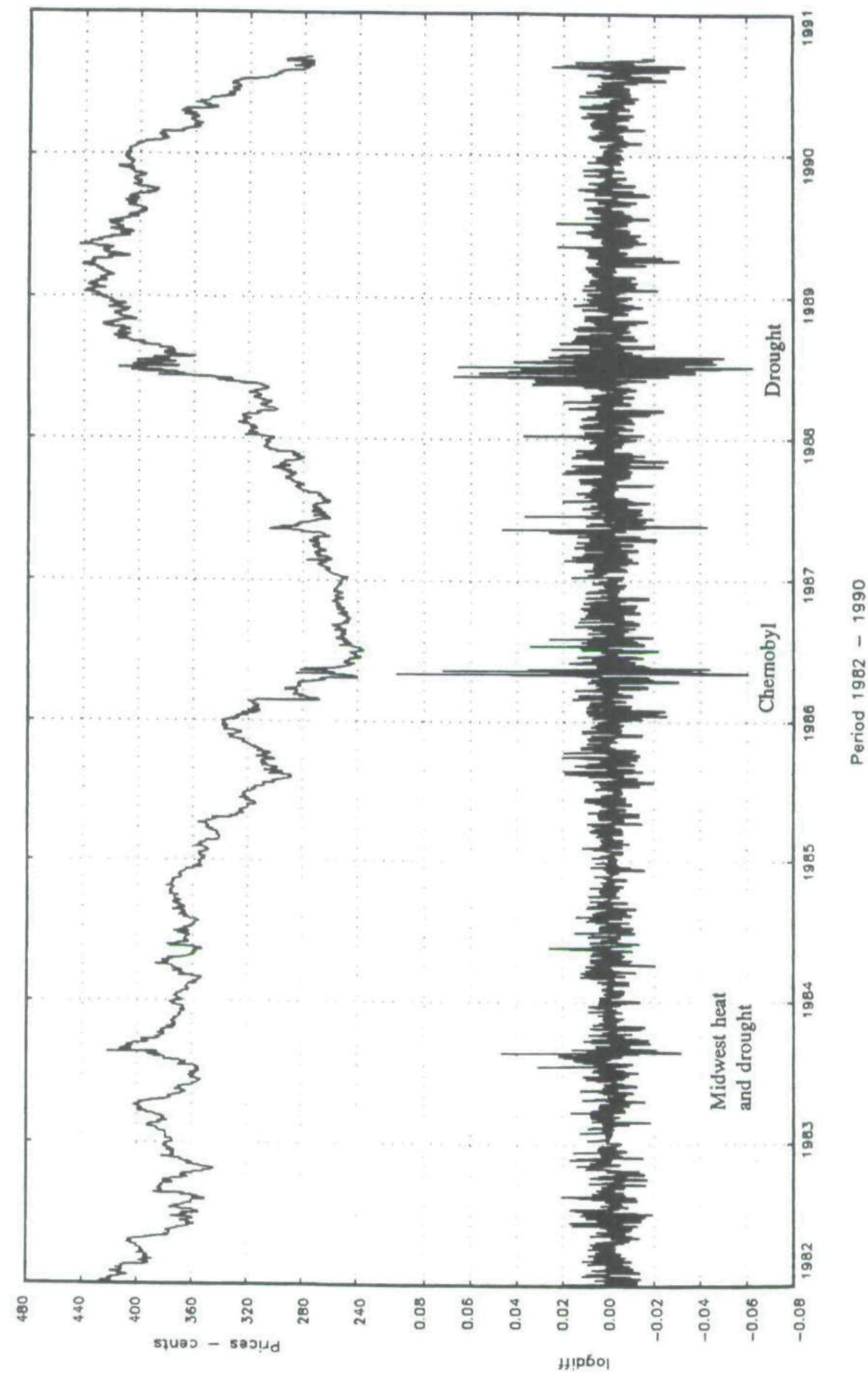

FIGURE 1

Kansas City wheat futures prices and log differences: 1982–1990.

TABLE I
Summary Statistics of Daily Kansas City Wheat Futures Prices from January 1982 through August 1990[a]

	Statistics
Sample size (T)	2191
Mean (μ)	−0.0109
Standard deviation (σ)	0.9774
Skewness[b]	0.6471[e]
Kurtosis[c]	11.1585[e]
D-statistic[d]	0.0717[e]

[a]Units are percentages. $R_t = [\ln(P_t) - \ln(P_{t-1})] \times 100$.
[b]Skewness is computed by $\sum_{t=1}^{T}(R_t - \mu)^3/(n-1)\sigma^3$.
[c]Excess kurtosis is computed by $\sum_{t=1}^{T}(R_t - \mu)^4/(n-1)\sigma^4 - 3$.
[d]The Kolmogorov–Smirnov Goodness-of-fit D-statistic. The critical value is $1.36/T^{1/2}$ at 5% significant level for T observations.
[e]Denotes the null hypothesis of normality (i.e., zero skewness and zero kurtosis) is rejected at a 5% level based on the critical values by Snedecor and Cochran (1980).

MODEL SELECTION AND VALIDATION

Procedure

Since the models are nested, the likelihood ratio test is used to select between the GARCH-t and the asymmetric GARCH-t models. Model selection between the asymmetric GARCH-t and the asymmetric EGARCH-t models are conducted by selecting the model with the

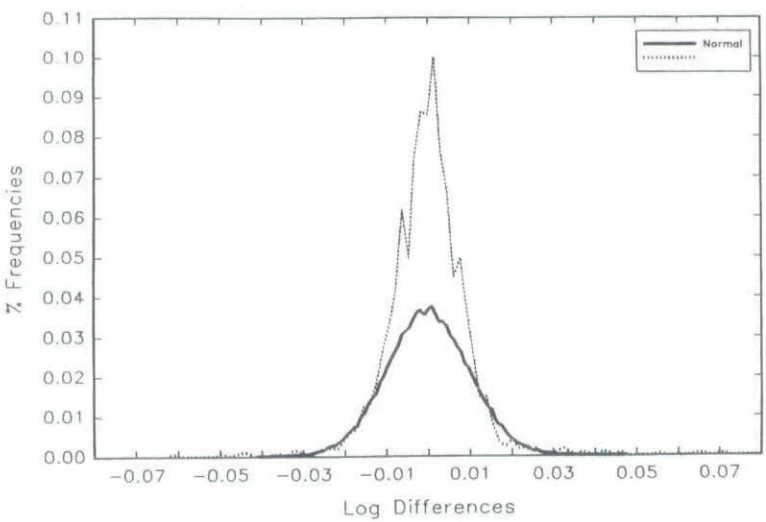

FIGURE 2
Empirical distribution of Kansas City wheat futures prices changes compared to the normal distribution with the same standard deviation.

higher Schwarz criteria. Schwarz criterion is useful for selecting among non-nested models with different numbers of parameters, because it penalizes the model with more parameters. Non-nested tests among some types of conditional heteroskedasticity have been developed [Higgins and Bera; Lee and Brorsen). None have been conducted for the GARCH and EGARCH models considered here. Non-nested tests can also be inconclusive. Therefore, the Schwarz criterion is used rather than trying to develop a non-nested test.

If the GARCH models are well-specified and fit the sample data, the standardized residual generated by the GARCH models should be i.i.d. normal or student. The Ljung–Box and McLeod–Li test statistics are used to test the null hypothesis of no serial dependence in the rescaled residuals, $(\hat{\varepsilon}_t/\hat{h}_t)$, and squared rescaled residuals, $(\hat{\varepsilon}_t/\hat{h}_t)^2$, of the selected model. The Brock–Dechert–Scheinkman (BDS) test [Brock et al. (1991)] is used to test the null hypothesis that $\{R_t\}$ is i.i.d.. The BDS statistic is based on the correlation integral. The statistic for raw data is asymptotically distributed as a standard normal random variable under the null hypothesis. However, Brock et al. (1991) have shown that the distribution of the BDS statistic is not standard normal when the data are GARCH residuals. Therefore, the test in this study is more accurate than the test by Yang and Brorsen (1992) which assumed the BDS statistic with GARCH residuals had a standard normal distribution. The tables in Brock et al. (p. 279) are used to obtain critical values of the test statistic. The Kolmogorov–Smirnov goodness-of-fit test is used to determine if the residuals have a t-distribution. The rescaled residuals are multiplied by $(\hat{\nu}/(\hat{\nu} - 2))^{1/2}$, where $\hat{\nu}$ is the estimated degrees of freedom of the model. This adjustment is needed because the variance of a t-distribution is $\hat{\nu}/(\hat{\nu} - 2)$. Yang and Brorsen's (1992) test is biased since they did not adjust the rescaled residuals.

Findings

Table II shows the estimated log-likelihoods and the tests of no asymmetry from the models of Kansas City wheat futures prices. In the asymmetric GARCH model, the total impact of past rising prices is not significantly different from that of past falling prices. However, the speed of adjustment for price rising is significantly different from that of price falling, implying significant asymmetries in mean returns. Asymmetry is also present in the mean equation of the asymmetric EGARCH model (Table II). The skewness term in the variance equation of the asymmetric EGARCH model is not significantly different from zero (Table II).

TABLE II

Estimated Log-Likelihoods and Tests of Asymmetry with Alternative Models of Daily Futures Prices of Kansas City Wheat

		Test Statistics of H_0: No Asymmetries		
		Mean[b]		
Model	*Maximized Log-Likelihood*[a]	*Total*	*Speed*	*Variance*[c]
Asymmetric GARCH(1, 1)-t	−2521.3	3.22	5.93[d,e]	na[e]
Asymmetric GARCH(2, 1)-t	−2515.4	3.33	5.86[d]	na
GARCH(2, 1)-t	−2520.6	na	na	na
Asymmetric GARCH(2, 2)-t	−2515.0	3.50	6.05[d]	na
Asymmetric GARCH(3, 1)-t	−2514.7	0.87	3.75[d]	na
Asymmetric EGARCH(1, 0)-t	−2523.5	8.08[d]	21.23[d]	1.39

[a]The sample has 2191 observations.
[b]Test statistics for the null hypothesis that there is no asymmetry in the mean equation.
[c]Test statistics of H_0: no asymmetry in the variance equation are the t-statistics of the parameter representing skewness [η in eq. (4)].
[d]Denotes rejection of the null hypothesis of no asymmetry at the 5% level.
[e]Not applicable.

Table III contains the test statistics used to select among models. Likelihood ratio statistics are used to select between nested models. Differences in Schwarz criteria are used to select between non-nested models. The asymmetric GARCH(2,1)-t process is selected and so its estimates are used in the simulation to obtain option premiums. Table IV reports estimates and test statistics of the asymmetric GARCH(2,1)-t

TABLE III

Test Statistics of Model Selection with Alterantive Models

Hypotheses		
Null	*Alternative*	*Statistics*
Asymmetric GARCH(1, 1)-t	Asymmetric EGARCH(1, 0)-t[b]	−3.29
Asymmetric GARCH(1, 1)-t	Asymmetric GARCH(2, 1)-t[a]	11.80[c]
GARCH(2, 1)-t	Asymmetric GARCH(2, 1)-t[a]	10.40[c]
Asymmetric GARCH(2, 1)-t	Asymmetric GARCH(2, 2)-t[a]	0.42
Asymmetric GARCH(2, 1)-t	Asymmetric GARCH(3, 1)-t[a]	1.40

[a]Likelihood ratio test statistic is obtained by $2T \times (L_1 - L_0)$, where $T (= 2191)$ is the number of observations, L_1 is the loglikelihood values under alternative hypothesis, and L_0 under null hypothesis.
[b]The statistic reported is the difference in Schwarz criteria which is obtained by $2T \times (L_1 - L_0) - (K_1 - K_0) \times \ln(T)$, where K_1 and K_0 are the number of parameters under alternative and null hypothesis, respectively.
[c]Denotes rejection of the null hypothesis in favor of the alternative hypothesis at the 5% significance level.

TABLE IV

Parameter Estimates and Test Statistics of the Asymmetric GARCH(2, 1)-t Process with Kansas City Wheat Futures Log Price Changes

	Estimated Coefficients	*(t-ratio)*		*Estimated Coefficients*	*(t-ratio)*
Mean			Variance		
Intercept	−0.125[a]	(−3.05)	Intercept	0.061	(1.74)
Lag 1 positive	0.027	(0.64)	Alpha	0.160[a]	(2.47)[b]
Lag 1 negative	0.151[a]	(3.69)	Beta1	0.190[a]	(2.27)[b]
Lag 2 positive	0.010	(0.26)	Beta2	0.592[a]	(7.25)[b]
Lag 2 negative	−0.134[a]	(−3.51)	D_{Mon}	0.048	(0.81)
Lag 3 positive	0.050	(1.23)	D_{Tue}	0.014	(0.22)
Lag 3 negative	−0.071	(−1.81)	D_{Wed}	−0.096	(−1.63)
D_{MON}	0.041	(0.88)	D_{Thu}	−0.039	(−0.70)
D_{TUE}	0.077	(1.68)	SIN252	0.006	(1.03)
D_{WED}	0.152[a]	(3.55)	COS252	−0.018[a]	(−2.47)
D_{THU}	0.036	(0.82)	SIN126	0.007	(1.21)
SIN252	−0.018	(−0.81)	COS126	−0.003	(−0.53)
COS252	0.061[a]	(2.79)	Maturity	−0.001	(−0.31)
SIN126	0.025	(1.19)			
COS126	−0.043[a]	(−2.09)			
Degrees of freedom					
ν	7.31[c]				
Wald F statistics					
Day of week in mean	3.66[a]				
Seasonality in mean	3.12[a]				
Day of week in variance	1.26				
Seasonality in variance	2.92[a]				
Ljung–Box and Mcleod–L1[d]					
$\varepsilon_t/h_t(12)$	13.66				
$\varepsilon_t^2/h_t^2(12)$	20.41				
BDS tests ($\varepsilon = \sigma$)[c]					
Raw data					
Dimension = 3	13.04[a]				
Dimension = 6	17.58[a]				
Dimension = 9	24.53[a]				
Rescaled data					
Dimension = 3	0.47				
Dimension = 6	−0.57				
Dimension = 9	−0.07				
Goodness-of-fit[f]					
D		0.013			

[a]Denotes the rejection of the null hypothesis at the 5% significance level, t-values are in parentheses.

[b]Since the GARCH terms are restricted to be positive, the null hypothesis lies on the boundary of the parameter space. Under the assumptions of Moran (1971), the t-statistic is distributed as a mixture of a degenerate distribution and a half t-distribution. Hypothesis tests can still be conducted in the usual fashion with t-tests. However, the t-ratio of α is computed as $t = e^{\hat{\alpha}}/(e^{\hat{\alpha}} s^2_{e^{\hat{\alpha}}} e^{\hat{\alpha}})^{1/2}$, where $s^2_{e^{\hat{\alpha}}}$ is the standard error of $e^{\hat{\alpha}}$, because the inequality constraint is imposed on the parameter α using an exponential transformation.

[c]The residuals are assumed to have a student-t distribution. The degrees of freedom of this student-t distribution is restricted to be greater than three.

[d]Both null hypotheses that ε_t/h_t are not autocorrelated and that ε_t^2/h_t^2 are not autocorrelated are tested with 12 degrees of freedom. Test statistics are distributed asymptotically as $\chi^2_{(12)}$ under the null hypothesis.

[e]H_0: The standardized residuals are i.i.d. The hypothesis test is based on Table F.4 in Brock et al. (p. 279).

[f]Kolmogorov–Smirnov D statistic. The critical value is $D_c = 1.36/T^{1/2} = 0.0299$ where T is the sample size.

model. The estimated GARCH terms are all positive and significant.[6] The sum of the GARCH terms (α, β_1, and β_2) is less than one which implies stationarity.[7] The effects of lagged rising prices are incorporated more quickly than those of lagged falling prices. Mean returns differ by day of the week, but variances do not. Significant seasonal patterns are revealed both in the mean and in volatility. The Ljung–Box and McLeod–Li tests (Table IV) do not detect any linear or second moment autocorrelations with the standardized data, which implies the GARCH-t process removed all the correlation in the first and second moments. The BDS statistics show that the null hypothesis of i.i.d. is rejected with the raw data (Table IV), implying that Kansas City wheat futures price changes are not i.i.d.. For the rescaled residuals, however, the BDS statistics do not identify nonlinear dependence. Using the Kolmogorov–Smirnov test, the null hypothesis that the GARCH rescaled residuals follow a student distribution is not rejected at the 5% significance level. Yang and Brorsen (1992) did not adjust the rescaled residuals by the degrees of freedom and found that a GARCH-t model would be rejected. Therefore, the adjustment is shown to be important.

IMPLICATIONS FOR OPTION PRICING

McBeth and Merville (1979) argued that the Black–Scholes formula overprices out-of-the-money options and underprices in-the-money options. However, Rubinstein (1985) argued that their conclusions did not always hold. Johnson and Shanno (1987) obtained numerical results for general cases in which the instantaneous variance obeys some stochastic processes. When volatility is a stochastic variable independent of stock price, Hull and White (1987) showed that the Black–Scholes formula overprices options that are at- or close-to-the money and underprices options that are deep in- and deep out-of-the-money.

Black's (1976) model specifics the option premium as a function of five underlying parameters: the current futures price (F_t), the exercise price of the option (X), the time to maturity of the option ($T - t$), the

[6]The parameters α, β_1, and β_2 are all restricted to be greater than or equal to zero. Thus the null hypothesis that the parameter is zero lies on the boundary of the parameter space. Wald-type tests of null hypothesis which lie on the boundary of the parameters space do not have the usual asymptotic normal distribution [Moran (1971)]. Also, the inequality constraint is imposed on the parameter α using an exponential transformation, so the t ratio of α is computed as $t = e^{\hat{\alpha}}/(e^{\hat{\alpha}} s^2_{e^{\hat{\alpha}}} e^{\hat{\alpha}})^{1/2}$, where $s^2_{e^{\hat{\alpha}}}$ is the standard error of $e^{\hat{\alpha}}$. Unfortunately, Wald tests are not invariant to nonlinear transformations. Thus, the hypothesis tests should be interpreted with caution.

[7]$\alpha + \beta_1 + \beta_2 < 1$ is a sufficient condition, although not a necessary condition. Most past research is not clear on this point.

risk-free rate of interest (r), and the variance of futures prices (σ^2). The Black premium (B) is

$$B = \begin{cases} e^{-r(T-t)}[X \times N(-d_2) - F_t \times N(-d_1) & \text{for put} \\ e^{-r(T-t)}[F_t \times N(d_1) - X \times N(d_2) & \text{for call} \end{cases} \tag{9}$$

where $d_1 = [\ln(F_t/X) + (\sigma^2/2)(T - t)]/\sigma(T - t)^{1/2}$, $d_2 = [\ln(F_t/X) - (\sigma^2/2)(T - t)]/\sigma(T - t)^{1/2}$, and $N(\ )$ is the normal cumulative density function.

The GARCH OPM cannot be solved in a closed form, since the stochastic volatility adds risk which cannot be diversified into a riskless hedge portfolio [Johnson and Shanno (1987); Hull and White (1987); Myers and Hanson (1993)]. Thus, the GARCH OPM approximates the premium by providing expected option prices at maturity with Monte Carlo integration. The GARCH OPM is an European option premium. Only European options are considered since estimates of American option premiums cannot be obtained with Monte Carlo methods [Hall (1989), p. 219].

Two sets of $T - t$ random numbers are generated: one from a t-distribution with $\hat{\nu}$ degrees of freedom and another from a standard normal distribution. Time is measured in number of trading days. Then, the futures prices F_t are simulated for $T - t$ periods using estimates from the selected model to get the futures price at maturity. Denoting this price at maturity $\{F_T\}_i$, the simulated option prices are

$$G = \begin{cases} e^{-r(T-t)}(1/n) \sum_{i=1}^{n} \max[X - \{F_T\}_i, 0] \text{ for call} \\ e^{-r(T-t)}(1/n) \sum_{i=1}^{n} \max[\{F_T\}_i - X, 0] \text{ for put} \end{cases} \tag{10}$$

where $n = 1000$ is the number of replications.

One efficient way to improve the accuracy of this calculation is to use the control variate technique [Hammersley and Handscomb (1964); Boyle (1977)]. This technique requires solving a problem which is similar to the one under consideration, but has an analytical solution. The solution of the simpler problem is used to increase the accuracy of the solution to the more complex problem. Essentially, a term is added to the GARCH option price, G is eq. (10). The term has expected value zero, but is negatively correlated with the error in G. The control variate is $B - B_{MC}$, where B is the analytical solution to Black's model in eq. (9) and B_{MC} is the Monte Carlo solution to Black's model. The same random number streams are used to compute G and B_{MC} and, thus, G and B_{MC}

are positively correlated. The GARCH option price calculated with the control variate technique (G^*) is

$$G^* = G + (B - B_{\mathrm{MC}}) \tag{11}$$

The calculation of premiums close to maturity takes a few seconds and premiums further from maturity take a few minutes. The GARCH option price calculation would be well suited for parallel processing. More replications than necessary are used in this study. In sum, calculation of GARCH option premiums would be practical with current technology.

MONTE CARLO STUDY

Procedure

A Monte Carlo study is conducted to determine the differences between Black's OPM and the GARCH OPM. Differences between Black's OPM and the GARCH OPM may be caused by not considering the observed conditional heteroskedasticity and non-normality in Black's OPM. The extent of the difference can be measured by the absolute difference ($B - G^*$) or by the percentage difference $\xi = (B - G^*)/G^*$. The differences for short-lived option premia are not the same as those for long-lived option premia. The differences are also different by how much the option is in the money or out of the money. Most commodity futures option exercise prices are within 10% of the actual price. Futures-exercise price ratios (F_t/X) of 0.90, 0.95, 1.0, 1.05, and 1.10 are considered. In the Monte Carlo study, exercise price is \$1.00. Differences are measured from six months through two weeks prior to maturity.

The average unconditional variance of the GARCH process $\hat{\alpha}_0/(1 - \sum_{i=1}^{q} \hat{\alpha}_i - \sum_{j=1}^{p} \hat{\beta}_j)$ is used as an initial volatility to generate conditional variances for G, and as the constant volatility for Black's analytical solution B and Black's Monte Carlo solution B_{MC}. The seasonality and the day of the week effects are not included in the Monte Carlo study. Thus, the difference in option premiums with the Black and GARCH models in the Monte Carlo study reflect only the different distributional assumptions. A mean percentage change of zero is used. Therefore, the asymmetry in the mean is not considered here. Maturities up to six months are considered to illustrate the model's predictions. But, of course, the model is only valid in predicting actual option premia which are within two or three months of maturity. The asymptotic t-statistics for the simulated differences are provided. These could be used to determine whether the reported pricing biases are significantly different from zero. They are the ratios of the simulated differences to the standard deviations of the differences.

Findings of Monte Carlo Study

In this section, the differences in option pricing between Black OPM and the GARCH OPM are calculated. Table V presents absolute and percentage differences between put option premiums with Black's OPM and the GARCH OPM. Black's OPM yields significantly lower premiums than the GARCH OPM for deep in- and deep out-of-the-money put options. Absolute differences increase as time to maturity increases in deep in- and deep out-of-the-money put options (Table V, Panel A). Percentage differences for deep in-the-money options also increase

TABLE V

Differences Between Black's Option Pricing and GARCH Option Pricing for Put Options When the True Process is GARCH

	Time to Maturity (months)					
	0.5	*1*	*1.5*	*3*	*4.5*	*6*
Panel A: Absolute differences[a]						
Deep in-the-money	−0.002	−0.008	−0.018	−0.030	−0.047	−0.059
($F_t/X = 0.90$)	(−0.20)	(−0.66)	(−1.09)	(−1.30)	(−1.71)	(−1.91)
In-the-money	−0.001	−0.007	−0.005	−0.019	0.059	−0.010
($F_t/X = 0.95$)	(−0.15)	(−0.59)	(−5.36)	(−0.88)	(2.39)	(−0.39)
At-the-money	0.051	0.052	0.078	0.070	0.069	0.040
($F_t/X = 1.0$)	(7.89)	(5.26)	(6.37)	(3.78)	(3.16)	(1.54)
Out-of-the-money	−0.021	−0.015	−0.017	0.008	0.061	0.024
($F_t/X = 1.05$)	(−5.19)	(−2.45)	(−1.81)	(0.55)	(3.46)	(1.14)
Deep Out-of-money	−0.009	−0.019	−0.036	−0.025	−0.028	−0.036
($F_t/X = 1.10$)	(−3.67)	(−5.35)	(−5.92)	(−2.58)	(−2.00)	(−1.99)
Panel B: Percentage differences (%)[b]						
Deep in-the-money	−0.02	−0.08	−0.18	−0.30	−0.46	−0.57
($F_t/X = 0.90$)	(−8.11)	(−9.85)	(−6.09)	(−4.08)	(−9.87)	(−0.16)
In-the-money	−0.03	−0.14	−0.10	−0.32	0.95	−0.17
($F_t/X = 0.95$)	(−14.88)	(−9.20)	(−8.83)	(−3.09)	(3.48)	(−0.83)
At-the-money	4.89	3.47	4.28	2.67	2.17	0.01
($F_t/X = 1.0$)	(0.40)	(0.58)	(0.41)	(0.91)	(1.01)	(0.96)
Out-of-the-money	−31.67	−6.73	−4.06	0.86	4.42	1.31
($F_t/X = 1.05$)	(−11.35)	(−0.43)	(−2.69)	(1.50)	(0.91)	(1.27)
Deep Out-of-money	−97.67	−64.92	−44.54	−9.05	−5.04	−4.20
($F_t/X = 1.10$)	(−5.58)	(−12.46)	(−12.95)	(−4.48)	(−0.63)	(−3.66)

Note: Asymptotic t-statistics are in parentheses.

[a]Black's option price minus GARCH option price when exercise price is set equal to $1.00. The absolute differences are measured in ¢/bushel.

[b][(Black's price − GARCH price)/GARCH price]*100.

as time to maturity increases, but those for deep-out-of-the money options decrease as time to maturity increases (Table V, Panel B). As time to maturity decreases, the time-value of deep out-of-the-money options decreases very quickly and eventually becomes zero. Therefore, deep out-of-the-money options close to maturity show extremely high percentage differences. At-the-money option premia are significantly higher with Black OPM than with the GARCH OPM. Percentage differences for at-the-money put options decrease as time to maturity increases.[8]

The simulation results confirm Hull and White's findings that the Black–Scholes model underprices in- and out-of-the-money options when stochastic volatility is present. Their argument that the Black–Scholes model overprices close-to-the-money options is also confirmed. The absolute differences are small and absolute differences between the two OPM's are larger for at-the-money options than for in- or or out-of-the-money options.

The asymptotic t-statistics (Table V) provide some evidence that the reported option pricing errors are not due to sampling errors. In particular, differences between Black's option price and the GARCH option price for deep out-of-the-money options are always significantly different from zero.

PREDICTING PREMIA

Procedure

To compare the performance of the GARCH OPM and Black's OPM, option prices are estimated for 1991 Kansas City wheat futures for two month periods prior to maturity for each March, May, July, September, and December contract. Both Black's OPM and GARCH OPM provide predictions conditional on $F_t, F_{t-1}, \ldots, F_{t-19}$. Thus, they predict today's premium based on today's information. The annualized risk-free interest rate is assumed constant during the simulation period at $r = 5.5\%$.[9] The estimated mean equation is not used in the simulation; instead, the expected percentage change used is zero.[10] In this out-of-

[8]When European options, are considered, put–call parity must hold. The results of the Monte Carlo study with call options are similar and, therefore, are not reported.
[9]During the simulation period, the range of the rate of return on Treasury bills is (0.052, 0.058).
[10]Because of the logarithmic transformation, the expectation is $.5h_t^2$ rather than zero (SAS/ETS p. 119). The difference between the approach used here and using $.5h_t^2$ is less than the rounding error in the tables.

sample simulation,[11] 20-day historical volatility,[12] which is seasonally adjusted,[13] is used to generate unconditional variances in the GARCH integration process G, in Black's analytical solution B, and in Black's Monte Carlo solution B_{MC}. Since the effect of seasonality is deterministic, it is calculated separately from the stochastic simulation and then added on at the end. The mean is again assumed to be zero; and, thus, asymmetry is only important in that it is considered when estimating the variance equation. The model is only used to predict nearby option premiums. Thus, the model is used to predict July option premiums only over the same time period that July futures contract data are used in estimating the GARCH model.

Results are given for both put and call options. The mean errors and root mean squared errors (RMSE) of Black's OPM and GARCH OPM are computed. Further, the Ashley–Granger–Schmalensee test is used to test if the mean squared error of Black's OPM is equal to that of the GARCH OPM. Statistical tests are based on White's heteroskedastic-consistent covariance matrix because of likely heteroskedasticity due to differences in maturity and on Newey and West's autocorrelation-consistent covariance matrix because the Ashley–Granger–Schmalensee test assumes independence.

Findings of Predicting Actual Premia

This section discusses which OPM is better in predicting actual premia. The ranges of futures prices during the simulation period, and exercise prices for in, at, and out-of-the-money options are shown in Table VI. Table VII shows the out-of-sample results over Dec. 1990 to Nov. 1991. A total of 2280 out-of-sample observations are provided. Except for out-

[11]The statistical model of Kansas City wheat futures price changes is estimated using data from Jan. 1982 through Sept. 1990. The Black and GARCH option premia are simulated for the trading of Dec. 1990 through Nov. 1991, so the simulation is out of sample.

[12]For the purpose of forecasting option premiums using Black's OPM, the implied volatility is a better choice of variance in Black's model than historical volatility because the implied volatility implicitly contains information about all market conditions including non-normality. However, since the objective is to test distributional assumptions between Black's OPM and the GARCH OPM, it is not appropriate to use implied volatility because then Black's OPM and the GARCH OPM would be conditioned on different information sets. It is theoretically possible, although not yet practical (because of computer costs), to calculate an implied volatility for the GARCH model. The GARCH implied volatility would not be the same as the implied volatility with Black model.

[13]The historical volatility is seasonally adjusted by multiplying it by the seasonality index which is calculated as$(\sum_{t=t_0}^{\tau} \hat{S}_t/(\tau - t_0 + 1)/(\sum_{t=t_0-19}^{t_0} \hat{S}_t/20)$, where t_0 is current date, τ is the day of maturity, and $\hat{S}_t$ is unconditional variance of day t. The unconditional variance is the expected value of eq. (6) calculated using estimated parameters.

TABLE VI

Ranges of Futures Prices During the Out of Sample Simulation Period and Strike Prices of In, At, and Out-of-the-Money Options for Each Contract[a]

Maturity	*March*	*May*	*July*	*September*	*December*
Panel A: Put option					
Price ranges	(2.55, 2.65)	(2.70, 2.90)	(2.85, 2.95)	(2.70, 3.00)	(3.10, 3.80)
Out-of-money	2.40	2.60	2.70	2.60	2.70
At-the-money	2.60	2.80	2.90	2.90	3.40
In-the-money	2.90	3.20	na[b]	3.10	3.80
Panel B: Call option					
Price ranges	(2.55, 2.65)	(2.70, 2.90)	(2.85, 2.95)	(2.70, 3.00)	(3.10, 3.80)
Out-of-money	2.90	3.20	3.20	3.30	3.60
At-the-money	2.60	2.90	2.90	2.90	3.40
In-the-money	2.40	2.60	2.70	2.60	2.70

[a]Units are in \$/bushel.
[b]In-the-money option is not available.

of-the-money call options, the mean error and root mean squared error (RMSE) of the GARCH OPM are smaller than those of Black's OPM. However, the differences are not large, especially in terms of RMSE. The Ashley–Granger–Schmalensee (AGS) test (Table VIII) finds the mean squared error of Black's OPM is significantly larger than that of GARCH OPM for in-the-money put as well as in-the-money call, and out-of-the-money call options. However, there is no winner for the at-the-money put, at-the-money call, and out-of-the-money call options. Therefore, the

TABLE VII

Out of Sample Forecasting Performance of Black and GARCH Option Pricing for 1991 Kansas City Wheat Options

	Black			GARCH		
Moneyness[a]	Out	At	In	Out	At	In
Panel A: Put option						
Mean error[b]	−0.60	−0.78	−0.09	−0.14	−0.75	0.05
Root mean squared error[b]	1.08	1.90	2.59	0.70	1.87	2.43
Panel B: Call option						
Mean error[b]	−0.34	−0.64	−0.10	−0.37	−0.62	0.004
Root mean squared error[b]	0.70	1.81	1.86	0.76	1.77	1.80

[a]The precise exercise prices for the in, at, and out-of-the-money options are given in Table VII.
[b]Mean errors and root mean squared errors are in cents per bushel.

TABLE VIII

Ashley–Granger–Schmalensee Test of the Out of Sample Forecast Performance of Black and GARCH Option Pricing for 1991 Kansas City Wheat Options

	β_1^a	β_2^a	F *Statistics*[b]	*Conclusion*	*Model Favored*
Panel A: Put option					
Out-of-money[c]	0.2233	0.4675[e]	12.77[e]	reject H_0	GARCH
	(1.58)	(4.84)			
At-the-money[c]	0.0054	0.0372[e]	0.34	not reject H_0	None
	(0.35)	(0.83)			
In-the-money[c]	0.7318[e]	0.0488	12.27[e]	reject H_0	GARCH
	(4.95)	(0.11)			
Panel B: Call option					
Out-of-money[d]	0.0401	0.0333	0.73	not reject H_0	None
	(1.09)	(0.99)			
At-the-money[c]	0.0101	0.0169	0.29	not reject H_0	None
	(0.76)	(0.36)			
In-the-money[c]	4.3970	0.1001	14.30[e]	reject H_0	GARCH
	(4.357)	(0.32)			

[a]The Ashley–Granger–Schmalensee test is based on the following regression results: $\Delta_t = \beta_1 + \beta_2(\Sigma_t - \overline{\Sigma}_t) + u_t$. Here,

$$\Delta_t = \begin{cases} e_t^B - e_t^G & \text{when } \overline{e}_t^B > \overline{e}_t^G, \\ e_t^G - e_t^B & \text{when } \overline{e}_t^G > \overline{e}_t^B. \end{cases}$$

where e_t^B is Black's option price minus actual optin price and e_t^G is GARCH option price minus actual price. $\Sigma_t = e_t^B + e_t^G$, assuming $\overline{e}_t^B > 0$ and $\overline{e}_t^G > 0$. $\overline{\Sigma}_t$, $\overline{e}_t^B$, and $\overline{e}_t^G$ are means of Σ_t, e_t^B, and e_t^G, respectively.

[b]If both β_1 and β_2 are negative, then F with one half of regular significance level can be used. However, if one of β_1 and β_2 is negative, then t test of the other coefficient should be used. If one of β_1 and β_2 is negative and significantly different from zero, then the test is inconclusive.

[c]H_0: Mean squared error of Black's OPM is equal to that of GARCH OPM,
H_1: Mean squared error of Black's OPM is greater than that of GARCH OPM.

[d]H_0: Mean squared error of Black's OPM is equal to that of Black's OPM,
H_1: Mean squared error of Black's OPM is greater than that of Black's OPM.

[e]Denotes significance at 5% level. Numbers in parentheses are t statistics.

GARCH OPM performs better than Black's OPM for in-the-money put and call options and out-of-the-money put options in this out-of-sample simulation. The out-of-sample results agree with the results from the Monte Carlo study in the previous section which shows little difference between GARCH OPM and Black OPM with at-the-money options.

SIMULATED TRADING

Procedure

This section considers which OPM gains more money in actual trading. The option trading simulation follows the methods of Hauser and Liu except that hypothesis tests are conducted with methods that do not assume homoskedasticity and normality. Hauser and Liu's method creates a riskless arbitrage position by taking positions in both options and futures markets. The ratio of positions in the two markets is determined by the option's delta which is defined as

$$h_p = \partial P/\partial F = -e^{-r(T-t)}N(-d_1) \text{ for put and}$$
$$h_c = \partial C/\partial F = e^{-r(T-t)}N(d_1) \text{ for call} \tag{12}$$

where h_p is the put option's delta and h_c is the call option's delta, $d_1 = \ln(F_t/X) + (\sigma^2/2)(T - t)]/\sigma(T - t)^{1/2}$, P and C are put and call option premia, and F, r, $T - t$, $N()$ are defined with eq. (9).

A position is established if the absolute difference between the predicted premium and the actual premium exceeds the actual premium by at least 5%. For example, if the predicted premium is less than the actual premium by 5%, then the option is assumed over-priced. With an over-priced put option, a short position in one option contract is taken and a short position in h_p underlying futures contracts is taken. With an over-priced call option, a short position in one option contract is taken and a long position in h_c underlying futures contracts is established also.

The daily net trading returns of each portfolio are

$$\pi_u = (L_{t+1} - L_t) - h_{Lt}(F_{t+1} - F_t) - TC$$
$$\pi_o = -(L_t - L_{t+1}) + h_{Lt}(F_{t+1} - F_t) - TC \tag{13}$$

where π_u and π_o are the return for under and over-valued options, respectively, and L_t is the premium on day t. With put options, $L_t = P_t$ and with call options, $L_t = C_t$. The futures price on day t is F_t. h_{Lt} is option's delta on day t ($h_{Lt} = h_p$ when the underlying option is a put, and $h_{Lt} = h_c$ when the underlying option is a call). TC is the trading cost which consists of brokerage cost and opportunity costs. The brokerage costs of a round turn trade are assumed to be \$50/contract in options and \$50/contract in futures. The margin for Kansas city wheat futures contract is assumed to be 5% of the contract value.

Trading returns from a random strategy are obtained also to estimate the standard error of trading returns under the null hypothesis of no information value. The random strategy is to trade options and underlying futures contracts based on the buying or selling signals given by a random deviate generator.[14] Standard deviations of each contract are obtained by 1000 replications of the random strategy. Asymptotic t values are calculated by dividing trading returns by the standard deviation of the corresponding random strategy.

Findings of Simulated Trading

Returns from trading of 1991 Kansas City wheat futures contracts and options are presented in Table IX. Option and corresponding futures contracts are traded based on whether the premium predicted with Black's OPM and the GARCH OPM is greater than or less than the actual premium. Trade is conducted for an in, at, and out-of-the-money put and call option for all five contract months. Daily trading returns are obtained for a two-month period prior to maturity of each contract.

Most returns are not significantly different from zero, which implies the market is efficient. Neither of the two OPMs dominates the other. When using the GARCH OPM, total after-cost returns for two months are positive for 11 out of 29 contracts. Nine contracts have positive returns after cost when using Black's OPM. However, the pairwise difference test for the total returns of all contracts shows there is no significant difference between returns of Black's OPM and returns of the GARCH OPM for both the before-cost return and after-cost return (Table IX). The pairwise differences are computed as (daily return by Black's OPM)-(daily return by the GARCH OPM). Negative signs of the t-statistics of the pairwise difference test imply that trading returns with the GARCH OPM are slightly higher than those with Black's OPM. Analysis of variance for a randomized block design shows differences in daily returns of the two OPMs are not statistically significant (Table X). The differences between Black's OPM and the GARCH OPM trading returns are much smaller than the differences Hauser and Liu found when using Black's OPM with different initial volatilities. This fact suggests that when trading options, what initial volatility to use may be a more important concern than what option pricing model to use.

[14]The Monte Carlo hypothesis testing method is a nonparametric method of measuring sampling variability which can be used when analytical approaches are not available [Noreen (1989)]. Uniform (0, 1) random numbers are generated for each trading day. If the random number of the day lies between 0 and x, all options and futures positions are cleared. x is the portion of days a neutral position is signaled by OPM. If the random number lies in the remaining first half, i.e. $(x, x + (1 - x)/2)$, the option is assumed to be under-priced. If the number lies in the remaining second half, i.e. $(x + (1 - x)/2, 1)$, the option is assumed to be over-priced.

TABLE IX

Returns from Trading 1991 KC Futures Wheat Options Based on Black and GARCH Option Pricing Models[a] and Pairwise Difference Tests

	Moneyness	Cost	March Black	March GARCH	May Black	May GARCH	July Black	July GARCH	September Black	September GARCH	December Black	December GARCH
Put	Out-of-money	Before[b]	7.56	7.26	211.38[g]	154.50[g]	119.33	119.33	155.78	335.11	5.00	5.00
			(0.19)[d]	(0.17)	(3.78)	(2.93)	(1.41)	(1.41)	(0.50)	(1.11)	(1.15)	(1.15)
		After[c]	−44.31	−94.95	−20.23	0.18	67.59	67.59	−12.02	−89.59	−44.96	−44.96
			(−0.47)	(−0.93)	(−0.18)	(0.002)	(0.57)	(0.57)	(−0.04)	(−0.28)	(−0.50)	(−0.50)
	At-the-money	Before	−100.15	−67.51	362.15	360.54	−11.93	−11.93	605.21	504.85	1487.00[g]	1582.20[g]
			(−0.37)	(−0.25)	(0.99)	(0.94)	(−0.03)	(−0.03)	(0.72)	(0.60)	(2.13)	(2.36)
		After	−653.62[g]	−675.98[g]	−101.21	−32.26	−290.95	−318.28	145.71	100.41	845.83	1101.90
			(−2.23)	(−2.31)	(−0.27)	(−0.80)	(−0.68)	(−0.74)	(0.17)	(0.11)	(1.20)	(1.63)
	In-the-money	Before	−281.53	−281.53	−287.33	132.67	na[e]	na	2040.90	2492.60	269.58	256.90
			(−1.92)	(−1.92)	(−1.05)	(0.60)			(1.31)	(1.55)	(0.63)	(0.55)
		After	−337.49	−337.49	−386.77	94.06	na	na	938.07	1641.20	−159.74	−172.33
			(−1.73)	(−1.73)	(−1.34)	(0.39)			(0.60)	(1.01)[f]	(−0.34)	(−0.34)
Call	Out-of-money	Before	64.38	64.38	−2.37	−13.13	113.29	113.29	41.14	70.72	−43.75	−279.26
			(1.40)	(1.40)	(−0.12)	(−0.66)	(1.52)	(1.52)	(1.13)	(1.86)	(−0.11)	(−0.72)
		After	14.26	14.26	−53.50	−465.88[g]	62.69	62.69	−113.03	−159.05	−648.56	−649.37
			(0.14)	(0.14)	(−0.61)	(−5.29)	(0.54)	(0.54)	(−1.18)	(−1.54)	(−1.60)	(−1.61)
	At-the-money	Before	318.55	151.82	−141.06	29.99	199.82	170.54	774.47	1369.00	−869.35	−1238.50
			(1.00)	(0.47)	(−0.38)	(0.08)	(0.48)	(0.42)	(0.95)	(1.68)	(−1.43)	(−1.64)
		After	−92.31	−276.39	−708.35	−251.48	−41.48	−70.76	106.34	737.83	−1277.3	−1568.50
			(−0.27)	(−0.77)	(−1.86)	(−0.66)	(−0.09)	(−0.17)	(0.13)	(0.89)	(−1.94)	(−2.00)

continued

TABLE IX (Continued)

Returns from Trading 1991 KC Futures Wheat Options Based on Black and GARCH Option Pricing Models[a] and Pairwise Difference Tests

		March		May		July		September		December	
Moneyness	Cost	Black	GARCH	Black	GARCH	Black	GARCH	Black	GARCH	Black	GARCH
In—the-money	Before	121.58	165.85	−144.43	−124.43	−401.80	−309.77	394.61	246.90	527.01	527.01
		(0.59)	(0.92)	(−0.35)	(−0.33)	(−0.74)	(−0.68)	(0.94)	(0.88)	(1.52)	(1.52)
	After	−56.61	−12.56	−334.72	−253.97	−608.68	−458.23	335.02	215.82	488.85	488.85
		(−0.22)	(−0.05)	(−0.70)	(−0.58)	(−1.08)	(−0.99)	(0.70)	(0.66)	(1.31)	(1.31)
Pairwise difference t-tests[f]	Before cost: −0.964										
	After cost: −1.198										

[a]Total returns from option trading for two months prior to maturity are reported. Units are in $/contract.
[b]Returns from trading before cost.
[c]Returns from trading after cost. Cost is brokerage cost plus opportunity cost.
[d]Numbers in the parentheses are t-values based on standard deviations of returns from random strategy. Random strategy is to trade options and futures contracts by the buying or selling signals given by a bootstrap method. Standard deviations of each contract is obtained by 1000 replications of random strategy.
[e]In-the-money put option is not available for 1991 KC July wheat futures contract.
[f]Tests are based on 29 observations of differences of (return from Black's OPM)-(return from the GARCH OPM) each in before cost and after cost.
[g]Denotes significance at 5% level.

TABLE X

Analysis of Variance Table of the Randomized Block Design for Comparison of the Daily Option Trading Returns for 1991 KC Wheat Futures Based on Black's and GARCH Option Pricing Models

Source	*Degrees of Freedom*	*Sum of Squares*	*Mean Sum of Squares*	*F-Values*	Pr > F
Total	2279	94860.56			
Among experimental units[a]	57	3514.95	61.67		
blocks[b]	28	3300.47	117.87	15.50	0.0001
treatments[c]	1	1.51	1.51	0.20[f]	0.6591
experimental errors[d]	28	212.97	7.61		
Within experimental units (sampling errors)[c]	2222	91345.33	41.11		

[a]Option trading for contracts with different maturity (March, May, July, September, and December) and moneyness (in, at, and out-of-the-money) which are based on Black OPM or GARCH OPM.
[b]Option trading for contracts with different maturity (March, May, July, September, and December) and moneyness (in, at, and out-of-the-money).
[c]Option trading by Black OPM or GARCH OPM.
[d]Experimental errors measure the variability of option trading returns within each treatment (Black OPM or GARCH OPM).
[e]Sampling errors measure the variability of the daily returns from option trading for each contracts. Since heteroskedasticity is present due to different time to maturity, feasible generalized least squares is used.
[f]Test of null hypothesis that treatment means (mean daily returns of the two OPMs) are the same. *F* value is the ratio of mean sum of squares of treatment to mean sum of squares of the experimental error.

CONCLUSIONS

This article introduces an asymmetric GARCH model that captures asymmetries in the mean equation and determines the most likely distribution of Kansas City wheat futures price changes among alternative autoregressive conditional heteroskedasticity models. The asymmetric GARCH(2, 1)-t with two lags of the conditional variance and one lag of the squared residuals, which considers asymmetry in the mean equation, is selected as the most likely model of Kansas City wheat futures price changes. The alternative models considered are the GARCH-t, the EGARCH-t, and the asymmetric EGARCH-t models.

A Monte Carlo study using the estimated asymmetric GARCH(2, 1)-t parameters shows that Black's model values deep out-of-the-money put options less than the GARCH option pricing model does. However, Black's option pricing model values at-the-money put option premiums higher than the GARCH option pricing model does. In the Monte Carlo study, differences between Black's model and the GARCH option model increase as time to maturity increases, which confirms Hull and White's findings. The GARCH option pricing model predicts actual option premiums of Kansas City wheat more accurately than Black's model for deep in-the-money put and call options and deep out-of-the-money put options. Actual trading returns with the GARCH OPM are slightly higher than those with Black's OPM, but the differences are not statistically significant. Most trading returns based on both option trading models are not significantly different from zero.

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