THE MISPRICING OF U.S. TREASURY CALLABLE BONDS

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INTRODUCTION

In today's complex market, borrowers' and investors' needs require the design of products which incorporate option-like features in their structure. In the fixed income securities area in particular, many instruments are embedded with such features. Examples of such securities are convertible bonds, callable bonds, putable bonds, and mortgage backed securities. Given the proliferation of these products, it is important to understand their structure and risk characteristics and develop pricing models which can replicate their pricing behavior accurately.

Representing a market of approximately $100 billion, callable Treasury bonds comprise about one fifth of the total Treasury bond market. Most of the currently outstanding issues of callable Treasury bonds were issued during the 1973–84 period with original maturities of 30 years. With the introduction of the STRIPS program in 1984 the Treasury stopped issuing callable bonds. However, a considerable number of the earlier issued bonds is still outstanding with the majority of them having coupon rates ranging from 7.0% to 14% and maturity dates from 1998 to 2014. Given the importance of the callable Treasury bond market,

This research was supported by a grant from the Faculty Research Initiative Program of the University of Michigan-Flint. I thank Phelim Boyle, Eliezer Prisman, and Yisong Tian along with seminar participants at the University of Waterloo, Wilfrid Laurier University, and York University for helpful comments and suggestions. The usual disclaimer applies.

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The Journal of Futures Markets, Vol. 15, No. 8, 861–879 (1995)

CCC 0270-7314/95/080861-19

the analysis of the pricing behavior of these instruments has recently received considerable attention. Longstaff (1992) and Edleson, Fehr, and Mason (1993) document an abnormal pricing behavior of certain callable issues, the appearance of negative implied option prices, which creates potential arbitrage opportunities for investors. Both studies, however, focus only on a small sample out of the total number of callable bonds outstanding during the time period that each study considered. Their sample is constrained to those callable instruments whose cash flows can be reconstructed by other Treasury instruments maturing at the same time as the callable bonds. Therefore, the question remains whether the erratic behavior of these callable bonds is a property of only the few bonds considered, or a property of the entire callable Treasury bond family. Furthermore, since no pricing model is assumed in these studies, the magnitude of the potential mispricing of these instruments is not really assessed.

The objective of this article is to address both of these issues. First, the puzzle of negative option values implied by the prices of U.S. Treasury callable bonds is investigated for the entire sample of callable bonds traded during the period from July 1989 to June 1993. It is found that negative implied option values persist only in relatively low coupon callable bonds while they are absent from higher coupon bonds. Furthermore, with the aid of a one-factor valuation model consistent with an observed term structure of interest rates, the magnitude of the mispricing is assessed. Overall results suggest that callable bond market prices are, in general, higher than prices generated by the model by approximately 1%.

The remainder of the article is organized as follows: the pricing characteristics of callable Treasury bonds are presented in the next section. The third section examines violations of the boundary condition implied by the pricing behavior of the callable bonds. Potential explanations of the mispricing and reasons which might prevent taking advantage of the arbitrage opportunities that exist are analyzed in the fourth and fifth sections. The magnitude of the mispricing is estimated in the last section.

PRICING CHARACTERISTICS

A long position in a callable Treasury bond can be considered as a portfolio of a long position in a straight bond with the same coupon and maturity, referred to hereafter as long bond, and a short position in a call option. The call option gives the U.S. Treasury the right to redeem the bonds at par on any of the semiannual coupon payment dates during the five-year period prior to the final maturity date of the bond. Four months notice must be given to bondholders before redemption. In other words,

$$B_{c,t} = B_{nc,t} - C \tag{1}$$

where

$B_{c,t}$ = the price of a callable bond with t years to maturity,

$B_{nc,t}$ = the price of an equivalent noncallable (long) bond, and

C = the value of the call option on the callable bond.

The callable bond can be viewed also as a portfolio of an equivalent noncallable bond which matures five years earlier than the callable bond, referred to hereafter as short bond, plus a short position in a series of ten put options with sequential expiry dates 6 months apart. Each put option gives the U.S. Treasury the right to extend the maturity of the bond for another six months. If one put option is not exercised, the remaining put options expires as well. In other words,

$$B_{c,t} = B_{nc,t-5} - P \tag{2}$$

where

$B_{c,t}$ = the price of a callable bond with t years to maturity,

$B_{nc,t-5}$ = the price of a noncallable (short) bond with $t - 5$ years to maturity, and

P = the value of the series of the put options on the callable bond.

Equations (1) and (2) and the nonnegativity of option prices imply

$$B_{c,t} \leq \mathrm{MIN}(B_{nc,t}, B_{nc,t-5}) \tag{3}$$

Longstaff (1992) documents a number of negative call option values implied by eq. (1) for five callable Treasury bonds for which he is able to replicate the cash flows for the equivalent long bond. These are the 7.5 August 1988–93, 7.00 May 1993–98, 8.50 May 1994–99, 8.00 August 1996–01, and 8.250 May 2000–05 bonds. The cash flows of the long bond are replicated by a portfolio of a noncallable Treasury bond and a strip both maturing at the same time as the callable Treasury bond. Using weekly observations for the period from June 1989 to September 1990, he finds that nearly two-thirds of the call values implied by the sample of callable bond prices are negative. An examination of potential factors contributing to this paradox does not provide an explanation. Edleson, Fehr, and Mason (1993), using the same cash flow replication methodology, extend Longstaff's results. They find that a considerable percentage of observed callable bond prices imply negative put prices for 11 callable bonds for which they are able to replicate the cash flows of the equivalent short bond. Jordan, Jordan, and Jorgensen (1993) suggest that the methodology used in the previous two studies, due to the differential tax treatment of the instruments used to reconstruct the cash flows of the callable bond, may lead to the appearance of a negative option value when the true value is positive. However, although the differential tax treatment could make an arbitrage strategy aiming to profit from the apparent mispricing unprofitable for a taxable investor, a tax-exempt aribtrageur could still generate arbitrage profits from negative option prices. As Longstaff (1992) suggests, the existence of even one trader in the market who can generate arbitrage profits at current market prices should drive option prices toward their appropriate values.

MARKET CALLABLE BOND PRICES AND THE BOUNDARY CONDITION

In this section the pricing behavior of all callable bonds traded between July 1989 and June 1993 is examined and any violations of the boundary condition (3) are documented. Monthly Treasury callable bond prices are from the *Wall Street Journal* for the 23 callable Treasury bonds outstanding during the period of the study. Maturities of these bonds range from August 1988–93 to November 2009–14. The flower bonds 4.250 August 87–92, 4.000 February 88–93, and 4.125 May 89–94 are excluded from the sample. Thus, 1078 total observations are included in the sample, 48 for each bond except the 7.5 August 1988–93 and 7.00 May 1993–98 bonds which were called by the Treasury before their actual maturity date.

The examination of condition (3) requires the specification of the term structure of interest rates at the time of valuation. Subsequently, the term structure can be used to derive the values of the short and long bonds and compare them to the market value of the corresponding callable bond. Various methods for the estimation of the term structure of interest rates have been proposed. Among the most widely used is the spline approach pioneered by McCulloch (1971, 1975). The approach is designed to resolve systematically the difficulties in term structure estimation caused by incomplete markets. By specifying the functional form of the discount function, the estimation of discount factors-spot rates is transformed into the estimation of parameters in the discount factors from a set of observed bond prices. Since there tend to be many more bonds at the short end than at the long end, splines are embedded in the discount function to increase the accuracy of estimation. A problem, however, exists with the availability of noncallable Treasury bond prices for a certain period of time. The maturity dates for 16 of the callable Treasury issues fall in the period between February 2007 and November 2014, with no other noncallable Treasury bonds maturing during the same period. The absence of data for such a considerable time period reduces the credibility of the estimated discount factors by any technique.

This article uses U.S. Treasury strips to establish an observed term structure of interest rates. U.S. Treasury strips are zero coupon bonds created from either the principal or individual coupons of U.S. Treasury bonds. In 1982, Merrill Lynch introduced a default-free zero coupon instrument by stripping and then selling separately the coupons and principal payments from Treasury bonds. In 1985, the U.S. Treasury introduced the STRIPS (Separate Trading of Registered Interest and Principal of Securities) program. The program allowed bond dealers to strip bonds with maturities greater than or equal to ten years which were issued after February 1985. Prices of strips maturing quarterly and spanning a period of 30 years have been published daily in the *Wall Street Journal* since 1989. The price structure of these strips defines a term structure of interest rates. The advantage of using strips is that the cash flows of both the short and long bonds corresponding to any callable bond can be replicated by a portfolio of strips and the values of both the short and long bonds are obtained as the aggregate value of the strip portfolio in each case. The following example demonstrates the construction of a replicating portfolio for a particular bond that assumes a 10% coupon bond maturing in exactly one year. The cash flows from this bond are \$5 in six months and \$105 in one year. Also assume that strips with a face value of \$100 mature in six months and one year. A replicating portfolio can be constructed by buying 5% of the strip maturing in six months and 105% of the strip maturing in one year. The replicating portfolio has exactly the same cash flows as the original 10% bond, and its value is equal to 5% of the value of the six month strip plus 105% of the value of the one year strip. Monthly closing bid and ask strip prices are from the *Wall Street Journal* for the period of July 1989 to June 1993. Due to some differences in the pricing structure between stripped coupons and stripped principal, only strips derived from coupon payments are considered.[1]

Table I, panel A, presents the results when observed market prices for each callable bond are compared to the values of the two portfolios of strips which exactly replicate the cash flows of the corresponding short and long bonds. The average of bid and ask prices are used for the callable bonds and the strips, and a par value of $100 for each bond is assumed. In total, there are 306 violations of the boundary condition (3) which represent 28.4% of the entire sample. The mean violation is $0.64, but almost 25% of the violations are more than $1.00. The maximum violation is $2.71.

Violations of condition (3) imply negative values for the options embedded in the callable bond. Table I, panel A, also suggests that the majority of the violations occur when the callable bond price exceeds the value of the corresponding short bond, which, in turn, implies negative put option values. There are 282, or 26.2% of the entire sample, violations of this nature with a mean violation of $0.67 and a maximum violation of $2.71. Only 65 times, or 6% of the entire sample, the price of the callable bond exceeds the value of the corresponding long bond, which, in turn, implies the existence of negative call values. The average violation in this case is $0.26, while the maximum violation is $0.99.

TABLE I

Violations of the Boundary Condition and the Nonnegativity of Option Prices

	Violations	*Mean*	*Max*	*75%*	*Med*	*25%*
Panel A: replication uses average of bid and ask prices (1078 observations)						
Boundary	306	0.64	2.71	0.98	0.46	0.20
Long	65	0.26	0.99	0.43	0.19	0.07
Short	282	0.67	2.71	1.05	0.47	0.23
Panel B: replication with bid–spread considered (1078 observations)						
Boundary	192	0.69	2.51	1.05	0.55	0.22
Long	28	0.29	0.83	0.42	0.27	0.14
Short	188	0.71	2.51	1.08	0.63	0.23

Long denotes the price of a noncallable bond with the same coupon and maturity as the corresponding callable bond. *Short* denotes the value of a noncallable bond with the same coupon as the corresponding callable bond and maturity five years shorter than the maturity of the corresponding callable bond. *Boundary* denotes the minimum of the values of the long and short bonds. A *violation* occurs if the value of the callable bond exceeds the value of either the long or the short bonds. Violations are reported in $'s and a $100 par value is assumed per bond.

[1] For a comprehensive discussion of the pricing differences and potential explanations for the phenomenon see Daves and Ehrhardt (1993).

In the absence of market imperfections, violation of condition (3) would allow an arbitrageur to generate arbitrage profits by selling the callable bond and buying the portfolio of strips which replicates the cash flows of the minimum of the equivalent short and long bonds. In reality, however, the Treasury bond market is not friction-free and these frictions impose certain costs on an arbitrageur. The bid–ask spread is probably the most serious of these frictions. The violations documented so far are based on the average of the bid and ask prices for both the callable bonds and strips. The obvious question is whether these violations are large enough to overcome bid–ask related transaction costs. Arbitrage profits implied by the violation of condition (3) are determined using the bid price for callable Treasury bonds and the ask price for strips. Results are also reported in Table I, panel B. The number of violations which are large enough to overcome the bid–ask spread is 192 or 17.81% of the entire sample. The average violation is \$0.69, the maximum violation of \$2.51, while 25% of the violations are greater than \$1.05.

Given the evidence, so far, of violations of condition (3), the pricing behavior of each individual callable bond is examined next. The number and magnitude of the violations are reported in Table II. Results suggest that the majority of the violations occurs for bonds with coupon rates 8.75 or less. The 7.625 Feb 02–07 and the 7.875 Nov 02–07, in particular, violate the condition 92% and 83% of the time, respectively, with an average violation of more than a dollar and maximum violations of \$2.71 and \$2.07, respectively. For all bonds with a coupon rate of 8.75% or lower, approximately 57% of the prices in the sample violate condition (3). This is consistent with Longstaff's (1992) findings as well as the findings of Edleson, Fehr, and Mason (1993). The samples in both of these studies consist of lower coupon bonds.

An interesting new result, however, is the absence of any significant number of violations of condition (3) for the higher coupon bonds.[7] For bonds with a coupon rate of 10% or higher, only 11 violations occur out of 528 observations, or 2% of the sample. This result is particularly intriguing since it documents a systematic difference between the pricing behavior of higher coupon callable bonds and that of the lower coupon ones. The puzzle of negative option values documented in the earlier

[7]Results in Table I could also suggest a maturity effect since it appears that the number and the magnitude of the violations reduce as time to maturity increases. However, regression results indicate that it is the size of the coupon which is tied to the magnitude of the mispricing and not the time to maturity.

TABLE II
Violations of Pricing Bounds

		Cash Flow Replication Using Avg of Bid–Ask Prices			Cash Flow Replication with Bid–Ask Margin Considered		
Bond	*Obs*	*Viol'ns*	*Mean*	*Max*	*Viol'ns*	*Mean*	*Max*
7.500 Aug 93	25	15	0.12	0.26	4	0.05	0.16
7.000 May 96	45	30	0.51	1.60	20	0.46	1.41
8.500 May 99	48	14	0.33	0.60	6	0.15	0.34
7.875 Feb 00	48	24	0.35	0.89	15	0.35	0.73
8.375 Aug 00	48	17	0.22	0.58	8	0.21	0.44
8.000 Aug 01	48	28	0.73	1.51	21	0.63	1.27
8.250 May 05	48	34	0.62	1.81	21	0.66	1.63
7.625 Feb 07	48	44	1.21	2.71	33	1.31	2.51
7.875 Nov 07	48	40	1.04	2.07	30	1.10	1.90
8.375 Aug 08	48	34	0.52	1.15	25	0.46	0.95
8.750 Nov 08	48	9	0.27	0.62	4	0.27	0.43
9.125 May 09	48	6	0.15	0.20	1	0.00	0.00
10.375 Nov 09	48	0	...	...	0		
11.750 Feb 10	48	1	0.35	0.35	0		...
10.000 May 10	48	0	...	...	0		...
12.750 Nov 10	48	1	0.20	0.20	0		
13.875 May 11	48	1	0.20	0.20	0		
14.000 Nov 11	48	1	0.68	0.68	1	0.43	0.43
10.375 Nov 12	48	1	0.01	0.01	0	...	
12.000 Aug 13	48	1	0.29	0.29	0		
13.250 May 14	48	2	0.37	0.56	1	0.30	0.30
12.500 Aug 14	48	2	0.74	0.91	1	0.31	0.31
11.750 Nov 14	48	1	0.53	0.53	1	0.27	0.27
Total	1078	306	0.64	2.71	192	0.69	2.51

literature is now tied to a coupon effect which seems to exist in the pricing behavior of callable bonds.

POTENTIAL EXPLANATIONS

Longstaff (1992) investigates potential explanations for the apparent mispricing of the callable bonds in his limited sample. Factors such as the Treasury's call policy, liquidity, and taxes fail to account for the appearance of the negative implied option values. Even more puzzling is the fact that this erratic pricing behavior is the property of only certain callable bonds and not others. In this section factors specifically associated with strip pricing, the cash flow reconstruction technique and the quality of the data used in the study are investigated as potential explanations of the violations of the boundary condition (3) and the discrepancy between the pricing behavior of low and high coupon bonds.

Strip Pricing and Term Structure Specification

The test in this study is actually a joint test of the efficiency of the callable Treasury bond market and the efficiency of the Treasury strip market. One can argue that the observed spot rate curve, as defined by a cross section of Treasury strips, does not necessarily coincide with a curve derived by the prices of other Treasury coupon bonds, mainly due to demand shifts and the differential tax treatment between coupon-bearing bonds and strips. Considerable arbitrage opportunities would exist in such cases. Government dealers could exploit these arbitrage opportunities by stripping or reconstituting the coupon bonds. It is this process which should prevent the price of a coupon bond from being materially different than the sum of the values of its stripped parts, to the extent that bid–ask spreads and other transaction related costs permit.

To verify this argument, the pricing relationship between strips and coupon bonds eligible for stripping and reconstitution is examined. Monthly closing prices for all Treasury bonds eligible for stripping and reconstitution are from the *Wall Street Journal* for the same time period used in the study of the callable bonds. Thus, 725 prices are collected for 21 bonds with coupons ranging from 7.125 to 10.625 and maturities from August 2015 to February 2023. The cash flows of these bonds are reconstructed in the same way as the cash flows of the callable bonds. If the value of the replicating strip portfolio is consistently lower than the market price of the replicated coupon bond, this might suggest that the apparent mispricing of certain callable bonds is the result of the pricing behavior of strips rather than of callable bonds. However, results suggest that, on average, the value of the strip portfolio exceeds the value of the corresponding coupon bond by $0.25. Furthermore, only 28 times, or 3.86% of the sample, the price of the bond exceeds the value of the strip portfolio by an average of $0.45. When the bid–ask spread is taken into consideration, the bond price exceeds the value of the strip portfolio only 8 times, or 1.1% of the sample, by an average of $1.19. There is no significant difference in the results between low and high coupon bonds.

Therefore, it can be argued that the pricing behavior of strips does not explain the mispricing of certain callable bonds, but further contributes to the puzzle because the value of the strip portfolio is, on average, greater than the value of the corresponding bond.

Taxes

In the absence of any taxes, the after-tax cash flows of the callable bond holder should be identical to the cash flows of the holder of the equivalent portfolio of strips. For a taxable investor, however, after-tax cash flows could be different. Although the specific tax rules are quite complex, the essence is the different tax treatment of coupon-bearing bonds selling at a discount, coupon-bearing bonds selling at a premium, and strips. The discount from par for both a strip and discount coupon bearing bond is amortized and the imputed interest accrues each year. However, while taxes on the imputed interest can be deferred until the discount bond is sold or matures, taxes on the strip have to be paid each year. The premium on a coupon-bearing bond selling above par is amortized over the life of the bond, and the amortization amount each year is deducted against the coupon income of the bond. There seems to be, therefore, a tax disadvantage of strips when compared to discount bonds, while strips and premium bonds are taxed symmetrically.[3]

Thus, replicating the cash flows of a discount coupon bearing bond with a portfolio of strips may result in a portfolio value which is lower than the value of the discount bond. Given the reconstruction technique, therefore, it might appear that bonds with lower coupon rates are overpriced. In the case of callable bonds this could appear as a violation of condition (3). As discussed in the previous section, stripping and reconstitution are expected to limit the size of the pricing differences for the bonds which are eligible for stripping and reconstitution. Callable bonds, however, are not eligible to be part of this process. And although the cash flows of the equivalent short or long bonds can be replicated with a portfolio of strips, this portfolio is not a perfect substitute for the callable bond. Therefore, the size of the apparent mispricing might be higher in this case. To find if there is a difference in the number and the size of violations, a sample consisting of the bonds with the most violations (bonds with coupon rates 8.75% or lower) is separated into two subsets: one with only premium prices and one with discount prices. Results are reported in Table III. Although the number of violations as a percentage of total observations is higher for the discount subset, there does not seem to be any significant difference in the magnitude of the violations. As a matter of fact, both the mean and the maximum violations are higher for the premium bonds. Furthermore, one can argue that even if tax-based factors are contributing to the observed mispricing of certain callable bonds, the existence of many institutional investors with a zero or low tax bracket in the Treasury bond market is expected to prevent mispricing from being material.

[3]For a comprehensive discussion of the differential tax treatment of the various issues see Jordan, Jordan, and Jorgensen (1995).

TABLE III

A Comparison Between Premium and Discount Bonds (Callable Bonds with Coupon Rates [illegible] 8.75)

	Premium	*Discount*
Observations	366	136
Violations	175 (47.81%)	108 (79.41%)
Mean violation	0.70	0.61
Standard deviation	0.61	0.52
Max violation	2.71	2.02

Quality of Data

The secondary market for Treasury securities is an over the counter market where a group of U.S. government securities dealers provide continuous bids on outstanding Treasury bonds.[3] Reported prices are, in general, prices quoted by these government dealers and do not represent actual transaction prices. However, a common assumption in studies of bond market data is that quoted bond prices are the true market clearing prices. Sarig and Warga (1989) argue that for not very liquid bonds traders have to guess the price which would have cleared an active market, had one existed. This might result in a systematic pricing discrepancy of certain bonds which do not trade as often. In their study, they examine discrepancies between quoted bond prices obtained from the CRISP bond tapes and quoted prices obtained from the Shearson Lehman Brothers database. Although they do not, or cannot, make inferences about the quality of the data in each data base, they suggest that discrepancies in prices is an indication that a quoted price might not be close to the true price. The mean discrepancy in bond quotes is found to be $.0037, with a standard deviation of $0.7907. The evidence is consistent with both sources of data erring in an unsystematic way, thereby introducing no upward or downward bias into the reported bond prices.

This study uses data collected from the *Wall Street Journal* for the period from July 1989 to June 1993. Prior to July 1990 bond data was supplied to the *Wall Street Journal* by Bloomberg Financial Markets, while from July 1990 and on data is supplied by the Federal Reserve Bank of New York, the same data source used by CRISP. Since the Federal Reserve receives quoted prices from all government dealers, it

[3] For more information see Fabozzi (1993).

is conceivable that prices reported in the *Wall Street Journal* and CRISP are of higher quality than those submitted by individual traders.

Additionally, this study examines whether the differences in the pricing behavior among certain callable bonds can be attributed to differences in the quality of data quoted due to liquidity factors. The proxies for liquidity that Sarig and Warga (1989) employed are used in this study. The proxies are the age of the bond, the amount of the bond outstanding, the bid–ask spread, and the number of runs, i.e., the number of times the quoted price has not changed from the previously quoted price. The age of the bond should be inversely related to its liquidity; the amount outstanding should be directly related to the bond's liquidity; the bid–ask spread should be inversely related to liquidity; and, the number of runs should be inversely related to liquidity, the idea being that the higher the number of runs, the higher the probability that no trade has occurred and the trader just repeats the previously quoted price.

Values of these proxies are developed for each bond, and results are reported in Table IV. The bid–ask spread before July 1990 when data was supplied by Bloomberg Financial Markets is, in general, larger than when data is supplied by the Federal Reserve Bank of New York. For most of the callable bonds, the typical bid–ask spread as supplied by the Federal Reserve is $0.125. In the study by Sarig and Warga (1989) a run occurred if the price of a bond did not change from the previous month's price. This study imposes a more restrictive definition—a run occurs if there is no change over the previous day's quoted price. Results in Table IV suggest that liquidity may be a factor for only the first three bonds in the sample. The rest do not appear to have substantial differences in liquidity. Although pricing violations seem to occur more frequently in older bonds, pricing behavior does not seem to be related to liquidity when liquidity is measured by the amount outstanding, the bid–ask spread, and the number of runs. As a mater of fact, the issue which more frequently and significantly violates the pricing bound, the 7.625 Feb 07 bond, seems to be at least as liquid as bonds issued later which do not experience an abnormal pricing behavior.

The reader must be aware that lack of liquidity does not necessarily suggest "poor" data. All it suggests is that the bond prices quoted by this particular source might be different than prices quoted by a different trader. To further validate the results in this study, replicated bond prices are compared to callable bond prices supplied by Lehman Brothers. Results obtained with data from the different source are very similar

TABLE IV

List of Bond Characteristics Which May Affect the Quality of Quoted Prices

Bond	*Obs'ns*	*Viol'ns*	*Mean*	*Issued*	*Amount*	*Spread*	*Runs*
7.500 Aug 93	25	15	0.12	8 73	0.93	22.6	2
7.000 May 98	45	30	0.51	5 73	0.46	35.5	3
8.500 May 99	48	14	0.33	5 74	0.86	30.0	4
7.875 Feb 00	48	24	0.35	2 75	2.05	13.0	2
8.375 Aug 00	48	17	0.22	8 75	2.54	13.5	2
8.000 Aug 01	48	28	0.73	8 76	0.74	17.9	1
8.250 May 05	48	34	0.62	5 75	2.07	15.0	2
7.625 Feb 07	48	44	1.21	2 77	2.69	14.0	1
7.875 Nov 07	48	40	1.04	11 77	1.23	14.0	1
8.375 Aug 08	48	34	0.52	8 78	1.34	14.2	1
8.750 Nov 08	48	9	0.27	11 78	3.56	14.1	
9.125 May 09	48	6	0.15	5 79	3.82	14.1	
10.375 Nov 09	48	0		11 79	3.17	14.2	1
11.750 Feb 10	48	1	0.35	2 80	1.68	14.4	
10.000 May 10	48	0		5 80	1.87	14.5	1
12.750 Nov 10	48	1	0.20	11 80	3.75	14.2	2
13.875 May 11	48	1	0.20	5 81	3.65	14.2	
14.000 Nov 11	48	1	0.68	11 81	4.21	13.7	
10.375 Nov 12	48	1	0.01	11 82	9.08	14.2	1
12.000 Aug 13	48	1	0.92	8 83	12.35	14.2	
13.250 May 14	48	2	0.37	5 84	4.65	14.2	
12.500 Aug 14	48	2	0.74	8 84	4.65	14.2	
11.750 Nov 14	48	1	0.53	11 84	5.15	14.5	

Issued denotes the date the bond was issued. *Amount* represents the amount [illegible] of the particular bond [illegible] during the sample period in billion $. *Spread* denotes the average bid-ask spread during the sample period in cents. *Runs* denotes the number of times the reported monthly price was unchanged over the previous one.

to the ones obtained earlier, and the difference in the pricing pattern between low and high coupon bonds persists.[7]

MARKET FRICTIONS AND ARBITRAGE

None of the factors analyzed in the previous section provide an adequate explanation for the existence of the apparent mispricing in the market of callable bonds. Furthermore, another puzzle remains: what prevents arbitragers from exploiting this pricing anomaly to its fullest extent? Reasons which can make arbitrage costly are discussed next.

The arbitrage opportunities documented in this article are true arbitrage for an individual who already owns the callable bond. In what is known as quasi arbitrage, the bondholder can sell the callable bond in the market and replace it with a portfolio of strips replicating the

[7] Although these results are not reported here, they are available from the author upon request.

cash flows of one of the equivalent short or long bonds. This is true for an investor who does not own the callable bond only if short selling is costless and the position is held until maturity or until the price of the callable bond corrects itself. However, since it takes place via a reverse repurchase agreement, short selling is not costless. Short selling costs can exceed arbitrage profits if the price anomaly does not disappear fast enough. As Cornell and Shapiro (1989) suggest, given a differential of three percentage points between the repo rate and the short-term interest rate, a dealer who takes a position when the theoretical arbitrage profit is \$3.00 will make money only if the profit potential returns to zero within 23 weeks. Beyond that date the dealer would lose money even if the anomaly subsequently disappeared. Given the uncertainty of the potential gains (or losses) when taking a position in a particular market, many players might be unwilling to exploit the arbitrage opportunities which exist in callable bonds. However, quasi arbitrage is still relatively costless and owners of callable bonds can easily exploit the pricing anomaly in certain callable bonds.

ESTIMATING THE MAGNITUDE OF THE MISPRICING

The violations of condition (3) documented above suggest that certain callable bonds can be overpriced; however, the magnitude of the mispricing needs to be estimated.

The Model

Although a callable Treasury bond can be viewed as a portfolio of a straight bond and an option written on this bond, the two components do not trade separately and, thus, they are not observable in the market place. Therefore, a model which prices both components of the issue concurrently needed. Vasicek (1977), Cox, Ingersoll, and Ross (1985), and Brennan and Schwartz (1979), among others, develop one- or two-factor term structure models for pricing bonds and other interest rate contingent claims. The disadvantage of these early models, however, is that they do not automatically fit an observed term structure. With the appropriate choice of parameters in each model, they can provide an approximate fit to many of the term structures observed. However, since option prices are very sensitive to changes in the price of the underlying asset, the straight bond in this case, a model which provides an exact fit to the observed term structure is needed.

The model is developed under the usual set of assumptions of continuous trading, no taxes, no transaction costs, and access to information for all investors. The dynamics of the term structure are captured by the movement of the short-term interest rate which is assumed to follow the mean-reverting Ornstein–Uhlenbeck process proposed by Vasicek (1977)

$$dr = g(l - r)dt + \sigma dz \tag{4}$$

where g, l, σ are positive constants and dz is a standard Gauss–Wiener process with mean zero and variance dt.

Assuming that the market price of the interest rate risk, $\lambda(t)$, is a function of time, the equivalent process in a risk neutral world becomes:

$$dr = [g(l - r) - \lambda(t)\sigma]dt + \sigma dz$$

or

$$dr = [\theta(t) - gr]dt + \sigma dz \tag{5}$$

where

$$\theta(t) = gl - \lambda(t)\sigma$$

Process (5) is an extension of the Vasicek (1977) model developed by Hull and White (1990), and it will be referred to as the Extended Vasicek model hereafter. Unless the functional form of the market price of interest rate risk, $\lambda(t)$, is known, the drift rate in process (5) is an unknown function of time. Hull and White (1993) develop a trinomial tree where in each step the function $\theta(t)$ is chosen in a way that the model provides an exact fit to the initial term structure of interest rates. The same numerical methodology is employed here to construct a tree which is used to price the callable Treasury bond. Although process (5) allows the likelihood of negative interest rates, it provides an exact fit to any observed term structure. The empirical application of the model requires the estimation of g and σ in processes (4) and (5), as well as the specification of the term structure of interest rates at a given time. To find the price of the callable Treasury bond, the trinomial tree can be used with the terminal condition that at maturity the value of the bond cannot exceed its face value. The existence of the embedded option also suggests that the value of the bond on any coupon payment date during the five year period prior to the bond's original maturity cannot exceed its call price which is equal to the par value of the bond for all callable Treasury issues.[6]

Parameter Estimation

A generalized version of Christie's (1982) maximum likelihood estimation procedure is used to estimate the parameters in process (4). A brief summary of the procedure appears in Appendix 1.[7] It provides simultaneous estimates of the parameters and the variance–covariance matrix. The data used for this purpose are the annualized yields on Treasury bills with approximately 90 days to maturity, observed on the last trading day of each month, for the period from November 1979 to December 1990. The resulting estimates are g = .574 (.3618), l = .0812 (.01811), and σ = .00954 (.00395). Numbers in parentheses denote asymptotic standard errors.

As mentioned earlier, the model requires the specification of the current term structure which is used as input in the development of the trinomial lattice. The term structure defined by the observed strip prices on the particular valuation date is used as an input in the model.

Results

The model is used to generate monthly closing prices for the 23 callable Treasury bonds traded during the period from July 1989 to June 1993. Table X presents differences between model-generated and market prices for both the total sample and two subsets of the entire sample: the first containing callable bonds with coupon rates greater than or equal to 10% and the second containing callable bonds with coupon rates lower than 10%. This differentiation seems necessary given the different pricing behavior of the two groups of bonds when conformance to the boundary condition was examined. Market price is defined as the average of the actual bid and ask prices for each bond. Table X also presents a comparison of the results derived by the Extended Vasicek model with the results obtained by an ad hoc procedure, referred to as the naive model hereafter. The naive model assigns as the value of the callable bond the upper limit of boundary condition (3), i.e., the minimum of the values of the equivalent long and short bonds.

[6] It is assumed that the bond can be called immediately, without the four months notice, on any coupon payment date during the five-year period prior to the maturity of the issue.

[7] Ogden (1987) uses the same methodology to estimate the parameters for a slightly different interest rate process.

TABLE V

Comparison of Model Generated Prices to Market Prices

Model	*RMSE*	*Mean Error*	*Max*	*75%*	*Med*	*25%*	*Min*
			All callable bonds in the sample (1078 observations)				
E.V.	1.4484	1.03	1.16	0.17	0.91	1.80	4.37
Naive	1.2148	0.50	7.45	0.99	0.56	0.09	2.71
			Callable bonds with coupon rates ≥ 10% (528 observations)				
E.V.	0.5383	0.21	1.09	0.18	0.17	0.58	1.79
Naive	1.4267	1.01	7.45	1.11	0.88	0.59	0.95
			Callable bonds with coupon rates < 10% (550 observations)				
E.V.	1.9587	1.81	1.16	1.26	1.78	2.32	4.37
Naive	0.9676	0.02	4.25	0.49	0.04	0.50	2.71

Error denotes model price minus market price. *E.V.* denotes the Extended Vasicek model. *Naive* denotes the model which determines the value of the callable bond as the minimum of the corresponding long and short bonds. A $100 par value per bond is assumed.

The objective of this comparison is to determine whether the more sophisticated Extended Vasicek model has any explanatory power with respect to callable bond market prices.

Overall, results suggest that for more than 75% of the sample market, prices exceed model-generated prices. On average, market prices exceed model generated prices by $1.03. The maximum by which a market price exceeds a model price is $4.37 while the maximum by which a model price exceeds a market price is $1.16. When the two subsets are considered, results are drastically different. For bonds with coupon rates greater than or equal to 10%, the mean pricing error is only $0.21. However, for more than 75% of the observations in the lower coupon bond sample, market prices exceed model prices by more than $1. For more than 25% of the observations in the lower coupon sample, market prices exceed model prices by $2.32 or more; while, on the average, market prices exceed model prices by $1.81.

A comparison of results from the Extended Vasicek model with results obtained by the naive model suggests that the latter provides a better fit to observed market prices in the overall sample. The root mean square error (RMSE) is used as a measure of fit. The mean square error (MSE) is defined as the sum of the squared errors divided by the number of observations. The MSE (or its square root: RMSE) is generally indicative of the lack of fit between model and market prices. The fact that the naive model provides a better fit is hardly surprising. Given the fact that 25% of the prices in the overall sample violate condition (3), any model, no matter how sophisticated it is, will always fail to provide good results and the naive approach will always seem to perform better. However, when the analysis focuses on the subset of the higher coupon bonds where violations of condition (3) are almost nonexistent, results suggest that the Extended Vasicek model has considerably more explanatory power than the naive approach.

CONCLUSION

This study examines the pricing behavior of U.S. Treasury callable bonds during the period from July 1989 to June 1993. Previous research by Longstaff (1992) and Edleson, Fehr, and Mason (1993) provides evidence of negative implied option values in the market of callable bonds. The empirical results in this article suggest that this pricing anomaly persists only in bonds with lower coupon rates. It is shown that low coupon callable bonds are, in general, overpriced by the market to the extent that their prices often imply negative values for the call and/or put options embedded in the bond. However, there is no support for the existence of such an anomaly in the market prices of callable bonds with relatively high coupon rates. An examination of certain factors which might affect the cash flow replication technique used in the study provides no explanation to the documented puzzle.

APPENDIX 1

Parameter Estimation

Given the process the interest rate, r, follows

$$dr = g(l - r)dt + \sigma dz$$

and denoting as Δr the change in r over a small interval of time, Δt, the log-likelihood function of a sample of size n is

$$l = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum[\Delta r - g(l - r)\Delta t]^2$$

where the summation is over all n. Maximizing the log-likelihood function with respect to g, l, and σ^2, the following equations are obtained:

$$g\Delta t\sum(l - r)^2 - \sum[\Delta r(l - r)] = 0$$

$$g\Delta t\sum(l - r) - \sum\Delta r = 0$$

$$n\sigma^2 - \sum[\Delta r - g(l - r)\Delta t]^2 = 0$$

The solution of this system of three nonlinear equations is obtained using the SAS procedure MATRIX. The procedure for obtaining an estimated variance–covariance matrix of the parameter estimates is described by Ogden (1987).

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