# SOLVING FOR OPTIMAL FUTURES AND OPTIONS POSITIONS USING A SIMULATION-OPTIMIZATION TECHNIQUE

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## INTRODUCTION

The issue of optimal hedging with commodity futures has been a focal point of many previous studies [Anderson and Danthine (1983), Antonovitz and Nelson (1988), Chavas and Pope (1982), Feder et al. (1980), Grant (1985), Holthausen (1979), Johnson (1959), Rolfo (1980)]. However, there are fewer studies of optimal hedging in the presence of both options and futures markets. With the assumptions that production is nonstochastic and there exists a linear basis relationship between futures and cash prices, Lapan et al. (1991) show that an expected utility maximizing producer will engage in options positions only for speculative purposes. Relaxing the assumption of nonstochastic production (but retaining the linear basis assumption), Sakong et al. (1993) demonstrate that the expected utility maximizing agent uses

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options also for hedging purposes. Casting the problem in a three-period framework, Lence et al. (1994) show that options are used for hedging purposes by forward-looking producers. However, they assume that production is nonstochastic and that there is no basis risk.

The modeling of optimal hedging is more complex when options are involved, because the distribution of returns on an options position is truncated at the strike price. Analytical solutions often require simplifying assumptions [as in Lapan et al. (1991), Sakong et al. (1993), and Lence et al. (1994)], while numerical integration approaches are problematic if several variables are random. The purpose of this article is to present a simple numerical procedure, a so-called simulation-optimization technique, to solve for the optimal futures and options positions for an expected utility maximizing farmer. The procedure is illustrated via application to a representative Midwest corn producer. Instead of making simplifying assumptions common to the above-cited theoretical literature, the producer in the example faces price and yield uncertainties, as well as basis risk.[1]

Selection of an optimal hedging strategy for a representative producer depends in part on the individual's subjective probability distribution about the course of prices and harvest output. In practice, time series forecasts are often used to approximate this subjective probability distribution [Antonovitz and Green (1990)]. When applying such a time series approach to commodity prices, however, it is important to allow for the possibility that the underlying stochastic process may be time-dependent. There is a body of evidence suggesting that movements in commodity prices tend to be characterized by distributions with time-dependent conditional variances [Baillie and Myers (1989), Myers (1990)]. The ARCH (Autoregressive Conditional Heteroskedastics) type model developed by Engle (1982) is used in this study to estimate random price distributions.

## THE EXPECTED UTILITY MAXIMIZATION MODEL

Consider a two-period production/marketing decision model where the producer plants a fixed acreage of corn in period 1 and plans to harvest

[1]Lence et al. (1994) contains an empirical section on optimal futures and options positions for a fat cattle producer. The simulation method there is different from the simulation-optimization procedure proposed in this study. The empirical analysis in Lence et al. (1994) is based on the optimization result derived from their theoretical model (with assumptions). The procedure proposed in this study is empirical in nature both in the simulation stage and in the optimization stage of the analysis.

and market the output in period 2. The producer faces output price uncertainty and random yield. To protect against an unfavorable end-of-period cash output price, the producer may engage in futures and options positions with contracts maturing at (or near) period 2. In choosing an options position, only put options are considered because a synthetic call can always be constructed through using futures and put options together. The random end-of-period profit, $\pi_2$, can be written as:

$$\pi_2 = P_2^c YA - WA + (P_2^f - P_1^f)q^f + \{\max[0, SP - P_2^f] - OP\}q^o \quad (1)$$

where $P^c$ and $P^f$ are cash and futures prices, respectively; $A$ is the fixed acreage of cropland; $Y$ is random yield per acre; $W$ is nonstochastic production costs per acre; $SP$ is the strike price of the put options and $OP$ is the options premium; $q^f$ and $q^o$ are futures quantity and options quantity, respectively, with a positive value denoting long position and negative value short position; and the subscript is used to denote time period.

The last term in eq. (1) reflects the fact that end-of-period profits for the options depend on whether the put options are in-the-money or out-of-the-money at maturity. Consider the case where the producer engages in a long put position; i.e., he buys put options. If the put options should become in-the-money at period 2 (i.e., $P_2^f < SP$), the producer exercises the put at the strike price ($SP$) and buys back the resulting futures position immediately at the prevailing market price ($P_2^f$). In this case, eq. (1) is equivalent to:

$$\pi_2 = P_2^c AY - WA + \left(P_2^f - P_1^f\right)q^f + (SP - P_2^f - OP)q^o \quad (2a)$$

On the other hand, if the put options become out-of-the-money at period 2 (i.e., $P_2^f > SP$), the producer who bought the puts would simply let the options expire and eq. (1) would become:

$$\pi_2 = P_2^c AY - WA + (P_2^f - P_1^f)q^f - OPq^o \quad (2b)$$

Based on the information available at the decision time, the producer maximizes the expected utility of the end-of-period profit by choosing both futures and options quantities. The optimization problem can be expressed as:

$$\max_{\{q^f, q^o\}} E[U(\pi_2)] = \int \int \int U(P_2^c, P_2^f, Y) G(P_2^c, p_2^f, Y)\, dP_2^c\, dP_2^f\, dY \tag{3}$$

where $G$ is the joint destiny function of the three random variables $P_2^c$, $P_2^f$, and $Y$. Differentiating eq. (3) with respect to $q^f$ and $q^o$, one obtains the following first-order conditions:

$$\begin{aligned} 0 &= E[U'(\cdot)(P_2^f - P_1^f)] \\ &= \int \int \int U'(\cdot)(P_2^f - P_1^f) G(P_2^c, P_2^f, Y)\, dP_2^c\, dP_2^f\, dY \end{aligned} \tag{4a}$$

$$\begin{aligned} 0 &= E[U'(\cdot)\{\max[0, SP - P_2^f] - OP\}] \\ &= \int \int \int U'(\cdot)\{\max[0, SP - P_2^f] - OP\} G(P_2^c, P_2^f, Y)\, dP_2^c\, dP_2^f\, dY \end{aligned} \tag{4b}$$

where $U'(\cdot) \equiv dU(\pi_2)/d\pi_2$. The second-order conditions are satisfied if one assumes that $U$ is strictly concave. Equations (4a) and (4b) can be used, in principle, to solve for the optimal futures and options positions. However, since the first-order conditions involve complicated integrals of $U'(\pi_2)$ with $\pi_2$ itself involving some truncation, there are no apparent analytical solutions to the problem (unless simplifying assumptions are made). Consequently, a numerical procedure is needed. To facilitate the numerical analysis, a negative exponential utility function is specified for example sake:

$$U(\pi_2) = -\exp(-\lambda \pi_2) \tag{5}$$

where $\lambda$ is the Arrow–Pratt absolute risk aversion coefficient.[2] The simulation-optimization procedure used in this study consists of the following three steps:

*Step 1: Stochastic Simulation.* Based on estimated parameters of the underlying distributions of the three random variables, use a random number generator to conduct a stochastic simulation for each of the random variables to generate a number (say, $T$) of states of nature for the variable. Then view each set of the $T$

[2]Since $\pi_2$ is truncated by the strike price, it is not normally distributed. Hence, a mean-variance model is not appropriate for the numerical analysis.

simulated values as a discrete representation of the underlying random variable, with each individual state of nature having a probability of $1/T$ of being realized.

*Step 2:* *Numerical Integration.* Based on the discrete realizations of the simulation, compute the end-of-period profit for each of the $T$ states of nature using eq. (1), and then sum over all states of nature with equal probability to arrive at a discrete representation of the expected negative exponential utility function:

$$E[U(q^f, q^o)] = \sum_{i=1}^{T}(1/T)[-\exp(-\lambda\pi_{2i})] \tag{6}$$

where $\pi_{2i}$ is the end-of-period profit under state of nature $i$.

*Step 3:* *Numerical Optimization.* Maximize the discrete expected negative exponential utility function in eq. (6) with respect to the choice variables, $q^f$ and $q^o$, using a nonlinear numerical optimization routine.

## AN ILLUSTRATION: THE CASE OF CORN PRODUCERS

As an example of the procedure, consider a producer who owns 400 acres of land on which he plants corn ($A = 400$). Based on a typical schedule for Midwest corn producers, the producer is assumed to plant his crops in the first week of April and harvest the corn in the second week of November. Hence, there are 32 weeks between planting and harvesting. In hedging against the price uncertainty associated with his future cash sale, the producer is assumed to take positions on December corn futures and/or at-the-money put options on December corn futures.[3] Once engaged, the producer is assumed to hold the positions until the crops are harvested.

### Estimation and Simulation of the Price Variables

A bivariate ARCH model is used to simulate the end-of-period distributions for December cash corn and December futures corn prices. The ARCH model allows the conditional variances to change over time as a function of past forecasting errors, while leaving the unconditional

[3]That is, the strike price is exogenously chosen as equal to the initial futures price (i.e., $SP = P_1^f$). The choice of strike price can be made endogenous to the model, at the expense of expanding the size of the numerical optimization model.

variances constant. The cash corn price data used in the estimation are Thursday's observations obtained by averaging the high and low quotes for the North-Central Iowa market. The December corn futures price data are Thursday's closing prices from the Chicago Board of Trade. The estimation period extends from the first week of 1985 through the last week of March 1990. The bivariate ARCH equation system is estimated via a maximum likelihood procedure using *RATS* [Doan (1992)]. The estimated system is reported below as eq. (7) and (8):

$$\underset{}{P_t^c} = \underset{(0.022)}{-0.3242} + \underset{(0.014)}{0.9585}P_{t-1}^c + \underset{(0.017)}{0.0529}P_{t-1}^f + \varepsilon_t^c \tag{7a}$$

$$P_t^f = \underset{(0.019)}{0.0423} + \underset{(0.017)}{1.0064}P_{t-1}^f - \underset{(0.015)}{0.0267}P_{t-1}^c + \varepsilon_t^f \tag{7b}$$

$$h_t^c = \underset{(0.0003)}{0.0024} + \underset{(0.102)}{0.5121}(\varepsilon_{t-1}^c)^2 \tag{8a}$$

$$h_t^{cf} = \underset{(0.0003)}{0.0015} + \underset{(0.084)}{0.3866}(\varepsilon_{t-1}^c \varepsilon_{t-1}^f) \tag{8b}$$

$$h_t^f = \underset{(0.0003)}{0.0028} + \underset{(0.084)}{0.3992}(\varepsilon_{t-1}^f)^2 \tag{8c}$$

where $H_t \equiv \left[\begin{smallmatrix} h_t^c & h_t^{cf} \\ h_t^{cf} & h_t^f \end{smallmatrix}\right]$ is the conditional variance–covariance matrix of $[\varepsilon_t^c, \varepsilon_t^f]'$, and $\varepsilon_t^c$ and $\varepsilon_t^f$ are the disturbance terms in the price equations. The standard deviations of the estimates are in parentheses. According to the specification of the bivariate ARCH model, the relationships between $\varepsilon_t^k$ (for $k = c$ and $f$) and $H_t$ are:

$$\varepsilon_t^c = (h_t^c)^{0.5} e_t^c \tag{9a}$$

$$\varepsilon_t^f = (a_t h_t^c)^{0.5} e_t^c + [(1 - a_t)h_t^f]^{0.5} e_t^f \tag{9b}$$

$$a_t \equiv (h_t^{cf})^2/(h_t^c h_t^f) \tag{9c}$$

where $e_t^c$ and $e_t^f$ are standard normals.

At any point in time, $t$, the producer's information set is assumed to contain the lagged values of $P_t^k$ and $\varepsilon_t^k$ appearing in the ARCH price equations. The decision point in the hedging analysis is chosen to be the week immediately following the estimation sample period (i.e., the first week of April 1990). Given the information set available at the decision time ($t = 0$), eq. (8) is used to compute components of $H_t$ for the first period ($t = 1$). An $e_t$ is then randomly drawn from a standard normal distribution and eq. (9) is used to compute the corresponding $\varepsilon_t^k$ ($t = 1$). The simulated $\varepsilon_t^k$ is then fed into eq. (7) to obtain a value for $P_t^k$ ($t = 1$).

The above forecasting procedure is applied recursively until a 32-step-ahead forecast is obtained (i.e., $P_t^k$, where $t = 32$). This price forecast is then defined as one possible state of nature for the end-of-period price. Now, through repeatedly drawing $e_t$ from a standard normal distribution and conducting the above recursive forecasting scheme, a desired number ($T$) of states of nature with equal probability ($1/T$) for futures and cash prices are obtained. Five hundred end-of-period cash prices and futures prices are simulated using the above procedure.

### Estimation and Simulation of the Yield Variable

A trend equation is estimated for the corn yield. Average yields for North-Central Iowa from 1950 through 1989 are from various issues of *Iowa Agricultural Statistics*. The yield equation is estimated as:

$$\begin{aligned} Y &= \underset{(3.48)}{48.704} + \underset{(0.15)}{2.193}\ \text{trend} + \mu \\ R^2 &= 0.86 \end{aligned} \tag{10}$$

The error term $\mu$ is normally distributed with zero mean and an estimated variance of 116.39 bushels per acre. Equation (10) is used to simulate the needed 500 discrete points for the yield distribution.

## THE OPTIMIZATION RESULTS

The estimated equations are used to obtain subjective price and yield distributions on which the optimization is based. The optimization is carried out for the crop years of 1990, 1991, and 1992. At the decision time of each crop year (first week of April), the producer is assumed to update the price and yield equations, using all the information available at the time. In other words, the equations are reestimated in 1991 and 1992, incorporating the previous year's observations.[4]

[4]The updated equations for 1991 and 1992 are, in general, similar to those for 1990, and can be obtained from the authors.

In addition to the price and yield distributions, other model parameters needed in the optimization are production costs per acre ($W$); the strike price ($SP$); the options premium ($OP$); the beginning-period futures price ($P_1^f$); and the Arrow–Pratt risk aversion coefficient ($\lambda$).[5] The production cost figure is based on estimates of the Extension Service at Iowa State University. The strike price, options premium, and beginning-period futures price are from the Chicago Board of Trade. The producer's risk aversion coefficient is specified parametrically, ranging from 0.00008 to 0.00010. The maximization problem is numerically solved using *MINOS* [Brooke et al. (1988)].

In solving for the optimal solution, three scenarios regarding the producer's choice of risk management tools are considered: (i) the producer uses only futures; (ii) the producer uses only put options; and (iii) the producer uses both futures and put options. The first scenario is denoted as the CF scenario, as only the cash and futures positions are involved. The second scenario is denoted as the CO scenario, as only the cash and options positions are involved. Finally, the third scenario is denoted as the CFO scenario, suggesting the engagement of cash, futures, and options positions. By comparing results among different scenarios, insights toward the optimal decision-making structure are obtained.

Before presenting the optimal futures and options positions, it is useful to examine the hedging effectiveness of the model. Following Lence et al. (1993), Table I reports ex ante returns and ex ante risks associated with the three hedging scenarios and the scenario under which no hedging is conducted (i.e., cash only).[6] Also reported in Table I are the ratios between corresponding ex ante risks and ex ante returns. The risk/return ratios under the cash only scenario are 1.71, 1.41, and 1.14, respectively, for 1990, 1991, and 1992. However, these ratios improve significantly as futures and/or options positions are added into the producer's portfolio, For example, when $\lambda$ is 0.00008, the 1990 risk/return ratio decreases from 1.71 to 0.82, 0.93, and 0.82, respectively, for the CF, CO, and CFO scenarios. For all the cases

[5]For simplicity, the model assumes no margin calls, commission fees, or other transaction costs.

[6]An ex ante return is calculated from the simulated price and yield distributions, given the optimal futures and/or options positions. An ex ante risk is the standard deviation of the corresponding ex ante return. Given that the objective function of the model is expected utility maximization, ex ante evaluation is appropriate. An ex post evaluation of the ex ante model may suffer small sample bias, unless the number of evaluations is large. In this study, the number of evaluations is three (1990, 1991, and 1992).

**TABLE I**
Ex Ante Return and Ex Ante Risk

| Level of Risk Aversion ($\lambda$) | Marketing Scenario[a] | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | CF | | | CO | | | CFO | | |
| | Ex Ante Return | Ex Ante Risk | Risk/Return Ratio | Ex Ante Return | Ex Ante Risk | Risk/Return Ratio | Ex Ante Return | Ex Ante Risk | Risk/Return Ratio |
| 1990 (Cash only: ex ante return 11,825, ex ante risk 20,218, risk/return ratio 1.71) | | | | | | | | | |
| 0.000080 | 24,122 | 19,862 | 0.82 | 21,349 | 19,945 | 0.93 | 24,122 | 19,862 | 0.82 |
| 0.000085 | 23,583 | 19,565 | 0.83 | 20,800 | 19,629 | 0.94 | 23,583 | 19,565 | 0.83 |
| 0.000090 | 22,988 | 19,265 | 0.84 | 20,487 | 19,465 | 0.95 | 22,988 | 19,265 | 0.84 |
| 0.000095 | 22,958 | 19,251 | 0.84 | 20,320 | 19,382 | 0.95 | 22,958 | 19,251 | 0.84 |
| 0.000100 | 22,803 | 19,178 | 0.84 | 19,841 | 19,165 | 0.97 | 22,803 | 19,178 | 0.84 |
| 1991(Cash only: ex ante return 13,908, ex ante risk 19,580, risk/return ratio 1.41) | | | | | | | | | |
| 0.000080 | 17,774 | 17,569 | 0.99 | 14,610 | 17,921 | 1.23 | 18,893 | 17,846 | 0.94 |
| 0.000085 | 17,675 | 17,517 | 0.99 | 14,605 | 17,924 | 1.23 | 19,316 | 17,947 | 0.93 |
| 0.000090 | 17,656 | 17,508 | 0.99 | 14,605 | 17,950 | 1.23 | 19,183 | 17,889 | 0.93 |
| 0.000095 | 17,545 | 17,457 | 0.99 | 14,535 | 17,977 | 1.24 | 19,091 | 17,857 | 0.94 |
| 0.000100 | 17,480 | 17,430 | 1.00 | 14,528 | 17,984 | 1.24 | 19,035 | 17,839 | 0.94 |
| 1992(Cash only: ex ante return 17,077, ex ante risk 19,448, risk/return ratio 1.14) | | | | | | | | | |
| 0.000080 | 22,522 | 17,724 | 0.79 | 19,090 | 17,636 | 0.92 | 22,552 | 17,485 | 0.78 |
| 0.000085 | 22,085 | 17,463 | 0.79 | 19,060 | 17,631 | 0.93 | 22,906 | 17,545 | 0.77 |
| 0.000090 | 21,971 | 17,405 | 0.79 | 19,014 | 17,633 | 0.93 | 22,791 | 17,504 | 0.77 |
| 0.000095 | 21,885 | 17,366 | 0.79 | 18,839 | 17,644 | 0.94 | 22,679 | 17,469 | 0.77 |
| 0.000100 | 21,866 | 17,357 | 0.79 | 18,805 | 17,650 | 0.94 | 22,641 | 17,471 | 0.77 |

[a]The CF scenario assumes that only cash and futures positions are involved; the CO scenario assumes that only cash and options positions are involved; and the CFO scenario assumes that all three positions are involved.

reported in Table I, ex ante risk decreases and ex ante return increases as the producer is allowed to engage in futures and/or options activities.

Table II presents the optimal market positions under various levels of risk aversion for the three hedging scenarios. In the case of the CF scenario, the negative futures quantities (appearing in the second column of Table II) indicate that the producer engages in a selling hedge, which is consistent with the fact that the futures position is used to hedge against output price uncertainty. The results also show that the futures quantity decreases as the level of risk aversion increases. Anderson and Danthine (1983) have shown that an expected utility maximizing producer's futures position can be decomposed into a hedging component and a speculative component, where the latter is a decreasing function of the degree of risk aversion. Given the simulated

**TABLE II**
Optimal Futures and Options Positions

| | *Marketing Scenario*[a] | | | | | |
|---|---|---|---|---|---|---|
| | *CF* | | *CO* | | *CFO* | |
| *Level of Risk Aversion* ($\lambda$) | *Futures Position (bushels)* | *Hedge Ratio* | *Options Position (bushels)* | *Hedge Ratio* | *Futures Position (bushels)* | *Options Position (bushels)* |
| 1990 | | | | | | |
| 0.000080 | −43,763 | 0.79 | 52,037 | 0.94 | −43,763 | 0 |
| 0.000085 | −41,850 | 0.75 | 49,041 | 0.88 | −41,850 | 0 |
| 0.000090 | −39,728 | 0.72 | 47,331 | 0.85 | −39,728 | 0 |
| 0.000095 | −39,620 | 0.71 | 46,419 | 0.84 | −39,620 | 0 |
| 0.000100 | −39,067 | 0.70 | 43,800 | 0.79 | −39,067 | 0 |
| 1991 | | | | | | |
| 0.000080 | −32,428 | 0.58 | 29,293 | 0.50 | −49,455 | −38,044 |
| 0.000085 | −31,594 | 0.56 | 29,079 | 0.50 | −48,225 | −36,737 |
| 0.000090 | −31,438 | 0.56 | 27,456 | 0.47 | −47,007 | −35,371 |
| 0.000095 | −30,502 | 0.54 | 26,171 | 0.45 | −46,179 | −34,709 |
| 0.000100 | −29,956 | 0.51 | 25,871 | 0.44 | −45,684 | −34,363 |
| 1992 | | | | | | |
| 0.000080 | −37,281 | 0.66 | 33,071 | 0.59 | −49,729 | −29,371 |
| 0.000085 | −34,290 | 0.61 | 32,577 | 0.58 | −48,450 | −27,999 |
| 0.000090 | −33,510 | 0.59 | 31,836 | 0.56 | −47,469 | −27,354 |
| 0.000095 | −32,924 | 0.58 | 28,960 | 0.51 | −46,581 | −26,957 |
| 0.000100 | −32,792 | 0.58 | 28,393 | 0.50 | −46,582 | −26,818 |

[a]The CF scenario assumes that only cash and futures positions are involved; the CO scenario assumes that only cash and options positions are involved; and the CFO scenario assumes that all three positions are involved. Notice that a positive quantity indicates long position and a negative value, short position.

output level,[7] the optimal hedge ratio is found to vary modestly from year to year. For example, when $\lambda$ is 0.00008, the optimal hedge ratio ranges from 0.58 to 0.79 for the period of 1990–1992.

In the case of the CO scenario, the positive options quantities (appearing in the fourth column of Table II) indicate that the producer buys put options to hedge the cash price uncertainty. Buying put options is consistent with the selling hedge strategy obtained in the CF scenario, as a put gives the buyer the right to a short futures position at the strike price. However, the optimal hedge ratio in the current CO scenario varies rather dramatically from year to year. For example, when $\lambda$ is 0.00008, the optimal hedge ratio is 0.94 for 1990, but only 0.50 and 0.59 for 1991 and 1992, respectively. Finally, the optimal hedge ratio is found to be a decreasing function of the risk aversion coefficient.

Now examine the results pertaining to the CFO scenario reported in the last two columns of Table II. The 1990 results differ drastically from the 1991 and 1992 results. Even though both futures and options are allowed in the CFO scenario, the optimal solution for 1990 does not involve options. Hence, the optimal futures quantities for 1990 in the CFO scenario coincide with those obtained in the CF scenario.

On the other hand, both futures and options positions enter into the optimal solutions for 1991 and 1992, with both quantities being negative. The negative futures quantities are consistent with the short futures positions taken in the CF scenario. However, the negative options quantities indicate that the producer sells put options, rather than buying puts as in the options hedging case (i.e., the CO scenario). As discussed before, part of the producer's short futures position is for speculative purposes, rather than hedging against cash price uncertainty. Hence, the above result can be interpreted to mean that the producer uses the short put position to protect the short speculative component of the futures position. In other words, the producer is speculating with a short straddle, which consists of combining a short futures with two short options.

The essence of speculating with a short straddle position can be made clear by considering explicitly the conditional nature of an options contract. If the futures price falls below the strike price, the buyer of the put would exercise the options and the producer who sold the put would incur a speculative loss. However, this speculative loss from

[7]The average simulated yields for 1990, 1991, and 1992 are 138.8, 140.7, and 141.0 bushels per acre, respectively. These figures suggest that the expected output levels are 55,520, 56,280, and 56,400 bushels, respectively, for 1990, 1991, and 1992.

options trading is compensated by a corresponding gain in the futures market, because the speculative component of the producer's short futures position benefits from the falling futures price. In this case, the options premium from selling puts augments the producer's revenue. On the other hand, if the end-of-period futures price should rise above the strike price, the producer suffers a loss in the speculative component of his short futures position. However, a futures price above the strike price means that the buyer of the put will forsake the options. Hence, the producer can use the options premium to compensate for the speculative loss in the futures market.

## CONCLUSIONS

This article presents a numerical procedure to solve the optimal futures and options positions for an expected utility maximizing producer facing price and yield uncertainty. Because the distribution of returns on an options position is truncated at the strike price, the first-order conditions of the maximization problem are not in a closed-form expression and a numerical procedure is needed. The procedure used in this study involves: stochastic simulation to arrive at a discrete representation of the underlying random variables; discrete numerical integration to arrive at a discrete representation of the expected utility function; and numerical optimization to arrive at the optimal futures and options quantities. The procedure is illustrated with an application to a representative Midwest corn producer.

The optimal solution under the scenario where only the futures market is allowed as a hedging tool shows that the producer engages in a short futures position. The optimal solution under the scenario where only the options market is used shows that the producer engages in a long put position. Finally, the optimal solution under the scenario where both futures and options are allowed shows that the producer, in general, engages in short futures and short put positions. This strategy suggests that the producer hedges his crops by the hedging component of the short futures position and then speculates using a short straddle, which is synthesized by combining a short put position with the speculative component of the short futures position.

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