A Statistical Model for the Relationship between Futures Contract Hedging Effectiveness and Investment Horizon Length

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INTRODUCTION

It is well known within the futures hedging literature that in-sample hedging effectiveness tends to increase as the investment horizon increases. The traditional explanation for this phenomenon is that noise in the market gets canceled-out in long investment horizons as the true underlying relationship between the spot and futures prices emerges. While intuitively appealing, the traditional explanation does not precisely define noise and offers little guidance on how to quantify the relationship between hedging effectiveness and investment horizon length.

The traditional methodology for determining hedging effectiveness is to obtain hedge ratios by regressing the change in the spot price on the change in the futures price. The degree of hedging effectiveness is measured by the R^2 statistic from this regression. The investment

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horizon length corresponds to the differencing interval in the regression methodology. Several studies have shown that hedging effectiveness tends to increase as the investment horizon increases, i.e., a monthly differencing interval will result in higher R^2 than will a weekly differencing interval [see Ederington (1979), Hill and Schneeweis (1981, 1982), Malliaris and Urrutia (1991a, 1991b), and Benet (1992)].

This study offers a statistical model for the relationship between in-sample hedging effectiveness and investment horizon length. The model is based on cointegration between spot and futures prices. Stock and Watson (1988) and Hylleberg and Mizon (1989) prove that when two series are cointegrated, they will share a stochastic trend and both series will be composed of permanent and transitory components.[1] Section three of this study shows that when a common stochastic trend drives the spot and futures prices, the degree of hedging effectiveness and the minimum variance hedge ratio depend on the investment horizon length. The degree of hedging effectiveness approaches one as the sampling interval increases. Intuitively, hedging effectiveness approaches one because, over long horizons, the shared permanent component ties the spot and futures series together and the effect of the transitory components becomes negligible. In the long-run the spot and futures prices are perfectly correlated.

Howard and D'Antonio (1991) and Lien and Luo (1993) also find that the optimal hedge ratio depends on the investment horizon length. The model presented in this study differs from those studies in several important respects. The Howard and D'Antonio (1991) results are driven by an assumption of autocorrelation in only the spot asset return series. While they do provide empirical evidence for their autocorrelation assumption, the assumption is not implied by market efficiency or any particular futures pricing model. The cointegration assumption used in this article is implied by market efficiency and the cost of carry and expectations models of futures pricing.

Lien and Luo (1993) also assume cointegration between the spot and futures price series. They use the error-correction representation implied by cointegration to derive multiperiod hedge ratios. This study uses the permanent/transitory representation implied by cointegration. The permanent/transitory decomposition offers several advantages over the error-correction mechanism. The permanent/transitory decomposi-

[1]Hylleberg and Mizon (1989) provide an excellent set of proofs of different statistical representations implied by cointegration. Among these are the common stochastic trends representation and the error-correction representation.

tion allows one to view the impact of the transitory and permanent components on spot and futures prices separately. This separation offers insight into the underlying forces that drive spot and futures prices. Also, Lien and Luo (1993) state that allowing for a bivariate GARCH (Generalized Autoregressive Conditional Heteroscedastic) complicates their model. The permanent/transitory model used in this study can easily be modified to incorporate GARCH processes. GARCH modeling has been shown to improve hedging effectiveness in Myers (1991) and Kroner and Sultan (1991). Lastly, this study provides in-sample and out-of-sample estimates of hedging effectiveness. Howard and D'Antonio (1991) do not apply their model to actual data and Lien and Luo (1993) do not provide any empirical estimates of the effectiveness of their multiperiod hedge ratios.

The permanent/transitory decomposition model is fit to five pairs of spot and futures series (the German mark, the Swiss franc, the Japanese yen, the S&P 500 index, and the municipal bond index). The canonical correlation decomposition technique of Tsay and Tiao (1990) is used to estimate the required parameters. Theoretical hedge ratios and in- and out-of-sample degrees of hedging effectiveness for each of the five pairs of contracts are calculated and compared to their regression counterparts. The major conclusions from the study are: (1) each hedging horizon length has a unique hedge ratio; (2) the long horizon hedge ratio converges to one; (3) in-sample hedging effectiveness converges to one as the hedging horizon length increases; and (4) the out-of-sample hedging effectiveness of the decomposition model declines with the investment horizon length but not as severely as with the standard ordinary least squares regression methodology.

In the following sections, the minimum variance hedge approach is presented and the relevant literature is reviewed; motivation is provided for the permanent/transitory decomposition and the theoretical results are derived; decomposition estimates for the five pairs of series studied are presented; the standard regression hedge ratios and degrees of hedging effectiveness are compared to their theoretical counterparts; and finally, concluding remarks are presented.

THE MINIMUM VARIANCE HEDGE

The emphasis of this study is on the behavior of the risk minimizing hedge ratio. The risk minimizing or minimum variance hedge ratio is equal to the covariance between the change in the spot price and the change in the futures price divided by the variance of the change in the futures price. Ederington (1979) was one of the first researchers to derive this result. The hedger's objective within this framework is to minimize the variance of the change in the value of the portfolio. Equation (1) formally presents the hedger's problem:

$$\begin{aligned}\text{Min Var}(\Delta HP_t) &= \text{Var}(\Delta S_t + N\Delta F_t) \\ &= \text{Var}(\Delta S_t) + N^2\,\text{Var}(\Delta F_t) + 2N\,\text{Cov}(\Delta S_t, \Delta F_t) \quad (1)\end{aligned}$$

where ΔHP_t is the change in the value of the hedge portfolio during period t; ΔS_t and ΔF_t are the change in the log of the spot and futures prices during period t; and N is the hedge ratio. The differencing interval for ΔS_t and ΔF_t is chosen to match the investment horizon. For example, if weekly data are used, then $\text{Var}(\Delta S_t)$ corresponds to the variance of weekly returns. Equation (2) solves for the minimum variance hedge ratio:

$$\begin{aligned}\frac{\partial\,\text{Var}(\Delta HP_t)}{\partial N} &= 2N\,\text{Var}(\Delta F_t) + 2N\,\text{Cov}(\Delta S_t, \Delta F_t) = 0 \\ N^* &= -\frac{\text{Cov}(\Delta S_t, \Delta F_t)}{\text{Var}(\Delta F_t)} \quad (2)\end{aligned}$$

where N^* denotes the minimum variance hedge ratio. The minimum variance hedge ratio is equal to the negative of the slope coefficient from an OLS (Ordinary Least Squares) regression of the change in the spot price on the change in the futures price.[2] While not explicitly stated in the standard regression methodology, N^* is the estimate of the hedge ratio corresponding to an investment horizon that matches the differencing interval of ΔS_t and ΔF_t.

The degree of hedging effectiveness, denoted E^*, typically is measured by the percentage reduction in the variance of the naked spot price changes.[3] Equation (3) shows the degree of hedging effectiveness:

$$E^* = \frac{\text{Var}(\Delta S_t) - \text{Var}(\Delta HP_t)}{\text{Var}(\Delta S_t)} = 1 - \frac{\text{Var}(\Delta HP_t)}{\text{Var}(\Delta S_t)} = \rho_{sf}^2 \quad (3)$$

[2]The exact specification and estimation technique of the OLS regression equation differs from study to study. The most common procedure is to regress the change in the spot price on the change in the futures price with some correction for autocorrelation. Dale (1981) estimates the regression with the levels of the variables. Myers (1991) and Kroner and Sultan (1991) use GARCH (Generalized Autoregressive Conditional Heteroscedastic) methodology to estimate time-varying conditional moments and construct N^* at each point in time.

[3]The reduction in variance is a statistically based measure. Another measure of hedging effectiveness is based on the distribution of returns. In a perfect hedge, the returns from the hedging strategy have mean zero and zero variance.

where ρ_{sf}^2 is the square of the correlation between the change in the spot and the futures prices.

Market practitioners are interested in factors that affect the optimal hedge ratio and the degree of hedging effectiveness. Ederington (1979), Hill and Schneeweis (1981, 1982), Malliaris and Urrutia (1991a, 1991b), and Benet (1992) document the fact that in-sample hedging effectiveness tends to increase as the investment horizon increases. Benet (1992) offers two explanations for the positive association between hedging effectiveness and the length of the hedging horizon. The economic rationale is that the arrival of information in the market resolves price uncertainty. More uncertainty is resolved, given a longer amount of time; the basis risk is reduced. The second justification is statistically based. Noise in the market tends to be canceled over time; the true underlying relationship between the spot and futures prices emerges in long investment horizons. Benet admits that neither of these arguments is particularly compelling. First, the notion of noise is not well defined, and it is not clear how the signal-to-noise ratio should behave as a function of the investment horizon. The true relationship between the spot and futures prices also may depend on the investment horizon. Malliaris and Urrutia (1991a, 1991b) also examine empirically the relationship between hedging effectiveness and investment horizon. They state that there are no theoretical guidelines to address whether longer horizon hedges should be more effective than shorter horizon hedges. Without an underlying theoretical basis, it is difficult to generalize the empirical results from the above studies to other futures contracts and time periods. The model below shows that the increase in hedging effectiveness follows from cointegration between the spot and futures prices.

THE DECOMPOSITION MODEL

Background

The model used in this study assumes that the spot and futures prices are cointegrated. Cointegration in futures markets does not necessarily occur in every instance; however, under many circumstances, cointegration between spot and futures prices would be expected on theoretical grounds and has been documented empirically. The theoretical arguments for cointegration between spot and futures prices are based on market efficiency, price convergence, and the stationarity of the cost of carry. Hakkio and Rush (1989) and Shen and Wang (1990) demonstrate that cointegration between the spot and futures prices is a necessary condition for market efficiency in an environment with no risk premium. A *simple market efficiency* model links the spot and futures prices through the equation:

$$S_t = a + bF_{t-1,t} + e_t \tag{4}$$

where S_t is the spot price at time t; $F_{t-1,t}$ is the price at time $t-1$ for the forward or futures contract maturing at time t; e_t is a mean zero error term; and a and b are constant coefficients [see Lai and Lai (1991)]. Simple efficiency requires $a = 0$, $b = 1$, and e_t to be stationary.[4] The stationarity of e_t implies that the spot and futures prices are cointegrated. Cointegration between the spot and futures prices is therefore a necessary condition for market efficiency when no risk premium is present. Even in the presence of a time-varying risk premium (a time-varying a coefficient), cointegration may still occur as long as the risk premium is stationary. Chowdhury (1991) and Lai and Lai (1991) argue that price convergence at maturity will lead to cointegration between the spot and futures prices. Lien and Luo (1993) discuss the relationship between cointegration and the cost of carry. They argue that with sufficient arbitrage, the difference between the spot and the futures prices is related to the cost of carry. A *cost of carry model* links the spot and futures prices through the equation:

$$F_t = S_t e^{(r_t + u_t - y_t)(T-t)} \tag{5}$$

where F_t is the futures price; S_t is the spot price; $(T - t)$ is the time to maturity; and r_t, u_t, and y_t are the riskless interest rate, storage costs, and convenience yield, respectively [see Hull (1993)]. Taking logs and rearranging, one obtains:

$$\log(F_t) - \log(S_t) = (r_t + u_t - y_t)(T - t) \tag{6}$$

The *convergence argument* for cointegration ties the two prices together as $(T - t) \rightarrow 0$. For any fixed $(T - t)$ or long $(T - t)$, the innovations in $\log(F_t) - \log(S_t)$ will be stationary (and hence F_t and S_t cointegrated) if r_t, u_t, and y_t are stationary or the three series are themselves cointegrated. Lien and Luo (1993) argue that, for short maturity contracts and low interest rates, a stationary cost of carry seems reasonable. For longer

[4]The qualifier, simple, is used here to indicate market efficiency in the most basic environment; one without a risk premium. In such an environment, the futures price cannot constantly over- or underpredict the future spot price.

maturity contracts, cointegration may still apply if the cost of carry is near zero due to a trade-off between convenience yield and storage costs.

In addition to the theoretical arguments, Seretis and Banack (1990) find that crude oil spot and futures prices in futures markets are cointegrated. Chowdhury (1991) uncovers evidence of cointegration between certain spot and futures prices in metals markets. Quan (1992) discovers cointegration in crude oil markets, while Lien and Luo (1993) find cointegration in foreign exchange markets.

Given the above theoretical arguments and empirical evidence, the spot and futures prices in this study are assumed to be cointegrated processes. Cointegration between the spot and futures prices results in a permanent/transitory representation for the two series. Stock and Watson (1988) and Hylleberg and Mizon (1989) prove that, if among n time series, there are r independent cointegrating relationships, then there is a common trends representation of the system, whereby each of the n series can be expressed as a linear combination of r random walks plus some transitory component. In the case of spot and futures prices, if the two series are cointegrated, then both the spot and futures price series can be expressed as the sum of a random walk (a permanent) and a transitory component.

In the following section, the spot and futures prices are modeled as the sum of a transitory component and a permanent component. The time series properties of the permanent/transitory decomposition have implications for the behavior of the hedge ratio and the degree of hedging effectiveness.

The Model

According to Stock and Watson (1988) and Hylleberg and Mizon (1989), when the log of the spot and futures prices (denoted S_t and F_t) are cointegrated, they can be described at time t by the following system of equations:

$$S_t = a_1 P_t + a_2 \tau_t \tag{7a}$$

$$F_t = b_1 P_t + b_2 \tau_t \tag{7b}$$

$$P_t = P_{t-1} + u_t \tag{7c}$$

$$\tau_t = \alpha \tau_{t-1} + v_t \tag{7d}$$

P_t and τ_t are permanent and transitory factors that drive the spot and futures prices. The data in eqs. (7a)–(7d) run from 1 to T, where the unit of time is arbitrary. P_t is a pure random walk and, as such, its innovations never vanish. a_1 and b_1 measure the response of the spot and futures prices to the permanent component. Similarly, a_2 and b_2 measure the response of the spot and futures prices to the transitory component. For tractability, the transitory component is assumed to be an AR(1) process where the innovations decay at a rate α, $0 \leq |\alpha| < 1$.

When the spot and futures markets are governed by eqs. (7a)–(7d), the minimum variance hedge ratio and the degree of hedging effectiveness will depend on the investment horizon length. The minimum variance hedge ratio for a k period investment horizon is equal to:

$$N^*(k) = \frac{a_1 b_1 k \sigma_u^2 + 2a_2 b_2 \left(\frac{(1 - \alpha^k)}{1 - \alpha^2)} \right) \sigma_v^2}{b_1^2 k \sigma_u^2 + 2b_2^2 \left(\frac{(1 - \alpha^k)}{(1 - \alpha^2)} \right) \sigma_v^2} \tag{8}$$

While the degree of hedging effectiveness, $E^* = \rho_{sf}^2$, is given by:

$$\rho_{sf}^{*2} = \frac{a_1 b_1 k \sigma_u^2 + 2a_2 b_2 \left(\frac{(1 - \alpha^k)}{(1 - \alpha^2)} \right) \sigma_v^2}{\sqrt{\left(a_1^2 k \sigma_u^2 + 2a_2^2 \left(\frac{(1 - \alpha^k)}{(1 - \alpha^2)} \right) \sigma_v^2 \right) \left(b_1^2 k \sigma_u^2 + 2b_2^2 \left(\frac{(1 - \alpha^k)}{(1 - \alpha^2)} \right) \sigma_v^2 \right)}} \tag{9}$$

[See the Appendix, Part A for a detailed derivation of eqs. (8) and (9).]

The parameters in eqs. (8) and (9) correspond to the unit of time in eqs. (7a)–(7d). For example, if the parameters in eqs. (7a)–(7d) correspond to weekly data, then σ_v^2, σ_u^2 are weekly variances, and setting $k = 1$ would correspond to the one-week hedge ratio and the one-week degree of hedging effectiveness. To calculate a six-month hedge ratio and degree of hedging effectiveness, one would set $k = 26$ in eqs. (8) and (9).

Equation (8) provides insight into the behavior of the minimum variance hedge ratio. There is no single true minimum variance hedge ratio; each investment horizon length (a given k) has its own distinct hedge ratio. To minimize portfolio variance, the investor should match the hedge ratio with the investment horizon length. Using the Ederington (1979) regression methodology, this would involve matching the differencing period to the investment horizon length. With a six-month investment horizon, one would need to take six-month price differences. The long differencing period presents one of two complications for the regression estimation technique: (1) if nonoverlapping data are used, sample sizes tend to be small; and (2) if overlapping differences are used, spurious statistical properties may be introduced into the series. Equation (8) avoids the problems associated with the regression methodology and gives the minimum variance hedge ratios for any arbitrary investment horizon.

Equation (8) leads to hedge ratio behavior that is consistent with the model found in Howard and D'Antonio (1991), but offers greater detail about the sources of the horizon dependency present in both models. In the Howard and D'Antonio (1991) model, the horizon dependency of the hedge ratio is determined exclusively by the size of the first order autocorrelation coefficient of the spot asset returns.[5] Equation (8) demonstrates that several factors influence the behavior of the hedge ratio: the relative size of the variance of the permanent and transitory components, σ_u^2 and σ_v^2; the degree of persistence of the transitory component, α; and the extent to which the spot and futures prices react to the underlying components.

Equation (10) shows that the long investment horizon hedge ratio can be found by taking the limit of $N^*(k)$ as k goes to infinity.

$$\lim_{K\to\infty} N^*(k) = \frac{a_1}{b_1} \tag{10}$$

The long investment horizon hedge ratio is related to the cointegrating vector between the spot and futures prices. This cointegrating vector is equal to $(1, -a_1/b_1)$. The limiting hedge ratio also is related to how the spot and futures prices react to the permanent component. $N^*(k)$ can differ substantially from a_1/b_1 for intermediate investment horizons. Price convergence between the spot and futures prices will constrain a_1 and b_1 to be the same sign; otherwise the spot and futures prices would drift in opposite directions over time.

The degree of hedging effectiveness also depends on investment horizon length in eq. (9). Equation (11) gives the limitation of $\rho^*(k)_{sf}^2$ as k goes to infinity.

$$\lim_{K\to\infty} \rho^*(k)_{sf}^2 = \frac{a_1 b_1 \sigma_u^2}{\sqrt{a_1^2 b_1^2 \sigma_u^4}} = 1 \tag{11}$$

[5]The Howard and D'Antonio (1991) hedge ratio is given by: $N^*(k) = -((1-\phi^k)/(1-\phi))((\mathrm{Cov}(\Delta S_t, \Delta F_t))/(\mathrm{Var}(\Delta F_t)))$, where ϕ is the first order autocorrelation coefficient in the spot return series.

For very long investment horizons, the correlation of the spot and futures price changes approaches one.[6] Intuitively, the rationale for eq. (11) is that the variance of the permanent component goes to infinity as the differencing interval, k, increases. As a consequence, the relative influence of the transitory component decreases for larger values of k. Even though the spot and futures prices have different loadings on the permanent component, their correlation will be one in the absence of any influence from the transitory component.

DECOMPOSITION METHODOLOGY AND ESTIMATION

Viewing the spot and futures prices as the sum of permanent and transitory components offers insight into the theoretical behavior of the hedge ratio and degree of hedging effectiveness. To compare the model to the regression methodology, however, requires a permanent/transitory decomposition of the spot and futures prices. To calculate the relationships for the theoretical hedge ratio and degree of hedging effectiveness empirically, one must obtain estimates of the parameters a_1, a_2, b_1, b_2, α, σ_u^2, and σ_v^2. These estimates require decomposition of the original spot and futures prices into their permanent and transitory components. The canonical correlation decomposition technique of Tsay and Tiao (1990) is used to estimate the required parameters. An outline of this procedure is provided in Part B of the Appendix.

The permanent/transitory model is fit to five pairs of spot and futures series: the German mark, the Swiss franc, the Japanese yen, the S&P 500 index, and the municipal bond index. The data are week-ending log prices collected from *Barron's* magazine. The futures prices are for the nearby contract. The spot exchange rates, the S&P 500 cash index, and the municipal bonds cash index are closing prices from New York stated in terms of the dollar. The futures exchange rates and the S&P 500 prices are the settlement prices from the Chicago Mercantile Exchange and are also in dollar terms. The futures price for the municipal bonds contract is the settlement price from the

[6]The limit can be seen by expanding the denominator and dividing the numerator and denominator by k. The highest ordered term in k for both the numerator and denominator is of order k^1. All terms vanish in the limit except the terms given in eq. (11). Mathematically, the limit approaches either $+1$ or -1. However, convergence of the spot and futures prices at maturity requires that the limit be $+1$.

Chicago Board of Trade.[7] Both in- and out-of-sample analyses are performed. The in-sample estimation is for the period, January 1990 to January 1991. The in-sample analysis is based on a comparison of the pattern of hedging effectiveness (as measured by R^2) using OLS and the decomposition technique. The out-of-sample analysis covers the period, February 1991 to January 1993, following the Malliaris and Urrutia (1991a) methodology. The out-of-sample measure of hedging effectiveness is the distribution of the returns on the hedged portfolio. OLS and the decomposition hedges are compared to a perfect hedge with zero mean and variance. A paired t-test is used to compare the differences of the two hedging techniques.

Table I presents the in-sample estimation results from the Tsay and Tiao (1990) decomposition. The technique decomposes the original spot and futures prices into permanent and transitory components. The column labeled P_t gives the loadings on the permanent component for the spot and futures prices (a_1 and b_1) for each of the five pairs of contracts. The column labeled τ_t gives the loadings on the transitory component for the spot and futures prices (a_2 and b_2) for each of the five pairs of contracts. The coefficient estimates are chosen to maximize the correlation between the particular component and the original underlying variable. This correlation and its significance are given in parentheses beneath each variable. Across the five contracts, all the permanent component loadings are statistically different from zero at the 1% level. The transitory component loadings are statistically different from zero at the 1% level for the Japanese yen. The transitory components of the municipal bond index are statistically different from zero at the 5% level, while the Swiss franc transitory components are statistically significant at the 10% level. Even though the transitory components for the S&P 500 and the German mark are not statistically different from zero, these components have an *economic* influence on the behavior of the decomposition hedge ratio and the degree of hedging effectiveness. Their influence is best seen by comparing the resulting theoretical measures of hedging effectiveness to their regression counterparts. This comparison is presented below. The alpha coefficient measures the speed at which shocks to the transitory component vanish.

[7]The Bond Buyer Municipal Index is composed of 40 federally tax-exempt, term municipal bonds. The bonds pay semiannual interest payments at a fixed coupon rate. For initial inclusion in the index, the bonds must be rated either "A-" or higher by Standard & Poor's or "A" or higher by Moody's and have a remaining maturity of at least 19 years. In addition, the bonds must be callable with call dates between seven and 16 years. For further details on the Bond Buyer Municipal Index, refer to the *Rules and Regulations* of the Chicago Board of Trade.

TABLE I

Permanent/Transitory Decomposition Parameter Estimates

	P_t	τ_t	α	σ_u^2	σ_v^2
S&P 500					
Spot	1.2379	0.1764			
	(0.933)[a]	(0.031)			
Futures	1.2389	0.9843			
	(0.930)[a]	(0.087)	0.5718	2.38607e − 04	1.4176e − 05
			(0.126)[d]		(16.83)[e]
German mark					
Spot	0.7595	−0.4002			
	(0.967)[a]	(−0.012)			
Futures	0.7597	0.9164			
	(0.970)[a]	(0.062)	0.1681	2.7863e − 04	5.2010e − 06
			(0.145)[d]		(53.57)[e]
Japanese yen					
Spot	1.0918	0.0808			
	(0.976)[a]	(−0.461)[a]			
Futures	1.0916	0.9967			
	(0.976)[a]	(−0.443)[a]	0.4426	1.9007e − 04	1.5991e − 06
			(0.127)[d]		(118.86)[e]
Municipal bond					
Spot	2.3076	0.4568			
	(0.915)[a]	(0.232)[b]			
Futures	2.3046	0.8896			
	(0.880)[a]	(0.295)[b]	0.4933	8.8738e − 06	8.7471e − 05
			(0.124)[d]		(0.101)
Swiss franc					
Spot	0.7133	−0.5988			
	(0.982)[a]	(0.217)[c]			
Futures	0.7159	0.8009			
	(0.982)[a]	(−0.198)[c]	0.1489	3.3379e − 04	1.1812e − 06
			(0.142)[d]		(282.58)[e]

P_t: The table entry corresponds to the loading on the permanent component. The number in parentheses below gives the correlation of the permanent component with the original series.
[a]Loading is significant at the 1% level.

τ_t: The table entry corresponds to the loading on the transitory component. The number in parentheses below gives the correlation of the transitory component with the original series.
[a,b,c]Loading is significant at the 1%, 5%, and 10% levels, respectively.

α: The speed of adjustment on the transitory component. The number below in parentheses gives the Dickey–Fuller standard error for the presence of a unit root.
[d]Significantly different from one at the 1% level.

σ_u^2: The variance of shocks to the permanent component.

σ_v^2: The variance of shocks to the transitory component. The number below in parentheses gives an $F_{50,50}$-statistic for the hypothesis $\sigma_u^2 > \sigma_v^2$.
[e]Significance at the 1% level.

All the coefficients are significantly different from one at the 1% level. This further supports the presence of a transitory component in both the spot and futures prices.

The permanent component coefficient estimates for the spot and futures series (a_1 and b_1) are virtually the same for each of the five pairs of contacts (the maximum difference is less than 0.5%). The similar loading on the permanent component for the spot and futures prices causes the long-run behavior of the spot and futures price series to be nearly identical. Consequently, for practical purposes, the long-run hedge ratio given by eq. (8) will approach one. The estimates in Table I and eqs. (8) and (9) are used below to generate horizon dependent hedge ratios and degrees of hedging effectiveness. These decomposition estimates are compared to their OLS counterparts in- and out-of-sample.

COMPARISON TO REGRESSION ANALYSIS

In-Sample Analysis

Using eqs. (8) and (9), the coefficients in Table I can be used to calculate theoretical estimates of $N^*(k)$ and $\rho^*(k)$. In this section, the hedge ratios and measures of hedging effectiveness are in-sample estimates for the period, January 1990 to January 1991. The next section compares out-of-sample measures of effectiveness. Figures 1–5 compare the theoretical estimates obtained from eqs. (8) and (9) to estimates obtained using regression analysis. The regression hedge ratio is the estimated slope coefficient obtained from regressing the k-period difference in the spot price against the k-period difference in the futures price. The regression degree of hedging effectiveness is the R^2 from the estimation. The differencing period, k, varies from one to 12 weeks, corresponding to a one- to 12-week investment horizon. Because the data are weekly, any differencing period greater than one uses overlapping observations. The regression estimates are corrected for possible autocorrelation resulting from the overlapping data using the AR(1) procedure (Cochrane–Orcutt) in the statistical package RATS. With the data in this study, the first-order autoregressive specification is sufficient to adjust for most of the presence of autocorrelation in the OLS estimations. In addition, the first-order autoregressive specification is used to make the OLS and decomposition methodologies most comparable. Serial correlation enters the decomposition model through the transitory component. For tractability, the transitory component in

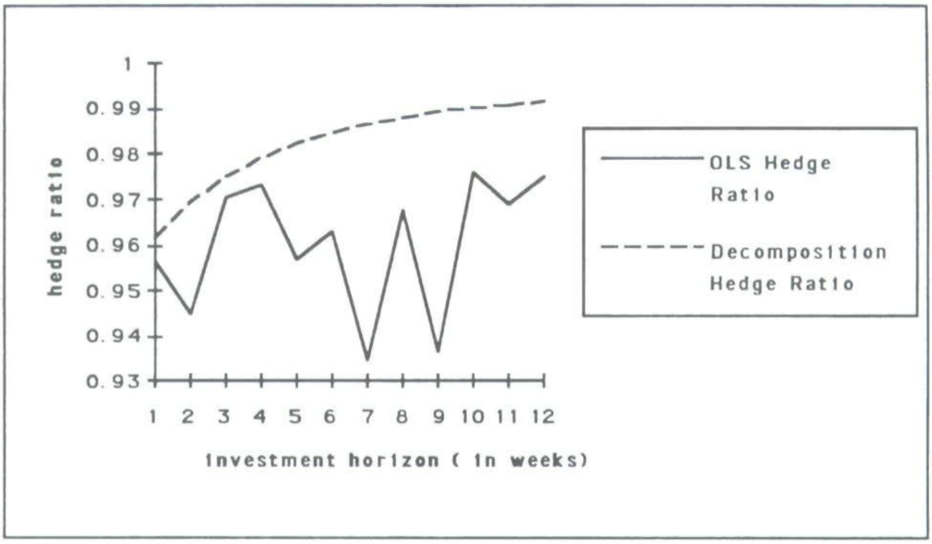

FIGURE 1a
In-sample comparison of regression and decomposition hedge ratios–S&P 500. Table entries corresponding to Figures 1–5 are available from the author upon request.

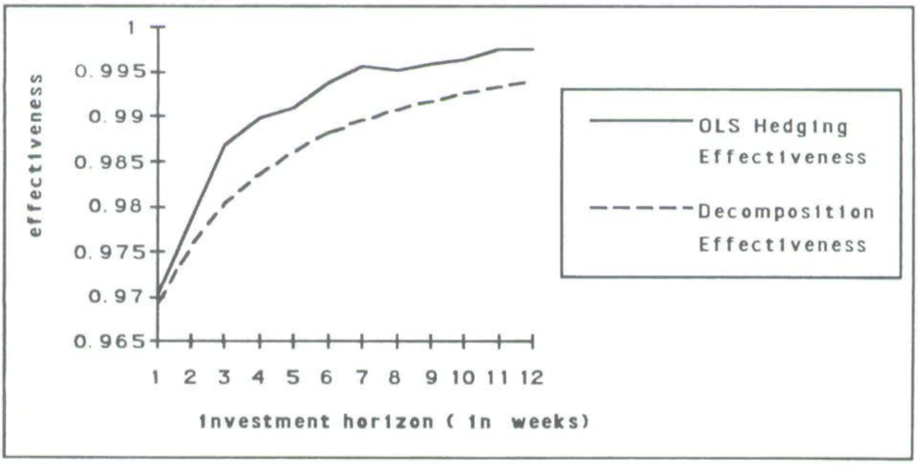

FIGURE 1b
In-sample comparison of regression and decomposition measures of hedging effectiveness–S&P 500.

the decomposition model is specified as an AR(1) process. Equations (8) and (9) explicitly adjust for the autocorrelation in the decomposition methodology.

Figures 1a–5a compare the regression hedge ratio and the hedge ratio derived from eq. (8). The solid line shows the regression hedge ratio. The dashed line gives the decomposition hedge ratio conditional

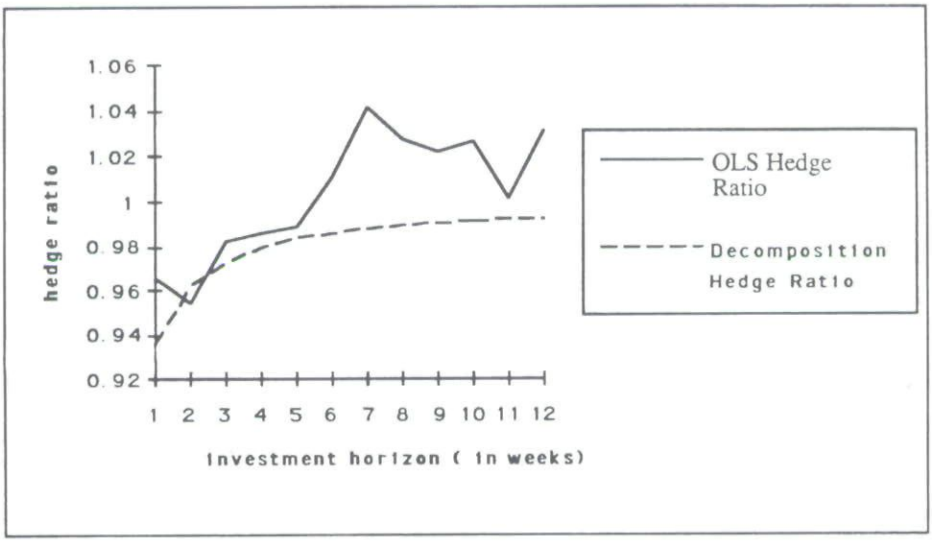

FIGURE 2a
In-sample comparison of regression and decomposition hedge ratios—German mark.

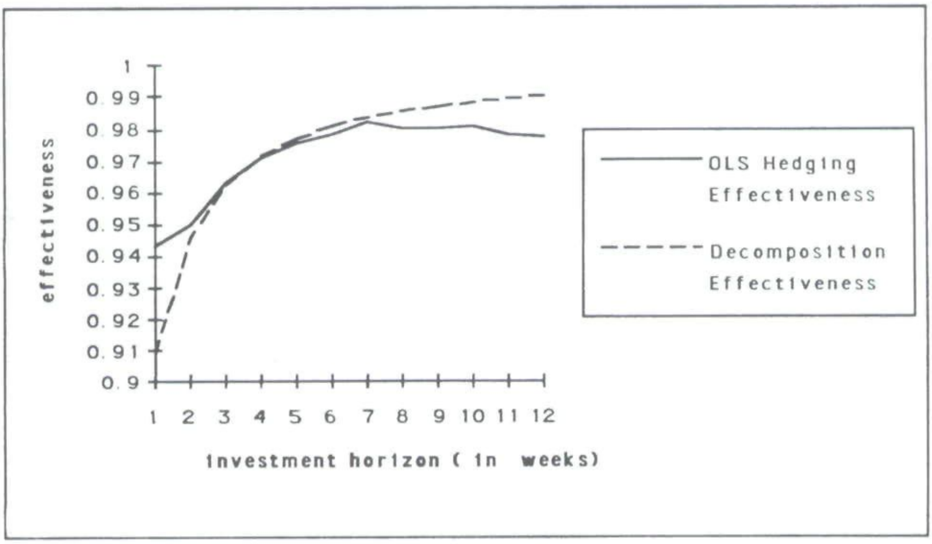

FIGURE 2b
In-sample comparison of regression and decomposition measures of hedging effectiveness—German mark.

on the estimates in Table I. The estimated regression hedge ratios vary in an unpredictable fashion as the investment horizon changes.[8] In

[8]The changing hedge ratio for each investment horizon is different from the *intertemporal* hedge ratio instability investigated in Malliaris and Urrutia (1991b), Geppert and Wilcox (1991), Myers (1991), and Kroner and Sultan (1991). In those studies, a particular hedging horizon is set and the hedge ratio estimates are compared for different points in time.

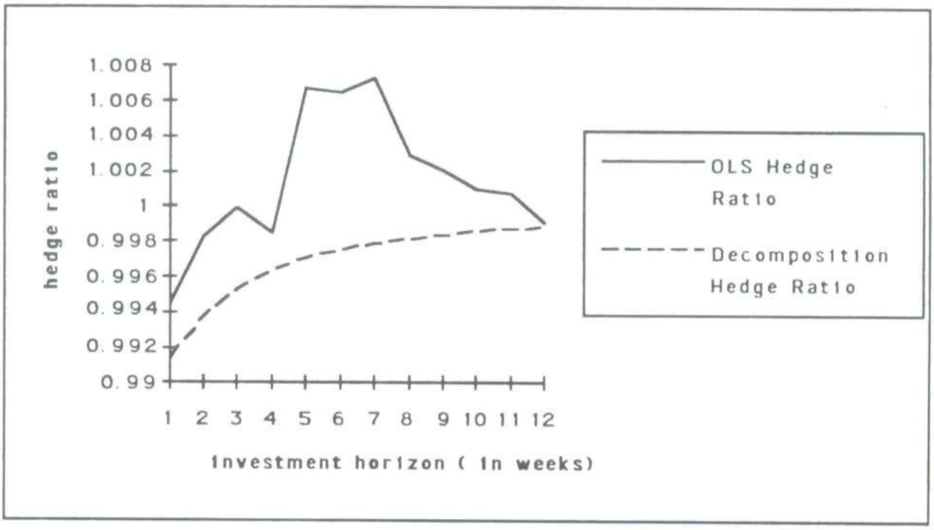

FIGURE 3a
In-sample comparison of regression and decomposition hedge ratios—Japanese yen.

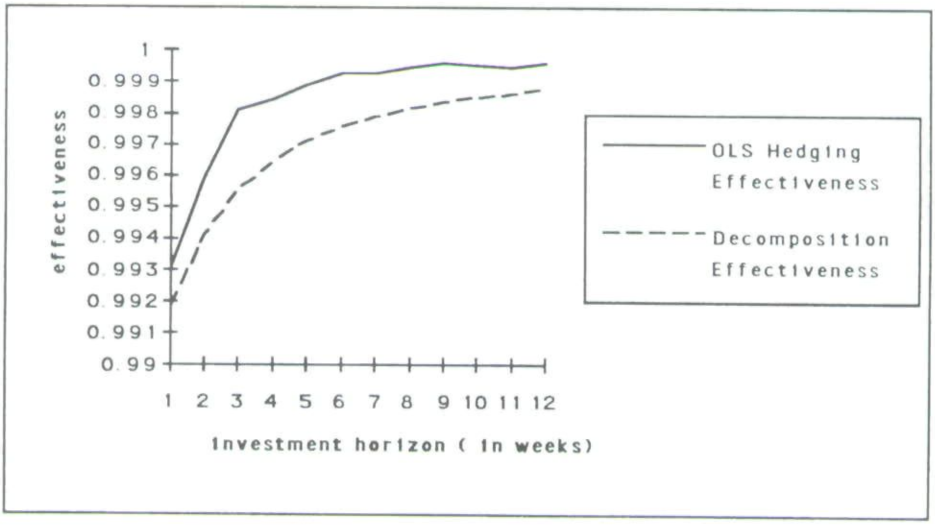

FIGURE 3b
In-sample comparison of regression and decomposition measures of hedging effectiveness—Japanese yen.

contrast, the decomposition hedge ratios increase monotonically at a decreasing rate, converging toward the long horizon hedge ratio of one.[9]

[9]Using weekly data, the hedge ratio approaches the limiting value very quickly. Even with a 12-week horizon, $k = 12$, the hedge ratio is approximately $+1$ for the five contracts investigated.

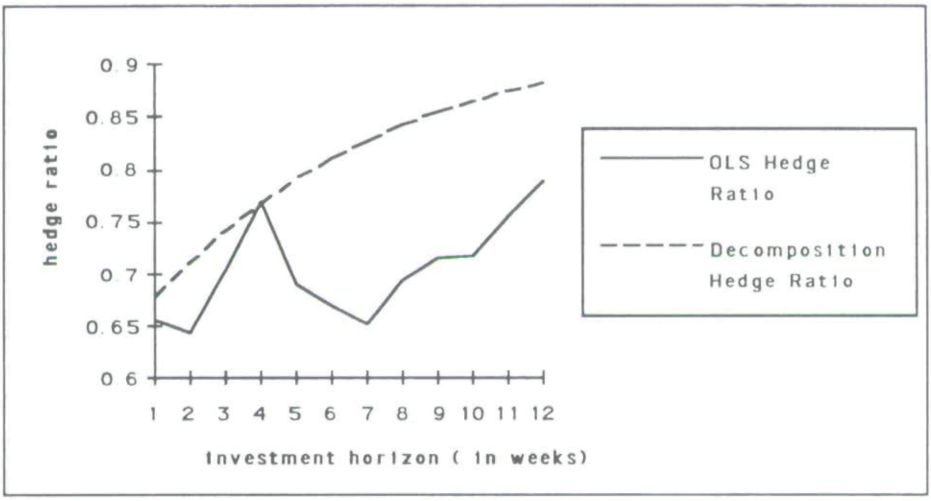

FIGURE 4a
In-sample comparison of regression and decomposition hedge ratios—municipal bond index.

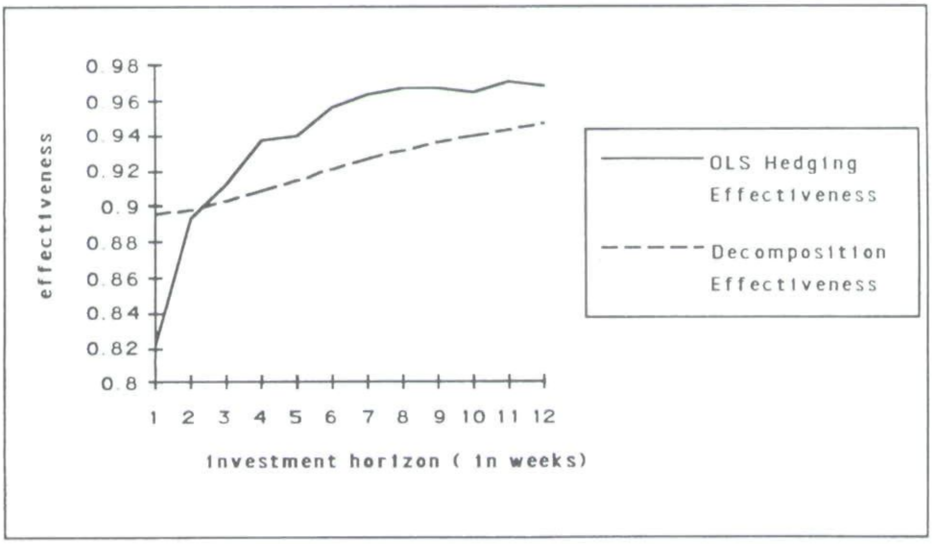

FIGURE 4b
In-sample comparison of regression and decomposition measures of hedging effectiveness—municipal bond index.

The decomposition model implies that over *a given* time interval, each hedging horizon has its own distinct hedge ratio.

Figures 1b–5b compare the *in-sample* hedging effectiveness of the regression hedge ratios and the decomposition hedge ratios for the 12 hedging horizons. For the regression hedge ratios, the degree of hedging effectiveness is the R^2 from the regression estimation.

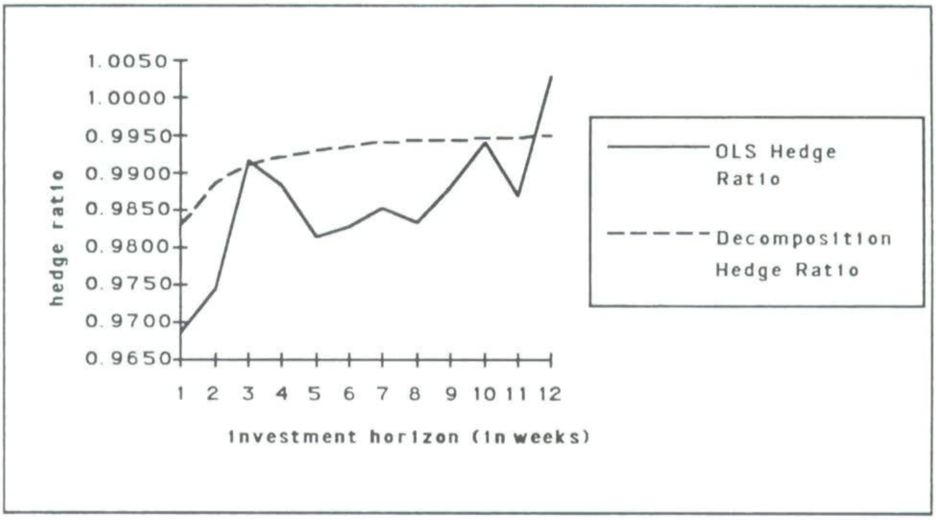

FIGURE 5a
In-sample comparison of regression and decomposition hedge ratios—Swiss franc.

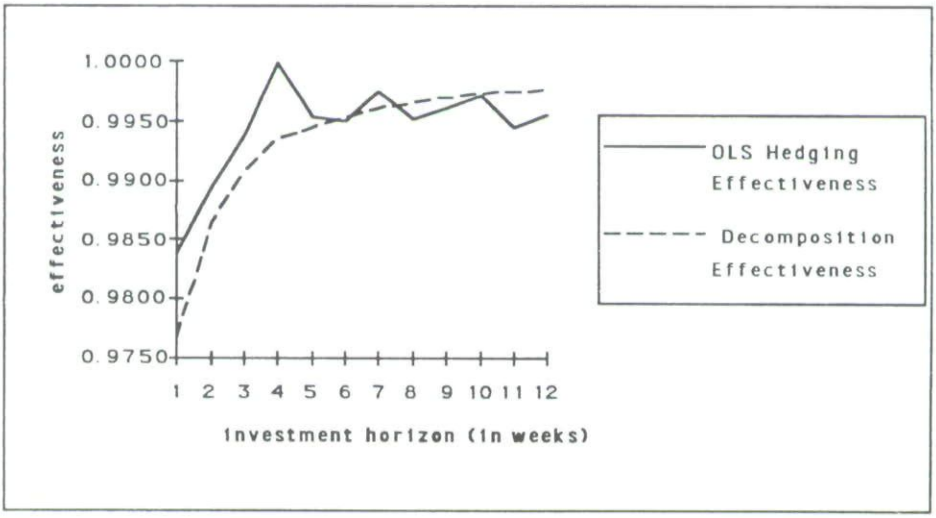

FIGURE 5b
In-sample comparison of regression and decomposition measures of hedging effectiveness—Swiss franc.

Equation (9) gives the degree of hedging effectiveness for the decomposition technique. The purpose of the *in-sample* analysis is to determine whether the presence of permanent and transitory components leads to the behavior of in-sample hedging effectiveness previously documented. As such, the *in-sample* comparison should not be viewed as a test of which method (regression or decomposition) provides greater in-sample effectiveness. Rather, the viewpoint should be whether the pattern of hedging effectiveness for different investment horizons is similar for the two methodologies. An out-of-sample comparison to show which method provides greater hedging effectiveness is given below. For all but the municipal bond index, the measures of hedging effectiveness for both the regression and the decomposition techniques exhibit the same pattern; in-sample hedging effectiveness increases (approximately) monotonically at a decreasing rate and converges toward one.

The regression technique offers no theoretical guidelines to address the behavior of the hedge ratio or hedging effectiveness for different hedging horizons. The evidence in Figures 1b–5b support the decomposition model as an *explanation* of the previously documented in-sample behavior of hedging effectiveness using OLS. Price convergence, simple market efficiency, and stationary carrying costs will lead to cointegration between the spot and futures prices. When the spot and futures prices are cointegrated, they will contain permanent and transitory components. The decomposition model demonstrates that the shared permanent component ties the spot and futures series together and, for long horizons, the effect of the transitory components becomes negligible. In the long-run the spot and futures prices are perfectly correlated.

Out-of-Sample Analysis

The above analysis provided a model for explaining in-sample hedge ratio behavior and hedging effectiveness. For practitioners, however, the concern is which method provides the greater out-of-sample hedging performance. Benet (1992) and Malliaris and Urrutia (1991a) document that, unlike in-sample hedging effectiveness, out-of-sample hedging effectiveness declines as the investment horizon increases. Each of those studies used hedge ratios obtained from OLS regression analysis and attribute the decline in out-of-sample hedging effectiveness to intertemporal hedge ratio instability. No attempt was made to investigate the potential sources of the underlying instability. This section analyzes the out-of-sample effectiveness of the decomposition methodology. The effect of the investment horizon is examined and a comparison is made to the regression methodology. The regression and decomposition hedge ratios estimated in the previous section for data covering January 1990 to January 1991 are used for the ex ante hedge ratios. Data from February 1991 to January 1993 are then used as the evaluation period. No updating of the hedge ratios is done for either the regression or decomposition methods. While actual hedgers would likely update their estimates using either technique, this approach provides a simple performance benchmark for comparison purposes.

Following Malliaris and Urrutia (1991a), the measure of hedging effectiveness used is the distribution of the returns on the portfolio given by eq. (12):

$$\Delta HP_t = \Delta S_t + N\Delta F_t \tag{12}$$

where ΔHP_t is the change in the value of the hedge portfolio during period t; ΔS_t and ΔF_t are the change in the log of the spot and futures prices during period t; and N is the hedge ratio. The returns from a perfect hedge would have zero mean and zero variance, as losses (gains) on the futures position should exactly offset gains (losses) on the spot position. Tables II–VI and Figures 6–10 present the out-of-sample hedging results over the two-year evaluation period. The mean ratio column in Tables II–VI gives the improvement of the decomposition methodology over the regression methodology, calculated as the ratio of the absolute value of the returns from the decomposition method to the absolute value of the returns from the regression method. A number smaller than one indicates superior performance by the decomposition method. The variance ratio is similarly calculated. (Table entries may not correspond exactly to those obtained through division due to rounding.)

TABLE II

Comparison of Out-of-Sample Hedging Effectiveness—S&P 500

Horizon in Weeks	*OLS Mean Return*	*Decomposition Mean Return*	*Mean Ratio*	*OLS Return Variance*	*Decomposition Return Variance*	*Variance Ratio*
1	0.0002	0.0016	8.0000	0.2591	0.2595	1.0018
2	−0.0156	−0.0020	0.1301	0.3169	0.3124	0.9857
3	−0.0204	−0.0163	0.8002	0.2993	0.2981	0.9960
4	−0.0316	−0.0248	0.7875	0.3038	0.3021	0.9945
5	−0.0627	−0.0273	0.4355	0.3372	0.3272	0.9701
6	−0.0705	−0.0335	0.4750	0.3046	0.2992	0.9824
7	−0.1420	−0.0398	0.2806	0.3147	0.2953	0.9386
8	−0.0901	−0.0435	0.4826	0.2814	0.2863	1.0173
9	−0.1836	−0.0494	0.2690	0.3035	0.2864	0.9436
10	−0.0943	−0.0537	0.5689	0.2840	0.2883	1.0151
11	−0.1272	−0.0585	0.4601	0.2638	0.2672	1.0131
12	−0.1236	−0.0693	0.5609	0.3213	0.3248	1.0109
Overall	−0.0800	−0.0347				
	t-stat.	14.296[a]				

The t-statistic is for a paired t-test across all investment horizons of the equality of mean returns using the two techniques.
[a]Significantly different at the 1% level.

TABLE III
Comparison of Out-of-Sample Hedging Effectiveness—German Mark

Horizon in Weeks	*OLS Mean Return*	*Decomposition Mean Return*	*Mean Ratio*	*OLS Return Variance*	*Decomposition Return Variance*	*Variance Ratio*
1	−0.0032	−0.0012	0.3852	0.2562	0.2590	1.0106
2	−0.0098	−0.0108	1.1044	0.3854	0.3833	0.9947
3	−0.0269	−0.0250	0.9321	0.4505	0.4538	1.0073
4	−0.0288	−0.0271	0.9403	0.4946	0.4973	1.0054
5	−0.0293	−0.0276	0.9419	0.4982	0.4988	1.0011
6	−0.0389	−0.0286	0.7346	0.4954	0.4803	0.9695
7	−0.0576	−0.0313	0.5432	0.5573	0.4676	0.8392
8	−0.0554	−0.0344	0.6207	0.5291	0.4710	0.8901
9	−0.0578	−0.0391	0.6776	0.5012	0.4545	0.9069
10	−0.0675	−0.0451	0.6685	0.4888	0.4278	0.8751
11	−0.0582	−0.0523	0.8978	0.3785	0.3722	0.9831
12	−0.0851	−0.0606	0.7122	0.3900	0.2991	0.7668
Overall	−0.0432	−0.0319				
	t-stat.	2.4001[a]				

The *t*-statistic is for a paired *t*-test across all investment horizons of the equality of mean returns using the two techniques.
[a]Significantly different at the 1% level.

Figures 6–10 indicate that out-of-sample hedging effectiveness (as measured by the mean return) tends to decrease as the hedging horizon increases. This pattern was also found in Benet (1992) and Malliaris and Urrutia (1991a). With the exception of the Swiss franc,

TABLE IV
Comparison of Out-of-Sample Hedging Effectiveness—Japanese Yen

Horizon in Weeks	*OLS Mean Return*	*Decomposition Mean Return*	*Mean Ratio*	*OLS Return Variance*	*Decomposition Return Variance*	*Variance Ratio*
1	0.0317	0.0313	0.9892	0.1603	0.1596	0.9956
2	0.0367	0.0358	0.9779	0.2138	0.2125	0.9942
3	0.0340	0.0329	0.9676	0.2175	0.2162	0.9943
4	0.0336	0.0329	0.9808	0.2106	0.2098	0.9962
5	0.0349	0.0317	0.9088	0.2175	0.2121	0.9752
6	0.0339	0.0307	0.9044	0.2259	0.2205	0.9765
7	0.0347	0.0309	0.8909	0.2171	0.2108	0.9710
8	0.0335	0.0314	0.9388	0.2226	0.2196	0.9862
9	0.0340	0.0323	0.9494	0.2205	0.2180	0.9884
10	0.0325	0.0313	0.9628	0.2201	0.2185	0.9930
11	0.0327	0.0316	0.9672	0.2233	0.2223	0.9953
12	0.0313	0.0311	0.9950	0.2460	0.2458	0.9994
Overall	0.0336	0.0320				
	t-stat.	3.6630[a]				

The *t*-statistic is for a paired *t*-test across all investment horizons of the equality of mean returns using the two techniques.
[a]Significantly different at the 1% level.

TABLE V
Comparison of Out-of-Sample Hedging Effectiveness—Municipal Bond Index

Horizon in Weeks	*OLS Mean Return*	*Decomposition Mean Return*	*Mean Ratio*	*OLS Return Variance*	*Decomposition Return Variance*	*Variance Ratio*
1	−0.0244	−0.0231	0.9455	0.1146	0.1148	1.0016
2	−0.0483	−0.0406	0.8407	0.2303	0.2197	0.9540
3	−0.0600	−0.0534	0.8907	0.2921	0.2837	0.9712
4	−0.0630	−0.0631	1.0019	0.3508	0.3509	1.0003
5	−0.0986	−0.0694	0.7042	0.4174	0.3731	0.8938
6	−0.1279	−0.0795	0.6218	0.4830	0.4078	0.8443
7	−0.1601	−0.0885	0.5530	0.5319	0.4148	0.7798
8	−0.1652	−0.0945	0.5723	0.4923	0.4040	0.8205
9	−0.1751	−0.1007	0.5751	0.4831	0.4102	0.8490
10	−0.1868	−0.0993	0.5317	0.4537	0.3741	0.8245
11	−0.1798	−0.1009	0.5613	0.4068	0.3563	0.8759
12	−0.1717	−0.1035	0.6030	0.3716	0.3467	0.9328
Overall	−0.1217	−0.0764				
	t-stat.	8.0420[a]				

The *t*-statistic is for a paired *t*-test across all investment horizons of the equality of mean returns using the two techniques.
[a]Significantly different at the 1% level.

the mean returns from the decomposition methodology are closer to zero than for the OLS regression methodology. For example, using the S&P 500 contract and a seven-week investment horizon, the mean return for the OLS hedged portfolio is −0.1420.

TABLE VI
Comparison of Out-of-Sample Hedging Effectiveness—Swiss Franc

Horizon in Weeks	*OLS Mean Return*	*Decomposition Mean Return*	*Mean Ratio*	*OLS Return Variance*	*Decomposition Return Variance*	*Variance Ratio*
1	0.0010	−0.0007	0.6798	0.4054	0.4134	1.0197
2	−0.0016	−0.0051	3.1360	0.5116	0.5150	1.0065
3	−0.0124	−0.0122	0.9839	0.5494	0.5494	1.0001
4	−0.0561	−0.0581	1.0367	0.3794	0.3794	0.9999
5	−0.0577	−0.0641	1.1297	0.3794	0.3772	0.9941
6	−0.0554	−0.0637	1.1495	0.3613	0.3617	1.0013
7	−0.0551	−0.0631	1.1434	0.3704	0.3725	1.0056
8	−0.0514	−0.0628	1.2220	0.3455	0.3488	1.0097
9	−0.0560	−0.0635	1.1342	0.3407	0.3452	1.0134
10	−0.0654	−0.0661	1.0107	0.3295	0.3303	1.0025
11	−0.0547	−0.0649	1.1868	0.2821	0.2869	1.1069
12	−0.0799	−0.0695	0.8698	0.2562	0.2431	0.9489
Overall	−0.0453	−0.0495				
	*t*stat.	3.1468[a]				

The *t*-statistic is for a paired *t*-test across all investment horizons of the equality of mean returns using the two techniques.
[a]Significantly different at the 1% level.

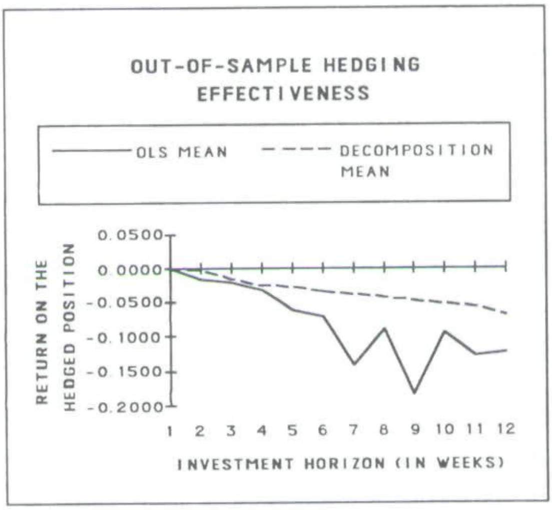

FIGURE 6
Out-of-sample hedging effectiveness—S&P 500.

For the decomposition hedged portfolio, the mean return is −0.0398. The decomposition hedge mean is only 28% the value of the OLS hedge mean. A paired t-test is performed to test the hypothesis that, overall, the mean returns for the decomposition hedged portfolio are different from the mean returns for the regression hedged portfolio. The paired test compares matched return pairs and corresponds to the comparison that actual hedgers would make. For the S&P 500, the overall mean return for the regression method is −0.0800, and −0.0347 for the

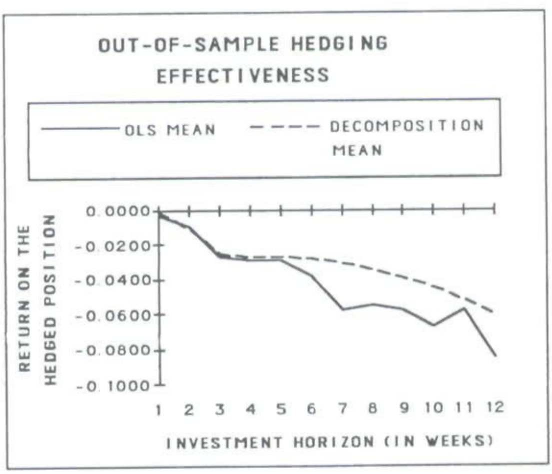

FIGURE 7
Out-of-sample hedging effectiveness—German mark.

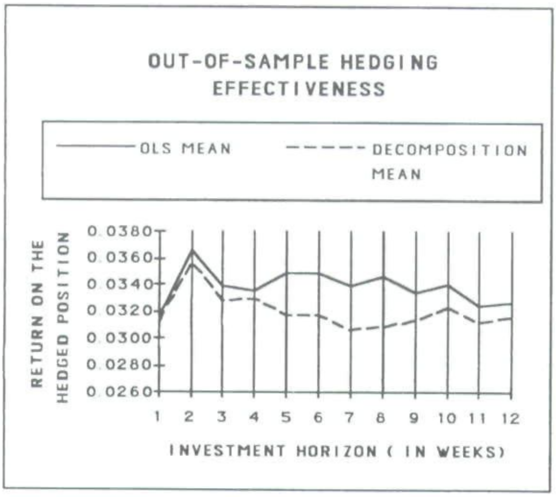

FIGURE 8
Out-of-sample hedging effectiveness—Japanese yen.

decomposition method. The t-statistic of 14.296 indicates that across all 12 horizons the decomposition mean is significantly different from the regression mean at the 1% level. The overall means are statistically different for all five contracts at the 1% level. For the five contracts, the decomposition procedure outperforms the OLS methodology in terms of mean return in 49 out of 60 cases. In addition to mean returns closer to zero, the decomposition return variances are smaller than the regression variances in 40 of the 60 cases.

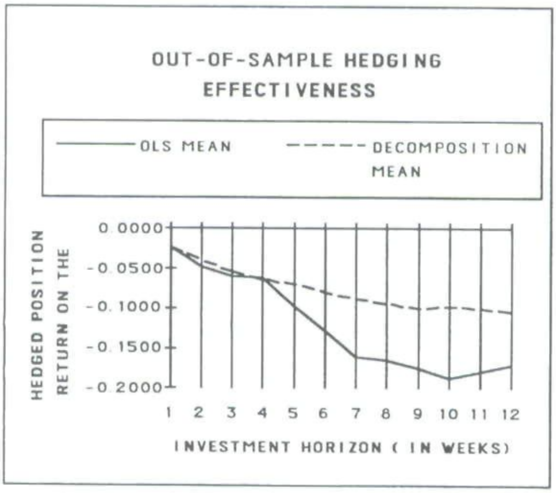

FIGURE 9
Out-of-sample hedging effectiveness—municipal bond index.

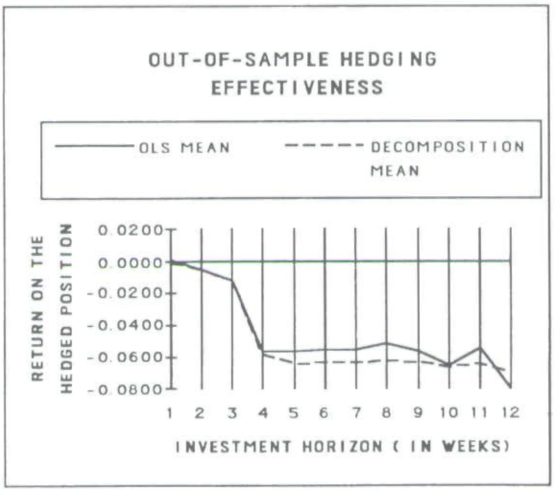

FIGURE 10
Out-of-sample hedging effectiveness—Swiss franc.

CONCLUSION AND FURTHER RESEARCH

This study derives a statistical model which can explain the well-documented observation that in-sample hedging effectiveness tends to increase with the hedging horizon. The model is based on cointegration between the spot and futures prices. Cointegration will occur in futures markets under several conditions. Cointegration between the spot and futures prices is a necessary condition for market efficiency when no risk premium is present. Also, price convergence and stationary carrying cost will lead to cointegration between spot and futures prices. When spot and futures prices are cointegrated, each price can be described in terms of permanent and transitory components. The presence of both permanent and transitory components in the spot and futures prices causes the hedge ratio and the degree of hedging effectiveness to depend on the investment horizon length. Over long horizons, the shared permanent component ties the spot and futures series together and the two prices will be perfectly correlated.

Five pairs of spot and futures series are investigated: the German mark, the Swiss franc, the Japanese yen, the S&P 500 index, and the municipal bond index. The data used are week-ending prices collected from *Barron's* magazine for the three-year period, January 1990 to January 1993.

By providing a theoretical basis for the behavior of the hedge ratio, the model allows the estimation of hedge ratios for any arbitrary investment horizon length and avoids the statistical problems associated with small samples or overlapping observations. The in-sample measures of hedging effectiveness for the decomposition model closely match the standard regression measures and follow the pattern documented elsewhere.

Out-of-sample hedging effectiveness tends to decrease as the hedging horizon increases for both the regression methodology and the decomposition methodology. However, the relationship is typically less severe for the decomposition methodology. With the exception of the Swiss franc and German mark, the decomposition methodology outperforms the standard regression methodology out-of-sample in terms of mean and variance. Across the five contracts, the decomposition procedure outperforms the regression methodology in terms of mean return in 49 out of 60 cases and in terms of variance in 40 of the 60 cases.

Although not investigated here, the decomposition model may have implications for the intertemporal stability of the hedge ratio at different investment horizons. Several studies have investigated the intertemporal stability of the minimum variance hedge ratios [see Malliaris and Urrutia (1991b), Geppert and Wilcox (1991), Myers (1991), and Kroner and Sultan (1991)]. Myers (1991) and Kroner and Sultan (1991) estimate models where the instability arises though time-varying second moments. Both studies use GARCH methodology to estimate time-varying conditional moments and construct the hedge ratio at each point in time. The decomposition model can easily be modified to incorporate GARCH methodology. The variance terms of the permanent and transitory components in eqs. (7c) and (7d) could be modeled as univariate GARCH processes and the predicted variance terms used as inputs into eqs. (8) and (9). The impact of this modification on out-of-sample hedging effectiveness is left for future research.

APPENDIX

Part A

Rewrite eqs. (7c) and (7d) in their moving average representations:

$$P_t = \sum_{i=0}^{\infty} u_{t-i} \tag{A1}$$

$$\tau_t = \sum_{i=0}^{\infty} \alpha^i v_{t-i} \tag{A2}$$

Substitute eqs. (A1) and (A2) into eqs. (7a) and (7b) and take k period differences of the resulting terms:

$$(S_t - S_{t-k}) = a_1 \sum_{i=0}^{k-1} u_{t-i} + a_2\left(\sum_{i=0}^{k-1} \alpha^i v_{t-i} - (1-\alpha^k)\sum_{i=0}^{\infty} \alpha^i v_{t-k-i}\right) \tag{A3}$$

$$(F_t - F_{t-k}) = b_1 \sum_{i=0}^{k-1} u_{t-i} + b_2\left(\sum_{i=0}^{k-1} \alpha^i v_{t-i} - (1-\alpha^k)\sum_{i=0}^{\infty} \alpha^i v_{t-k-i}\right) \tag{A4}$$

For a k period investment horizon, the hedger wishes to minimize the variance of the k period difference in its spot position. To calculate the minimum variance hedge ratio for a k period investment horizon, one needs to first calculate the variances of $(S_t - S_{t-k})$ and $(F_t - F_{t-k})$ and also their covariance. Equations (A5)–(A7) give the required variances and covariance terms.

$$E\{(S_t - S_{t-k})(S_t - S_{t-k})\} = a_1^2 k\sigma_u^2 + 2a_2^2\left(\frac{(1-\alpha^k)}{(1-\alpha^2)}\right)\sigma_v^2 \tag{A5}$$

$$E\{(F_t - F_{t-k})(F_t - F_{t-k})\} = b_1^2 k\sigma_u^2 + 2b_2^2\left(\frac{(1-\alpha^k)}{(1-\alpha^2)}\right)\sigma_v^2 \tag{A6}$$

$$E\{(S_t - S_{t-k})(F_t - F_{t-k})\} = a_1 b_1 k\sigma_u^2 + 2a_2 b_2\left(\frac{(1-\alpha^k)}{(1-\alpha^2)}\right)\sigma_v^2 \tag{A7}$$

Equation (8) is obtained by substituting (A7) and (A6) into eq. (1). Equation (9) is obtained by substituting eqs. (A5)–(A7) into eq. (3).

Part B

Let x_t be an N-dimensional time series indexed from 1 to T. Compute the matrix A defined by eq. (B1).

$$A = \left(\sum_{t=2}^{T} x_t x_t'\right)^{-1}\left(\sum_{t=2}^{T} x_t x_{t-1}'\right)\left(\sum_{t=2}^{T} x_t x_{t-1}'\right)^{-1}\left(\sum_{t=2}^{T} x_t x_t'\right) \tag{B1}$$

The left two bracketed terms of A can be estimated as the coefficient matrix from a matrix regression of x_{t-1} on x_t. The right two bracketed terms of A are the coefficient matrix from a matrix regression of x_t on x_{t-1}. After obtaining A, the next step is to calculate the orthogonal eigenvalue/eigenvector decomposition of A. Arrange the eigenvalues, denoted $\lambda_1, \lambda_2, \ldots, \lambda_N$, in descending order and let $w_1, w_2, \ldots, w_N$ be the corresponding eigenvectors. The original N series are transformed into N new orthogonal series, $y_{1t} \ldots y_{Nt}$, called canonical variates according to $y_{it} \equiv w_i' x_t$. The series y_{it} can be interpreted as the ith most persistent transformation of the original series.

The canonical variate approach allows a convenient way to decompose the series x_t into permanent and transitory components. Define the matrix $W \equiv [w_1, \ldots, w_N]'$ which implies that $y_t = W \cdot x_t$. Also define β_{ij} as the ijth element of $C \equiv W^{-1}$. Then the original series, x_{it}, is equal to:

$$x_{it} = \sum_{j=1}^{N} \beta_{ij} y_{jt} \tag{B2}$$

If there are q cointegrating relationships in the original N series, then the first $(N - q)$ canonical variates $(y_1, \ldots, y_{N-q})$ will be random walks (permanent components) and the remaining q canonical variates will be stationary (transitory components). x_{it} is equal to the sum of the permanent component and the transitory component, as seen in eq. (B3).

$$x_{it} = \sum_{j=1}^{q} \beta_{ij} y_{jt} + \sum_{j=1+q}^{N} \beta_{ij} y_{jt} \tag{B3}$$

where the first summation is the permanent component and the second summation is the transitory component. In this study, $N = 2$ and $q = 1$. x_{1t} is the spot asset price, and x_{2t} is the corresponding futures price. The coefficients in eqs. (7a)–(7d) correspond to $\beta_{11} = a_1$, $\beta_{12} = a_2$, $\beta_{21} = b_1$, $\beta_{22} = b_2$, $y_{1t} = P_t$, $y_{2t} = \tau_t$. The coefficient α is obtained by estimating the OLS regression given in eq. (B4).

$$y_{2t} = \alpha y_{2t-1} + v_t \tag{B4}$$

The variance term σ_v^2 is obtained from the residuals in eq. (B4), while the variance term σ_u^2 can be calculated from the variance of $(y_{1t} - y_{1t-1})$.

BIBLIOGRAPHY

Benet, B. A. (1992): "Hedge Period Length and Ex-Ante Futures Hedging Effectiveness: The Case of Foreign-Exchange Risk Cross Hedges," *The Journal of Futures Markets,* 12:163–175.

Chowdhury, A. R. (1991): "Futures Market Efficiency: Evidence from Cointegration Tests," *The Journal of Futures Markets,* 11:577–589.

Dale, C. (1981): "The Hedging Effectiveness of Currency Futures Markets," *The Journal of Futures Markets,* 1:77–88.

Ederington, L. H. (1979): "The Hedging Performance of the New Futures Markets," *Journal of Finance,* 34:157–170.

Geppert, J. M., and Wilcox, S. E. (1991): "The Hedge Ratio: An Exact Test of the Random Walk Hypothesis," *Journal of the Midwest Finance Association,* 20:96–106.

Hakkio, C. S., and Rush, M. (1989): "Market Efficiency and Cointegration: An Application to the Sterling and Deutschemark Exchange Market," *Journal of International Money and Finance,* 8:75–88.

Hill, J., and Schneeweis, T. (1981): "A Note on the Hedging Effectiveness of Currency Futures," *The Journal of Futures Markets,* 1:660–663.

Hill, J., and Schneeweis, T. (1982): "The Hedging Effectiveness of Foreign Currency Futures," *The Journal of Financial Research,* 5:95–104.

Howard, C. T., and D'Antonio, L. J. (1991): "Multiperiod Hedging Using Futures: A Risk Minimization Approach in the Presence of Autocorrelation," *The Journal of Futures Markets,* 11:697–710.

Hull, J. C. (1993): *Options, Futures, and Other Derivative Securities."* New York: Prentice Hall, pp. 68–69.

Hylleberg, S., and Mizon, G. E. (1989): "Cointegration and Error Correction Mechanisms," *The Economic Journal,* 99:113–125.

Kroner, K. F., and Sultan, J. (1991): "Foreign Currency Futures and Time Varying Hedge Ratios," Working Paper.

Lai, K. S., and Lai, M. (1991): "A Cointegration Test for Market Efficiency," *The Journal of Futures Markets,* 11:567–575.

Lien, D., and Luo, X. (1993): "Estimating Multiperiod Hedge Ratios in Cointegrated Markets," *The Journal of Futures Markets,* 13:909–920.

Malliaris, A. G., and Urrutia, J. L. (1991a): "The Impact of the Lengths of Estimation Periods and Hedging Horizon on the Effectiveness of a Hedge: Evidence from Foreign Currency Futures," *The Journal of Futures Markets,* 11:271–289.

Malliaris, A. G., and Urrutia, J. L. (1991b): "Tests of Random Walk of Hedge Ratios and Measures of Hedging Effectiveness for Stock Indexes and Foreign Currencies," *The Journal of Futures Markets,* 11:55–68.

Quan, J. (1992): "Two-Step Testing Procedure for Price Discovery Role of Futures Prices," *The Journal of Futures Markets,* 12:139–149.

Myers, R. J. (1991): "Estimating Time Varying Optimal Hedge Ratios on Futures Markets," *The Journal of Futures Markets,* 11:39–53.

Seretis, A., and Banack, D. (1990): "Market Efficiency and Cointegration: An Application to Petroleum Markets," *Review of Futures Markets,* 9:372–380.

Shen, C. H., and Wang, H. R. (1990): "Examining the Validity of a Test of Futures Market Efficiency: A Comment," *The Journal of Futures Markets,* 10:195–196.

Stock, J. H., and Watson, M. W. (1988): "Testing for Common Trends," *Journal of the American Statistical Association,* 83:1097–1107.

Tsay, R., and Tiao, G. (1990): "Asymptotic Properties of Multivariate Nonstationary Processes with Applications to Autoregressions," *Annals of Statistics,* 18:220–250.