PREDICTING STOCK MARKET VOLATILITY: A NEW MEASURE

JEFF FLEMING
BARBARA OSTDIEK
ROBERT E. WHALEY

INTRODUCTION

The CBOE Market Volatility Index (VIX) is an average of S&P 100 option (OEX) implied volatilities. As such, it represents a market-consensus estimate of future stock market volatility.[1] The computation and dissemination of VIX on a real-time basis offers practitioners and academics an important new source of information. Practitioners, for

This research was supported by the Futures and Options Research Center at the Fuqua School of Business, Duke University. We gratefully acknowledge the helpful comments and suggestions of Fischer Black, Mark Rubinstein, and two anonymous referees. We also thank participants at the University of Pennsylvania, the University of Texas at Dallas, and the University of Waterloo/KPMG Peat Marwick Thorne seminars, as well as attendees of the 1993 Conference on Financial Innovation: 20 Years of Black/Scholes and Merton (Duke University) and the 1994 Berkeley Program in Finance, Ojai Valley, California.

[1] Since OEX options are the most actively traded index options, VIX promises to be the U.S. stock market volatility "standard" upon which derivative contracts may be written. A variety of option or embedded-option security positions are sensitive to changes in expected stock market volatility. Volatility derivatives, therefore, could be used to offset this exposure. Whaley (1993) describes the economic benefits of volatility derivatives as well as some risk management applications.

■ *Jeff Fleming is an Assistant Professor of Administrative Science at Rice University.*

■ *Barbara Ostdiek is an Assistant Professor of Administrative Science at Rice University.*

■ *Robert E. Whaley is the T. Austin Finch Foundation Professor of Business Administration at the Fuqua School of Business, Duke University.*

The Journal of Futures Markets, Vol. 15, No. 3, 265–302 (1995)

CCC 0270-7314/95/030265-38

example, face a variety of day-to-day decisions such as asset allocation, covered-call writing, and portfolio insurance, all of which require an up-to-the-minute estimate of future stock market volatility. On the other hand, academics are interested in studying temporal patterns in expected return and risk. Changes in the level of VIX represent changes in conditional stock market volatility. By virtue of the index construction, this conditional volatility measure is market-determined, forward-looking, and has a constant (one-month) forecast horizon.

The usefulness of the volatility index as a measure of expected volatility depends on its behavior. This study investigates the statistical properties of VIX and evaluates its predictive power. The assessment proceeds along three levels. First, the time-series history of VIX is examined to provide a description of its univariate properties and seasonalities. Second, the study considers the consistency of VIX with the empirical literature regarding conditional volatility, such as the temporal relationship between volatility and stock market returns. Finally, the article evaluates the performance of VIX as a forecast of stock market volatility.

The evidence reported in this study indicates that VIX is well behaved with little evidence of seasonality. Daily changes in VIX display a slight first-order autocorrelation. For weekly changes, significant mean reversion is detected. VIX also has a strong negative and asymmetric association with contemporaneous stock market returns. And, regarding forecast performance, the index exhibits a strong relationship to future realized stock market volatility. On the basis of this evidence, the volatility index indeed appears to be a useful proxy for expected stock market volatility.

The methodology and findings of this study are presented as follows. The second section describes the construction of VIX. The third section examines the CBOE's Market Volatility Index over a seven-year period (1986–1992).[2] Properties of daily and weekly changes in VIX are documented and the relationship between VIX changes and S&P 100 index returns is investigated. The fourth section focuses on intraday and intraweek seasonalities in VIX changes. The performance of VIX as a stock market volatility forecast is considered in the fifth section. The final section provides a summary of the major findings of the article.

[2]This historical VIX series is constructed ex post (rather than in real-time) from the CBOE's MDR system and was made available to researchers by the CBOE when the volatility index was introduced in February 1993.

COMPOSITION OF THE CBOE MARKET VOLATILITY INDEX

The idea of creating a volatility index from option prices emerged soon after the introduction of exchange-traded options in April 1973. Gastineau (1977), for example, created a "volatility-index" by averaging the implied volatilities of the at-the-money call options of 14 stocks. Cox and Rubinstein (1985, Appendix 8A) refined the Gastineau procedure by including multiple call options on each stock and by weighting the volatilities in such a manner that the index is at the money and has a constant time to expiration.

The CBOE Market Volatility Index captures the spirit of these earlier efforts and extends the concept in two important ways. First, VIX is based on index options rather than stock options. While the "average" level of individual stock volatilities may be of passing interest, market participants are more concerned with portfolio risk—the level of risk after the idiosyncratic risks of the individual stocks have been diversified away. Second, VIX is based on the implied volatilities of both call and put options. This not only increases the amount of information incorporated into the index but also mitigates concerns regarding call/put option clienteles and possible mismeasurements of the reported index level and the short-term interest rate. Since the focus of this study is to evaluate VIX as a measure of future stock market volatility, the study begins with a description of the index's construction.

Implied Volatility Computation

The CBOE Market Volatility Index is constructed from the implied volatilities of eight OEX options.[3] Computing these component implied volatilities requires three types of information: (i) an option valuation model, (ii) the values of the model's determinants (except for volatility), and (iii) an observed option price. With respect to the model, VIX construction is based on the Black–Scholes (1973)/Merton (1973b) option valuation framework. Since the OEX options are American-style, the cash-dividend adjusted binomial method is used to compute the component implied volatilities.

Within the Black–Scholes/Merton model, option value is determined by the current index value, the option's exercise price and time to expiration, the riskless interest rate, and the amount and timing of the anticipated cash dividends paid during the option's life. To generate the

[3]A more detailed description of VIX construction is provided in the appendix in Whaley (1993).

historical VIX series, 1986–1992, the CBOE used actual cash dividends to proxy for anticipated dividends.[4] The source of the dividend data prior to June 1988 is from Harvey and Whaley (1992a). The source of the dividend data from June 1988 through December 1992 is from the *S&P 100 Information Bulletin*. The interest rate is the effective yield (based on the average bid/ask discount) of the T-bill whose maturity most closely matches the option expiration and has at least 30 days to maturity. The source of the T-bill discounts is *The Wall Street Journal*. Of the three remaining option determinants, the exercise price and time to expiration are known. The reported S&P 100 index level is used to proxy for the current S&P 100 index value.[5] The CBOE automatically records into its Market Data Retrieval (MDR) system the reported index level whenever an OEX option trades or has price quotes issued.

The third type of information required to compute the component implied volatilities is observed option prices. Here, the midpoint of the most recent bid/ask price quote is used. Using actual transaction prices would induce negative first-order autocorrelation in implied volatility changes as option prices bounce randomly between bid and ask levels. The implied volatility for each option series, then, is determined by equating the theoretical option value, given the procedures outlined above, with the most recent bid/ask price midpoint.

Trading-Day Adjustment

The implied volatilities for the OEX option series used in computing VIX are stated in trading days rather than calendar days. If the time to option expiration were measured in calendar days, the implied volatility would be a rate per calendar day. The return variance over a weekend (from Friday close to Monday close), then, should be three times greater than it is over any other pair of trading days (say, Monday close to Tuesday close). Empirically, however, weekend volatility is approximately the same as volatility during trading days.[6] For this reason, each (calendar-day) implied volatility is adjusted to a trading-day basis. First, based on

[4]The real-time VIX computation uses a forecast of future dividends to compute the component implied volatilities. The historical series for these forecasts were unavailable when the CBOE introduced the index.

[5]To avoid the problems associated with extreme staleness of the reported index level, computation and dissemination of VIX generally begins at 9:00 AM (CST), 30 minutes after the stock market has opened. Also, since the stock market closes at 3:00 PM (CST), 15 minutes prior to the OEX option market, VIX computation ends at 3:00 PM each day.

[6]French and Roll (1986) estimate that the weekend return variance for all NYSE and AMEX stocks is only 10.7% greater than the trading day variance. For the largest market capitalization quintile (which includes all S&P 100 stocks), the weekend return variance is only 8.2% higher.

the number of calendar days to expiration, N_c, the number of trading days, N_t, is computed as

$$N_t = N_c - 2 \times \text{int}(N_c/7) \tag{1}$$

since expiring OEX options always cease trading on a Friday (except for a rare holiday). Next, the implied volatility rate is multiplied by the ratio of the square root of the number of calendar days to the square root of the number of trading days, that is,

$$\sigma_t = \sigma_c\left(\frac{\sqrt{N_c}}{\sqrt{N_t}}\right) \tag{2}$$

where $\sigma_t(\sigma_c)$ is the annualized trading-day (calendar-day) implied volatility.

This trading-day adjustment of implied volatilities is different than simply using the number of trading days in valuing the option. The option's time to expiration parameter affects valuation not only through total volatility, but also through the expected rate of appreciation in the index level and through the length of time over which the option's expected payoff is discounted to the present. Both of these latter considerations are more appropriately measured using calendar days.

Weighting the Component Implied Volatilities

The implied volatilities of the eight near-the-money, nearby, and second nearby OEX call and put options provide the inputs for constructing the volatility index. Some definitions and notation will facilitate the discussion of the weighting procedure used to determine VIX. The nearby OEX series is defined as the series with the fewest, but with at least eight, calendar days to expiration. The second nearby OEX series is the series of the following contract month. To represent the component implied volatilities, subscript the volatility rate with two terms, separated by a comma, e.g., $\sigma_{c,1}$. The first subscript is either c or p, depending on whether the implied volatility is computed from a call or a put. The second subscript is either "1" or "2," depending on whether the implied volatility is computed from the first (nearby) or second (second nearby) option series. Finally, denote the exercise price just below (above) the current index level, S, as $X_b(X_a)$. This notation is used as a superscript, e.g., σ^{X_b}.

The weighting of the component implied volatilities has three steps. First, the implied volatilities of the call and the put in each of the four categories of options are averaged. $\sigma_1^{X_b}$, for example, denotes the average of the call and put implied volatilities for the nearby options series with the lower exercise price, that is, $\sigma_1^{X_b} \equiv (\sigma_{c,1}^{X_b} + \sigma_{p,1}^{X_b})/2$. The other three averages are denoted $\sigma_2^{X_b}$, $\sigma_1^{X_a}$, and $\sigma_2^{X_a}$. Averaging the call and put implied volatilities mitigates possible bias due to staleness in the observed index level. When the market rises quickly during the trading day, for example, the most recently observed stock prices for some component stocks may lag their true values. This, in turn, causes the reported S&P 100 index level to lag its true value. OEX options, on the other hand, trade more frequently than the S&P 100 basket of stocks. Consequently, an OEX call (put) implied volatility, computed from the reported S&P 100 index level during a rising market is upward- (downward-) biased. Since the upward (downward) bias of the call implied volatility is approximately equal to the downward (upward) bias of the put implied volatility, the effect of infrequent trading of index stocks on the level of VIX is mitigated.[7]

Second, "at-the-money" implied volatilities are created by interpolating between the nearby and second nearby average implied volatilities,

$$\sigma_1 = \sigma_1^{X_b}\left(\frac{X_a - S}{X_a - X_b}\right) + \sigma_1^{X_a}\left(\frac{S - X_b}{X_a - X_b}\right) \tag{3a}$$

and

$$\sigma_2 = \sigma_2^{X_b}\left(\frac{X_a - S}{X_a - X_b}\right) + \sigma_2^{X_a}\left(\frac{S - X_b}{X_a - X_b}\right) \tag{3b}$$

In this way, VIX is always "at-the-money."[8]

Finally, a constant time to expiration is maintained. The nearby and second nearby at-the-money volatilities are weighted to create a constant 30-calendar day (22-trading day) time to expiration. Letting N_{t_1} and N_{t_2} represent the number of trading days to expiration of nearby and second nearby contracts, the CBOE Market Volatility Index is

$$\text{VIX} = \sigma_1\left(\frac{N_{t_2} - 22}{N_{t_2} - N_{t_1}}\right) + \sigma_2\left(\frac{22 - N_{t_1}}{N_{t_2} - N_{t_1}}\right) \tag{4}$$

[7]This averaging also mitigates the possible effects of mismeasurement in the riskless rate. Thanks are due Mark Rubinstein for pointing this out.

[8]The linear interpolation of in-the-money and out-of-the-money implied volatilities to create an at-the-money implied volatility implicitly assumes that the "volatility smile" is well approximated by a line. For the small range over which the interpolation is made, this approximation is reasonable. [See, for example, Dumas, Fleming, and Whaley (1994).] Similarly, the nearby and second nearby implied volatilities are interpolated to create a constant 30-calendar day (22-trading day) time to expiration. Hence, the degree of bias in VIX as an estimate of the 30-day volatility rate depends upon the linearity of the term structure of volatility between these two nearby maturities.

The CBOE uses this procedure to compute VIX on a real-time basis every minute of the trading day. With this construction, VIX provides investors with an up-to-the-minute assessment of the market's perception of future volatility. This study now turns to investigating the empirical properties of changes in the level of VIX.

DAILY AND WEEKLY VIX MOVEMENTS

This study examines daily and weekly movements of the CBOE Market Volatility Index during the seven-year sample period, January 1986 through December 1992. This section begins by describing VIX and S&P 100 index movements during the period and their behavior surrounding some important economic events. The univariate properties of VIX changes are then discussed, together with VIX cross-correlations with the S&P 100 index.

Historical Account

Historical data are available on the CBOE Market Volatility Index since the beginning of 1986. Figure 1 plots the weekly closing index levels against the absolute magnitude of weekly S&P 100 returns. Several periods of relatively high volatility and large stock market moves stand out. The October 1987 stock market crash is accompanied by an end-of-week volatility level in excess of two and a half times the next largest historical level. (Figure 1 is truncated. VIX closed the week at 98.81 on October 23, 1987, after reaching its historical high of nearly 173 in the last hour of trading on October 20.) As the figure shows, it took several months for VIX to return to pre-crash levels. The index declined somewhat erratically throughout 1988, attaining a plateau around 20 by early 1989 before spiking up during the October 1989 mini-crash. (The index opened on Monday, October 16, 1989, at 51.71 but closed the day at 24.29.) The index then remained somewhat erratic for several months before settling around 20 just before the Iraq invasion of Kuwait in August 1990. News of the invasion brought a large stock market move (the S&P 100 lost nearly 15% that week) as well as a sharp increase in VIX (from 20 to 35). After falling sharply to the low 20's at the end of 1990, the index again spiked when Allied forces attacked Iraq in January 1991. During the remainder of 1991 and in 1992, the index fell to historically low levels with a moderate spike in late 1991 and another

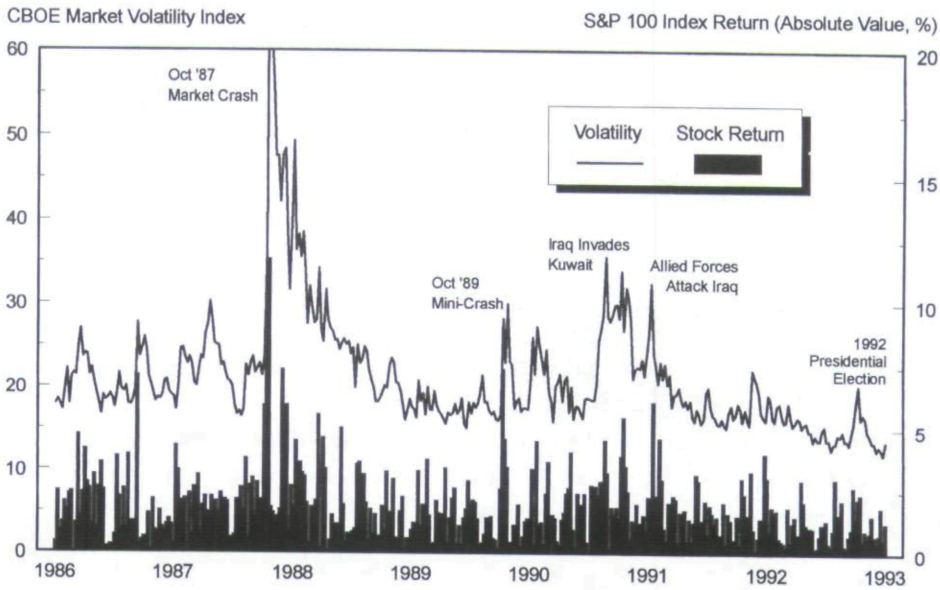

FIGURE 1.
Weekly closing CBOE market volatility index levels and S&P 100 index return absolute values during the period from 1986 through 1992. The data points for each index reflect the closing level for the last day of each week. The S&P 100 returns are reported on the basis of the log index relative multiplied by 100.

in November 1992 around the U.S. presidential election before closing the year under 15.

During the period covered by Figure 1, spikes in the volatility index are usually accompanied by large moves, up or down, in the stock index level. There are exceptions, however. In mid-1987 and again in November 1992, for example, VIX spiked sharply during periods when S&P 100 index movement remained fairly moderate.

Statistical Properties of the Volatility Index

The empirical analysis of the volatility index is based on volatility "changes" (i.e., $\Delta\mathcal{V}_t \equiv \mathcal{V}_t - \mathcal{V}_{t-1}$, where $\mathcal{V}_t$ denotes the level of the volatility index at time t) rather than levels. Three considerations dictate this. First, the variable of interest for academics and practitioners is changes or innovations to expected volatility. They want to know how changes in expected volatility influence changes in security valuation. Second, if stock prices follow a random walk, estimation of the relationship between the stock and volatility indexes in levels may be spurious. Third, VIX levels also appear to be near-random walk. Daily VIX levels have a first-order autocorrelation of nearly 97%. Although a unit root can be rejected for this series, such high autocorrelation may affect inference in finite samples.

The univariate properties of the volatility index are considered first. Summary statistics are calculated for daily and weekly VIX changes over the entire sample period and for calendar-year subperiods. In each case, results are presented with and without the 1987 and 1989 market crashes. The 1987 crash period is defined, in most analyses, as October 19–30, 1987, and the 1989 mini-crash period is defined as October 13–16, 1989.

Table I summarizes the properties of daily closing volatility index changes. The mean daily changes range from −0.0827% in 1988 to 0.0243% in 1990. The mean volatility change over the entire sample period is −0.0026. Apparently, over the seven-year period, market volatility did not drift in one direction or another. The standard deviation of the daily volatility changes (volatility of the volatility) also is fairly stable, ranging from 0.7421 in 1992 to 1.8014 in 1988. Not surprisingly, including the 1987 crash observations increases the 1987 standard deviation of volatility changes by more than a factor of five, from 1.6013 to 8.8886 on a daily basis.

Table I also provides the autocorrelation structure of the daily volatility index changes for one through three lags. Unlike the mean and standard deviation, the autocorrelation structure of VIX changes varies quite substantially from year to year. The first-order autocorrelations, for example, range from −0.301 in 1988 to 0.094 in 1987 without the crash. Over the noncrash sample, the first- and second-order coefficients, −0.073 and −0.104, reveal a significant negative autocorrelation. This degree of correlation, however, is much smaller than the autocorrelation reported in Harvey and Whaley (1991) for individual S&P 100 options. Based on 3:00 PM prices, they report an autocorrelation structure of −0.33 and −0.13 (−0.33 and −0.09) for daily volatility changes implied by the nearby at-the-money call (put).

The smaller autocorrelations for VIX relative to those for individual OEX implied volatilities are attributable to index construction. First, VIX is a time-to-expiration weighted average of the volatilities implied by the nearby and second nearby options. This weighting places less weight on the near-expiration implied volatilities, which tend to be more volatile as expiration approaches. Second, the component implied volatilities are computed using bid/ask price midpoints rather than transaction prices. This eliminates the spurious negative correlation induced by bid/ask bounce. Finally, the index is an average of eight option prices which reduces the rebounding effect on the implied volatility following a day

TABLE I

Statistical Properties of Daily Closing CBOE Market Volatility Index Level Changes[a]

		Volatility Index Changes					S&P 100 Index Returns					Cross Correlations[b]				
Period	*Obs.*	*Mean*	*SD*	$\rho(1)$	$\rho(2)$	$\rho(3)$	*Mean*	*SD*	$\rho(1)$	$\rho(2)$	$\rho(3)$	−2	−1	0	+1	+2
Noncrash Sample[c]																
1986–1992	1755	−0.0185	1.2891	−0.073[d]	−0.104[d]	−0.038	0.0004	0.0101	0.008	−0.036	−0.036	0.072[d]	0.092[d]	−0.615[d]	0.006	0.086[d]
1986	252	0.0025	0.7497	0.067	−0.047	0.012	0.0005	0.0097	0.036	−0.059	−0.033	0.087	0.057	−0.305[d]	−0.114	0.088
1987	242	0.0091	1.6013	0.094	−0.124[d]	−0.023	0.0005	0.0132	0.064	−0.008	−0.024	0.071[d]	−0.019	−0.553[d]	0.021	0.095[d]
1988	253	−0.0827	1.8014	−0.301[d]	−0.091	0.007	0.0004	0.0120	−0.080	−0.058	−0.061	0.062	0.233[d]	−0.714[d]	0.100	0.083
1989	249	−0.0425	0.9256	0.006	−0.138[d]	−0.096	0.0011	0.0076	−0.009	−0.013	−0.048	0.139[d]	0.184[d]	−0.521[d]	−0.046	0.068
1990	253	0.0243	1.5592	0.011	−0.143[d]	−0.089	−0.0002	0.0106	0.079	−0.096	−0.054	0.111	0.012	−0.791[d]	−0.029	0.094
1991	252	−0.0134	1.2167	−0.039	−0.063	−0.045	0.0009	0.0095	−0.027	0.012	−0.004	0.023	0.101	−0.616[d]	−0.017	0.107
1992	254	−0.0261	0.7421	−0.146[d]	−0.059	−0.083	0.0001	0.0064	−0.100	−0.013	−0.023	0.021	0.121	−0.615[d]	−0.023	0.031
Entire Sample																
1986–1992	1769	−0.0026	3.5562	−0.065[d]	−0.320[d]	0.061[d]	0.0004	0.0123	−0.014	−0.085[d]	−0.027	0.220[d]	−0.046	−0.677[d]	0.110[d]	0.110[d]
1987	253	0.0820	8.8886	−0.057	−0.348[d]	0.072	0.0001	0.0222	−0.003	−0.138[d]	−0.020	0.317[d]	−0.120	−0.806[d]	0.161[d]	0.142[d]
1989	252	−0.0045	1.2036	−0.136[d]	−0.040	0.014	0.0009	0.0089	−0.108	0.025	−0.007	0.037	0.217[d]	−0.675[d]	0.113	0.018

[a]The mean, standard deviation (SD), and autocorrelations (ρ) for the first three lags are provided for volatility index changes and S&P 100 index returns along with the cross-correlations between volatility index changes and S&P 100 index returns.

[b]Correlations for negative (positive) lags denote the correlation between volatility index changes and past (future) S&P 100 returns.

[c]The noncrash sample excludes index observations during the periods 10/19/87–10/30/87 and 10/13/89–10/16/89.

[d]Identifies correlations significant at the 5% level where the standard error is calculated as $1/\sqrt{T}$.

on which temporary demand or supply pressure influences the price of a single option.

Table I also summarizes the properties of daily S&P 100 index returns. Most notably, the autocorrelation for one through three lags is negligible except when the October 1987 crash data is included. The effect of infrequent trading on close-to-close S&P 100 index returns appears to be insignificant.

The final five columns of Table I summarize the cross-correlation structure between VIX changes and S&P 100 index returns. The relationship between changes in volatility and changes in the stock market is important for a number of reasons. The Sharpe (1964)/Lintner (1965) and Merton (1973a) capital asset pricing models predict that stock prices will fall as expected volatility rises.[9] In addition, Black (1976) and Christie (1982) suggest that leverage can induce an inverse relationship between stock returns and changes in future volatility. When stock prices fall (relative to bond prices), leverage increases and causes expected volatility to increase.[10,11] If the volatility index proxies for future stock market volatility, VIX movement should be inversely related to the contemporaneous stock market return.

The estimated contemporaneous correlation between changes in expected volatility and S&P 100 returns reported in Table I is large and negative across the sample and within each subperiod. The contemporaneous cross correlation in the noncrash sample is -0.615. As the theory predicts, stock prices fall (rise) when expected volatility rises (falls). The correlation reverses sign and dampens out quickly to more moderate levels at the noncontemporaneous lags. Slight positive correlation shows up between the current change in expected volatility and both past and future stock index returns, but the stronger of the two effects appears between VIX changes and past stock market returns.

Table II summarizes the statistical properties of weekly volatility changes. The weekly data exhibit patterns in mean and standard deviation similar to those apparent in the daily data. The mean weekly index

[9] Merton's theory motivates many recent AutoRegressive Conditional Heteroskedasticity In Means (ARCH-M) models of stock market volatility. See, for example, French, Schwert, and Stambaugh (1987), Bollerslev, Engle, and Wooldridge (1988), and Nelson (1989, 1991).

[10] Black (1992) argues that the inverse relationship between stock prices and volatility also may be attributable to changes in operating leverage. For example, a downward revision in the firm's prospects without a commensurate reduction in costs (other than interest) causes the firm's operating leverage to increase and the stock price to fall.

[11] More recent studies, such as French, Schwert, and Stambaugh (1987) and Schwert (1989, 1990), find an even stronger relationship between returns and volatility than suggested by the leverage arguments of Black and Christie.

TABLE II

Statistical Properties of Weekly CBOE Market Volatility Index Level Changes[a]

		Volatility Index Changes					S&P 100 Index Returns					Cross Correlations[b]				
Period	*Obs.*	*Mean*	*SD*	$\rho(1)$	$\rho(2)$	$\rho(3)$	*Mean*	*SD*	$\rho(1)$	$\rho(2)$	$\rho(3)$	−2	−1	0	+1	+2
Noncrash Sample[c]																
1986–1992	359	−0.1164	2.3531	−0.259[d]	0.008	−0.074	0.0025	0.0215	−0.145[d]	0.029	−0.047	0.033	0.205[d]	−0.464[d]	0.114[d]	−0.048
1986	51	0.0267	1.6168	−0.099	0.049	0.074	0.0025	0.0207	−0.130	0.052	−0.195	−0.066	0.226	−0.245	−0.047	−0.170
1987	49	−0.2000	2.9332	−0.211	0.120	−0.091	0.0041	0.0296	−0.213	0.078	−0.103	0.112	0.187	−0.221	0.235	−0.021
1988	52	−0.4027	2.3321	−0.243	−0.034	−0.163	0.0020	0.0231	−0.299[d]	0.046	−0.031	0.062	0.419[d]	−0.651[d]	0.051	0.032
1989	50	−0.1218	1.7452	−0.264	−0.017	−0.082	0.0049	0.0170	−0.065	−0.315[d]	0.172	0.124	0.207	−0.163	−0.222	0.287[d]
1990	53	0.0909	3.1973	−0.336[d]	0.009	0.089	−0.0008	0.0239	−0.167	0.099	−0.059	−0.064	0.241	−0.730[d]	0.212	−0.142
1991	52	−0.0983	2.7785	−0.300[d]	−0.068	−0.254	0.0041	0.0208	0.051	−0.043	0.112	0.106	0.022	−0.559[d]	0.182	−0.092
1992	52	−0.1160	1.2426	−0.215	−0.039	−0.194	0.0008	0.0129	−0.126	0.014	−0.300[d]	−0.082	0.150	−0.609[d]	0.077	−0.128
Entire Sample																
1986–1992	364	−0.0127	7.8903	−0.113[d]	−0.314[d]	0.007	0.0018	0.0286	−0.059	−0.135[d]	−0.025	0.280[d]	0.079	−0.724[d]	0.051	0.190[d]
1987	52	0.3944	20.2702	−0.101	−0.341[d]	0.013	0.0003	0.0579	−0.003	−0.249	−0.026	0.385[d]	0.056	−0.859[d]	0.051	0.275[d]
1989	52	0.0144	2.0478	−0.209	0.149	−0.123	0.0040	0.0181	−0.119	−0.189	0.183	0.008	0.258	−0.336[d]	−0.159	0.096

[a]The mean, standard deviation (SD), and autocorrelations (ρ) for the first three lags are provided for volatility index changes and S&P 100 index returns along with the cross-correlations between volatility index changes and S&P 100 index returns. The weekly dataset is formed by selecting the observations which occur on the first trading day of each week of the sample.

[b]Correlations for negative (positive) lags denote the correlation between volatility index changes and past (future) S&P 100 returns.

[c]The noncrash sample excludes index observations during the periods 10/19/87–10/30/87 and 10/13/89–10/16/89.

[d]Identifies correlations significant at the 5% level where the standard error is calculated as $1/\sqrt{T}$.

changes range from −0.4027 in 1988 to 0.0909 in 1990, approximately five times their corresponding levels in Table I. The volatility of the weekly volatility index changes ranges from a low of 1.2426 in 1992 to 3.1973 in 1990. Again, including the crash periods sharply affects the volatility of the volatility index. The standard deviation of 1987 volatility changes, including the October market crash, is over 20—more than six times greater than the largest standard deviation reported for the noncrash years.

Table II also provides the autocorrelation structure of the weekly volatility index changes. The weekly changes have significantly stronger negative first-order autocorrelation, −0.259 over the noncrash period, than evidenced in daily changes, −0.073. Unlike the daily autocorrelation, the weekly autocorrelation is negative in all years. The magnitude of these lag one autocorrelation coefficients, relative to those in Table I, reflect the persistence of longer-term stock market volatility. Negative first-order autocorrelation also appears in the weekly S&P 100 index returns. This autocorrelation, however, largely dissipates by the second lag.

The strong temporal association between volatility changes and stock market returns, reported in Table I for daily observations, is also present in Table II for weekly observations. Over the noncrash sample, the contemporaneous cross-correlation is −0.464. This relationship is fairly strong in all years. Positive relationships between current expected volatility and past-week and future-week stock returns also are evident. The correlation between the current volatility change and the lagged and lead stock market returns, respectively, are 0.205 and 0.114. The cross-correlations estimated for the other noncontemporaneous lags, however, exhibit greater variability from year to year and are generally insignificant.

Co-Movement of the Volatility Index with Stock Market Returns

A number of researchers have detected intertemporal relationships between stock market prices and expected volatility. This section focuses more explicitly on these relationships. In particular, VIX is used as a proxy for expected volatility and the S&P 100 proxies for the stock market in validating several stylized facts reported in previous research with alternative volatility proxies. First, Black (1976) and Christie (1982) documented a strong negative association between stock market returns and changes in ex post future volatility. French, Schwert, and Stambaugh (1987) provided similar evidence using changes in expected stock market volatility. Second, Schwert (1989, 1990) found asymmetry in the relationship between returns and expected volatility, that is, the increase in expected volatility corresponding to a given negative stock market return is larger than the decrease in expected volatility corresponding to a similar size positive return. Third, French, Schwert, and Stambaugh reported a positive relationship between the predictable component of stock market volatility and expected stock returns. If VIX represents the market volatility prediction, changes in the volatility index may be positively related to *future* S&P 100 returns. Finally, to the extent VIX is mean reverting, the strong negative contemporaneous relationship between stock returns and expected volatility may induce a slight positive association between changes in expected volatility and *past* stock market returns.

The degree to which these past findings are reflected in the relationship between VIX and the S&P 100 index can be assessed with the following multivariate regression of the volatility change on two lag, two lead, and the contemporaneous stock market returns, as well as the magnitude (absolute value) of the contemporaneous return,

$$\Delta \mathcal{V}_t = \alpha + \sum_{i=-2}^{2} \beta_{S,i} R_{S,t+i} + \beta_{|S|} |R_{S,t}| + \epsilon_t \tag{5}$$

This specification focuses on the intertemporal relationship between stock market returns and changes in expected volatility. A significant negative (positive) $\beta_{S,i}, i = -2, \ldots, 2$, coefficient indicates that increases in expected volatility at time t are accompanied by stock market declines (increases) at time $t + i$. A significant positive (negative) $\beta_{|S|}$ coefficient indicates that stock market moves, independent of their direction, are accompanied by increases (decreases) in expected volatility.[12] The sum of $\beta_{|S|}$, and $\beta_{S,0}$, therefore, measures the asymmetry of the relationship between changes in expected volatility and stock market returns.

Several factors may influence the estimation of regression (5). First, the reported S&P 100 index level is based on the last trade prices of the index stocks. Since stocks do not trade continuously, reported index levels will always be stale relative to true index levels. Consequently, stock index returns, measured over short intervals, will be positively autocorrelated and the estimated contemporaneous correlation

[12]Coefficients on lead and lag absolute stock index returns are predominantly insignificant, so they are not included in regression (5).

between changes in VIX and S&P 100 returns will be reduced. Table I, however, reports that the autocorrelation in daily S&P 100 returns is insignificant, indicating that infrequent trading effects are negligible. Second, the estimation of (5) may be affected by the correlation between R_S and $|R_S|$. Empirically, however, the correlation between R_S and $|R_S|$ is inconsequential (i.e., -5.7%). Finally, Table I suggests that autocorrelation and heteroskedasticity in volatility changes may influence estimation. To allow for these residual properties, the t-statistics provided for regression (5), and for all subsequent regressions in this study, are based on heteroskedasticity and autocorrelation consistent standard errors using Hansen's (1982) method of moments estimation and Parzen weights [Gallant (1987), p. 533]. Since the length of residual autocorrelation in regression (5) is unknown, Andrews's (1991) method of automatic bandwidth selection is employed.

Table III reports the regression (5) results for daily VIX changes and S&P 100 returns. In the noncrash sample, the coefficient of the contemporaneous, signed change is large and negative, -75.55, with a t-statistic of -14.21. This coefficient is consistent with the cross-correlation analysis reported in Table I and confirms the strong negative contemporaneous relationship between changes in expected volatility and stock market returns. The lag one and two and lead two coefficients are positive and significant but much smaller in magnitude than the contemporaneous coefficient. So, while a positive relationship exists between changes in expected volatility and past and future stock returns, the relationship is dominated by the negative contemporaneous association. Nonetheless, the sign and magnitude of these coefficients offer some support for the hypothesized noncontemporaneous relationships described above. Finally, the $\beta_{|S|}$ coefficient is large and positive, 30.11, with a t-statistic of 3.87. Apparently, a positive relationship exists between the size of a stock market move and the contemporaneous change in expected stock market volatility.

The estimates of $\beta_{|S|}$ reported in Table III indicate a significant asymmetry in the relationship between volatility changes and contemporaneous stock market returns. If the stock market return is positive, the coefficient impacting the change in volatility is $\beta_S^+ = \beta_{S,0} + \beta_{|S|}$, or -45.43, across the noncrash sample. A stock market increase is expected to accompany a decrease in the volatility index. On the other hand, if the stock return is negative, the coefficient is $\beta_S^- = \beta_{S,0} - \beta_{|S|}$, or -105.66. A stock market decline is expected to accompany an increase in volatility. The difference in the magnitudes of the coefficients (β_S^- is more than two times the size of β_S^+), however, clearly indicates

TABLE III

Intertemporal Relationship between Daily CBOE Market Volatility Index Level Changes and S&P 100 Index Returns[a]

		Parameters							
Period	*Obs.*	$\hat{\alpha}$	$\hat{\beta}_{S,-2}$	$\hat{\beta}_{S,-1}$	$\hat{\beta}_{S,0}$	$\hat{\beta}_{S,+1}$	$\hat{\beta}_{S,+2}$	$\hat{\beta}_{\lvert S\rvert}$	$\overline{R}^2$
Noncrash Sample[b]									
1986–1992	1747	−0.2208	9.5196	14.0795	−75.5455	2.0667	8.0575	30.1124	0.4081
		(−4.32)	(3.30)	(3.12)	(−14.21)	(0.60)	(2.81)	(3.87)	
1986	252	−0.1819	5.1406	6.2303	−20.0682	−6.5091	4.4249	26.3378	0.1475
		(−3.05)	(1.01)	(0.83)	(−3.12)	(−1.75)	(1.14)	(2.86)	
1987	238	−0.2623	16.3850	5.1979	−57.5143	8.1536	10.6520	25.1166	0.2735
		(−2.16)	(1.98)	(0.61)	(−6.55)	(0.98)	(1.31)	(1.63)	
1988	253	−0.4410	8.1098	30.1161	−100.4044	7.8048	6.8231	45.9079	0.5869
		(−3.73)	(1.25)	(2.37)	(−8.39)	(0.95)	(1.08)	(3.35)	
1989	245	−0.2014	16.6966	24.4043	−62.8410	−4.7843	11.6726	32.0117	0.3356
		(−2.88)	(2.45)	(4.13)	(−7.71)	(−0.76)	(1.56)	(2.81)	
1990	253	−0.2259	7.6415	11.7328	−114.5914	4.3701	1.6730	28.3287	0.6403
		(−2.18)	(1.23)	(1.86)	(−11.39)	(0.77)	(0.31)	(2.09)	
1991	252	−0.1955	6.1050	10.2497	−83.0250	−5.8522	14.3468	32.8888	0.4172
		(−1.58)	(0.91)	(0.96)	(−6.00)	(−0.71)	(2.04)	(1.70)	
1992	254	−0.1475	3.9267	6.0358	−73.1161	−9.4160	−0.0178	26.2737	0.3950
		(−2.69)	(0.60)	(0.96)	(−8.83)	(−1.34)	(0.00)	(2.46)	
Entire Sample									
1986–1992	1769	−0.7996	67.1272	−0.4420	−165.5825	26.0177	15.2251	107.1235	0.5721
		(−2.12)	(1.75)	(−0.04)	(−4.27)	(1.89)	(1.23)	(2.00)	
1987	253	−1.5793	132.5014	−14.6419	−241.2580	53.5455	21.3915	141.0650	0.7773
		(−3.38)	(3.39)	(−0.84)	(−6.47)	(3.68)	(1.47)	(2.87)	
1989	252	−0.3284	13.8265	29.6450	−83.8694	3.1146	11.4113	56.4292	0.5603
		(−3.36)	(2.24)	(5.31)	(−7.95)	(0.59)	(2.04)	(3.31)	

[a]Parameter estimates are presented for the regression of daily volatility index changes on two lag, two lead, and the contemporaneous S&P 100 index returns, as well as the magnitude (absolute value) of the contemporaneous return [regression (5)]. *t*-statistics are provided in parentheses and are based on Hansen's (1982) heteroskedasticity and autocorrelation consistent method of moments estimation using Andrews's (1991) method of automatic bandwidth selection to identify the length of autocorrelation. $\overline{R}^2$ is the coefficient of determination, adjusted for 7 degrees of freedom.

[b]The noncrash sample excludes index observations during the periods 10/19/87–10/30/87 and 10/13/89–10/16/89.

asymmetry. Negative stock market moves are associated with changes in the volatility index that are much larger than those associated with positive stock market moves of similar size.

The regression (5) results for weekly VIX changes and S&P 100 returns are presented in Table IV. These results are similar in sign to the daily results but, for signed returns, only the contemporaneous and lag one coefficients are significant. The contemporaneous coefficient for the noncrash sample, -51.78 (t-statistic = -6.41), reveals a strong relationship between volatility and returns at the weekly level. The $\beta_{|S|}$ coefficient, 39.53 (t-statistic = 3.52), indicates a positive relationship also exists between the size of a weekly stock market move and the contemporaneous weekly change in expected stock market volatility. β_S^+ and β_S^-, as defined above, show an even more exaggerated asymmetric relationship between the direction of the stock market move and the change in expected volatility than apparent in the daily data. For the weekly regression, $\beta_S^+ = -12.25$ and $\beta_S^- = -91.31$, nearly seven times the size of β_S^+.

The documented relationship between VIX and stock index returns calls into question the reasonableness of using the Black–Scholes (constant volatility) option valuation framework. Its use, however, can be defended on two grounds. First, there is no commonly accepted alternative model. While the Cox and Ross (1976) constant elasticity of variance (CEV) model captures the inverse relationship between returns and volatility, it does not account for asymmetry in the relationship nor does it account for changing behavior in the relationship through time. Second, even if an alternative specification were available, estimation is practically impossible. Relatively error-free option pricing data are available only for at-the-money options. Estimating more than one option valuation parameter, however, requires option prices for a wide range of exercise prices.

INTRADAY AND INTRAWEEK VOLATILITY INDEX EFFECTS

Finding seasonalities in stock returns is commonplace in the empirical finance literature. Among them are intraday effects such as those documented by Wood, McInish, and Ord (1985), where return volatility appears higher at the open and the close of trading than it does mid-day, and intraweek effects such as the day-of-the-week effect first documented by Cross (1973), where average stock returns from Friday close to Monday close are negative. Since implied volatility is based on, among other things, a stock index level, it may be the case that patterns

TABLE IV

Intertemporal Relationship between Weekly CBOE Market Volatility Index Level Changes and S&P 100 Index Returns[a]

		Parameters							
Period	*Obs.*	$\hat{\alpha}$	$\hat{\beta}_{S,-2}$	$\hat{\beta}_{S,-1}$	$\hat{\beta}_{S,0}$	$\hat{\beta}_{S,+1}$	$\hat{\beta}_{S,+2}$	$\hat{\beta}_{\lvert S\rvert}$	$\bar{R}^2$
Noncrash Sample[b]									
1986–1992	351	−0.6622	9.7225	14.6725	−51.7806	2.3657	−4.6733	39.5294	0.2786
		(−3.69)	(1.86)	(1.99)	(−6.41)	(0.43)	(−0.77)	(3.52)	
1986	51	−0.5933	−0.3579	17.8419	−19.5634	−11.3825	−4.7039	43.4411	0.1437
		(−1.86)	(−0.04)	(1.80)	(−1.52)	(−1.38)	(−0.46)	(2.13)	
1987	45	−1.0030	15.2152	2.2957	−15.7150	17.4212	−8.5607	38.7154	−0.0021
		(−1.91)	(1.22)	(0.09)	(−1.48)	(1.61)	(−0.49)	(2.41)	
1988	52	−0.5671	18.4246	30.3138	−60.8558	−9.0142	−0.3278	11.2933	0.4566
		(−2.00)	(2.00)	(2.02)	(−4.75)	(−1.18)	(−0.03)	(0.75)	
1989	46	−1.7707	17.2260	13.9513	−26.5206	−17.6619	36.7919	106.4155	0.3667
		(−5.02)	(1.48)	(1.44)	(−2.22)	(−1.76)	(2.81)	(6.25)	
1990	53	−1.1013	7.6948	20.2207	−86.5321	4.5324	−0.9285	61.6884	0.5792
		(−2.23)	(0.91)	(1.48)	(−6.38)	(0.55)	(−0.09)	(2.41)	
1991	52	−0.2270	7.1066	9.8035	−83.9868	24.6817	−19.4493	25.6398	0.3100
		(−0.39)	(0.56)	(0.59)	(−4.41)	(0.98)	(−1.26)	(0.60)	
1992	52	−0.5704	−6.2643	6.5136	−61.4774	−1.8602	−7.0131	48.7456	0.4061
		(−3.11)	(−0.72)	(0.68)	(−5.25)	(−0.22)	(−0.69)	(2.61)	
Entire Sample									
1986–1992	364	−2.5666	79.2777	31.2923	−139.8820	13.5171	17.4244	143.2953	0.6934
		(−3.37)	(2.40)	(1.32)	(−5.53)	(0.89)	(1.14)	(2.97)	
1987	52	−5.5701	128.2781	45.9112	−158.3691	37.7500	8.7468	189.8963	0.8349
		(−4.05)	(3.93)	(1.47)	(−5.59)	(2.36)	(0.28)	(3.92)	
1989	52	−1.8604	9.1373	19.3003	−47.8466	−19.8697	22.6584	131.9104	0.5568
		(−5.16)	(0.89)	(2.14)	(−4.22)	(−2.35)	(1.81)	(7.17)	

[a]Regression (5) is estimated using weekly volatility index changes and weekly S&P 100 index returns. The weekly observations are selected from the closing levels on the first trading day of each week. See notes to Table III regarding parameter estimation.

[b]The noncrash sample excludes index observations during the periods 10/19/87–10/30/87 and 10/13/89–10/16/89.

also appear in the volatility index. This section investigates intraday and intraweek patterns in the volatility index.

Intraday Volatility Index Behavior

To identify intraday patterns in the volatility index, minute-by-minute volatility index levels are examined. First, the mean volatility index level is computed for each day and is used to construct minute-by-minute deviations from the daily mean. The minute-by-minute deviations are then averaged longitudinally across all days during the sample period. Figure 2 displays the average deviation by time of day.

In Figure 2, the volatility index begins the day about 10 basis points above the daily mean. The index then slowly decays until about 2:00 PM, when it quickly drops about 25 basis points below the mean. The slow decay until 2:00 may be an artifact of index construction. The number of days to expiration used to compute the VIX component implied volatilities remains constant throughout the trading day, even though the time to expiration does, in fact, decrease. To illustrate the effect this may have on the volatility index, consider the value of a

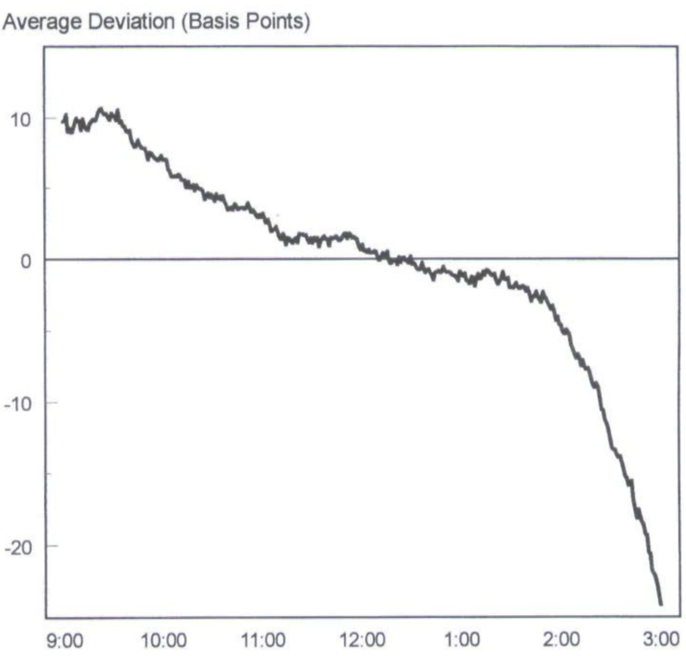

FIGURE 2.
Average intraday deviations in CBOE market volatility index levels. The average deviations from the daily mean volatility index level are plotted for each minute of the trading day. The averages are computed for the noncrash sample period from 1986 through 1992 which excludes 10/19/87–10/30/87 and 10/13/89–10/16/89.

30-day, at-the-money OEX call at 9:00 AM. If the S&P 100 index is 400, the interest rate is 6%, the dividend yield rate is 3%, and volatility is 20%—the theoretical value of an American-style call is 9.615. Now, suppose the only thing that changes between 9:00 AM and 2:00 PM is that the time to option expiration shortens by five hours. The value the call using $30 - 5/24 = 29.79$ days to expiration is 9.579. If 9.579 is the call's price, but the implied volatility is still computed using 30 days to expiration, the implied volatility becomes 19.91%. In other words, because the time to expiration parameter is not continuously adjusted during the trading day, the implied volatility must decrease to pick up the reduction in the option's time premium. Under this explanation, the reduction in the volatility index should be smooth throughout the day, as is generally shown in Figure 2 (until 2:00 PM), and should be approximately 10 basis points since the parameter values used in the illustration above closely match the average parameters realized during the sample period.

Figure 2 also shows that the volatility index drops by about 25 basis points after 2:00 PM. This, too, may be attributable to integer measurement of days to expiration. Since market makers have no control over the erosion in option time premium overnight, they may choose to mark down option prices prior to the market close on day t to reflect the number of days to expiration for the following day, $t + 1$. Using the parameters from the illustration above, this means the call option price should fall throughout the trading day from 9.615 (at 30 days) to 9.446 (at 29 days). If the call price is 9.446 at the end of the day and the number of days to expiration used is 30, the implied volatility is 19.62%, approximately 38 basis points below its true level. Not coincidentally, perhaps, the observed drop from the beginning to the end of the day in Figure 2 is about 35 basis points.

Although this illustration provides an explanation for the intraday patterns observed in the volatility index, it also appears to signal the possibility of arbitrage trading profits, particularly between 2:00 PM and 3:00 PM. But, consider the magnitude of the profits signaled in the illustration. At 3:00 PM, the option would be $30 - 6/24 = 29.75$ days from expiration and its value would be 9.573. Arbitrage profits seem to exist because, by 3:00 PM, the option appears to be valued at 9.446 with a time to expiration of 29 days. The economic magnitude of the disparity, however, is just $9.573 - 9.446 = 0.127$, almost exactly an eighth. Transaction costs, including those encountered in futures trades to complete the delta-neutral arbitrage, would seem to render these profits economically insignificant.

Despite the economic insignificance of the end-of-day decline in option prices, the decline implies a statistically significant effect on the volatility index. This significance can be shown in a regression of hourly deviations from the mean (DEV) on hourly dummy variables,

$$\mathrm{DEV}_t^{\mathcal{V}} = \sum_{i=1}^{7} \beta_i D_{i,t} + \epsilon_t \tag{6}$$

The coefficients β_i, $i = 1, \ldots, 7$, represent the average deviation from the mean daily volatility for each hour during the trading day. The regression results are reported in Table V.

Table V confirms the results depicted in Figure 2 by showing that the intraday deviations from the mean are quite persistent in a statistical sense. At 9:00 AM, for example, the coefficient estimate is 0.0972 with a t-statistic of 6.46. The volatility index starts the day about 10 basis points above the daily mean. By the end of the trading day, the volatility index is significantly below the daily mean. At 3:00 PM, the coefficient estimate is -0.2418, with a t-statistic of -14.43. The time-of-day pattern evidenced by the volatility index, then, is a systematic decline throughout the day as the time premiums of the component options erode.

It would appear that a more precise construction which accounts for the deterministic time decay could remove the intraday trend in VIX. The decay, however, is not constant throughout the day. Although the time to expiration appears to decline by a full day from 9:00 to 3:00, most of the decay occurs in the final hour. The steeper decline in the index during the final hour indicates the OEX option traders do not simply account for the overnight time decay proportionally throughout the trading day. Instead, the time erosion is not impounded in option prices until traders believe there is a high probability that they will hold a given position overnight. Because the intraday trend in VIX is not a constant function of time, correcting it with a time to expiration adjustment would prove difficult.

Intraweek Volatility Index Behavior

Intraweek behavior is displayed in Figure 3. The average closing volatility index level is computed across the days of the sample for both the *trading-day* and *calendar-day* volatility indexes. Recall from the description of VIX construction that the component implied volatilities are adjusted from calendar days to trading days to reflect the fact that total weekend volatility is approximately the same as trading day volatility. Figure 3 demonstrates what would happen if

TABLE V

Intraday Effects in CBOE Market Volatility Index Levels[a]

Period	Obs.	Coefficients (Hour of Day) 9:00	10:00	11:00	12:00	1:00	2:00	3:00	$\overline{R}^2$
Noncrash Sample[b]									
1986–1992	12,286	0.0972	0.0729	0.0292	0.0062	−0.0090	−0.0461	−0.2418	0.0457
		(6.46)	(7.38)	(4.07)	(0.82)	(−1.01)	(−4.78)	(−14.43)	
1986	1,767	0.1309	0.0682	0.0425	0.0223	−0.0343	−0.0563	−0.2894	0.1582
		(4.79)	(4.44)	(3.58)	(1.94)	(−2.66)	(−3.54)	(−11.70)	
1987	1,689	0.1674	0.1110	0.0158	−0.0270	−0.0295	−0.0954	−0.2643	0.0395
		(2.71)	(3.30)	(0.63)	(−0.84)	(−1.08)	(−2.88)	(−5.15)	
1988	1,771	0.0521	0.0408	0.0395	0.0190	−0.0120	−0.0241	−0.2395	0.0215
		(1.35)	(1.54)	(1.79)	(0.83)	(−0.51)	(−0.78)	(−3.40)	
1989	1,749	0.0670	0.0223	−0.0050	−0.0024	0.0343	0.0284	−0.1852	0.0405
		(1.95)	(1.06)	(−0.35)	(−0.16)	(1.94)	(1.27)	(−6.84)	
1990	1,768	0.0710	0.1167	0.0472	0.0278	0.0295	−0.0777	−0.3366	0.0535
		(1.49)	(3.41)	(1.92)	(1.19)	(0.75)	(−2.85)	(−7.00)	
1991	1,764	0.1204	0.0880	0.0346	−0.0037	−0.0392	−0.0585	−0.2053	0.0482
		(3.45)	(3.63)	(2.06)	(−0.21)	(−2.13)	(−2.14)	(−5.07)	
1992	1,778	0.0766	0.0644	0.0291	0.0062	−0.0123	−0.0401	−0.1723	0.0557
		(3.09)	(2.85)	(2.05)	(0.65)	(−0.97)	(−2.51)	(−6.07)	
Entire Sample									
1986–1992	12,358	0.1236	0.0734	0.0200	−0.0176	−0.0224	−0.0551	−0.2234	0.0135
		(5.24)	(5.73)	(2.10)	(−0.96)	(−1.54)	(−4.00)	(−6.53)	
1987	1,747	0.2734	0.1190	−0.0360	−0.1819	−0.1165	−0.1513	−0.1559	0.0033
		(2.48)	(1.80)	(−0.72)	(−1.51)	(−1.35)	(−2.03)	(−0.74)	
1989	1,763	0.1481	0.0177	−0.0175	−0.0144	0.0273	0.0223	−0.1643	0.0110
		(1.53)	(0.77)	(−1.05)	(−0.84)	(1.46)	(0.94)	(−3.24)	

[a]Parameter estimates are presented for the regression of hour-by-hour deviations from the daily mean volatility index level on hourly dummy variables [regression (6)]. The coefficient estimates represent the average deviation for each hour of the trading day. *t*-statistics are provided in parentheses and are based on Hansen's (1982) heteroskedasticity and autocorrelation consistent method of moments estimation using Andrews's (1991) method of automatic bandwidth selection to identify the length of autocorrelation. $\overline{R}^2$ is the coefficient of determination, adjusted for 7 degrees of freedom.

[b]The noncrash sample excludes index observations during the periods 10/19/87–10/30/87 and 10/13/89–10/16/89.

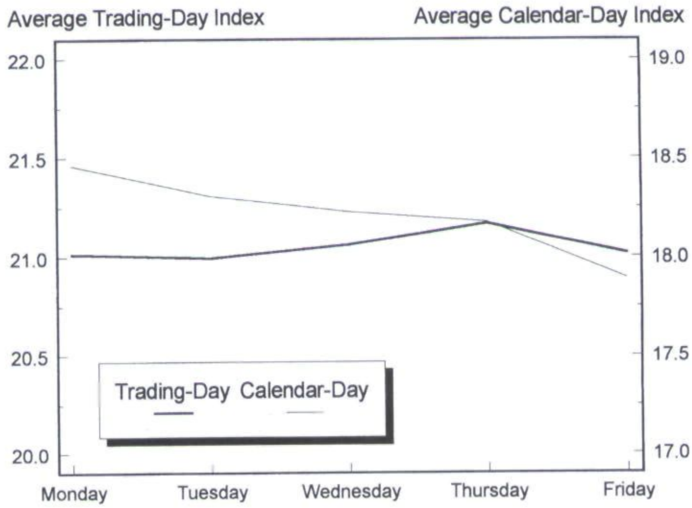

FIGURE 3.
Average intraweek CBOE market volatility index levels. The average volatility index levels computed for the sample period from 1986 through 1992 (excluding 10/19/87–10/30/87 and 10/13/89–10/16/89) are plotted for each day of the week. Curves are provided for the trading-day volatility index and the calendar-day volatility index.

no adjustment is made.[13] Note, among other things, that the scale on the right shows that calendar-day volatilities are considerably lower than trading-day volatilities.

Figure 3 shows the importance of the trading-day adjustment. When calendar days are used, the average index level starts high on Monday and systematically falls during the week. The total drop in the calendar-day index is about 70 basis points, with about half of that amount occurring on Friday. Such behavior is perplexing, since there is little economic reason to believe expected market volatility should decline throughout the week. The average trading day index, on the other hand, displays a more sensible pattern, remaining fairly stable at a level of about 21%. The average level increases slightly on Wednesday and again on Thursday; however, the total size of the increase is less than 20 basis points.

Tables VI and VII provide a closer scrutiny of the day-of-the-week effects in intraweek VIX changes shown in Figure 3. The tables report results for the regressions of VIX changes on five day-of-the-week

[13]The calendar-day volatility index is not publicly available. It was computed when VIX was designed to demonstrate the virtues of using the trading-day adjustment.

TABLE VI

Day-of-the-Week Effects in CBOE Market Volatility Index Level Changes[a]

		Coefficients (Day of Week)					
Period	*Obs.*	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*	$\overline{R}^2$
Noncrash Sample[b]							
1986–1992	1755	−0.1836	0.0598	0.0363	0.1180	−0.1357	0.0059
		(−2.44)	(1.12)	(0.66)	(1.67)	(−1.60)	
1986	252	−0.2857	0.1188	0.0165	0.2445	−0.0876	0.0419
		(−2.92)	(1.40)	(0.16)	(2.18)	(−0.77)	
1987	242	−0.0172	0.1912	−0.2322	0.0233	0.0832	−0.0089
		(−0.07)	(0.94)	(−1.33)	(0.09)	(0.32)	
1988	253	−0.4319	−0.2010	0.3904	0.0682	−0.2735	0.0101
		(−1.60)	(−1.46)	(2.19)	(0.28)	(−0.75)	
1989	249	−0.0493	−0.0316	0.0656	0.2220	−0.4294	0.0394
		(−0.41)	(−0.25)	(0.71)	(1.66)	(−2.76)	
1990	253	−0.2080	−0.0141	−0.0596	0.3814	0.0129	−0.0002
		(−0.76)	(−0.08)	(−0.36)	(1.66)	(0.06)	
1991	252	−0.1286	0.2188	−0.0410	−0.0736	−0.0538	−0.0059
		(−0.68)	(1.94)	(−0.26)	(−0.46)	(−0.25)	
1992	254	−0.1561	0.1367	0.1008	−0.0390	−0.1896	0.0158
		(−1.56)	(1.38)	(0.99)	(−0.40)	(−1.67)	
Entire Sample							
1986–1992	1769	0.1663	−0.0072	−0.1914	0.1504	−0.1208	−0.0007
		(0.48)	(−0.10)	(−0.98)	(1.30)	(−1.32)	
1987	253	2.4702	−0.3163	−1.8071	0.2524	−0.0561	0.0081
		(1.06)	(−0.77)	(−1.39)	(0.37)	(−0.21)	
1989	252	−0.1323	0.0071	0.0656	0.2220	−0.1963	−0.0008
		(−0.92)	(0.05)	(0.71)	(1.66)	(−0.71)	

[a]Parameter estimates are presented for the regression of daily volatility index changes on day-of-the-week dummy variables [regression (7)]. *t*-statistics are provided in parentheses and are based on Hansen's (1982) heteroskedasticity consistent method of moments estimation. Andrews's (1991) method of automatic bandwidth selection identifies the length of residual autocorrelation as zero. $\overline{R}^2$ is the coefficient of determination, adjusted for 5 degrees of freedom.
[b]The noncrash sample excludes index observations during the periods 10/19/87–10/30/87 and 10/13/89–10/16/89.

dummy variables,

$$\Delta \mathcal{V}_t = \sum_{i=1}^{5} \beta_i D_{i,t} + \epsilon_t \tag{7}$$

where $\Delta \mathcal{V}_t$ is the close-to-close change in volatility and $D_{i,t}$ is a dummy variable for each day of the week, Monday through Friday. Table VI contains the regression results for VIX computed on the basis of trading days and Table VII contains the results for VIX computed using calendar days.

The estimated day-of-the-week coefficients confirm the patterns illustrated in Figure 3. The Table VI results, for example, show little

TABLE VII

Day-of-the-Week Effects in Calendar-Day Volatility Index Level Changes[a]

		Coefficients (Day of Week)					
Period	*Obs.*	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*	$\overline{R}^2$
Noncrash Sample[b]							
1986–1992	1755	0.4056	−0.0406	−0.1094	−0.0442	−0.2751	0.0364
		(6.27)	(−0.85)	(−2.31)	(−0.73)	(−3.78)	
1986	252	0.2773	0.0067	−0.1112	0.0755	−0.2236	0.0525
		(3.36)	(0.09)	(−1.26)	(0.79)	(−2.29)	
1987	242	0.6704	0.0382	−0.3714	−0.1569	−0.1268	0.0458
		(3.29)	(0.22)	(−2.44)	(−0.71)	(−0.57)	
1988	253	0.3034	−0.2696	0.1698	−0.1178	−0.4167	0.0136
		(1.31)	(−2.17)	(1.11)	(−0.57)	(−1.33)	
1989	249	0.4583	−0.0892	−0.0704	0.0641	−0.5102	0.1170
		(4.46)	(−0.75)	(−0.89)	(0.56)	(−3.82)	
1990	253	0.4712	−0.1122	−0.2149	0.1610	−0.1678	0.0197
		(2.00)	(−0.69)	(−1.51)	(0.82)	(−0.87)	
1991	252	0.3867	0.0996	−0.1635	−0.1918	−0.1892	0.0289
		(2.31)	(1.00)	(−1.20)	(−1.38)	(−1.02)	
1992	254	0.2873	0.0417	−0.0173	−0.1438	−0.2816	0.0652
		(3.30)	(0.46)	(−0.20)	(−1.69)	(−2.91)	
Entire Sample							
1986–1992	1769	0.7134	−0.0980	−0.3079	−0.0181	−0.2631	0.0117
		(2.38)	(−1.49)	(−1.81)	(−0.18)	(−3.34)	
1987	253	2.8551	−0.3948	−1.7435	0.0292	−0.2545	0.0223
		(1.43)	(−1.11)	(−1.55)	(0.05)	(−1.11)	
1989	252	0.3823	−0.0573	−0.0704	0.0641	−0.3067	0.0274
		(3.05)	(−0.47)	(−0.89)	(0.56)	(−1.28)	

[a]The calendar-day volatility index level is computed using the same procedure used to compute VIX, but without the adjustment to trading days. Parameter estimates are presented for the regression of daily calendar-day volatility index changes on day-of-the-week dummy variables [regression (7)]. See notes to Table VI regarding estimation of the standard errors. $\overline{R}^2$ is the coefficient of determination, adjusted for 5 degrees of freedom.

[b]The noncrash sample excludes index observations during the periods 10/19/87–10/30/87 and 10/13/89–10/16/89.

change in the volatility index from day to day. Only the Monday dummy variable is significant in the overall noncrash sample, with the volatility index falling by less than 20 basis points from Friday close to Monday close. On the other hand, the results for the volatility index using calendar days (Table VII) show pronounced seasonality. Volatility increases significantly from Friday to Monday and decreases significantly from Tuesday to Wednesday and Thursday to Friday. On the basis of these results, the trading-day adjustment to implied volatility is clearly warranted and it removes almost all of the intraweek seasonality that is seen without the adjustment.

Finally, the influence of stock index futures and option expiration days is worth noting. A great deal of public controversy has been focused on the volatility of stock index returns on "triple-witching" days when stock index futures, options, and futures options expire. Stoll and Whaley (1987, 1991), for example, document increased volatility on the days surrounding expirations. To investigate the magnitude of expiration-day effects on VIX, regression (7) is estimated with additional dummy variables to identify the expiration day, the day before expiration, and two days before expiration.[14] Table VIII reports the regression results. While the estimations vary considerably from year to year, the overall noncrash sample indicates that VIX falls by 45 basis points the day before expiration and by 49 basis points on expiration day. One interpretation of these results is that index futures and option expiration days are truly high volatility days. Since VIX represents the average volatility rate over the next 30 days, it should not be surprising that the average rate falls as the high volatility rate of an expiration day is dropped from the average and is replaced by the volatility rate of a typical day. This evidence is consistent with Day and Lewis (1988).

THE VOLATILITY INDEX AS A FORECAST OF STOCK MARKET VOLATILITY

The previous two sections focus on characterizing the movements of VIX. This section evaluates how well VIX forecasts subsequent, realized volatility. This interpretation of the volatility index may appear, at first, inconsistent with the option valuation model which determines VIX. In particular, the Black–Scholes/Merton framework assumes a known and constant volatility rate. The true volatility rate, on the other hand, is unknown and time-varies. This contradiction requires that the implied volatility be related to some notion of an expected average volatility. For example, for the volatility processes described in Hull and White (1987) (the volatility risk premium is either zero or constant), Feinstein (1989) demonstrates that the implied volatility approximates the market expectation of the average volatility over the life of the option.[15] The level of VIX, then, because it is based on the implied volatilities of OEX options, represents an estimate of expected S&P 100 index (or "stock market") volatility.

The analysis of VIX as a forecast of stock market volatility proceeds in two steps. The first step is the consideration of the bias in VIX

[14]Post-expiration dummy variables were also considered in the expiration-day regression but were insignificant and are not presented.

[15]Feinstein (1989) showed that the approximation is most accurate for at-the-money and near-expiration options.

TABLE VIII

Expiration Day Effects in Volatility Index Level Changes[a]

		Coefficients (Day of Week and Expiration)								
							Expiration Day			
Period	*Obs.*	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*	−2	−1	0	$\overline{R}^2$
Noncrash Sample[b]										
1986–1992	1755	−0.1836	0.0603	0.0579	0.2263	−0.0212	−0.0837	−0.4543	−0.4856	0.0141
		(−2.44)	(1.13)	(0.96)	(2.95)	(−0.24)	(−0.58)	(−2.63)	(−2.18)	
1986	252	−0.2857	0.1188	0.1138	0.3278	0.0013	−0.4213	−0.3403	−0.3705	0.0588
		(−2.92)	(1.40)	(1.14)	(2.37)	(0.01)	(−1.45)	(−1.72)	(−1.75)	
1987	242	−0.0172	0.1831	−0.3266	−0.0305	0.0741	0.4074	0.2360	0.0388	−0.0183
		(−0.07)	(0.91)	(−1.72)	(−0.10)	(0.36)	(0.81)	(0.48)	(0.05)	
1988	253	−0.4319	−0.2010	0.4278	0.1985	0.0433	−0.1619	−0.5535	−1.3467	0.0228
		(−1.60)	(−1.46)	(2.03)	(0.74)	(0.10)	(−0.43)	(−0.93)	(−1.82)	
1989	249	−0.0493	−0.0316	0.1398	0.3821	−0.3789	−0.3214	−0.6804	−0.2102	0.0546
		(−0.41)	(−0.25)	(1.40)	(3.27)	(−2.46)	(−1.39)	(−1.68)	(−0.46)	
1990	253	−0.2080	−0.0141	−0.0449	0.6226	0.2241	−0.0626	−1.0251	−0.8974	0.0162
		(−0.76)	(−0.08)	(−0.23)	(2.64)	(0.86)	(−0.19)	(−1.73)	(−1.97)	
1991	252	−0.1286	0.2188	0.0562	0.0892	−0.0539	−0.4128	−0.6784	0.0006	−0.0023
		(−0.68)	(1.94)	(0.34)	(0.65)	(−0.36)	(−0.96)	(−1.38)	(0.00)	
1992	254	−0.1561	0.1301	0.0307	0.0044	−0.0627	0.3452	−0.1541	−0.5654	0.0363
		(−1.56)	(1.31)	(0.34)	(0.04)	(−0.53)	(1.14)	(−0.86)	(−2.28)	
Entire Sample										
1986–1992	1769	0.1663	−0.0083	−0.2349	0.2654	−0.0030	0.2038	−0.4855	−0.5043	−0.0008
		(0.48)	(−0.11)	(−0.95)	(1.88)	(−0.03)	(0.74)	(−2.33)	(−2.22)	
1987	253	2.4702	−0.3596	−2.2863	0.2105	−0.0951	2.2491	0.1784	0.1738	−0.0015
		(1.06)	(−0.88)	(−1.43)	(0.25)	(−0.41)	(1.47)	(0.20)	(0.21)	
1989	252	−0.1323	0.0071	0.1397	0.3821	−0.0754	−0.3214	−0.6804	−0.5138	0.0085
		(−0.92)	(0.05)	(1.40)	(3.27)	(−0.23)	(−1.39)	(−1.68)	(−0.95)	

[a]Parameter estimates are presented for a regression of daily volatility index changes on day-of-the-week and expiration day dummy variables. The expiration day dummy variables identify the expiration day (0), the day before expiration (−1), and two days before expiration (−2). See notes to Table VI regarding estimation of standard errors. $\overline{R}^2$ is the coefficient of determination, adjusted for 8 degrees of freedom.

[b]The noncrash sample excludes index observations during the periods 10/19/87–10/30/87 and 10/13/89–10/16/89.

as a forecast, that is, the degree to which VIX is below or above the subsequent realizations. Unbiasedness, while desirable, is not a crucial property since forecast bias can be corrected if the degree of bias is known or can be estimated. The second step is the examination of the orthogonality of the VIX forecast error with other available information. If VIX incorporates all available information, then additional parameters will not improve the forecasting of future volatility. In this section, the analysis of the relationship between the volatility index and future stock market volatility uses the time-series methodology developed in Fleming (1994).

The volatility index is constructed for a constant 30-calendar-day horizon (22 trading days) and, therefore, should represent the expectation of stock market volatility during the following month. The unbiasedness and orthogonality tests, then, require an estimate of the volatility realized during the following month. In this section, the estimation of realized volatility is based on a constant 28-calendar-day horizon which closely corresponds to the 30-calendar-day interval on which VIX is based. At the same time, a 28-day realized volatility avoids the measurement problems attributable to intraweek and weekend effects in stock market returns because a 28-day horizon nearly always includes an equal number of stock returns from each day of the week. Specifically, for both unbiasedness and orthogonality tests, the realized volatility rate is given by

$$\mathcal{V}_t^* = \left[\left(\frac{19}{2} \right)^{\frac{1}{2}} \frac{\Gamma\left(\frac{19}{2} \right)}{\Gamma\left(\frac{20}{2} \right)} \right] \left(\widehat{\mathcal{V}^{*2}}_t \right)^{\frac{1}{2}} \tag{8}$$

where

$$\widehat{\mathcal{V}^{*2}}_t = \frac{1}{19} \sum_{i=1}^{20} \left[\ln\left(\frac{S_{t+i}}{S_{t+i-1}} \right) - \overline{R}_t \right]^2$$

is the sample variance and $\overline{R}_t$ is the mean S&P 100 index return over the interval from t to $t + 20$.[16] The term in the squared brackets of (8) corrects for the bias attributable to using the square root of the sample variance as an estimator for the standard deviation.[17] With a constant

[16] $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx$ is the gamma function.

[17] This correction factor is identical to the correction provided in Cox and Rubinstein (1985, p. 256), and is based on the assumption that the stock index level is lognormally distributed. For the results provided below, the effect of the correction is fairly small. For example, for a volatility based on 20 returns, the correction factor is 1.013.

28-calendar-day interval, 20 daily returns are involved in estimating $\mathcal{V}_t^*$; this explains the constants 20 (N) and 19 ($N - 1$).[18]

In conducting the unbiasedness and orthogonality tests, an AR(1) model of volatility will serve as the standard for evaluating the forecast performance of VIX. Poterba and Summers (1986) and Stein and Stein (1991), for example, argue that volatility follows an AR(1). In addition, Canina and Figlewski (1993) suggest that an AR(1) specification using the historical volatility rate dominates the OEX implied volatility as a forecast of realized volatility. The historical volatility rate, defined as $\mathcal{V}_{h,t} = \mathcal{V}_{t-20}^*$, is the realized volatility over the preceding 28 calendar days (20 trading days). In terms of notation, $\mathcal{V}_{i,t}$ will be used to denote either VIX (i.e., i = VIX) or historical volatility (i.e., $i = h$).

Unbiasedness Tests

The unbiasedness of a proxy such as $\mathcal{V}_{i,t}$ for, say $\mathcal{V}_t^*$, is frequently assessed with the regression,

$$\mathcal{V}_t^* = \alpha + \beta \mathcal{V}_{i,t} + \epsilon_t \tag{9}$$

Unbiasedness requires that the estimates of α and β be indistinguishable from zero and one, respectively. The volatility index and the computed realized and historical volatility levels, however, are near random walk processes. As a result, eq. (9), specified in volatility levels, potentially represents a spurious regression in finite samples. Subtracting the lagged independent variable from both sides of the regression provides a testable relationship,

$$\mathcal{V}_t^* - \mathcal{V}_{i,t-1} = \alpha + \beta(\mathcal{V}_{i,t} - \mathcal{V}_{i,t-1}) + \epsilon_t \tag{10}$$

that is free of the spurious regression problem. To account for possible residual heteroskedasticity and autocorrelation, Hansen's (1982) generalized method of moments (GMM) is used to estimate the vector,

$$\frac{1}{T}\sum_{t=1}^{T}(\mathcal{V}_t^* - \mathcal{V}_{i,t-1} - \alpha - \beta(\mathcal{V}_{i,t} - \mathcal{V}_{i,t-1}))(1, \mathcal{V}_{i,t} - \mathcal{V}_{i,t-1})' \tag{11}$$

Since vector (11) involves two equations and two unknowns, the estimates of α and β are identically equal to those in eq. (10). GMM estimation of eq. (11), however, provides consistent covariance matrix

[18] $N = 20$ is used for expositional convenience. During intervals in which holidays exist, N would be less than 20. The empirical results are based on the actual number of trading days in the 28-calendar-day interval.

estimation and facilitates a framework for conducting the orthogonality tests. Parzen weights, with a lag length of 19 (the length of the overlap between consecutive daily volatilities), are used to estimate the covariance matrix.

The first two lines for each sample in Table IX summarize the unbiasedness regression results for the volatility index and the historical volatility. The CS^2-statistics provide a Wald test of the joint restrictions on α and β,

$$CS^2 = \begin{pmatrix} \hat{\alpha} \\ 1 - \hat{\beta} \end{pmatrix}' \hat{\Omega}^{-1} \begin{pmatrix} \hat{\alpha} \\ 1 - \hat{\beta} \end{pmatrix} \overset{a}{\sim} \chi_2^2 \tag{12}$$

TABLE IX

Relationship between the CBOE Market Volatility Index and Future S&P 100 Index Volatility[a]

	Noncrash Sample (1679 Days)[b]						
	α	t_α	β	$t_{1-\beta}$	$\overline{R}^2$	CS^2	OI^2
Unbiasedness Tests[c]							
Volatility index	−0.0584	−15.19	0.6283	4.30	0.0315	291.00	—
Historical volatility	−0.0028	−0.65	0.6314	2.25	0.0186	6.38	—
Orthogonality Tests[d]							
Volatility index	0.0148	0.98	0.6483	4.51	0.4530	323.99	0.24
Historical volatility	0.0710	3.93	0.4802	4.10	0.2738	17.11	17.87

	Entire Sample (1770 Days)						
	α	t_α	β	$t_{1-\beta}$	$\overline{R}^2$	CS^2	OI^2
Unbiasedness Tests[c]							
Volatility index	−0.0513	−5.58	0.6842	5.51	0.0449	188.53	—
Historical volatility	−0.0009	−0.09	0.8928	0.48	0.0215	0.27	—
Orthogonality Tests[d]							
Volatility index	0.0536	3.37	0.5047	6.68	0.1519	93.11	0.05
Historical volatility	0.0796	6.91	0.4931	6.79	0.0815	56.96	6.42

[a]The table considers the unbiasedness and orthogonality of VIX and the 28-day historical volatility as a forecast of the S&P 100 volatility realized over the following 28 calendar days. For each set of results, *t*-statistics report the significance of departures from zero and one, respectively, for α and β. The *t*-statistics are based on Hansen's (1982) method of moments estimation of the covariance matrix, using Parzen weights and a residual autoregressive lag length of 19. $\overline{R}^2$ is the coefficient of determination, adjusted for 2 degrees of freedom.

[b]The noncrash sample excludes index observations during the periods 9/21/87–11/30/87 and 9/15/89–11/10/89.

[c]The unbiasedness tests involve estimation of regression (11). The unbiasedness test statistic is CS^2, which is a Wald test that α and β, respectively, are nondifferent from zero and one. Under the null, CS^2 is asymtotically distributed χ_2^2.

[d]The orthogonality tests are based on regression (13), using the historical volatility as an instrument in the VIX regression, and using VIX as an instrument in the historical volatility regression. The orthogonality test statistic, OI^2, is the Hansen (1982) test of overidentifying restrictions and is asymptotically distributed χ_1^2 under the null of orthogonality.

where $\hat{\Omega}$ is the GMM estimate of the covariance matrix of the parameter estimates, $\hat{\alpha}$ and $\hat{\beta}$. On the basis of the CS^2-statistics, the unbiasedness of the volatility index as a forecast of realized S&P 100 volatility is strongly rejected. In fact, the average difference between the volatility index and the subsequent, realized stock market volatility is 584 basis points. The historical volatility incurs a weaker, but significant, unbiasedness rejection. Its unconditional bias is just 28 basis points. Of course, the difference in means between the historical and realized measures is low by construction. Since the two samples are simply offset by 20 trading days, the difference in means is confined to the effect of differences between the first 20 historical volatility observations and the last 20 realized volatility observations.

Several factors may contribute to the forecast bias of the volatility index. First, the option pricing model may be misspecified. This may produce systematic bias in implied volatility levels while still producing an estimate that is highly correlated with the true implied volatility. Second, the estimation of the component implied volatilities ignores the wildcard option imbedded in the OEX option. Ignoring the wildcard option biases the level of VIX upward. Fleming and Whaley (1994) estimate that the wildcard option imbedded in an at-the-money OEX option may bias implied volatility, on average, by about 60 basis points. Third, the calendar-day to trading-day adjustment employed to account for weekend volatility may overcompensate for lower weekend volatility. The absence of day-of-the-week effects in the trading day index, however, indicates that the magnitude of this adjustment is appropriate. Fourth, the infrequent trading of S&P 100 stocks may bias downward the realized volatility estimate. The low autocorrelation in daily S&P 100 returns reported in Table I, however, indicates that this would not account for a large share of the bias in VIX. Finally, a portion of the upward bias in VIX may result from the over-pricing of OEX options. This finding is consistent with the evidence reported in Longstaff (1993), for OEX call options, and Fleming (1994), for both calls and puts. Fleming, however, also found that the degree of forecast bias in OEX implied volatilities is not large enough to generate arbitrage trading profits after transaction costs.

While the forecast bias of the volatility index is strong, it is not problematic if the bias is constant and/or its magnitude is known. Given the constant 30-calendar-day horizon of the index, the bias may be reasonably constant. Below, a bias-adjusted VIX which assumes constancy is evaluated as a forecast. The more interesting result of the unbiasedness tests, though, involves the explanatory power of the volatility index relative to historical volatility. The $\overline{R}^2$ statistics reported in Table IX indicate that VIX provides a much stronger relationship with realized volatility than does historical volatility. The orthogonality tests below confirm this evidence.

Orthogonality Tests

In this section, the orthogonality of the volatility index and historical volatility are measured relative to one another. For example, the orthogonality of the volatility index would be rejected if its forecast error covaries with historical volatility. If this were the case, historical volatility could be used to supplement VIX to provide a better composite forecast of realized volatility. Orthogonality is assessed with GMM estimation of the vector,

$$\frac{1}{T}\sum_{t=1}^{T}(\mathcal{V}_t^* - \alpha - \beta\mathcal{V}_{i,t})(1, \mathcal{V}_{i,t}, \mathcal{V}_{-i,t})' \tag{13}$$

where $\mathcal{V}_{-i,t}$ represents the alternative forecast to $\mathcal{V}_{i,t}$. Recall that eq. (11), for the unbiasedness test, is specified in differences as a precaution against the spurious regression problem. Inference regarding orthogonality using eq. (13), however, is unaffected by the spurious regression problem, as is documented in Fleming (1994). Since eq. (13) remains specified in levels, Table IX provides the results of the basic regression stated in both levels (orthogonality) and differences (unbiasedness).

Unlike eq. (11), the system in (13) is overidentified with three equations and two unknowns and cannot be set equal to zero. The closeness of the solution to zero, however, directly reflects the orthogonality of forecast $\mathcal{V}_{i,t}$ to the alternative forecast, $\mathcal{V}_{-i,t}$. The Hansen (1982) test of overidentifying restrictions measures the closeness to zero and provides a formal orthogonality test. In Table IX, this test statistic is labeled OI^2 and is asymptotically distributed χ_1^2 under the null hypothesis of orthogonality.

The final two lines for each sample in Table IX summarize the orthogonality tests. As with the direct unbiasedness tests, unbiasedness is again rejected (on the basis of the CS^2-statistics) for both the volatility index and the historical volatility. Again, the $\overline{R}^2$ statistics indicate that the explanatory power of the volatility index dominates that of the historical volatility. The orthogonality test results provide further confirmation of this evidence. Orthogonality of the volatility index as a forecast of realized stock market volatility is not rejected ($OI^2 = 0.24$) when the historical volatility is available in the specification. However, when the test is reversed and the volatility index is included as an instrument in the estimation of the historical volatility system, strong orthogonality rejections result ($OI^2 = 17.87$). On the basis of this comparison, the volatility index dominates the historical volatility as a forecast of stock market volatility.

The results in Table IX are consistent with the recent literature regarding the forecast quality of OEX implied volatilities. Fleming (1994) provided a more complete set of orthogonality tests using the implied volatility of at-the-money, nearby expiration OEX options. Those tests included instruments such as stock market returns, interest rate variables, and the ARCH-based parameters. While several instruments induced orthogonality rejections for the historical volatility, none produced rejection for the implied volatility. Day and Lewis (1992) provided a more direct comparison between OEX implied volatilities and weekly GARCH and EGARCH forecasts, finding that the implied volatility contributes information incremental to that contained in GARCH-based forecasts. They also found, however, that in-sample GARCH forecasts contribute information incremental to the implied volatility. Out-of-sample tests, though, provided inconclusive orthogonality evidence. Given these results and the fact that VIX is a function of the component OEX implied volatilities, the results reported in Table IX are not surprising.

Bias-Adjusted Volatility Index Forecasts

The evidence of the volatility index's explanatory power and orthogonality demonstrate that the index represents a quality forecast of future stock market volatility, provided its upward bias is constant or estimable. Two factors suggest that the VIX forecast bias may be reasonably well behaved. First, VIX represents the interpolation of eight OEX implied volatilities to maintain a constant moneyness level and a constant 30-calendar-day horizon. Because these parameters are fixed, VIX does not incur the usual time-variation in implied volatilities resulting from moneyness and time-to-expiration effects. Second, a portion of the VIX forecast bias is attributable to the OEX wildcard option which is ignored in computing the VIX component implied volatilities. Fleming and Whaley (1994) show that the wildcard's value is approximately linear in moneyness and time to expiration. Since VIX is always constructed from options straddling the money and straddling 30 days to expiration, and with an equal weight on calls and puts, the wildcard's effect on the VIX forecast bias should remain fairly constant.

To the extent the VIX forecast bias is relatively constant, a naive adjustment based on a rolling average of past forecast errors may sufficiently correct the bias. The forecast performance of two naive adjustments is considered below. A 253-day bias adjustment is computed by averaging the most recently realized year (or 253 trading days) of daily forecast biases; a 20-day bias adjustment uses the most recent month of daily biases. Because these adjustments are made using only information available at the time of the VIX observation, the *bias-adjusted* VIX levels represent out-of-sample forecasts of stock market volatility.[19]

Table X summarizes the forecast performance of the bias-adjusted VIX, as well as the unadjusted VIX and historical volatility over the same sample period. The unadjusted VIX and historical volatility results provide a control and are not qualitatively different from those reported in Table IX. The bias-adjusted VIX results, on the other hand, reflect evidence regarding the constancy of the VIX forecast bias. For each adjustment, the unconditional forecast bias (α) is just 4 basis points. This measure of bias is tautological, however, since the mean error of a rolling average adjusted VIX goes to zero, by construction, as the sample size increases.

The more meaningful assessment of the bias adjustments concerns the reaction of the $\overline{R}^2$ and the conditional unbiasedness statistic, CS^2. For the 20-day adjustment, for example, the CS^2-statistic falls from 252.25 to 8.97, but the explanatory power of VIX is reduced from 0.0356 to 0.0250. Although the 20-day adjustment removes bias, it entails a dramatic loss of information regarding future volatility. The lower (but, still significant) CS^2-statistic of 15.56 indicates that the 253-day adjustment also removes a large degree of the bias. Unlike the 20-day adjustment, however, the 253-day adjustment retains the explanatory power of VIX ($\overline{R}^2 = 0.0341$). The performance of the 253-day bias adjustment, which is a fairly constant adjustment since it is based on a long history of past biases, suggests the forecast bias of VIX is also fairly constant. The 253-day-bias-adjusted volatility index behaves similar to the unadjusted VIX, but with a lower magnitude of forecast bias.

[19]The VIX forecast at time t is of the volatility realized over the next 20 trading days (i.e., $t+1, \ldots, t+20$). The most recent past forecast bias available at time t, then, is from the VIX forecast at $t-20$. The 253-day bias adjustment, for example, averages the biases for the 253 trading days ending at $t-20$.

TABLE X

Relationship between the Bias-Adjusted CBOE Market Volatility Index and Future S&P 100 Index Volatility[a]

	Noncrash Sample (1407 Days)[b]					
	α	t_α	β	$t_{1-\beta}$	$\overline{R}^2$	CS^2
Unbiasedness Tests[c]						
Unadjusted volatility index	−0.0601	−14.26	0.6327	4.12	0.0356	252.25
Historical volatility	−0.0032	−0.65	0.6435	1.90	0.0187	4.73
253-day-bias-adjusted volatility index	0.0004	0.09	0.6389	3.89	0.0341	15.56
20-day-bias-adjusted volatility index	0.0004	0.08	0.6184	3.00	0.0250	8.97
	Entire Sample (1498 Days)					
	α	t_α	β	$t_{1-\beta}$	$\overline{R}^2$	CS^2
Unbiasedness Tests						
Unadjusted volatility index	−0.0515	−4.80	0.6856	5.37	0.0456	157.01
Historical volatility	−0.0009	−0.07	0.9068	0.41	0.0216	0.19
253-day-bias-adjusted volatility index	−0.0002	−0.02	0.6887	5.27	0.0431	51.47
20-day-bias-adjusted volatility index	0.0001	0.00	0.6919	1.76	0.0282	3.32

[a]The table considers the out-of-sample performance of two bias-adjusted volatility indexes as a forecast of the realized 28-day S&P 100 volatility. The bias-adjusted index is constructed by subtracting the average of the most recently realized 253 (or 20) daily forecast biases from the current VIX level. See notes to Table IX regarding estimation of standard errors.
[b]The noncrash sample (and the bias adjustment during this sample) excludes index observations during the periods 9/21/87–11/30/87 and 9/15/89–11/10/89.
[c]See Table IX notes for an explanation of the unbiasedness tests and the definition of CS^2.

SUMMARY

The CBOE Market Volatility Index (VIX) offers the market place and academic researchers a new measure of stock market volatility. This study describes how the index is constructed from the implied volatilities of eight S&P 100 index options, and then assesses its behavior over a seven-year period. Univariate time-series properties and seasonalities of the index are documented as well as the temporal relationship between the volatility index and stock market returns. Finally, the performance of VIX as a forecast of future stock market volatility is evaluated.

The time-series description shows that daily and weekly VIX changes are reasonably well behaved. The first-order autocorrelation of daily changes is significantly negative, although its magnitude is small (−0.07) and much lower than is observed for individual OEX implied volatilities. For weekly changes, the negative first-order autocorrelation is more pronounced (−0.26), perhaps reflecting mean reversion in the index over this longer interval of time.

The investigation of seasonalities reveals a slight intraday decline in the volatility index, with the index then falling sharply at the end of the day. This downward trend appears to be induced in the volatility index by assuming a constant time to expiration throughout the trading day while OEX option traders appear to price intraday time decay. The magnitude of the trend is about 35 basis points. However, it does not occur at a constant rate throughout the day. The analysis of intraweek patterns reveals no seasonalities in VIX, with the index trading within a range of about 20 basis points throughout the week. These descriptions of the time-series properties of the volatility index should be useful to researchers and practitioners in developing methods for valuing derivatives on the volatility index.

The volatility index also exhibits a strong temporal relationship with stock market returns. A large negative contemporaneous correlation exists between VIX changes and S&P 100 index returns, suggesting an inverse relationship between expected volatility and stock market prices. Furthermore, the relationship shows significant asymmetry. *Negative* stock market moves are accompanied by larger absolute changes in expected volatility than are *positive* market moves. VIX, to a much lesser extent, also exhibits a positive association with noncontemporaneous (both lag and lead) stock market returns.

Finally, VIX performs well as a volatility forecast, demonstrating a strong relationship with future realized stock market volatility. While the index is not *unbiased,* it is a significantly more accurate proxy for expected stock market volatility than is the commonly used first-order autoregressive volatility model. Because it imbeds market expectations, VIX appears to be a useful instrument for forecasting volatility.

BIBLIOGRAPHY

Andrews, D. W. K. (1991): "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometrica,* 59:817–858.

Black, F. (1976): "Studies of Stock Price Volatility Changes," *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economics Section,* pp. 177–181.

Black, F. (1992): "Living Up to the Model," in *From Black–Scholes to Black Holes: New Frontiers in Options,* London: Risk Magazine Ltd.

Black, F., and Scholes, M. (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* 81:637–659.

Bollerslev, T., Engle, R. F., and Wooldridge, J. M. (1988): "A Capital Asset Pricing Model with Time Varying Covariances," *Journal of Political Economy,* 96:116–131.

Canina, L., and Figlewski, S. (1993): "The Informational Content of Implied Volatility," *Review of Financial Studies,* 6:659–681.

Christie, A.A. (1982): "The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects," *Journal of Financial Economics,* 10:407–432.

Cox, J.C., and Ross, S.A. (1976): "The Valuation of Options for Alternative Stochastic Processes," *Journal of Financial Economics,* 3:145–166.

Cox, J.C., and Rubinstein, M. (1985): *Options Markets.* Englewood Cliffs, NJ: Prentice-Hall.

Cross, J. (1973): "The Behavior of Stock Prices on Friday and Monday," *Financial Analysts Journal,* 29:67–69.

Day, T.E., and Lewis, C.M. (1988): "The Behavior of the Volatility Implicit in the Prices of Stock Index Options," *Journal of Financial Economics,* 22:103–122.

Day, T.E., and Lewis, C.M. (1992): "Stock Market Volatility and the Information Content of Stock Index Options," *Journal of Econometrics,* 52:267–287.

Dumas, B., Fleming, J., and Whaley, R.E. (1994): "Implied Volatility Smiles: Empirical Tests," Working Paper, The Fuqua School of Business, Duke University.

Feinstein, S. (1989): "The Black–Scholes Formula is Nearly Linear in σ for At-the-Money Options; Therefore Implied Volatilities from At-the-Money Options are Virtually Unbiased," Working Paper, Federal Reserve Bank of Atlanta.

Fleming, J. (1994): "The Quality of Market Volatility Forecasts Implied by S&P 100 Index Option Prices," Working Paper, Jones Graduate School of Administration, Rice University.

Fleming, J., and Whaley, R.E. (1994): "The Value of Wildcard Options," *Journal of Finance,* 49:215–236.

French, K.R., and Roll, R. (1986): "Stock Return Variances: The Arrival of Information and the Reaction of Traders," *Journal of Financial Economics,* 17:5–26.

French, K.R., Schwert, G.W., and Stambaugh, R.F. (1987): "Expected Stock Returns and Volatility," *Journal of Financial Economics,* 19:3–29.

Gallant, A.R. (1987): *Nonlinear Statistical Models.* New York: Wiley.

Gastineau, G.L. (1977): "An Index of Listed Option Premiums," *Financial Analysts Journal,* 30:70–75.

Hansen, L.P. (1982): "Large Sample Properties of Generalized Method of Moment Estimators," *Econometrica,* 50:1029–1054.

Harvey, C.R., and Whaley, R.E. (1991): "S&P 100 Index Option Volatility," *Journal of Finance,* 46:1551–1561.

Harvey, C.R., and Whaley, R.E. (1992a): "Dividends and S&P 100 Index Option Valuation," *The Journal of Futures Markets,* 12:123–137.

Harvey, C.R., and Whaley, R.E. (1992b): "Market Volatility Prediction and the Efficiency of the S&P 100 Index Option Market," *Journal of Financial Economics,* 30:33–73.

Hull, J., and White, A. (1987): "The Pricing of Options on Assets with Stochastic Volatilities," *Journal of Finance,* 42:281–300.

Lintner, J. (1965): "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics,* 47:13–37.

Longstaff, F. A. (1993): "Martingale Restriction Tests of Option Pricing Models," Working Paper, Anderson Graduate School of Management, UCLA.

Merton, R. C. (1973a): "An Intertemporal Capital Asset Pricing Model," *Econometrica,* 41:867–888.

Merton, R. C. (1973b): "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science,* 4:141–183.

Nelson, D. B. (1989): "Modeling Stock Market Volatility Changes," *Proceedings of the 1989 Meetings of the American Statistical Association, Business and Economics Section,* pp. 93–98.

Nelson, D. B. (1991): "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica,* 59:347–370.

Poterba, J. M., and Summers, L. H. (1986): "The Persistence of Volatility and Stock Market Fluctuations," *American Economic Review,* 76:1142–1151.

Schwert, G. W. (1989): "Why Does Stock Market Volatility Change over Time?" *Journal of Finance,* 44:1115–1154.

Schwert, G. W. (1990): "Stock Volatility and the Crash of '87," *Review of Financial Studies,* 3:77–102.

Sharpe, W. F. (1964): "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance,* 19:425–442.

Stein, E. M., and Stein, J. C. (1991): "Stock Price Distributions with Stochastic Volatility: An Analytic Approach," *Review of Financial Studies,* 4:727–752.

Stoll, H. R., and Whaley, R. E. (1987): "Program Trading and Expiration-Day Effects," *Financial Analysts Journal,* 43:16–28.

Stoll, H. R., and Whaley, R. E. (1991): "Expiration-Day Effects: What Has Changed?" *Financial Analysts Journal,* 47:58–72.

Whaley, R. E. (1993): "Derivatives on Market Volatility: Hedging Tools Long Overdue," *Journal of Derivatives,* 1:71–84.

Wood, R., McInish, T., and Ord, J. K. (1985): "An Investigation of Transaction Data for NYSE Stocks," *Journal of Finance,* 40:723–739.