THE WELFARE COSTS OF *ARKANSAS BEST*: THE INEFFICIENCY OF ASYMMETRIC TAXATION OF HEDGING GAINS AND LOSSES

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INTRODUCTION

Internal Revenue Service policy regarding the taxation of hedging gains and losses has changed dramatically over the past five years. Prior to the Supreme Court's 1988 decision in the *Arkansas Best* case, the IRS taxed gains and losses on corporate hedge transactions symmetrically. If the item hedged generated an ordinary gain or loss, the income on the hedge position was ordinary as well. For example, a firm hedging interest expense through the sale of Eurodollar futures could aggregate the gains and losses on the futures position with the actual interest paid and other ordinary income items to determine its tax liability. Thus, if interest rates rose, the increase in interest expense would largely off-set the effect of the futures position's profit on the firm's taxes.

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In 1990, after the *Arkansas Best* case, the IRS ruled that gains and losses must be taxed asymmetrically. In other words, gains and losses on a hedged position (e.g., Eurodollar futures) were taxed as capital income even if the item hedged (e.g., interest expense) was ordinary. In 1993, the IRS changed policy again restoring symmetric treatment to hedge taxation.

This article demonstrates that asymmetric taxation is inefficient because it introduces a new source of deadweight loss. Specifically, after-tax profits are riskier under asymmetric taxation than symmetric taxation. Since many firms hedge precisely *because* they are averse to risk in their after-tax income, the claimants and customers of these firms bear higher costs under asymmetric taxation. These costs result from higher probabilities of financial distress, less efficient capital structures, poorer risk sharing, and more acute agency problems. Because of these distortions, symmetric taxation is more efficient than asymmetric taxation. That is, given a symmetric corporate tax rate and an asymmetric rate that generate identical expected corporate tax revenues, aggregate after-tax risk-adjusted profits are higher in the symmetric tax regime. Thus, the *Arkansas Best*-based tax policy was less efficient than the symmetric hedge taxation policy.

The size of the welfare loss resulting from asymmetric taxation depends upon the motivations for hedging. This article demonstrates that firms are risk neutral with respect to after-tax income may hedge nonetheless to minimize taxes. Asymmetric taxation eliminates this form of hedging, but this has no efficiency implications because it merely results in a transfer of wealth from the hedging firms to the government (i.e., to other taxpayers).

Other firms hedge to reduce agency and transactions costs. Such hedges enhance efficiency. The firms that hedge for these purposes are averse to after-tax income risk. Such firms incur higher costs, on average, when hedging gains and losses are taxed asymmetrically. Thus, to the extent that firms hedge to reduce resource costs (rather than tax costs), asymmetric taxation can cause appreciable welfare losses. The theory derived herein implies that to raise a given amount of taxes from firms, even if some hedges are tax motivated, it would be preferable to charge a higher corporate tax rate and tax all hedges symmetrically rather than tax hedges asymmetrically at a lower rate.

The following section presents a simple model of hedging with two types of firms—firms that are neutral to after-tax profit risk, and firms that are averse to such risk. Four main results are demonstrated.

A MODEL OF THE WELFARE EFFECTS OF HEDGE TAXATION

Consider an economy with two types of firms. The "type 1" firms are neutral to risk in their after-tax income. That is, their objective functions are linear in after-tax income. These firms maximize expected after-tax income. In contrast, the "type 2" firms are strictly risk averse in after-tax income. Their preferences are concave in after-tax income and they maximize their expected utility. For simplicity and without loss of generality, assume that the type 2 firms have identical utility functions $U(Y)$, where Y is income, $U'(Y) > 0$, and $U''(Y) < 0$. The concavity of the utility function implies that type 2 firms are risk averse. That is, their expected utility is higher, the lower the variability of their income. There are N_1 type 1 firms and N_2 type 2 firms.

At first blush the assumption of risk averse firms appears ad hoc. The Modigliani–Miller theorem implies that in the absence of taxes, transactions costs, or other frictions, firms should be risk neutral. If such frictions exist, however, firms may increase the expected value of their after-tax cash flows by reducing the riskiness of these flows. As a result, they are effectively risk averse.[1]

Economists have identified three primary sources of transaction-cost induced sources of risk aversion. First, firms with debt or other fixed obligations (e.g., labor contracts) may incur extraordinary costs of financial distress if their income falls below some critical level. They avoid this cost if profit remains above this threshold. These costs are, therefore, a non-linear function of the firm's income and this non-linearity can make the firm risk averse. To see why, consider two firms with identical expected pre-tax incomes, but the first firm has a positive probability of falling into financial distress (and incurring the associated costs) while the second does not. This could occur if the first firm has debt, while the other is equity financed, or because the first firm's profits are more variable. In this case, the expected after-tax, after-financial distress income of the first firm is less than that of the second firm because it incurs this extraordinary cost with positive probability. Thus, even though the expected incomes, exclusive of distress costs, of the two firms are identical, the market value of the second firm exceeds that of

[1]Smith and Stulz show how various frictions can make firms effectively risk averse. The Modigliani–Miller theorem states that firm financing decisions are irrelevant in zero transactions cost world. Hedging is a financing decision because it involves the issue of financial claims against a firm. Thus, according to Modigliani–Miller, firms have no incentive to engage in hedging to reduce risk in the absence of taxes, transactions costs, or agency costs. Put another way, firms are risk neutral in the absence of these frictions.

the first. As a result, a reduction in the dispersion of the first firm's after-tax income makes its claimants better off because this reduces the probability that the firm will incur financial distress costs. The firm is, therefore, effectively risk averse.

Second, firms can reduce agency costs by concentrating ownership to a small number. However, this typically requires the owners to invest a large fraction of their wealth in the firm. If these individuals are risk averse, they require a risk premium to bear any increase in the variability of the firm's value, *even if this risk is non-systematic*. That is, it is costly to use concentrated ownership to mitigate agency problems because of the resulting inefficiencies in risk sharing. If it is possible to shift risk (through futures markets, for instance) while retaining concentrated ownership, all claimants can be made better off. Similar results obtain if management compensation is tied to firm performance to align their incentives with the interests of shareholders. Risk averse managers demand a salary premium to bear the risk that is inherent in this compensation scheme. That is, their expected compensation must be higher if their pay is tied to risky firm performance than if they receive a flat salary. A reduction in the riskiness of corporate performance reduces the size of the necessary premium and thereby reduces the average compensation shareholders must pay to management. This increases shareholder expected returns. Again, it is accurate to characterize the firm's claimants as averse to riskiness in its after-tax income because a rise in risk increases the expected value of some cost.

Third, a firm with debt outstanding that suffers an unexpected increase in its debt to equity ratio may fail to invest in positive net present value projects because its bondholders (rather than the shareholders) reap a substantial portion of the value of the investment when the firm is highly leveraged. By reducing risk, the firm can reduce the probability of such an unexpected increase in its leverage, thereby mitigating the underinvestment problem. This preserves the value of the firm's future investment options, which redounds to the benefit of current shareholders. Therefore, firms with substantial investment options (e.g., R&D intensive firms) may be risk averse even if they are only modestly leveraged.

It is important to recognize that in the presence of these frictions, firms that reduce the riskiness of their after-tax cash flows incur lower expected resource costs. For example, firms that reduce the probability of financial distress incur the associated costs less frequently. Similarly, the owners of closely held firms require a smaller risk premium, the lower the dispersion in the income of these firms. Reducing risk also allows firms to undertake certain actions that would be prohibitively expensive in a riskier environment. For example, firms that can reduce the riskiness of their cash flows can take on more debt; this can enhance efficiency because debt has some desirable incentive effects [Jensen (1986); Grossman and Hart (1984); Myers and Majluf (1984); Diamond (1989)]. Thus, in the presence of transactions costs, risk reducing measures often enhance efficiency and increase the wealth of corporate claimants.

In sum, it is legitimate to assume that some firms are averse to risk in their after-tax income.[2] A concave utility function is simply a reduced-form characterization of the objective function of a risk averse firm.[3] This utility function is mathematical shorthand that summarizes the effect of the underlying economic frictions that determine a firm's attitudes toward risk. Therefore, it is not an ad hoc assumption, but is instead firmly grounded in an understanding of modern corporate finance theory. Moreover, the sources of this risk aversion have important implications for the welfare effects of hedging, and thus for the welfare effects of hedge taxation. The remainder of the analysis explores this issue in detail.

Each firm in the model economy has two sources of ordinary income. One source is riskless, and equals I_i for firm i. The other source of income is risky, and equals y_i for firm i. For simplicity and without loss of generality, each firm has no sources of capital income.

There exists a perfect hedge (such as a futures contract) for each risk. That is, each firm can purchase or sell a contingent claim with a price that is perfectly correlated with its risky source of ordinary income. The expected value of this payoff equals 0. Without loss of generality one can assume that the payoff to the hedge instrument for firm i equals y_i.[4] For simplicity, assume that firms incur no transactions costs when hedging.

[2] It is possible that firms trade derivatives to increase risk due to an agency problem. For example, highly leveraged firms managed in the interest of shareholders may expropriate bondholders by increasing risk, perhaps through derivatives trading. Deterring derivatives trading by such firms through the use of asymmetric taxation would actually be welfare enhancing. However, such trading could also be deterred by establishing appropriate hedge accounting rules and by including convents in debt agreements that restrict derivative trading to legitimate hedging activities in accordance with the accounting rules.

[3] Goldberg (1990) and Varian (1990) both argue that a firm utility function is essentially a reduced-form representation of the structural factors that determine the attitude of a firm's claimants towards risk.

[4] The numerical simulation analysis described in the appendix and reported in Pirrong (1993a, 1993b) demonstrates that the results that follow hold even in the absence of perfect correlation between the risky income and the hedging instrument. Those articles also show that the implications of the formal model derived herein continue to hold even if (a) I_i is risky, and (b) the firm has other sources of capital income.

The objective of this analysis is to compare two tax regimes. The first regime taxes gains and losses on different legs of a hedge transaction symmetrically. That is, both the income from the risky asset and the gain or loss on the hedge position are ordinary income, and hedging firms aggregate them for tax purposes. In the symmetric regime, the tax rate on ordinary income equals τ^s if ordinary income is positive; the marginal tax rate equals 0 if income is negative. Thus, in the symmetric regime a firm i that buys β units of the hedge instrument (i.e., chooses a "hedge ratio" equal to β) pays a tax equal to[5]

$$T_i^s = \tau^s \max[0, I_i + y_i + \beta y_i]$$

The second regime taxes gains and losses on different legs of a hedge position asymmetrically. The gains and losses on the hedge position are taxed as capital gains income, but all other profits are taxed as ordinary income. The marginal tax rate for positive (non-positive) capital and positive (non-positive) ordinary income in this regime both equal τ^a (0). For simplicity, the loss limit on capital income is ignored. The results derived below hold even if a firm can deduct some finite amount of capital losses against ordinary income. Thus, under asymmetric taxation, total taxes for a firm i that chooses a hedge ratio equal to β equal

$$T_i^a = \tau^a \max[0, I_i + y_i] + \tau^a \max[0, \beta y_i]$$

Finally, assume that the government must raise $R_{1,2}$ dollars in expected revenue from corporate taxation. This assumption is intended to permit determination of the efficiency effects of the different tax regimes while holding expected tax revenues constant. (This assumption means that the tax authority is risk neutral).

First consider the incentives of type 1 and type 2 firms to hedge in the two tax regimes. With symmetric taxation, if a type 1 firm i chooses a hedge ratio β_i, its expected profit equals

$$\begin{aligned} E(\Pi_i^s) &= E\{I_i + y_i + \beta_i y_i - \tau^s \max[0, I_i + y_i + \beta_i y_i]\} \\ &= I_i - E\{\tau^s \max[0, I_i + y_i + \beta_i y_i]\} \qquad (1) \end{aligned}$$

In this expression, Π_i^s is firm i's after-tax profit in the symmetric regime. Note that the firm's tax payment, $E\{\tau^s \max[0, I_i + y_i + \beta_i y_i]\}$, equals the value of an option on the firm's income. Because the tax function is convex, Jensen's inequality implies that $E\{\tau^s \max[0, I_i + y_i + \beta_i y_i]\} >$

[5]A firms sells the hedging instrument if $\beta < 0$.

$\tau^s I_i$ if $\beta_i \neq -1$; whereas, $E\{\tau^s \max[0, I_i + y_i + \beta_i y_i]\} = \tau^s I_i$ if $\beta_i = -1$. Thus, under symmetric taxation, type 1 firms completely hedge their *pre-tax* income risk even though they are neutral to *after-tax* income risk, because reducing the riskiness of their pre-tax cash flows reduces the expected value of tax payments. This result essentially replicates an implication of Smith and Stulz (1986).

Given symmetric taxation, if a type 2 firm j chooses a hedge ratio β_j, its expected utility equals

$$E[U(\Pi_j^s)] = EU(I_j + y_j + \beta_j y_j - \tau^s \max[0, I_j + y_j + \beta_j y_j]) \quad (2)$$

It is again straightforward to show that firm j chooses $\beta_j = -1$. This is true for two reasons. First, as was shown above for the type 1 firms, this hedge ratio maximizes j's expected profit by minimizing its expected tax payment. Second, when $\beta_j = -1$, the firm's after-tax income is riskless. Thus, by hedging the firm maximizes expected after-tax income and minimizes risk. Since $U' > 0$ and $U'' < 0$, $\beta_j = -1$ which maximizes firm j's expected utility.

The foregoing discussion implies the following result.

Result 1. Both type 1 and type 2 firms choose $\beta = -1$ in the symmetric tax regime. That is, both types hedge completely if hedging gains and losses are taxed symmetrically.

The intuition behind this result is as follows. Under symmetric taxation, firms can hedge all risks, including the risk attributable to tax payments. This is beneficial to both kinds of firms. Since the expected value of tax payments is equal to the value of an option on a firm's profits and the value of this option is positively related to the riskiness of the firm's pre-tax profits, both type 1 and type 2 firms have an incentive to eliminate this riskiness through complete hedging. In addition, type 2 firms incur higher transactions and agency costs on average, the greater the riskiness in their tax payments. By hedging away the variability of their tax payments, these firms can reduce expected transactions and agency costs and, thereby, increase their expected utility.

Now consider the hedging activity of the two types of firms in the asymmetric tax regime. In this case, a type 1 firm i earns the following expected profit if it chooses a hedge ratio equal to β_i

$$\begin{aligned} E(\Pi_i^a) &= E\{I_i + y_i + \beta_i y_i - \tau^a \max[0, I_i + y_i] - \tau^a \max[0, \beta_i y_i]\} \\ &= I_i - \tau^a E \max[0, I_i + y_i] - \tau^a E \max[0, \beta_i y_i] \end{aligned} \quad (3)$$

In (3) Π_i^a is firm i's after-tax profit in the asymmetric tax regime. The first two terms on the second line of (3) do not depend upon β_i. However, $\tau^a E \max[0, \beta_i y_i] > 0$ unless $\beta_i = 0$. The firm pays higher taxes on average (and has a lower expected after-tax income as a result) if it hedges than if it does not. This implies

Result 2. Under asymmetric taxation type 1 firms never hedge, i.e., type 1 firms choose $\beta_i = 0$.[6]

The foregoing result obtains because in an asymmetric regime, hedging actually *increases* the expected value of tax payments. In the asymmetric regime, a type 1 firm is short two tax options to the government if it hedges. It is short only one of these tax options if it does not hedge. To minimize the expected value of its obligations to the government, the type 1 firm rationally chooses not to hedge.

Although asymmetric taxation makes type 1 firms worse off if $\tau^a = \tau^s$, this does not create a deadweight loss because the additional revenue that asymmetric taxation generates allows the government to reduce the amount of revenue needed dollar for dollar from other taxpayers. That is, an asymmetric regime transfers wealth from type 1 firms to other taxpayers and government claimants.

Asymmetric taxation has far more detrimental effects for type 2 firms. If a type 2 firm j chooses a hedge ratio β_j, under asymmetric taxation its expected utility equals.

$$E[U(\Pi_i^a)] = EU(I_j + y_j + \beta_j y_j - \tau^a \max[0, I_j + y_j] - \tau^a \max[0, \beta_j y_j]) \tag{4}$$

[6]This result implies that if tax minimization is the dominant motive for hedging, the introduction of asymmetric taxation should lead to a dramatic decline in the number of firms that hedge. It is possible to show that type 2 firms may still hedge under an asymmetric tax regime for efficiency enhancing reasons. In that case, one would not expect to observe such a large decline in the number of firms that hedge.

Result 2 needs to be qualified somewhat due to the fact that some hedging instruments may not be subject to asymmetric taxation. In particular, prior to the issuance of the new rules on hedge taxation, it appeared that the IRS intended to permit firms to treat gains and losses on swap transactions as ordinary even though economically equivalent futures or options transactions were to be taxed as capital. This tax discrimination between functionally identical transactions implies that firms that hedge for tax purposes may substitute swaps for futures, options, and cash market hedges rather than eschew hedging altogether. Thus, Result 2 can be re-written as "Type 1 firms will never use hedge instruments subject to asymmetric taxation."

If $\tau^a = \tau^s$, $E[U(\Pi_j^a)] < E[U(\Pi_j^s)]$. This is true because: (a) for any β_j, $E(\Pi_j^a) < E(\Pi_j^s)$, and (b) Π_j^a is riskier than Π_j^s. Part (a) holds because $\max[0, I_j + y_j + \beta_j y_j] < \max[0, I_j + y_j] + \max[0, \beta_j y_j]$ for all I_j, y_j, and $\beta_j \neq 0$. Put another way, the firm's expected income declines because it is now short a portfolio of options, rather than a single option on a portfolio. Merton (1973) demonstrates that this portfolio of options is costlier than the option on the portfolio.

The reasoning behind part (b) of the statement is as follows. Recall that under symmetric taxation, the firm's after-tax income is riskless if $\beta_j = -1$. Under asymmetric taxation, however, the firm's after tax income necessarily depends upon y_j and is, therefore, risky. To see why, assume $\beta_j < 0$. (An identical argument holds for $\beta_j > 0$.) Three cases are relevant: (1) $0 \geq y_j \geq -I_j, 2) y_j \leq -I_j, 3) y_j \geq 0$. In case 1

$$\Pi_j^a = y_j(1 + \beta_j - \tau^a - \tau^a \beta_j) + (1 - \tau^a) I_j \equiv \Pi_{j1}^a(\beta_j, y_j)$$

In case 2

$$\Pi_j^a = y_j(1 + \beta_j - \tau^a) \equiv \Pi_{j2}^a(\beta_j, y_j)$$

Finally, in case 3

$$\Pi_j^a = y_j(1 + \beta_j - \tau^a \beta_j) + (1 - \tau^a) I_j \equiv \Pi_{j3}^a(\beta_j, y_j)$$

There is no β_j such that the right-hand sides of these three equations are equal for all y_j. That is, it is impossible to choose a β_j such that $\Pi_{j1}^a(\beta_j, y_j) = \Pi_{j2}^a(\beta_j, y_j) = \Pi_{j3}^a(\beta_j, y_j)$ for all realizations of y_j. Thus, because y_j is stochastic, for any choice of β_j, the after-tax income of a type 2 firm necessarily varies randomly. The firm's after-tax income is variable because it cannot hedge the variation in its tax payments if hedging gains and losses are taxed asymmetrically.

Since the firm is risk averse, if $\tau^a = \tau^s$ the increase in riskiness and the decrease in expected income that results from asymmetric taxation makes type 2 firms worse off than they are under symmetric taxation. Indeed, even if the taxing authority were to choose a $\tau^a < \tau^s$ such that $E(\Pi_j^a) = E(\Pi_j^s)$, risk averse firm j would still strictly prefer symmetric taxation because (a) its after-tax income is riskier under asymmetric taxation, and (b) expected transactions and agency costs are greater due to the greater the riskiness in the firm's income. This implies:

Result 3. For a choice of τ^s and τ^a such that $E(\Pi_j^a) = E(\Pi_j^s)$ for type 2 firm j, the firm strictly prefers symmetric taxation because its income is riskier under asymmetric taxation.

The intuition behind this result is as follows. Under symmetric taxation, gains (losses) on the underlying position off-set losses (gains) on the hedge position for tax purposes. Thus, the firm can hedge its tax risk. In contrast, under asymmetric taxation gains (losses) on the underlying position do *not* off-set losses (gains) on the hedge position. Therefore, in the asymmetric regime the firm *cannot hedge its tax risk*. Tax payments are variable. Since the firm is averse to after-tax income risk due to the existence of economic frictions, this variability of tax payments makes the firm worse off in the asymmetric tax regime even if average tax payments are held constant across regimes. The firm is worse off because the tax risk increases expected transactions and agency costs. These higher costs result from higher probabilities of financial distress, inefficient ownership and capital structures, and the inefficient exercise of investment options.

The foregoing demonstrates clearly the potentially perverse effects of asymmetric taxation of hedging gains and losses. Most importantly, for a given amount of revenue raised from a particular type 2 hedging firm, this form of taxation makes firms that hedge because they are averse to after-tax income risk worse off. That is, asymmetric taxation creates a unique form of deadweight loss because it forces firms that incur real costs to bear risk to incur more uncertainty than they do under symmetric taxation.

This suggests that asymmetric taxation is an inferior form of taxation because of dollar of tax revenue generated from type 2 firms under this form of taxation costs more than a dollar, due to inefficient risk bearing. Under the assumptions of this analysis, a dollar of revenue generated by symmetric taxation costs a dollar. It is possible to prove this conjecture rigorously under certain simplifying assumptions.

Specifically, approximate firm j's utility by its certainty equivalent income:

$$CE_j = E(\Pi_j) - 0.5\chi_j\sigma_j^2 \tag{8}$$

where σ_j^2 is the variance of firm j's after-tax profit (assuming that it has chosen its utility maximizing hedge ratio); the parameter χ_j is the coefficient of absolute risk aversion for firm j; $\chi_j > 0$ for type 2 firms;

and $\chi_j = 0$ for type 1 firms.[7] This approximation of the firm's utility function takes into account the fact that higher risk imposes higher costs on type 2 firms.

Assume that the tax authority desires to maximize the total certainty equivalent income of the type 1 and type 2 firms while raising $R_{1,2}$ dollars in expected tax revenue from them. The authority chooses a tax rate that maximizes:

$$TCE = \sum_{i=1}^{N_1} CE_i + \sum_{j=1}^{N_2} CE_j \tag{9}$$

subject to the constraint that total expected tax payments from these $N_1 + N_2$ firms equal $R_{1,2}$. The first sum is the total certainty equivalent income earned by the type 1 firms and the second sum is the total certainty equivalent income earned by the type 2 firms.

Assume that the tax authority determines that under asymmetric taxation, the tax rate that maximizes total certainty equivalent after-tax income subject to the revenue constraint equals τ^{a*}. Given this tax rate, the expected after-tax profit of firm k equals $E[\Pi_k^a(\tau^{a*})] = I_k - T_k^a(\tau^{a*})$, where $T_k^a(\tau^{a*})$ is the expected tax payment of firm k. Note that $R_{1,2} = \sum_{i=1}^{N_1} T_i^a(\tau^{a*}) + \sum_{j=1}^{N_2} T_j^a(\tau^{a*})$. Under asymmetric taxation,

[7]Certainty equivalent income is a second-order Taylor approximation of expected utility. It is not exact unless firms have quadratic utility functions, or profits are normally distributed. Under more general conditions, firms' utility functions depend upon higher order moments of wealth than the variance. However, under the assumptions made in the article that (a) perfect pre-tax hedges exist and (b) other income is riskless, the certainty equivalence measure of utility gives the same ordering and the same results as a more exact measure of utility. This is true because firm profit under symmetric taxation is riskless, which holds expected profit constant.

The simulation analysis described in the appendix can be used to determine the robustness of the results to the order of the approximation when (a) and (b) do not hold. Specifically, assuming the type 2 firms have exponential utility, that I_j is risky, and that futures returns are imperfectly correlated with $y_j + I_j$, the optimal hedge ratios are calculated using numerical methods. The higher order moments of firm profits under symmetric and asymmetric tax regimes are then simulated. In these simulations, profits exhibit more negative skew and greater kurtosis under asymmetric taxation. Since risk averse agents with consistent preferences dislike variance and kurtosis, and like positive skew [see Ingersoll (1990, p. 41)], this implies that if hedge ratios for other utility functions are similar to those determined for the exponential utility function, a fourth order expansion of the utility function would give the same results as the second order approximation used here. That is, firms require a higher expected income to compensate for the greater negative skew and the greater kurtosis that asymmetric taxation produces. Therefore, given two taxes that raise the same revenue (and thus leave total expected profits unchanged), aggregate risk-adjusted income is lower under asymmetric taxation. Indeed, the second order approximation used in the derivation in the text would actually understate the welfare loss arising from asymmetric taxation because it does not reflect the disutility arising from more negative skew and greater kurtosis.

the total expected after-tax profit $E(\Pi_T^a)$ earned by the $N_1 + N_2$ firms equals

$$\begin{aligned} E(\Pi_T^a) &= \sum_{i=1}^{N_1} E[\Pi_i^a(\tau^{a*})] + \sum_{j=1}^{N_2} E[\Pi_j^a(\tau^{a*})] \\ &= \sum_{i=1}^{N_1}[I_i - T_i^a(\tau^{a*})] + \sum_{j=1}^{N_2}[I_j - T_j^a(\tau^{a*})] \\ &= \sum_{i=1}^{N_1} I_i + \sum_{j=1}^{N_2} I_j - R_{1,2} \end{aligned} \tag{10}$$

Thus, defining (a) TCE^a as the total certainty equivalent income under asymmetric taxation and (b) σ_{ja}^2 as the variance of firm j's after-tax profits in this tax regime

$$TCE^a = E(\Pi_T^a) - 0.5 \sum_{j=1}^{N_2} \chi_j \sigma_{ja}^2 \tag{11}$$

where $\sigma_{ja}^2 > 0$ for all type 2 firms j.

Now consider the symmetric taxation case. Here, if the tax authority chooses a tax rate equal to τ^s, total expected after-tax income $E(\Pi_T^s)$ equals

$$\begin{aligned} E(\Pi_T^s) &= \sum_{i=1}^{N_1} E[\Pi_i^s(\tau^s)] + \sum_{j=1}^{N_2} E[\Pi_j^s(\tau^s)] \\ &= \sum_{i=1}^{N_1}[I_i - T_i^s(\tau^s)] + \sum_{j=1}^{N_2}[I_j - T_j^s(\tau^s)] \\ &= \sum_{i=1}^{N_1} I_i + \sum_{j=1}^{N_2} I_j - R_{1,2} \end{aligned} \tag{12}$$

where $T_k^s(\tau^s)$ is the expected tax payment from firm k. The last line follows due to the constraint that expected total revenue raised, given a tax rate of τ^s equals $R_{1,2}$. Thus, denoting the total certainty equivalent income given symmetric taxation by TCE^s, and defining the variance of firm j's after tax income in the symmetric regime as σ_{js}^2,

$$TCE^s = E(\Pi_T^s) - 0.5 \sum_{j=1}^{N_2} \chi_j \sigma_{js}^2 \tag{13}$$

A comparison of (8) and (10) reveals that $TCE^s > TCE^a$. This is true because (7) and (9) imply that $E(\Pi_T^s) = E(\Pi_T^a)$, and because the analysis preceding Result 3 implies $\sigma_{ja}^2 > \sigma_{js}^2 = 0$ for all j.[8] This produces

Result 4. Holding corporate tax revenues constant across tax regimes, total certainty equivalent income is higher under symmetric taxation than under asymmetric taxation.

This result strongly suggests that symmetric taxation welfare dominates asymmetric taxation because the aggregate risk-adjusted income of individuals holding claims on firms subject to the corporate tax is higher under symmetric taxation. Even if some firms are worse off in a symmetric regime, other firms reap an even larger benefit. In theory, it is therefore possible to use lump sum transfers from those who gain from symmetric taxation to compensate the losers and still leave the winners better off.

With a single tax rate for all firms, it is not possible to show in general that symmetric taxation makes *all* corporate claimants better off in the absence of lump sum transfers from winners to losers. That is, in the real world where lump sum transfers are impractical, symmetric taxation may not strictly Pareto dominate asymmetric taxation, given the revenue constraint. Some firms (most likely the risk neutral ones) may be made better off by the reduction in the tax rate in the asymmetric regime even though other firms lose even more due to inefficient risk bearing. However, if the *owners* of claims on corporations are well diversified, it is possible that all *investors* are made worse-off by a government-revenue-neutral move to an asymmetric tax regime.

To see why, assume that each individual investor holds the market portfolio and risk free bonds. This assumption is consistent with some asset pricing models, such as the CAPM. Note that risk is costly in this model because firms incur higher agency and transactions costs, the greater the amount of risk they bear. Because it inflates these costs, asymmetric taxation reduces expected after-tax payoffs to investors in these firms. Asymmetric taxation therefore causes the wealth of each investor holding the market portfolio to fall by an amount proportional to

[8]The simulation analysis described in the appendix, and reported in Pirrong (1993a, 1993b) imply that the inequality holds even if the equality does not. The simulations show that if no perfect hedges exist, or if the other sources of income I_j are risky but not hedgeable, then $\sigma_{ja}^2 > \sigma_{js}^2 > 0$. Thus, even if there are no perfect hedges, or if the firm bears risks it cannot hedge, symmetric taxation still dominates asymmetric taxation.

the present value of the resulting decline in certainty equivalent income. Thus, although some *firms* may benefit from asymmetric taxation and the concomitant reduction in the corporate tax rate, this form of taxation may reduce the wealth of *each investor* because (a) each holds a position in all firms and (b) the aggregate certainty equivalent profits of the firms that are adversely affected by asymmetric taxation fall by a larger amount than the total increase in profits of the firms that benefit from it. As a result, under certain assumptions about the behavior of firm owners, symmetric hedge taxation *strictly* Pareto dominates asymmetric hedge taxation because all investors are better off under symmetric taxation than asymmetric taxation.

Some may claim that the assumption of complete investor diversification is invalid in this context. This is a legitimate criticism since hedging may enhance efficiency because incomplete diversification economizes on agency costs. Two points deserve mention, however. First, agency cost economizing is not the sole reason for hedging. Some firms with well-diversified shareholders hedge to reduce expected financial distress and underinvestment costs. Second, *any* investor diversification tends to off-set the inter-firm distributive effects that may prevent symmetric taxation from strictly Pareto dominating asymmetric taxation. Taking these two facts together, the number of investors made worse-off by a government-revenue-neutral symmetric tax is likely to be very small relative to the number of investors made better off by asymmetric taxation.

The importance of these welfare effects cannot be slighted. Asymmetric taxation of hedge gains and losses reduces aggregate investor wealth because it forces risk averse firms to bear more risk for the same amount of expected revenue raised. As a consequence, the claimants of these firms bear higher costs in the form of bankruptcy and other financial distress costs, less efficient capital and ownership structures, more acute agency problems, and under-investment in positive net present value investments. These are real costs to the economy. Thus, although asymmetric taxation eliminates one form of hedging that merely produces a transfer between some firms and the government, it also reduces the efficiency of risk bearing in the economy. This produces a deadweight loss.

This analysis has an important implication: In order to raise a given amount of revenue from corporate taxpayers, it is more efficient to do so with a higher corporate tax rate through symmetric taxation of hedging gains and losses, than with a lower corporate tax rate through asymmetric taxation. This is true as long as *some* firms hedge for efficiency enhancing reasons. Thus, even if hedging allows firms to reduce their expected tax payments and the tax authority deems it necessary to raise more revenue from corporate tax payers, it is better to generate this additional revenue by raising the corporate tax rate and allowing firms to off-set hedging gains and losses for tax purposes, than to do so by taxing firms asymmetrically.

CONCLUSIONS AND EXTENSIONS OF THE ANALYSIS

This article presents a model that shows that asymmetric taxation of hedging gains and losses reduces the wealth of claimants of firms subject to the corporate tax. This is true because hedgers bear more risk in the asymmetric tax regime. Firms that hedge because they are averse to risk in their after-tax income are worse-off as a result. These companies include highly leveraged firms (e.g., banks and other financial intermediaries), firms with valuable investment options (e.g., banks and other financial intermediaries), firms with valuable investment options (e.g., R&D intensive companies), and closely held corporations and partnerships (e.g., commodity merchants). Claimants of these firms bear higher expected transactions and agency costs in an asymmetric hedge tax regime because they cannot hedge the risk of tax payments.

The model demonstrates that: (1) both types of firms hedge in a symmetric tax regime; (2) only the risk averse firms hedge in an asymmetric tax regime; (3) risk averse firms strictly prefer symmetric taxation even if the tax authority adjusts tax rates so as to keep the expected profits of these firms constant; and (4) it is always possible to find a symmetric corporate tax rate that generates a higher total after-tax certainty equivalent income for the firms than any given asymmetric tax, while producing identical revenue for the government. This last result implies that symmetric taxation is welfare superior to asymmetric taxation because total investor wealth is higher under symmetric taxation. Under certain assumptions, moreover, it is possible to show that symmetric hedge taxation strictly Pareto dominates asymmetric taxation. Specifically, if all investors hold the market portfolio and riskless bonds, all investors are better off under symmetric taxation.

One potential extension of this work is to endogenize firm pre-tax profits. The model of this article assumes that pre-tax profits are exogenous; in particular, they do not depend upon the tax rate. To raise a given amount of corporate tax revenue, it is necessary to impose a higher rate in a symmetric tax regime than an asymmetric one. It is possible that this higher marginal rate could induce a larger distortion in investment decisions than would occur in an asymmetric tax regime. The resulting deadweight losses could off-set the deadweight loss that results from greater after-tax profit variability inherent in the asymmetric tax regime. It is unlikely, however, that the investment distortion arising from the higher marginal tax rate in a symmetric tax regime outweighs the risk tax distortion. Three facts support this conjecture.

First, the analysis of Gordon (1985) and Gordon and Wilson (1986) implies that the distortions in investment decisions arising from the corporate income tax are minor. This result obtains because, although this tax reduces the returns from investment, it also reduces the discount rate investors use because the government shares in the risk. The lower discount rate off-sets the effect of the lower after-tax cash flows. For reasonable values of time preferences and the marginal product of capital, the net effect of the corporate income tax on investment is small.

Second, the proof of Result 4 sets tax rates so that the corporate tax revenue raised in the asymmetric regime equals the revenue raised in the symmetric regime. Thus, the average corporate tax on investment is the same in the two regimes. According to the theory of Bulow and Summers (1984), the average tax rate approximates the welfare costs of capital/corporate taxation when the assumptions of the Gordon and Gordon and Wilson analysis do not hold. This occurs when the government does not share in the risk of deviations between actual depreciation rates and the ex ante depreciation schedules used to calculate corporate income taxes.

Third, and perhaps most importantly, Fellingham and Wolfson (1985) show that a convex tax code penalizes investments in risky assets. This distorts the allocation of capital between risky investments and riskless ones. In a symmetric tax regime, even risk neutral, type 1 firms can hedge to mitigate the effects of this feature of the tax code. In an asymmetric tax regime, hedging actually exacerbates this distortion. As a result, less investment in risky positive-net-present-value projects takes place in an asymmetric tax regime than in a symmetric one. This distortion inherent in asymmetric taxation may be more costly than any distortions arising from a higher tax rate in a symmetric regime.

In conclusion, the model of this article demonstrates that asymmetric taxation of hedging gains and losses creates a unique form of deadweight loss. Firms that use derivative products such as futures, options, and forward contracts to reduce risk incur higher transactions and agency costs when the government taxes the gains and losses on these transactions asymmetrically. Put another way, asymmetric taxation impairs the efficiency of risk bearing in the economy. This analysis implies that the Internal Revenue Service's recent policy (rooted in the *Arkansas Best* decision) of treating gains and losses on hedging instruments as capital and the gains and losses on the hedged transactions as ordinary was inefficient, and that the present policy of symmetric taxation makes the claimants of hedging firms better off.

APPENDIX

The analysis in the text assumes that (a) there is only one risky component of ordinary income and (b) the price of the hedging instrument is perfectly correlated with that risky source of income. It is intractable to determine analytically how asymmetric taxation affects the variability of a firm's income and, therefore, its expected utility when these assumptions are changed. A simulation analysis demonstrates, however, that the conclusion drawn in the text—namely, that asymmetric taxation makes risk averse firms worse-off—is robust to changes in these assumptions.

As before, assume that y is the hedgeable component of a firm's income, but that the firm also earns I in non-hedgeable ordinary income and that I is variable. Define $z = y + I$. Moreover, the profit on a hedging instrument is f, where the correlation between f and z is less than one. In addition, z and f are jointly normally distributed, with probability density $n(z, f)$. This specification is sufficiently flexible to allow imperfect correlation between y and f and a risky I that is imperfectly correlated with f.

The firm chooses a position equal to β_s in the hedge instrument under symmetric taxation, and β_a under asymmetric taxation. Then the firm's expected utility is:

$$EU = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(\Pi) n(z, f)\, dz df \tag{A.1}$$

where

$$\Pi = \Pi_s = z + \beta_s f - \tau \max[0, z + \beta_s f]$$

if the firm is subject to symmetric taxation, and

$$\Pi = \Pi_a = z + \beta_a f - \tau \max[0, z] - \tau \max[0, \beta_a f]$$

if it is taxed asymmetrically.

The simulations assume that $U(.)$ is the exponential utility function, and $\tau = 1/3$. Given this utility function and tax rate and based on (a) an assumed variance–covariance matrix for z and f and (b) assumed values for the means of these variables, (A.1) is solved numerically to find the choice of the β_s that maximizes the firm's expected utility under symmetric taxation and the value of β_a that maximizes the firm's expected utility under asymmetric taxation. The integrals in (A.1) are estimated using 15 point Gauss–Legendre quadrature. The Newton–Raphson method is used to solve for the optimal β_a and β_s.

This exercise is repeated for a number of values for the means, variances, and covariances of z and f. For all values simulated, the firm's expected utility is lower under asymmetric taxation. Moreover, the first four moments of Π_s and Π_a are calculated. In each case simulated, Π_a has a lower mean, a higher variance, a lower coefficient of skewness, and a higher coefficient of kurtosis than Π_s. The behavior of the higher order moments implies that even if the tax rate in an asymmetric regime were low enough (relative to its value in the symmetric regime) to ensure that the firm earned the same expected (i.e., mean) profit in each regime, the firm's expected utility would be lower in the asymmetric regime. This is true because a higher variance, higher kurtosis, and more negative/less positive skewness make a risk averse firm worse-off. [See Ingersoll (1987)]. Thus, the simulation results show that asymmetric taxation is inefficient, even under weaker assumptions about the correlation between the profit on a hedging instrument and the profit on hedgable risks and the existence of non-hedgable sources of income.

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