IMPORT AND HEDGING UNCERTAINTY IN INTERNATIONAL TRADE

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INTRODUCTION

Import or export decisions are made, in general, under conditions of uncertainty about changes in the exchange rate during the period relevant to the decisions. Also, an importing firm faces uncertainty about the imported commodity's price and an exporting firm faces uncertainty about the price in its export market. Many of the internationally traded commodities and most goods do not have futures or forward markets. Thus, hedging the uncertain price in such markets is impossible. However, the major currencies are traded both in futures and forward markets. This enables importing or exporting agents to hedge their risks in the forward or futures markets using foreign exchange rate contracts.[1] The effect of forward exchange markets on the level of trade is discussed in Ethier (1973) and Baron (1976). The first article concentrates on the mean-variance trade off while the second article emphasizes the role of forward markets in planning output and trade.

Studies on hedging in the presence of price uncertainty [see Danthine (1978), and Holthousen (1979), among others] have shown a separation between production and hedging, between production

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[1] It is known that foreign currencies are among the few commodities for which futures and forward markets coexist where contracts in both markets are actively traded. [See Stein (1984) p. 231.]

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and the level of uncertainty, and between production and the degree of risk aversion. Other articles [see, e.g., Kawai and Zilcha (1986); Paroush and Wolf (1986); Stein (1984); and Anderson and Danthine (1983)] have shown that, in the presence of basis risk, the separation properties do not hold.[2]

More recently, Kerkvliet and Moffett (1991) derived the optimal hedge of an uncertain future currency cash flow. The optimal hedge is found to be determined, amount others, by the variance of the expected cash flow and the correlation of the future cash flow with actual exchange rate movements. The optimal hedge is also addressed in Paroush and Wolf (1992), in their research on the derived demand. Their article explores the significant role of production technology in setting the effect of the firm's exogenous parameters on the employment of inputs and hedging. It establishes an explicit relationship among derived demand for inputs, basis risk, attitude towards risk, futures-spot price link, and hedging. Specifically, they find that the upward bias and the downward bias of the futures price with respect to the expected spot price affect the size of the demand for factors of production. Further, they determine that the presence of basis risk determines the impact that input prices, product and futures prices, attitude towards risk and price volatility of the uncertain input have on the derived demand for inputs.

The purpose of this article is to examine import, production, and hedging decisions under joint input price and exchange rate uncertainties. It is different from Kawai and Zilcha (1986) in its focus and in the results. Kawai and Zilcha concentrate on the separation theorem and full double hedging, and examine the effect of introducing a forward market. In contrast, this article compares the effect of an additional source of risk on import and hedging and examines the effect of risk on hedging and import. It is different from Kerkvliet and Moffett (1991) in that it provides a different and more detailed analysis of the characteristics of the hedging and importing decisions. Therefore, the present work may be viewed as an extension of the works of Paroush and Wolf (1992), Kerkvliet and Moffett (1991), Kawai and Zilcha (1986), and Baron (1976) in that it addresses the import and hedging decisions of an importing firm under multiplicative sources of uncertainty. However, it differs from these articles as well as from the others listed above in its framework, its approach, its focus and methodology, and in its results.

[2]Basis is defined as the difference between the futures and the spot prices. Basis risk is defined as the risk that upon maturity of the futures contract its price will be different from the spot price due to the divergence in timing, location or quality of the deliverable commodity compared to the one produced.

The article analyzes the optimal production and hedging decision of a firm that imports inputs and operates in a competitive industry. The sensitivity of this decision is analyzed with respect to the model's parameters under a flexible exchange rate regime. Most of the results and conclusions apply also to an exporting firm under similar conditions. One of the findings is that the effects of risk aversion and the variance of the exchange rate on hedging depend on the nature of the equilibrium price structure, i.e., upon whether one faces a contango or backwardation, as well as on the initial magnitude of hedging. Another finding is that, under input price and exchange rate uncertainties, hedging is lower than it is under just exchange rate uncertainty if the input price in the first case is higher than that in the second case.

The issues addressed in this article and its findings are important to many countries for which the foreign trade sector is a significant component of the gross domestic product (GDP). Many firms produce products that are vital to the growth rate of the GDP. Some of the inputs used in the production process are imported. For example, the automotive industry in Japan imports steel, a major input in the production process of automobiles. The transportation segment of the U.S. industry uses imported oil from overseas. The computers industry imports memory chips from overseas. There are many more examples in almost every segment of the economy that are effected in one way or another by the level of uncertainty of importing companies and their ability to hedge some of the risks.

In recent years a growing number of countries initiated pilot studies to examine the feasibility of introducing futures markets to allow hedging. Countries such as Mexico, Spain, Italy, Taiwan, Korea, and Israel, to name but a few, are actively pursuing the possibility of introducing a number of futures contracts such as index futures and foreign currency futures contracts. It is of utmost interest to these countries in particular and to any other importing country, to learn about the possible effects of introducing a futures contract that will allow an importing firm to hedge its risk. Of course, the exact effect on import, production and hedging depends on parameters such as the risk aversion of the importing firm and the volatility of the foreign exchange rates.

THE MODEL

Suppose that there exists a competitive, importing firm that maximizes its expected utility, v, of the stochastic terminal wealth, $\tilde{w}$, with respect to the imported input, x, used in the production function, $f(x)$ (where

$f' > 0; f'' < 0$), and a futures position, h, in the foreign exchange futures market. The hedging–h, in this article–is in terms of the local currency rather than a quantity of an underlying commodity. For example, if the firm is located in the U.S. and imports its input from Japan, it hedges in futures contracts, of dollars per yen. Thus, h could be something like \$100,000. For an exchange rate of 0.0095 dollars per yen this hedging represents about 10,526,316 yen. In this article's setting, there is no futures contract for the underlying imported input (as is the case in many real life situations). The firm can only hedge in the foreign currency futures market.[3] The foreign exchange spot price, $\tilde{g}$ and the foreign price of the imported input, $\tilde{p}_x$, are presumed to be mean reverting stochastic variables as follows:

$$\tilde{g} = \overline{g} + \sigma\tilde{\epsilon}; \quad \tilde{p}_x = \overline{p}_x + \gamma\tilde{\eta}; \quad E(\epsilon) = 0; \quad E(\eta) = 0;$$
$$E(\epsilon^2) = 1; \quad E(\gamma^2) = 1$$

where E is the expected value operator. The parameters $\overline{g}$ and $\overline{p}_x$ are the expected foreign exchange spot prices and the expected foreign price of the imported input, respectively, σ and γ are the standard deviations of the foreign exchange spot price and the foreign price of the imported input, respectively. It is also assumed that $\tilde{p}_x$ and $\tilde{g}$ are independent, which implies:[4]

$$E(\eta\epsilon) = 0 \quad \text{and} \quad E(\eta^2\epsilon^2) = 1$$

Rather than deal with the complexities resulting from imperfect correlation between spot and futures prices, and to concentrate on production and hedging behavior, basis risk is not considered.[5] This implies that the unknown future spot and futures exchange rates are identical. The wealth equation then becomes:

$$\tilde{w} = pf\{x\} - \overline{g}x\overline{p}_x - h(b - \overline{g}) + h\sigma\tilde{\epsilon} - x(\overline{p}_x\sigma\tilde{\epsilon} + \overline{g}\gamma\tilde{\eta} + \sigma\gamma\tilde{\epsilon}\tilde{\eta}) \tag{1}$$

[3] Many firms find themselves in real life unable to find futures contracts for the specific commodity they need to hedge. This scenario assumes that no futures contracts are available on the underlying imported input. However, there is a futures contract available on the exchange rate. Thus, the hedging firm can partially hedge the risk.

[4] This assumption is reasonable since, in general, most factors affecting the exchange rate are different from those affecting just one internationally traded commodity.

[5] Basis risk in the context of hedging is discussed extensively in articles such as Paroush and Wolf (1992) and Paroush and Wolf (1986), among others.

where p is the product price and b is the deterministic futures exchange rate in period one. The first order conditions are:

$$v_x = (pf' - \bar{g}\bar{p}_x)E(u') - \bar{g}\gamma E(u'\epsilon) - \bar{p}_x\sigma E(u'\epsilon) - \sigma\gamma E(u'\epsilon\eta) = 0 \quad (2)$$

$$v_h = (\bar{g} - b)E(u') + \sigma E(u'\epsilon) = 0 \quad (3)$$

Using a Taylor expansion around the expected wealth $\overline{w}$, where:

$$\overline{w} = pf(x) - x\bar{p}_x - h(b - \bar{g})$$

one obtains the following equalities for small risks:

$$E(u'\epsilon)/E(u') = -(h - x\bar{p}_x)\sigma R; \quad E(u'\eta)/E(u') = R\gamma\bar{g}x; \quad E(u'\epsilon\eta)/E(u') = \sigma\gamma Rx$$

where $R = -u''/u'$ is the Arrow–Pratt measure of absolute risk aversion which is assumed to be constant.[6] The substitution of the equations above in the first order conditions yields eqs. (4) and (5).

$$pf' - \bar{g}\bar{p}_x - \bar{g}^2\gamma^2 Rx + \sigma^2\bar{p}_x R(h - x\bar{p}_x) - \sigma^2\gamma^2 Rx = 0 \quad (4)$$

$$\bar{g} - b - \sigma^2 R(h - x\bar{p}_x) = 0 \quad (5)$$

COMPARATIVE STATICS OF IMPORT AND HEDGING

The effect of the model's parameters on hedging and import are examined in this section. Two propositions are introduced. The first one is on the effect of the variance of the exchange rate and risk aversion on import. The second proposition compares import and hedging in the presence of two (multiplicative) sources of uncertainty to the import and hedging under exchange rate uncertainty only.

[6]These equalities are based on the following approximations:

$$u'\{\tilde{w}\} \sim u'\{\overline{w}\} + (\tilde{w} - \overline{w})u''\{\overline{w}\} \quad \text{and} \quad E(u'\{\tilde{w}\}) \sim u'\{\overline{w}\}.$$

From eq. (5), the hedging process is derived as shown in eq. (6).

$$h = (\overline{g} - b)/\sigma^2 R + x\overline{p}_x \tag{6}$$

The hedging level is a function of the futures price structure, i.e., upward bias (backwardation) or downward bias (contango) of the expected spot price with respect to the futures price ($\overline{g} - b$), as well as the variance of the foreign currency exchange rate σ^2, and the degree of risk aversion R. When there is an upward bias, the higher the bias the higher is the hedging. The higher the variance of the foreign exchange price and the higher the degree of risk aversion the lower the hedging. These relationships can be verified from the first term in eq. (6). The expected cost of the imported input directly affects the level of hedging. The higher the expected price of the imported input the higher will be the level of hedging. Upon substitution of eq. (6) in eq. (4) one obtains, after some manipulations, eq. (7).

$$f' = \overline{p}_x b/p + x\gamma^2 R(\overline{g}^2 + \sigma^2)/p \tag{7}$$

Due to declining marginal productivity, the higher the marginal productivity, the lower the level of production. Thus, the effect of the model parameters on production can be verified from eq. (7). The production is inversely related to the determined futures price in the first period date b, inversely related to the expected spot price in the second period $\overline{g}$ and inversely related to the variance of the foreign exchange rate $- \sigma^2$.

The variances of exchange rates are used in many real life cases as measures of risks in the foreign currencies markets. It is then important to identify the effect of a change in the variance of the exchange rate on import and hedging. Using an econometric model where the utility function can be estimated along with the degree of risk aversion, the results may be useful in assessing the direction of change and even quantifying the change in import.[7]

Based on eqs. (6) and (7) the following proposition is derived that addresses the affect of the risk parameters on import hedging:

Proposition I. *The Variance of the exchange rate, σ^2, and risk aversion, R, have a negative effect on imports. Their effect on hedging, however, depends upon whether one faces price backwardation or contango, i.e. $(\overline{g} - b) > 0$, or < 0, respectively.*

[7] With the HARA (hyperbolic absolute risk aversion) utility function with appropriate risk parameters, it is possible to simulate the above-mentioned change. This is, however, beyond the scope of the article, and is offered merely as a suggestion for future research.

Proof. Using the second order conditions, the following results are used as a proof to the proposition:

$$\text{sign}(\partial x/\partial R) = \text{sign} - \sigma^2\gamma^r Rx(\overline{g}^2 + \sigma^2) < 0$$

$$\text{sign}(\partial h/\partial R) = \text{sign } pf'' = (\overline{g} - b)R - h\sigma^2\gamma^2 R(\overline{g}^2 + \sigma^2)$$

$$\text{sign}(\partial x/\partial \sigma^2) = \text{sign} - \sigma^2\gamma^2 R^2 x < 0$$

$$\text{sign}(\partial h/\partial \sigma^2) = \text{sign}(pf'' - R\gamma^2\overline{g}^2)(\overline{g} - b)/\sigma^2 - hR^2\gamma^2\sigma^2 \quad \text{Q.E.D.}$$

As can be seen from the proof above, the effect is clearly negative in backwardation. The reason for it is that, in backwardation, the firm overhedges, i.e., $h > x\overline{P}_x$. The amount of overhedging $h - x\overline{P}_x$ can be viewed as speculation. Thus, as the risk parameters increase, there is less incentive to speculate and therefore h declines. In contango, the effect on hedging is ambiguous. Since the firm underhedges initially, the effect of the risk parameters is to increase the hedging level. However, due to a lower input demand there is less need for hedging. If the second effect is stronger (weaker) that the first one, hedging will decline (increase). In comparison to Kerkvliet and Moffett (1991), this analysis provides an examination of the effect of the two risk parameters (not just one), i.e., the variance of the exchange rate and the risk aversion under different pricing structures. As the proposition above indicates, it is important to distinguish between a market in contango versus one in backwardation. A change in one of the risk parameters will have a different impact on the import of the input under different price structures.

Proposition II. *Import of the input and hedging under input price and exchange rate uncertainty are lower than those under just exchange rate uncertainty, if the expected input price under the multiplicative uncertainty, $\overline{p}_x^u$, is greater than or equal to the input price under just exchange rate uncertainty, p_x^c.*

Proof. Under exchange price uncertainty only, when input price is certain, the first order conditions yield $f'^c = bp_x^c/p$. The term on the right-hand side of this equation is smaller than or equal to the first term on the right-hand side of eq. (7) where there are two sources of uncertainty. Since the second term on the right-hand side of eq. (7) is positive; then, $f'^u > f'^c$, which implies that $x^u < x^c$. Based on the last inequality, it is verified from eq. (7) and $h^c = (\overline{g} - b)/\sigma^2 R + xp_x^c$, which is the optimal hedging when the input price in non-stochastic, that $h^u < h^c$. Q.E.D.

This proposition shows that, under the condition of $\overline{p}_x^u \geq p_x^c$, an additional source of uncertainty implies lower hedging. The explanation for this result is that the reduction in the derived demand for the input, which implies lower import, reduces the level of hedging. A priori, one would expect an increase in hedging when an additional source of uncertainty is added (ceteris paribus). However, the demand for the input is adjusted downward and, therefore, the negative "income effect" surpasses the positive "substitution effect" which causes hedging to decline. The portion of the proposition above concerning input relationship, i.e., $x^u < x^c$ resembles somewhat the results of Baron (1970), Leland (1972), and Sandmo (1971). This article, however, unlike the above three articles, shows that this property holds under a different framework and scenario, i.e., in this study's setting, there is uncertainty due to the random exchange rate, even when the inputs' price is certain. The case termed uncertain has a multiplicative form of two sources of uncertainty: the input's price and the exchange rate. Furthermore, in this model there is an additional market: the futures market. Thus, not only is a competitive firm reducing its output when an additional source of uncertainty is added, but it also reduces its hedging. Therefore, the proposition is an enhancement of the existing theory of the behavior of a competitive firm under uncertainty in the presence of a futures market.[8]

Another result is that, in the presence of a multiplicative form of two random variables, the separation property does not hold and the hedging and import (production) decisions are not independent of each other. Also, the import (production) decision depends on the degree of risk aversion and the variances of the exchange rate and the input's price. The Table of Comparative Statics (Table I) summarizes the effect of the model parameters on import and hedging.

SUMMARY

This study presents a model in which a competitive and risk averse importing agent, who faces a multiplicative form of uncertainty, maximizes expected utility by choosing an optimal level of an imported input and an optimal hedge in the futures market. It is found that in the presence of two sources of uncertainty, i.e., foreign exchange rate uncertainty and input's price uncertainty in a multiplicative form, when $\overline{p}_x^u \geq p_x^c$, the

[8]The conditions for the hedging under input price uncertainty to be higher than that under input price certainty are that $x^u < b(p_x^c - \overline{p}_x^u)/(\gamma^2 R(\overline{g}^2 + \sigma^2)$ and $\overline{p}_x^u < p_x^c$.
Under these conditions, $x^u > x^c$ (Since $f'^u < f'^c$) and, therefore, $h^u > h^c$. The derivation of these conditions is based on eqs. (6) and (7) and follows the same logic as that of the proof in the proposition.

TABLE I
A Table of Comparative Statics

Variable Parameter	x	h
p	$\text{sign}(\partial x/\partial p) = \text{sign } f'\sigma^2 R > 0$	$\text{sign}(\partial h/\partial p) = \text{sign } \sigma^2 \bar{p}_x R f' > 0$
$\bar{p}_x$	$\text{sign}(\partial x/\partial \bar{p}_x) = \text{sign } - b\sigma^2 R < 0$	$\text{sign}(\partial h/\partial \bar{p}_x) =$ $\text{sign } - \sigma^2 R[(xpf'' + \bar{p}_x b) - x\gamma^2 R(\bar{g}^2 + \sigma^2)]$
$\bar{g}$	$\text{sign}(\partial x/\partial \bar{g}) = \text{sign } - 2\bar{g}\gamma^2 R x < 0$	$\text{sign}(\partial h/\partial \bar{g}) =$ $\text{sign } \sigma^2\gamma^2 R(1 + \bar{g}^2/\sigma^2 - 2\bar{p}_x R\bar{g}x) - pf''$
R	$\text{sign}(\partial x/\partial R) =$ $\text{sign } - \sigma^2\gamma^2 R x(\bar{g}^2 + \sigma^2) < 0$	$\text{sign}(\partial h/\partial R) =$ $\text{sign } pf''(\bar{g} - b)R - h\sigma^2\gamma^2 R(\bar{g}^2 + \sigma^2)$
σ^2	$\text{sign}(\partial x/\partial \sigma^2) =$ $\text{sign } - \sigma^2\gamma^2 R^2 x < 0$	$\text{sign}(\partial h/\partial \sigma^2) =$ $\text{sign}(pf'' - R\gamma^2\bar{g}^2)(\bar{g} - b)/\sigma^2 - hR^2\gamma^2\sigma^2$
γ^2	$\text{sign}(\partial x/\partial \gamma^2) =$ $\text{sign } - \sigma^2 R^2 x(\bar{g}^2 + \sigma^2) < 0$	$\text{sign}(\partial h/\partial \gamma^2) =$ $\text{sign } - \sigma^2 R^2 x\bar{p}_x(\bar{g}^2 + \sigma^2) < 0$

derived demand for input and hedging levels are both lower compared to those in the presence of just exchange rate uncertainty. Also, when the two sources of uncertainty above exist simultaneously, the separation property does not hold, i.e., the production decision is not independent of the hedging decision and the risk parameters, while under just the exchange rate uncertainty the separation property holds. The effect of the exchange rate variance or the effect of risk aversion depends on the futures-expected spot price relationship. In backwardation, where $\bar{g} - b > 0$, the effect is negative; while in contango, where $\bar{g} - b < 0$, it is indeterminate.

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