PRICE LIMITS AS AN EXPLANATION OF THIN-TAILEDNESS IN PORK BELLIES FUTURES PRICES

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Knowing the price distribution is important in financial economics. Equilibrium models such as the capital asset pricing model, option pricing models, and portfolio models of asset allocation are based on assumptions about the distribution of prices.

Much research has attempted to approximate the return generating process in futures markets. Two important findings have emerged. First, the unconditional distribution of most futures returns is leptokurtic [e.g., Cornew, Town, and Crowson (1984); Hall, Brorsen, and Irwin (1989)]. A leptokurtic, or fat-tailed, distribution has more observations around the mean and in the extreme tails than a normal distribution. In practice, this implies that extremely high or low returns are more likely than with a normal distribution. Second, most futures returns are conditionally heteroskedastic [e.g., Connolly (1989); Fujihara and Park (1990); Yang and Brorsen (1993)]. This implies that current return variability can be explained by past information. Further, the conditional heteroskedasticity guarantees unconditionally

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leptokurtic returns [Engle, (1982)]. These findings are important in determining option premiums [e.g., Hull and White (1987); Myers and Hanson (1993)] or optimal hedge ratios [e.g., Myers and Thompson (1989)].

While most futures prices' unconditional distributions show fatter tails than a normal distribution, Yang and Brorsen (1994) show that the unconditional distribution of pork bellies futures price changes are thin-tailed; and, even more interestingly, these prices follow a GARCH process. These seemingly contradictory findings suggest the simple GARCH process cannot model the platykurtic pork bellies futures price movements.[1]

A distinct feature of the pork bellies futures market is that pork bellies prices are extremely volatile and hit price limits more frequently than prices in other markets. Brennan (1986) argues that price limits in futures markets are a partial substitute for margin requirements, which ensure contract performance by controlling default risk. However, the limits prohibit futures prices from adjusting fully to new information and reduce the number of observations in the tails. This may cause thin-tailed distributions since price changes are restricted by limits.

The objective of this study is to determine whether the thin-tailedness in pork bellies futures price changes results from price limits. For the test, a statistical model of the sample data is first developed and consistently estimated by incorporating the price limits in estimation. Due to price limits, the dependent variable is truncated, and ignoring it provides biased and inconsistent parameter estimates. This study models a two-limit tobit process with conditional heteroskedasticity. Such a process is called a "Tobit-GARCH" model.

Monte Carlo hypothesis testing procedures [Noreen (1989)] are used to test the null hypothesis that the pork bellies futures prices are generated by a Tobit-GARCH model. The null hypothesis of a GARCH model is also tested. The GARCH model is also simulated with price limits for comparison.

Past research examining price limit effects includes Khoury and Jones (1984) and Ma and Sears (1989). Sutrick (1993) examines alternative ways to estimate equilibrium prices that are subject to limit moves. Kodres (1993) examines the effect of price limits on tests of whether futures markets provide unbiased forecasts of foreign exchange spot prices. Kodres has estimated a Tobit-GARCH model.

[1] Yang and Brorsen (1994) find that even a detailed GARCH-t model can be rejected as a model of pork bellies futures price changes.

The estimation procedure used here is simpler than Kodres'. It also goes beyond Kodres by properly adjusting the GARCH equation for truncation in the mean process. No previous study has tested whether price limits can explain the thin-tailedness in futures markets. The results of this study are important for option pricing models and optimal hedge ratios, which require exact understanding of price dynamics.

UNDERLYING MODEL WITHOUT PRICE LIMITS

Let p_t^* be the daily equilibrium futures price. Suppose the data generating process of the futures price changes is

$$\begin{aligned} r_t^* &= x_t\beta + e_t^* \\ e_t^* \mid \Theta_{t-1} &\sim N(0, h_t^2) \end{aligned} \tag{1}$$

where $r_t^* = \log(p_t^*) - \log(p_{t-1}^*)$, x_t is a vector of predetermined variables which may include lagged endogenous variables, and β is the corresponding vector of parameters. e_t^* is the disturbance term, which follows a normal distribution with mean zero and variance h_t^2 conditional on past information Θ_{t-1}. This is the GARCH process and the unconditional distribution of e_t^* is leptokurtic [Bollerslev (1986)].

Day-of-the-week dummies are included to capture the different return and volatility for each day of the week. Autocorrelation in the mean equation, if ignored, causes inconsistent estimates in the presence of conditional heteroskedasticity. Akaike's (1981) information criterion suggests one lagged dependent variable. Following Bollerslev (1986), a GARCH (1, 1) process is used for the conditional variance equation. The model in eq. (1) becomes

$$\begin{aligned} r_t^* &= \alpha_0 + \alpha_1 r_{t-1}^* + \alpha_2 D_T + \alpha_3 D_W + \alpha_4 D_T + \alpha_5 D_F + e_t^* \\ h_t^2 &= \beta_0 + \beta_1 e_{t-1}^{*\,2} + \beta_2 h_{t-1}^2 + \beta_3 D_T + \beta_4 D_W + \beta_5 D_T + \beta_6 D_F \end{aligned} \tag{2}$$

where D is the dummy variable for each day-of-the-week (e.g., $D_M = 1$ if Monday and 0 otherwise, and so on; D_T, Tuesday; D_W, Wednesday; D_H, Thursday; D_F, Friday).

MODEL WITH PRICE LIMITS

The dependent variable, r_t^* in eq. (2) is an unobservable latent variable because of the price limits. Thus, the model is not directly estimable with conventional estimation methods. The observed return is truncated due to the price limits. The limited nature of the truncated dependent variable should be considered to obtain consistent estimates of the parameters.

Because of the price limits, p_t^* is not directly observable. Instead, the observed prices (p_t) are

$$\begin{aligned} p_t &= p_{t-1} + L \quad && \text{if } p_t^* \geq p_{t-1} + L \\ &= p_t^* && \text{if } p_{t-1} - L < p_t^* < p_{t-1} + L \\ &= p_{t-1} - L && \text{if } p_t^* \leq p_{t-1} - L \end{aligned} \tag{3}$$

where L is the limit that the price on day t can move from the previous day's closing price.

Let $LO_t = \log(p_t^*) - \log(p_t)$ be the price movement left over due to the price limits on day t. With no limit move, $p_t^* = p_t$ and $LO_t = 0$. LO_t is positive if the price hits the upper limit and is negative if the price hits the lower limit. Then, the observed return, r_t, can be expressed through a simple manipulation as

$$r_t = \log(p_t) - \log(p_{t-1}) = r_t^* - LO_t + LO_{t-1} \tag{4}$$

LO_{t-1} appears to capture the truncation on day $t - 1$. Both LO_t and LO_{t-1} are nonzero if the price hits a limit on two consecutive days t and $t - 1$. Substituting eq. (4) into the mean equation of (2) yields

$$\begin{aligned} r_t + LO_t = \alpha_0 + \alpha_1 r_{t-1} + \alpha_1(LO_{t-1} - LO_{t-2}) + LO_{t-1} \\ + \alpha_2 D_T + \alpha_3 D_W + \alpha_4 D_T + \alpha_5 D_F + e_t^* \end{aligned} \tag{5}$$

LO_{t-1} appears in the right-hand side since it is predetermined on day t. If the left-over can be measured exactly, the parameters can be estimated as a GARCH process. However, the left-over is not observable (in fact, this makes the true return unobservable) and causes a measurement-errors-in-variables problem. To cope with the econometric problem, a proxy for the left-over can be used.

The model in eq. (5) includes measurement errors in both sides of the equation. Following Maddala (1977), Kodres (1993) notes that using a proxy to correct errors in both the dependent and independent variables compounds the econometric problem and, therefore, uses a proxy constructed with the weighted spot price for the measurement errors in regressors and a tobit model for the limited dependent variable. This study follows Kodres's approach and uses a two-limit tobit model in which errors follow a GARCH process.

It should be noted that e_t^* in eq. (5) is also truncated and unobservable when the limited dependent variable is modeled as a tobit process (in contrast to using a proxy for the left-over in the dependent variable). Thus, e_{t-1}^* in the variance equation in (2) is also truncated.[2] From eqs. (1) and (4),

$$e_t^* = r_t + LO_t - LO_{t-1} - x_t\beta = e_t + LO_t \tag{6}$$

and the variance equation in (2) becomes

$$\begin{aligned} h_t^2 = \beta_0 + \beta_1(e_{t-1} + LO_{t-1})^2 + \beta_2 h_{t-1}^2 + \beta_3 D_T + \beta_4 D_W \\ + \beta_5 D_T + \beta_6 D_F \end{aligned} \tag{7}$$

As in Kodres (1988), information from the spot market is used for the proxy. However, since no appropriate adjusting factor is suggested for the pork bellies markets, this study allows the factor to be estimated together with other parameters. The left-over variable is constructed as

$$LO_t = \gamma_0 d_t + \gamma_1 d_t[lc_t - lp_t] \tag{8}$$

where d_t is one if the p_t^* hits a limit on day t and is zero otherwise. lc_t is the log cash price and lp_t is the log futures price, and γ's are factors adjusting the discrepancy between the futures and spot markets. The left-over variable is not subject to upper or lower limits. A symmetric response is assumed. Substituting eq. (8) into (5) and (7) yields the model to be estimated as

$$\begin{aligned} r_t + LO_t &= \alpha_0 + \alpha_1 r_{t-1} + \gamma_0 d_{t-1} + \gamma_1 d_{t-1}(lc_{t-1} - lp_{t-1}) \\ &\quad + \alpha_1[\gamma_0(d_{t-1} - d_{t-2}) + \gamma_1(d_{t-1}(lc_{t-1} - lp_{t-1}) \\ &\qquad - d_{t-2}(lc_{t-2} - lp_{t-2}))] \\ &\quad + \alpha_2 D_T + \alpha_3 D_W + \alpha_4 D_T + \alpha_5 D_F + e_t^* \\ h_t^2 &= \beta_0 + \beta_1[e_{t-1} + \gamma_0 d_{t-1} + \gamma_1 d_{t-1}(lc_{t-1} - lp_{t-1})] \\ &\quad + \beta_2 h_{t-1}^2 + \beta_3 D_T + \beta_4 D_W + \beta_5 D_T + \beta_6 D_F \end{aligned} \tag{9}$$

[2] Kodres (1993) does not correct for the measurement error in e_{t-1} in the variance equation.

Kodres (1993) uses a bivariate normal distribution in the likelihood function to model more than two consecutive limit moves. The resulting likelihood function implies that unexplained returns (i.e., error terms) of two consecutive days are correlated if the previous day's price also hits a limit. Intuitively, information remaining from the previous consecutive limit days affects the current "mean" price path if it is still important. Further, if the proxy variable represents the left-over correctly, special treatments for multiple day limit moves are not necessary.[3]

The Tobit-GARCH Model

The mean equation in (5) can be rewritten as

$$r_t + LO_t = z_t\delta + e_t^* \tag{10}$$

where $z_t\delta = x_t\beta + LO_{t-1}$. Since e_t^* is assumed normally distributed, the probabilities of upper, no, and lower limit moves are, respectively,

$$\begin{aligned} \text{prob(upper)} &= \text{prob}(r_t - z_t\delta < e_t^*) = 1 - \Phi[(r_t - z_t\delta)/h_t] \\ \text{prob(no)} &= 1/h_t\phi[(r_t - z_t\delta)/h_t] \\ \text{prob(lower)} &= \text{prob}(r_t - z_t\delta > e_t^*) = \Phi[(r_t - z_t\delta)/h_t] \end{aligned} \tag{11}$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are, respectively, the distribution and density functions of the standard normal. Therefore, the log-likelihood function of the Tobit-GARCH model is

$$\begin{aligned} L = {} & \Sigma_t UL_t[1 - \Phi((r_t - z_t\delta)/h_t)] + \Sigma_t LL_t\Phi((r_t - z_t\delta)/h_t) \\ & + \Sigma_t(1 - UL_t)(1 - LL_t)[(2\pi h_t^2)^{-1/2}\exp(-(r_t - z_t\delta)^2/2h_t^2)] \end{aligned} \tag{12}$$

where $UL_t = 1$ if the price hits the upper limit and 0 otherwise, and $LL_t = 1$ if the price hits the lower limit and 0 otherwise. If all of UL_t and LL_t are zero, then the log-likelihood function in (12) becomes that of the GARCH process. When UL_t or LL_t is not zero but ignored in the likelihood function, then the GARCH estimates would be biased, even asymptotically.

Since the first order derivatives of the Tobit-GARCH likelihood function are highly nonlinear, an iterative numerical method must be

[3]For completeness, this study also models multiple limit moves in the left-over variable. The variables representing the multiple limit moves are not significant at all.

used. This study uses the algorithm developed by Berndt et al. (1974) and is written in FORTRAN. This quasi-Newton method estimates the Hessian at each iteration using the cross-product of the first derivatives.

TEST PROCEDURES

The estimated model is simulated with and without price limits. Thus, simulated values of r_t and r_t^* are obtained.[4] Under the null hypothesis that the Tobit-GARCH model is the true model, the moments of actual and simulated r_t should be the same. The testing procedure is a Monte Carlo hypothesis testing procedure and uses the first four moments as the test statistics.

The approach obtains the distribution of the test statistics under the null hypothesis (i.e., the Tobit-GARCH model) using Monte Carlo simulation. First, samples of r_t and r_t^* of size equal to the original data are obtained with the estimated parameters and stochastic simulation. The initial conditions are the same as with the original data. Then, the first four moments of both simulated r_t and r_t^* are computed. These steps are repeated 1000 times to obtain 1000 estimates of the four moments of both r_t and r_t^*. The distribution of each of the four moments under the null hypothesis is then obtained using the 1000 estimates of the four moments.

The Monte Carlo hypothesis testing procedure is a nonparametric approach, which makes no assumption about the distributions of the test statistics. The 1000 estimates of each moment are sorted by ascending order to construct confidence intervals. The Tobit-GARCH model is well calibrated if the four moments of truncated returns (i.e., r_t) contain those of the actual data within 95% or less confidence intervals. Finally, if the observed thin-tailedness of the return in eq. (1) is due to the price limits, then the kurtosis of r_t should be less than zero (i.e., thin-tailed) while that of r_t^* should be larger than zero (i.e., fat-tailed).

The GARCH process is accepted in the literature as a promising model of most financial price changes. However, Kodres (1993) is the only other study using a GARCH model which takes price limits into account. To demonstrate the consequence of ignoring price limits, two sets of returns are generated from the simple GARCH (1, 1) model: one set which completely ignores the price limits and another set which

[4]Allowing price limits in simulation should be distinguished from that in estimation. In estimation the price limits enter into the log-likelihood function of the Tobit-GARCH model. In simulation the simulated price is truncated if its change is larger than the limits, and the left-over is added to the next day's price change.

allows the price limits in simulation. Using the same procedure of validating the Tobit-GARCH model, the null hypothesis that the pork bellies futures price data can be appropriately modeled as a GARCH process is tested.

DATA DESCRIPTION

This study considers the pork bellies futures market. Thin-tailedness is found also in other livestock futures markets in which price limits are frequently binding [Yang and Brorsen (1994)].[5] However, the percentage of limit moves is the largest with pork bellies.

The price changes are defined as the first differences in daily logarithmic closing prices. To maintain a continuity of data and minimize differences in the maturity of contracts, the data set consists of the changes in logarithmic prices of futures contracts until the third Tuesday of the month prior to delivery, after which the logarithmic changes in the next nearest delivery month are used. Since differences are taken before splicing, no outlier is created at rollover. The data are from the Dunn & Hargitt Commodity Data Bank. This is a commercial database that is routinely checked for errors and thus should be reasonably error free.

Table I reports the summary statistics of the sample data. The sample period for the analysis is January, 1985 to December, 1988. The sample data are not skewed but have thinner tails than a normal distribution at the 1% significance level. During the sample period, the closing price is at the limits 142 of 1012 trading days or 14% of the time. The upper limit is hit 65 times and the lower limit is hit 76 times.

TABLE I

Summary Statistics of Pork Bellies Futures Price Changes

					Limit Moves	
N	*Mean*	*Variance*	*Skewness*	*Kurtosis*	*Upper*	*Lower*
1012	−0.080	3.744	−0.037	−0.553[a]	65	76

Note: Kurtosis is the relative measure. If normal, it is 0. $R_t = [\log(p_t) - \log(p_t - \Delta p_t)] \cdot 100$. It is multiplied by 100 to avoid the scaling problem and to express the return as a percentage term. The daily price limit is two cents per pound (or $800 per contract) above or below the previous day's closing price.

[a]Denotes rejection of the null hypothesis of a normal distribution at the 5% level.

[5]The distributions of feeder cattle, live cattle, and lumber futures prices are also thin-tailed.

TABLE II
Estimated Tobit-GARCH and GARCH Models

	Tobit-GARCH		*GARCH*	
		Mean		
Intercept	-0.366^{a}	(-2.49)	-0.291^{a}	(-2.24)
r_{t-1}	0.077^{a}	(2.07)	0.069^{a}	(2.14)
D_T	0.180	(0.85)	0.125	(0.67)
D_W	0.447^{a}	(2.11)	0.369^{a}	(1.97)
D_H	0.336	(1.57)	0.239	(1.29)
D_F	0.486^{a}	(2.38)	0.408^{a}	(2.22)
D_{t-1}	-0.175	(-1.41)		
$D_{t-1}(lc_{t-1} - lp_{t-1})$	0.165	(0.19)		
		Variance		
Intercept	0.945^{a}	(2.02)	0.597	(1.32)
$(e_{t-1} + LO_{t-1})^2$	0.054^{b}	(2.87)	0.048^{a}	(2.27)
h_{t-1}^2	0.933^{b}	(33.17)	0.909^{b}	(18.14)
D_T	-0.524	(-0.64)	-0.324	(-0.44)
D_W	-0.969	(-1.21)	-0.414	(-0.55)
D_H	-0.700	(-1.09)	-0.631	(-1.00)
D_F	-1.935^{a}	(-2.54)	-0.842	(-1.22)
Log-likelihood	-2071.447		-2080.474	
Day-of-the-week effects[c]:				
Mean		27.80^{b}		21.65^{a}
Variance		6.82		7.52

Note: Numbers in parentheses are t-values.
[a]Significance at the 5% level.
[b]Significance at the 1% level.
[c]Log-likelihood ratio test statistics for the joint hypotheses of the day-of-the-week effects (Ho: $D_i = 0$, for all i = Tuesday, Wednesday, Thursday, and Friday).

RESULTS

Estimated Results

Table II reports the estimated Tobit-GARCH and GARCH models. The GARCH model is estimated without the left-over variable because the model does not consider the price limits.

In the Tobit-GARCH model, the estimated ARCH and GARCH terms are significant at the 5% levels and indicate wide-sense stationarity [Bollerslev (1986)]. Monday returns (intercept in the mean equation) are the lowest and Friday returns are the highest followed by Wednesday returns. These results confirm the "Black Monday" or weekend anomaly in the pork bellies futures market. Though not significant as a whole from the log-likelihood ratio test, the day-of-the-week effect is indicated also in volatility. Monday returns are the most volatile while Friday returns show the least variability.

Neither γ_0 nor γ_1 is statistically significant at conventional levels. In addition, the log-likelihood test statistic for the joint hypothesis (not reported) is 2.5 indicating that the lagged left-over variables are not important in explaining the mean process. These results imply that the market responds so quickly and information left from the previous limit days has little or no effect on today's closing price.[6] While considering the truncation in the dependent variable is important, worrying about inclusion of the left-over term in the mean and variance equations is shown to be relatively unimportant.

Ignoring the limited nature of the dependent variable would lead to biased and inconsistent estimates. The GARCH process tends to understate the parameter estimates (Table II), i.e., all parameter estimates are smaller in absolute magnitude than those of the Tobit-GARCH counterparts. The standard errors from the GARCH process are also larger than those from the Tobit-GARCH estimation. The results imply that when price limits are important, ignoring them provides biased inferences.

Hypothesis Test Results and Model Validation

The simulated results for r_t and r_t^* from the Tobit-GARCH model are reported in Table III. The simulated r_t contains all of the first four moments of the original data within the 95% confidence interval. This is also true for the number of limit moves. Thus, the Tobit-GARCH model cannot be rejected as a model of pork bellies futures prices. Except for variance and skewness, the other two moments are even included within 50% (25 percentile to 75 percentile) or less intervals. The percentile of the actual skewness is 17.3%. However, the simulated measures of skewness contain zero at the 44.6 percentile. This implies that the skewness generated by the Tobit-GARCH model is not statistically different from zero. The result is consistent with the original data, which distribute symmetrically around zero (Table I). The Tobit-GARCH model is well calibrated and successfully approximates the sample data generating process.

The average kurtosis of simulated returns with price limits (r_t) is −0.778 indicating thin-tailedness, and, in 858 out of 1000 cases

[6]The results may result also from the failure of the cash price as a proxy for the equilibrium futures price. The pork bellies futures market may not be as closely tied to the cash market as other futures markets.

TABLE III

Percentiles of the Four Moments of the Simulated Returns from the Tobit-GARCH Models

					Limits	
	Mean	*Variance*	*Skewness*	*Kurtosis*	*Upper*	*Lower*
Tobit-GARCH without price limits (r_t^*)						
0% (min.)	0.446	4.888	0.700	0.317		
5%	0.253	6.103	0.185	−0.029		
10%	0.215	6.532	0.131	0.059		
25% (Q1)	0.150	7.274	0.076	0.213		
50% (median)	0.076	8.480	0.001	0.430		
75% (Q3)	0.013	9.801	0.057	0.735		
90%	0.055	11.476	0.132	1.132		
95%	0.090	12.860	0.174	1.437		
100% (max.)	0.347	24.226	0.426	6.224		
Actual	48.3%	OB	37.4%	OB		
Zero	53.2%		50.7%	7.0%		
Tobit-GARCH with price limits (r_t)						
0% (min.)	0.417	1.465	−0.601	1.655	5	10
5%	0.222	2.428	0.081	1.485	21	31
10%	0.187	2.846	0.056	1.390	31	41
25% (Q1)	−0.128	3.831	−0.024	1.170	63	72
50% (median)	0.062	4.897	0.006	0.778	111	118
75% (Q3)	0.010	6.265	0.038	0.261	171	173
90%	0.024	7.574	0.076	0.140	229	223
95%	0.044	8.289	0.102	0.412	253	246
100% (max.)	0.100	17.650	0.246	3.938	343	306
Actual	42.2%	23.3%	17.3%	60.6%	27.1%	26.6%
Zero	80.0%		44.6%	85.8%		

Note: OB denotes out of bounds.

(85.8%) the returns have kurtosis less than zero. On the other hand, the median kurtosis of simulated r_t^* is 0.430 indicating leptokurtosis. Further, the percentile of the normal kurtosis (zero) is 7.0%, and, in 70 out of 1000 cases, r_t^* has kurtosis greater than zero. The hypothesis that r_t^* follows unconditionally a normal distribution is rejected against the alternative hypothesis of leptokurtosis at the 7.0% significance level.

In sum, these results suggest that the distribution of daily pork bellies futures price changes would have had fat tails without price limits. The thin-tailedness observed in the original data results from the price limits.

TABLE IV

Percentiles of the Four Moments of the Simulated Returns from the GARCH Models

	Mean	*Variance*	*Skewness*	*Kurtosis*
		GARCH without price limits		
0% (min.)	0.280	2.595	0.476	−0.321
5%	0.172	3.129	−0.156	0.153
10%	0.146	3.224	−0.122	0.094
25% (Q1)	0.108	3.403	0.065	0.002
50% (median)	0.068	3.620	−0.005	0.120
75% (Q3)	0.025	3.848	0.052	0.272
90%	0.021	4.122	0.107	0.442
95%	0.051	4.260	0.141	0.557
100% (max.)	0.124	4.797	0.308	1.931
Actual	43.2%	65.0%	35.9%	OB
Zero	85.3%		53.1%	25.1%
		GARCH with price limits		
0% (min.)	0.244	1.283	0.171	−1.404
5%	0.159	2.022	0.082	−1.182
10%	0.137	2.285	−0.063	−1.078
25% (Q1)	0.099	2.622	−0.029	0.849
50% (median)	0.058	2.962	0.004	0.536
75% (Q3)	0.017	3.256	0.035	0.284
90%	0.016	3.482	0.073	−0.121
95%	0.034	3.622	0.093	0.002
100% (max.)	0.099	4.171	0.203	0.739
Actual	34.6%	97.7%	20.7%	48.9%
Zero	83.6%		46.0%	94.5%

Note: OB denotes out of bounds.

What would be the effect of ignoring the price limits when they are important? The simulated returns from the GARCH models are summarized in Table IV. The first panel shows the first four moments of data generated from the GARCH model without accounting for price limits in simulation. This is what most research seems to consider. The results indicate that the simulated returns contain the first three moments, mean, variance, and skewness within the 95% confidence interval. However, this model unambiguously overstates kurtosis. All kurtosis coefficients of the GARCH simulated returns are larger than that of the original data. Therefore, the GARCH model can be rejected as a model of pork bellies futures price changes.

For comparison the same statistics are reported for the data generated from the model that does not consider price limits in estimation but includes them in the simulation. The second panel of Table IV indicates that the actual mean, skewness, and kurtosis are contained within the 95% confidence interval. However, the GARCH model generates variances biased downward, that is, 977 of 1000 variances are less than that of the original data. In sum, the simple GARCH model which does not consider price limits in estimation generates overstated kurtosis when the limits are not included in simulation and understated variance when the limits are included in simulation.

SUMMARY AND CONCLUSIONS

This study supports the hypothesis that price limits cause the thin-tailedness observed in the distribution of pork bellies futures prices. The Tobit-GARCH model, when simulated without price limits, generates leptokurtic returns. However, when the price limits are included in the simulation, the distribution is platykurtic. The institutional restriction of price limits explains the observed thin-tailedness in daily pork bellies futures price changes.

The Tobit-GARCH model is not rejected as a model of pork bellies futures prices. This is not the case for the simple GARCH process. The GARCH model is a special case of the Tobit-GARCH model. Conceptually, the Tobit-GARCH model is suggested (over the GARCH) for markets with price limits. This is even true for prices that have unconditionally leptokurtic distributions. Pork bellies represent an extreme case with price limits being hit 14% of the time. For some markets, price limits may be hit so seldom that ignoring them makes no discernible difference. Yet, econometric theory suggests the Tobit-GARCH model for consistent estimates. The Tobit-GARCH model should certainly be used for price series that are more frequently restricted by price limits and have unconditionally thin-tailed distributions.

The Tobit-GARCH model is likely to be more important in option pricing than in testing hypotheses. Hypothesis tests (Table II) yield the same conclusions with the GARCH and Tobit-GARCH models. However, stochastic simulations show that the moments of the two models can be quite different, which suggests that considering price limits could lead to more accurate option pricing models.

The option pricing model by Myers and Hanson (1993) should consider the Tobit-GARCH model for commodities that hit limits frequently. The key to pricing options is estimating variance. The predicted variances of the returns from the GARCH process (median = 3.62) are much smaller than those for the true but unobservable returns r_t^* (median = 8.48).

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