A RISK-RETURN MEASURE OF HEDGING EFFECTIVENESS: A SIMPLIFICATION

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INTRODUCTION

The four distinct approaches to hedging, as summarized by Gray and Rutledge (1971), can still describe most recent theoretical and empirical works.[1] In the portfolio-based modeling approach, Howard and D'Antonio (1984, 1987), Nelson and Collins (1985), and Chang and Shanker (1987), among others, advocate a single parameter measure of hedging performance that takes into account both expected returns and risk. The model is built on the risk–return separation, making it useful to individuals with different degrees of risk aversion.

The first article of Howard and D'Antonio (HD) has become one of the most referenced in this subset of futures research. Chang and Shanker contribute by pointing out the limiting cases for HD. This article further contends that HD have inappropriately defined a μ_f, the "rate of return" on futures, in the derivation of the optimal hedge ratio. As a result, the hedge ratio contains an initial futures to spot price

[1]An exception includes the Mean-Gini Approach, such as the model proposed by Cheung, Kwan, and Yip (1990).

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The Journal of Futures Markets, Vol. 15, No. 1, 39–44 (1995)

CCC 0270-7314/95/010039-06

ratio that makes the choice of historical data and estimation awkward. This article shows how the HD model can be simplified and shows how the optimal Hedge Ratio can be more easily estimated.

AN UNNECESSARY TERM IN THE OPTIMAL HEDGE RATIO

HD attempt to highlight the role of rate of return in futures hedging. However, the notation of the rate of return on futures is a tricky one. Since no cash is paid up front, futures traders lack a basis to evaluate investment performance. The initial margin deposited is not an "investment" since the trader retains the ownership of the deposit, be it cash or any legitimate substitute put up as a margin. Therefore, it is not surprising to find that different approaches have been suggested while none of them has received universal acceptance. For example, Reilly (1989, pp. 968) computes rates of return on the initial margin, while Figlewski (1984 and 1985) and Cheung, Kwan, and Yip (1990) use the initial futures price. It is this latter approach that HD use to define the expected futures return μ_f. However, this definition causes problems in that it results in an awkward choice of historical data when one attempts to estimate the optimal hedge ratio. To illustrate, consider the rate of return on the hedged position in HD

$$r_h = \frac{Q_s(P_{s1} - P_{s0}) + Q_f(P_{f1} - P_{f0})}{Q_s P_{s0}} \tag{1}$$

HD rearrange this as

$$r_h = r_s + r_0 b^* r_f$$

where P_{st} represents the spot price at time t, P_{ft} represents the futures price at time t, Q_f is the quantity of futures, and Q_s is the quantity of spot,

$$r_s = \frac{P_{s1} - P_{s0}}{P_{s0}}$$

$$r_0 = \frac{P_{f0}}{P_{s0}}$$

$$b^* = \frac{Q_f}{Q_s}$$

$$r_f = \frac{P_{f1} - P_{f0}}{P_{f0}}$$

The resulting optimal hedge ratio is given by

$$b^* = \frac{\Gamma_0 - r_{sf}}{r_0 \pi_0 (1 - \Gamma_0 r_{sf})}$$

where

$$\Gamma_0 = \frac{\alpha_0}{\pi_0} = \frac{\mu_f}{\sigma_f} \Big/ \frac{\mu_s - i}{\sigma_s}$$

μ_f and μ_s are the expected values of r_s and r_f,

σ_f and σ_s are the standard deviations of r_s and r_f,

$i =$ the riskfree rate,

$\alpha_0 = \mu_f / \mu_s - i$

$\pi_0 = \sigma_f / \sigma_s$

$$r_{sf} = \frac{\sigma_{sf}}{\sigma_s \sigma_f}$$

the correlation coefficient of r_s and r_f.

If the above μ_f is dropped and an alternative r_s is defined, then (1) can be straightforwardly expressed as

$$r_h = r_s + b^* r_a \tag{3}$$

where

$$r_a = \frac{(P_{f1} - P_{f0})}{P_{s0}}$$

and

$$\mu_a, \sigma_a$$

are the expected value and the standard deviation, respectively.

With this alternative definition, it can be shown that the resulting b^* from the problem formulated by HD is as follows (see Appendix)

$$b^* = \frac{\Gamma - r_{sa}}{\pi(1 - \Gamma r_{sa})} \tag{4}$$

where

$$\pi = \sigma_a/\sigma_s$$

$$r_{se} = \frac{\sigma_{sa}}{\sigma_s \sigma_a}$$

$$\Gamma = \frac{\alpha}{\pi} = \frac{\mu_a}{\sigma_a} \Big/ \frac{\mu_s - i}{\sigma_s}$$

$$\alpha = \mu_a/\mu_s - i$$

Comparing (2) and (4), the notable difference is the presence of r_0 in (2), although both are ex ante, or mathematically, identical. The r_0, as defined in HD, is the ratio of current price per unit for the futures and spot. Since it is a common practice for hedgers to estimate their hedge ratios from historical data, the existence of r_0 in (2) will cause difficulties concerning which particular value of P_{f0} and P_{s0} to choose. For example, suppose one has historical data of 52 weekly returns, he or she can compute average rates of return and standard deviations for use. However, which futures and spot prices within the 52 weeks should be chosen to compute r_0 is an open question. Equation (4), on the other hand, relieves this problem of choice.[2]

One may argue that it is better to use the current futures/spot price ratio and leave the estimations of other parameters in (2) to historical data. This procedure has problems, too. For example, if a firm learns that it needs to initiate an identical hedging problem only several days after it has initiated a hedge, using futures/spot price ratios from different days in the two hedges will result in different hedge ratios.[3] Apparently, there is no reason to change an estimate of the optimal hedge ratio within just a few days.

CONCLUSION

This article points out that HD inappropriately define a rate of return on futures in their derivation of the optimal hedge ratio. It makes the estimation of the optimal hedge ratio difficult when one attempts to use historical data. The suggested simplification will benefit practitioners as well as empirical researchers.

[2] The initial futures to spot price ratio does have the advantage of providing the information on the initial basis.

[3] This is not an uncommon case. For example, a firm may need to hedge two consecutive foreign exchange cash inflows when the foreign exchange market is extremely volatile.

APPENDIX

Since

$$r_h = r_s + b^* r_a \tag{A1}$$

Therefore,

$$E(r_h) = \mu_s + b^* \mu_a$$
$$\mathrm{Var}(r_h) = \sigma_s^2 (1 + b^* \pi^2 + 2b^* \pi r_{sa})$$

where

$$\pi = \sigma_a / \sigma_s$$
$$r_{sa} = \frac{\sigma_{sa}}{\sigma_s \sigma_a}$$

Therefore,

$$E(r_h) - i = \mu_s - i + b^* \mu_a$$
$$= (\mu_s - i)(1 + \alpha X_n)$$

where

$$\alpha = \frac{\mu_a}{(\mu_s - i)}$$

Following HD, one can maximize

$$\Theta_h = \frac{E(r_h) - i}{\sigma(r_h)}$$
$$= \frac{\Theta_s (1 + \alpha X_n)}{I} \tag{A2}$$

where

$$\Theta_s = \frac{(\mu_s - i)}{\sigma_s}$$
$$I = (1 + X_n^2 \pi^2 + 2b^* \pi r_{sa})^{1/2}$$

Differentiating Θ_h with respect to X_n, and setting the result equal to zero results in

$$\Gamma I^2 = (1 + \alpha X_n)(b^* \pi + r_{sa})$$

where

$$\Gamma = \frac{\alpha}{\pi} = \frac{\mu_a}{\sigma_a} \Big/ \frac{\mu_s - i}{\sigma_s}$$

Finally, substituting I into the above and rearranging, one obtains

$$b^* = \frac{\Gamma - r_{sa}}{\pi(1 - \Gamma r_{sa})} \tag{A3}$$

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