EXPORTING AND HEDGING DECISIONS WITH A FOWARD CURRENCY MARKET: THE MULTIPERIOD CASE

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INTRODUCTION

Foreign exchange rates have been volatile since the fall of the Bretton Woods system, or fixed exchange rate system, in the 1970s. This volatility has led to greater commodity price variability [Kofman and Viaene (1990)] and has had an effect on the patterns of international trade [Thursby and Thursby (1985)]. Thus, forward currency markets have become an important exchange rate stabilizing and risk management mechanism for firms involved in international trade by supplying exporting firms with the necessary markets to hedge against exchange rate risk.

Several studies have shown, using a static framework, that when a forward currency market exists, a risk averse firm's production decision is independent of the firm's attitude toward risk as well as its subjective probability of the future spot exchange rate [see, e.g., Ethier (1973); Baron (1976); Katz and Paroush (1979); Paroush

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and Wolf (1992)].[1] These studies have also demonstrated that the firm's optimal hedge is a direct function of the bias of the forward currency market. Specifically, a risk averse firm will over (fully, under) hedge when the forward currency rate is greater than (equal to, less than) the expected spot exchange rate. Finally, these studies have shown that the introduction of an unbiased forward currency market leads to an increase in the firm's optimal export level.

Several authors have extended the separation result by demonstrating that it also holds: for a perfectly competitive producer who faces both exchange rate and output price risk when the firm has access to both a forward currency market and a futures commodity market [Kawai and Zilcha (1986)]; for a firm that is not an expected utility maximizer or is not risk averse [Safra and Zilcha (1986)]; for a price discriminating firm that sells its produce in both the domestic and world markets [Eldor and Zilcha (1987); Broll and Zilcha (1992)]; and, for a risk averse firm which faces multiple sources of revenue risk. However, as Katz (1984) shows, the separation result is not valid when the forward currency market is not perfectly competitive. Broll and Wahl (1992) extend the separation result to the case where the firm hedges its static exchange rate risk with a currency option contract and show that the firm fully hedges its export revenues if and only if the risk premia of the currency put is equal to the risk premia of the call options. In a more dynamic framework, Zilcha and Eldor (1991) use a two period model to demonstrate that the separation result is not valid for a competitive exporting firm that chooses its capital level once and for all and that the firm tends to over hedge in both time periods when the exchange rates are positively correlated over time and the forward currency market is unbiased.

The purpose of this study is to investigate, using a multiperiod framework, the effects of introducing a perfectly competitive and possibly biased forward currency market on the export and hedging decision of a risk averse firm that exports its non-stochastic production and faces uncertain exchange rates.[2] In light of the empirical evidence which rejects the unbiasedness hypothesis [see, e.g., Hodrick (1981, 1987), and Modjtahedi (1991)], the model does not, a priori, impose unbiasedness

[1] This separation result has also been derived for a firm operating in a closed economy which faces output price uncertainty and has access to a forward-futures commodity market [see, e.g., Holthausen (1979) and Feder, Just, and Schmitz (1980)].

[2] The assumption that the firm is risk averse is not new. Several studies have suggested that managers may be risk averse and, hence, seek to minimize total firm risk [Helfat (1988)].

and allows for the existence of a contango and normal backwardation forward currency markets. The results obtained are applicable to an importing firm since the importer's decision problem is, basically, the same as the exporter's once the agent's profits have been redefined. Further, the results do not rely on the assumption of perfect competition in the international commodity market as long as the firm remains a price taker in the forward currency market.[3]

The structure of the article is as follows. The second section presents the general model and derives the optimal export level. The optimal multiperiod hedging decisions are analyzed in the third section.

OPTIMAL EXPORTING DECISIONS

Consider a risk averse producer who determines its optimal export level and forward currency hedge prior to the realization of the uncertain spot exchange rate. It is assumed that the firm maximizes her/his expected von-Neumann Morgenstern utility of uncertain wealth, where uncertain wealth is defined as the present value of the future stream of uncertain profits.[4] Stochastic profits at time t are given by the difference between the firm's random revenue, due to the export of its product and the forward currency sales, and its deterministic costs of production. Formally, the producer's decision problem can be represented as

$$\underset{\{q_t, y_t\}}{\text{Max}} \quad EU\left(\sum_{t=1}^{T} \delta^{t-1}\tilde{\pi}_t\right)$$

s.t.

$$\tilde{\pi}_t = \tilde{e}_t p_t q_t + (e_t^f - \tilde{e}_t)y_t - C(q_t) \quad \forall\, t \qquad (1)$$

where δ represents the firm's discount factor for random profits earned in period t; p_t denotes the international price of the commodity at time t;

[3]For example, the separation result holds as long as the firm's decision problem is linear in the stochastic exchange rate and forward exchange rate. This linearity exists whenever the forward currency market is perfectly competitive and is independent of the structure of the international commodity market [Broll and Zilcha (1992)].

[4]The assumption that the producer maximizes his/her expected utility of uncertain wealth implies that s/he is forward looking; that is, uncertainty of future periods may modify actual optimal decisions. It is important to note that this choice of maximand affects the producer's optimal multiperiod hedge but not his/her production and export decisions. An alternative to this decision framework, is one where it is assumed that the producer maximizes the expected present value of his/her von-Neumann Morgenstern utility of each period's random profits (i.e., $\sum_{t=1}^{T} EU(\tilde{\pi}_t)$). This maximand implies that the producer determines period's t optimal export and hedging decisions without taking into consideration future random effects. Thus, the multiperiod model would be reduced to a sequential series of static decision problems since the production process is assumed to be time separable, and the properties of the multiperiod model would be the same as those of the static model.

$\tilde{e}_t$ and e_t^f are the stochastic spot exchange rate and the forward exchange rate from date $t-1$ to date t, respectively; q_t is the firm's optimal export level committed to before the resolution of uncertainty (i.e., before $\tilde{e}_t$ is known); and x_t represents the forward sale of foreign currency by the firm in date $t-1$ to be delivered in date t. The cost function, $C(q_t)$, is assumed, as in most models, to be non-decreasing and convex in q_t.

Assuming an interior solution exists for all t, the necessary and sufficient first order conditions[5] associated with (1) are

$$E\{(\tilde{e}_t p_t - C'(q_t)) U'(\Pi^*) \mid \Omega_t\} = 0 \tag{1a}$$

$$E\{(e_t^f - \tilde{e}_t) U'(\Pi^*) \mid \Omega_t\} = 0 \tag{1b}$$

where (*) denotes the optimal levels of the choice variables when a forward currency market exists; Π^* represents the present discounted value of profits; Ω_t denotes the producer's information set at time t; Ω_t is composed of all available information including past realizations of prices, exchange rates, and export and hedging decisions.

Equations (1a) and (1b) imply that the firm determines its optimal output according to the following rule

$$e_t^f p_t = C'(q_t^*) \tag{2}$$

that is, the optimal export level is determined by equating the firm's marginal cost to the certain domestic price established by the forward exchange rate. Equation (2) establishes that the Separation Theorem holds for the multiperiod case when all inputs can be optimally adjusted at all time periods. Therefore, the firm's optimal volume of export is independent of the producer's utility function and of the probability distribution of $\tilde{e}_t$. That is, the producer's level of exports is completely determined by the world price, forward exchange rate, and production costs at time t.

Hence, by participating in the forward currency market, the producer is capable of eliminating the uncertainty s/he faces. This result relies on the time separability of the production process, since, as Zilcha and Eldor (1991) show, the firm is unable to separate its production and hedging decisions if an input, such as capital, is chosen once and for all in the first period. Furthermore, it is interesting to note that this extension of the static Separation Theorem is independent

[5] The sufficient second order condition for a maximum of (1) over (q_t, x_t) is satisfied since $C(q_t)$ is assumed to be convex in q_t.

of whether the firm takes into account future uncertainties when determining today's optimal export levels or does not take them into account. That is, it is independent of whether the maximand is the expected utility of uncertain wealth or the present discounted expected utility of each period's uncertain profits. This multiperiod separation result is also independent of whether the forward currency market is biased or unbiased, as long as the firm participates in the biased forward currency market.[6] However, the direction and magnitude of the bias affects the difference in exports relative to the case when the firm has no access to forward currency markets.

When the firm has no access to a forward currency market, its optimal export level is determined by

$$E\{(\tilde{e}_t p_t - C'(\hat{q}_t))U'(\hat{\Pi})|\Omega_t\} = 0 \tag{3}$$

where (^) represents the optimal levels when no forward markets exist (i.e., $x_t = 0\ \forall\ t$). Equation (3) can be rewritten as

$$\mathrm{Cov}(U'(\hat{\Pi}), \tilde{e}_t p_t - C'(\hat{q}_t)|\Omega_t) + E(\tilde{e}_t p_t - C'(\hat{q}_t)|\Omega_t)E(U'(\hat{\Pi})|\Omega_t) = 0 \tag{4}$$

which implies

$$E(\tilde{e}_t p_t - C'(\hat{q}_t)|\Omega_t) = E(e_t|\Omega_t)p_t - C'(\hat{q}_t) > 0 \tag{5}$$

since $\mathrm{Cov}(\cdot)|\Omega_t) < 0$ and $U(\cdot)$ is concave and monotonic. On the other hand, the optimal export level derived in the presence of forward currency markets [eq. (2)] can be rewritten as

$$(E(e_t|\Omega_t) - \rho_t)p_t - C'(q_t^*) = 0 \tag{6}$$

where ρ_t, the risk premium of the forward currency market, is defined as $\rho_t = E(\tilde{e}_t|\Omega_t) - e_t^f$.

Comparing eqs. (5) and (6) implies that the introduction of a forward currency market may: (a) increase the firm's optimal volume of exports when the forward currency market is unbiased or a contango (i.e., $\rho_t \leq 0$); (b) increase the producer's export levels if the forward currency market is characterized by normal backwardation ($\rho_t > 0$)

[6] A forward currency market is said to be unbiased if the risk premium, defined as $\rho_t = E(e_t|\Omega_t) - e_t^f$, is zero, that is, when the expected future spot price is equal to the forward exchange rate. On the other hand, the forward currency market is considered biased when the risk premium is non zero. This situation is referred to as a contango (normal backwardation) when $\rho_t < (>) 0$.

and the bias is bounded from above by $\rho_t^m = -\mathrm{Cov}(U', \tilde{e}_t)/EU'$; or (c) reduce the optimal exports under normal backwardation forward currency markets when $\rho_t > \rho_t^m$.

Therefore, the introduction of a forward currency market does not induce an unambiguous increase in the volume of exports of the firm. The effect on the volume of trade depends on both the sign and magnitude of the risk premium. When the forward currency market is unbiased or a contango, the introduction of a forward currency market leads to an unambiguous increase in the level of exports. This effect is greater for the contango case than for the unbiased. However, when the forward currency market is characterized by normal backwardation, the results are not so clear and, in fact, the introduction of forward currency market may lead to a decrease in the firm's export volumes. Only when the forward market's risk premium is lower than the upper bound ρ_t^m will the export levels increase. When the bias in the forward currency market is greater than ρ_t^m, one obtains the perverse result that with access to a forward currency market at time t, the firm decreases its volume of trade.

OPTIMAL HEDGING DECISIONS

The firm's optimal hedging rules derived from a multiperiod framework differ, in general, from those derived by other studies. This contrast is due to the effect that the stochastic spot exchange rate at time t has on the future conditional expectation of the exchange rate. When an agent uses all of the information available in his/her expectation formation, the conditional expected exchange rate at time $t+1$ depends on the realization of the exchange rate at time t and, thus, independence of the random exchange rates is a special case.[7] Proposition 1, proven in the appendix, characterizes the firm's optimal hedging decision when exchange rates are positively correlated over time.[8]

Proposition 1. *Assuming that the exchange rates are positively correlated over time, that is, $e_t(\tilde{e}_{t-1})$ is increasing in $\tilde{e}_{t-1}$, then (a) the firm fully hedges in the last period and over hedges in all other time periods when the forward currency market is unbiased; (b) the firm over hedges in all time periods if the forward currency market is a contango; and (c) the firm's*

[7] In general $E(\tilde{e}_t | \Omega_t) = e_t(e_{t-1}, e_{t-2}, \ldots, e_{t-n})$. However, for tractability it is assumed that $E(\tilde{e}_t | \Omega_t) = e_t(e_{t-1})$.

[8] Positively (negatively) correlated exchange rates over time are a characteristic of countries whose inflation rate are consistently greater (lower) than their trading partner's inflation rate.

hedging decisions are ambiguous at time periods except in the last when it under hedges under normal backwardation forward currency markets.

This result establishes that the producer determines his/her optimal hedging behavior incorporating the effect that the stochastic spot exchange rate may have on future profits.[9] Under the assumption of positive correlation between the spot rates and future profits, the firm tends to over hedge even when the forward currency market is unbiased. Only in the special case of serial independence of the exchange rates does the firm fully hedge when the unbiasedness hypothesis of the forward currency market is satisfied. This result generalizes the hedging behavior derived for unbiased forward markets when the firm fully hedges the value of its exports [see, e.g., Feder, Just, and Schmitz (1980) and Kawai and Zilcha (1986) for the static case and Kawai (1981) for the dynamic case].

The basic implication of this behavior is that the classification of a firm as a pure speculator or pure hedger cannot be based solely on its optimal hedging pattern. When the forward currency markets are unbiased, the firm is not speculating when it over hedges at time t. The over hedge is an attempt to attenuate the effect of the random spot exchange rate on future profits and, in this context, the firm could be considered a speculative hedger. When the producer faces a forward currency market that is biased, however, the hedging decision can be separated into two elements: a speculative hedger and a pure speculator. As a pure speculator, the firm speculates by selling (buying) forward currency contracts when the risk premium is negative (positive). Thus, with a contango forward currency market, the pure speculative component of the hedging behavior reinforces the speculative hedging element, leading to a greater over hedge relative to the unbiased forward market case. For the case of normal backwardation, however, the speculative hedging element implies that the firm should over hedge while the pure speculative element implies that the firm should buy forward currency contracts. The relative magnitude of these opposing forces determines whether the firm will over hedge, fully hedge, or under hedge when the risk premium is positive.

The same procedure can be used to analyze the case when the exchange rates are negatively correlated over time. The results for this

[9] Howard and D'Antonio (1991), assuming a first order auto regressive stochastic process, present empirical evidence that an agent tends to over hedge in order to hedge against later period spot holdings.

case show that the producer tends to: under hedge in all time periods under normal backwardation forward currency markets; under hedge in all time periods except in the last when they fully hedge if the forward currency market is unbiased. When the forward market is a contango, the optimal hedging pattern is ambiguous for all t except the last when the firm over hedges.

CONCLUDING REMARKS

The results show that when a firm with a time separable technology has access to a perfectly competitive forward currency market, the separation between exporting and hedging decisions holds at all time periods. That is, the firm determines its optimal volume of exports based on the world price, production costs, and forward currency rate. Hence, trading decisions are independent of the distribution of the stochastic spot exchange rates. This implies that exchange rate stabilizing policies will not have an impact on the producer's export decision when a forward currency market is already established. These exchange rate stabilizing policies affect, however, the firm's forward currency hedging decision since they have an effect on the conditional expectations of future spot exchange rates. More importantly, the introduction of a forward currency does not always lead to an increase in the volume of exports. It is possible, when the forward market is characterized by normal backwardation (i.e., $E(\tilde{e}_t \mid \Omega_t) > e_t^f$) that the optimal level of exports of the firm will decrease relative to the case when there are no forward currency markets. The magnitude of the risk premium determines whether the trade volume will increase or decrease at time t.

Additionally, the optimal hedging plan over time consists of two elements: speculative hedging and pure speculating. In the special case when forward markets are unbiased, only the first element is present and the agent will over (under) hedge at time t when the exchange rates are positively (negatively) correlated over time in order to hedge against present and future effects of this period's uncertain spot exchange rate. On the other hand, both elements of the hedging decision are present when the forward market is biased. Additional to the element of speculative hedging, the firm acts as a pure speculator by selling (buying) forward currency contracts when the risk premium is negative (positive) and exchange rates are positively correlated over time. The relative magnitude of these two elements determines the net hedging behavior when the forward currency market is biased.

APPENDIX

Proof of Proposition 1:

The producer's optimal hedging decision is derived from eq. (1b) which can be rewritten as

$$\mathrm{Cov}(U'(\Pi^*), \tilde{e}_k \mid \Omega_t) = -\rho_k E(U'(\Pi^*) \mid \Omega_k) \quad \forall\, k \tag{A.1}$$

this relationship implies that the optimal hedge at time t, y_t^*, is determined such that

$$\mathrm{Cov}(U'(\Pi^*), \tilde{e}_t \mid \Omega_t) \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0 \quad \text{when } \rho_t \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} 0 \tag{A.2}$$

where

$$E(\Pi^* \mid \Omega_k) = \sum_{t=1}^{k-1} \delta^{t-1} [e_t p_t q_t^* + [e_t^f - e_t] y_t^* - C(q_t^*)]$$

$$+ \sum_{t=k}^{T} \delta^{t-1} [\tilde{e}_t p_t q_t^* + [e_t^f - \tilde{e}_t] y_t^* - C(q_t^*)] \tag{A.3}$$

Therefore, given (A.2), the firm's optimal hedging decision at time k is determined according to the following rule:

$$\frac{\partial E(\Pi^* \mid \Omega_k)}{\partial \tilde{e}_k} \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 0 \quad \text{when } \rho_k \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} 0 \tag{A.4}$$

For $k = T$, the optimal hedge is

$$y_T^* \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} p_T q_T^* \quad \text{when } \rho_T \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} 0 \tag{A.5}$$

Hence, the firm will fully hedge in the last period only if the forward currency market is unbiased and will over (under) hedge when the forward market is a contango (normal backwardation).

When $k = T - 1$, however, (A.4) implies

$$\frac{\partial E(\Pi^* \mid \Omega_{T-1})}{\partial \tilde{e}_{T-1}} = \delta^{T-2} [p_{T-1} q_{T-1}^* - y_{T-1}^*] + \delta^{T-1} \frac{\partial E(\tilde{e}_T \mid \tilde{e}_{T-1})}{\partial \tilde{e}_{T-1}} y_T^* \tag{A.6}$$

which leads to the following optimal hedge

$$\begin{aligned} y_{T-1}^* &> p_{T-1}q_{T-1}^* \quad \text{when } \rho_{T-1} \leq 0 \\ y_{T-1}^* &\begin{Bmatrix} \geq \\ < \end{Bmatrix} p_{T-1}q_{T-1}^* \quad \text{when } \rho_{T-1} > 0 \end{aligned} \tag{A.7}$$

since, by assumption $\bar{e}_T(\tilde{e}_{T-1})$ is increasing in $\tilde{e}_{T-1}$. The hedging rule represented by (A.7) states that the firm will over hedge in period $T - 1$ when the forward currency market is unbiased or a contango. However, the optimal hedge when the risk premium is positive is ambiguous; the firm may under, fully, or over hedge.

Following the same procedure for any $0 < k < T$ yields

$$\begin{aligned} (\Pi^* \mid \Omega_k) = & \sum_{t=1}^{k-1} \delta^{t-1}[e_t p_t q_t^* + [e_t^f - e_t]y_t^* - C(q_t^*)] \\ & + \delta^{k-1}[\tilde{e}_k p_k q_k^* + [e_k^f - \tilde{e}_k]y_k^* - C(q_k^*)] \\ & + \sum_{t=k+1}^{T} \delta^{t-1}[[\bar{e}_t(\tilde{e}_{t-1}) - \rho_t]y_t^* - \tilde{e}_t\xi_t - C(q_t^*)] \end{aligned} \tag{A.8}$$

where $\xi_t = y_t^* - p_t q_t^*$ denotes a slack variable. Thus, (A.4) implies that the optimal hedge for $0 < k < T$ is

$$\begin{aligned} y_k^* &> p_k q_k^* \quad \text{when } \rho_k \leq 0 \\ y_k^* &\begin{Bmatrix} \geq \\ < \end{Bmatrix} p_k q_k^* \quad \text{when } \rho_k > 0 \end{aligned} \tag{A.9}$$

Therefore, the firm will over hedge in all time periods when the risk premium is negative; will over hedge in all time periods except in the last when it fully hedges if the forward market is unbiased; and will over, fully, or under hedge in all time periods except in the last where it under hedges when the risk premium is positive.

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