Multiperiod Hedging in the Presence of Conditional Heteroskedasticity

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INTRODUCTION

The conventional approach to estimating the (one-period) optimal hedge ratio is to use the slope coefficient from a simple regression of spot prices on futures prices. Criticisms of this method focus on analytical and statistical properties of the estimate. Whereas the slope coefficient reflects the ratio of the unconditional covariance to the unconditional variance (of futures prices), the optimal hedging rule requires conditional moments that depend on the information available at the time the hedging decision is made. Myers and Thompson (1989) suggest that the correct ratio may be derived once the relevant information variables are incorporated into the regression model. Viswanath (1993) adopts this approach and finds the current basis to be a relevant information variable in determining the hedge ratio. The resulting regression model is similar to Fama and French (1987). These articles essentially argue

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that the intercept in the simple regression model should be a function of conditioning information variables.

Bell and Krasker (1986) show analytically that not only the intercept but the slope coefficient is a function of conditioning information variables. The method is adopted in Cita and Lien (1992), Lien and Luo (1993), and Lien (1993b). As the slope coefficient reflects the optimal hedge ratio, this approach directly models the ratio as a function of conditioning variables. Statistically speaking, the above criticisms pertain to the misspecification of the functional form or/and the omittence of explanatory variables in the regression equation. Park and Bera (1987) and Park (1991) argue that a Box–Cox transformation should be applied to both spot and futures prices in the simple regression. Yet it is a difficult to interpret the resulting slope estimate as the optimal hedge ratio. The former article also argues that the error term in the linear regression model may be conditionally or unconditionally heteroskedastic, and a more efficient estimator may be derived. Nonetheless, this approach assumes a constant optimal hedge ratio.

Instead of applying the regression method, the optimal hedge ratio can be estimated from the second moments of the joint process of spot and futures prices. Traditionally, the second moment matrix is assumed to be constant over time. Anderson (1985) and Fackler (1986) present evidence that commodity price volatility changes as markets move through cycles of high and low uncertainty about future economic conditions. In general, heteroskedasticity is expected in the price process. While unconditional heteroskedasticity is found in several markets [see Fackler (1986), Mathews and Fackler (1989)], research interest centers mostly on conditional heteroskedasticity; in particular, autoregressive conditional heteroskedasticity (ARCH) and its generalized version, GARCH [for a recent survey, see Bollerslev, Chou, and Kroner (1992)]. The advantage of the GARCH specification is that conventional conditional density assumptions (i.e., that is normal or student-t) allow for time dependent conditional variances and leptokurtosis in the unconditional distribution. The latter two properties have strong empirical support.

Using a mean squared error criterion, Pagan and Schwert (1990) and Akgiray (1989) find GARCH models dominate several competitive models including ARMA, nonparametric, and Markov switching ones in various financial markets. Adopting a quadratic utility function, West, Edison, and Cho (1993) show that GARCH models produce the highest utility, on average, in foreign exchange markets. A mean squared error criterion also favors GARCH, but not as sharply. Empirical determination of optimal hedge ratios within the bivariate GARCH framework is described in Cecchetti, Cumby, and Figlewski (1988); Baillie and Myers (1991); Kroner and Sultan (1991); Myers (1991); and Sephton (1993). Their results indicate that the standard assumption of a time-invariant optimal hedge ratio is inappropriate. Nonetheless, the better statistical performance of GARCH models over linear models does not warrant better hedging performance. In fact, none of the above work finds significant improvement of GARCH hedging over conventional simple hedging.

All of the above works consider only one-period hedging decisions. With daily settlement in futures markets, it is natural to set the period (i.e., unit) of time equal to one day. Realism suggests that the representative hedger's planning horizon covers multiple periods. Thus, to more fully consider both the usual hedging practice and the influence of daily resettlement, a multiperiod arrangement seems more appropriate. Howard and D'Antonio (1991) analyze multiperiod hedging decisions when spot prices are autocorrelated. Mathews and Holthousen (1991) provide a similar analysis for a general homoskedastic price process. Lien (1993a) and Lien and Luo (1993) derive multiperiod hedge ratios when spot and futures prices are cointegrated. In these setups, the single period hedge ratio is constant over time. The main purpose of this article is to incorporate the two important factors, multiperiod planning horizon and conditional heteroskedasticity of prices and hedge ratios, into the hedging decision framework. Lien (1993b) adopts the regression method of Bell and Krasker (1986) to proceed. This article uses bivariate GARCH modeling for estimates of multiperiod hedge ratios. The multiperiod GARCH hedging performance is then evaluated.

This article utilizes a basic multiperiod model which is similar to Howard and D'Antonio (1991). The optimal multiperiod hedge ratio is derived and then conditional heteroskedasticity is introduced into the joint price process. The mean process is described by an error correction model while the GARCH specification is applied to model the conditional heteroskedasticity. These two properties help characterize multiperiod hedge ratios in detail. A two-period model is examined to illustrate the influence of conditional heteroskedasticity. Analytically, the first-period hedge ratio may be decomposed into four components reflecting the usual one-period ratio, the effects of spot price autocorrelation, of futures price autocorrelation, and of conditional heteroskedasticity. These results are applied to major foreign exchange markets for empirical determination of hedge ratios. For each case, better statistical performance for GARCH modeling is established. Time-invariant hedge ratios are inappropriate. Following Baillie and Myers (1991), both ex-ante and ex-post evaluation methods are applied to examine hedging performance. While GARCH hedge ratios perform a little better then three alternative hedge strategies (no hedge, constant hedge, and error-correction hedge) in ex-ante evaluation, the ex-post performance clearly favors the constant or error-correction hedge. These results extend conclusions reached in the one-period framework studies [e.g., Myers (1991); Kroner and Sultan (1991); Sephton (1993)] to the mutliperiod case.

THE BASIC MODEL

Consider a hedger who has a planning horizon of T periods. At time t, $t = 0, 1, \ldots, T$, the hedger holds a spot position S_t on a specific commodity. The sequence $\{S_o, S_1, \ldots, S_T\}$ is assumed to be generated by an exogenous, stochastic process.[1] Thus, at time t all future spot positions $S_{t+1}, S_{t+2}, \ldots$, and S_T are treated as exogenous random variables. The expectation of a given future spot position may change over time. That is, if $E_t(S_r)$ denotes the expectation of S_r at time t, where $r > t$, then it is possible that $E_{t+1}(S_{t+j}) \neq E_t(S_{t+j})$ for some $j > 0$. However, the changing process is assumed to be independent of the price history and previous hedging decisions. At a given time t, to reduce risk exposure, the hedger may choose to participate in the futures market by holding the futures position $F_t = b_t S_t$, where b_t is the hedge ratio. The hedging decisions are made sequentially. That is, at time t, b_t is determined by using all the information available. Let p_t and f_t denote the spot and futures prices at t, respectively. Then the price history up to time t, $\{p_o, f_o, p_1, f_1, \ldots, p_{t-1}, f_{t-1}\}$ is part of the information available to the hedger.

The initial wealth of the hedger is given by W_o. Ignoring interest rates and transaction costs, market operations lead to the following end-of-period wealth:

$$
\begin{aligned}
W_T &= W_o + \sum_{t=0}^{T-1} S_t[(p_{t+1} - p_t) + b_t(f_{t+1} - f_t)] \\
&= W_o + \sum_{t=0}^{T-1} S_t[\Delta p_{t+1} + b_t \Delta f_{t+1}] \qquad (1)
\end{aligned}
$$

[1] This section generalizes the results of Lien (1992) by allowing future spot positions to be random variables.

where Δ denotes the difference operator.[2] That is, $\Delta p_{t+1} = p_{t+1} - p_t$ and $\Delta f_{t+1} = f_{t+1} - f_t$. A pure hedger whose primary objective is to minimize the variance of the end-of-period wealth by sequentially choosing optimal hedge ratios is considered next.[3] The problem may be solved backwards using the dynamic programming technique.

Specifically, at time $T-1$, the conditional variance of W_T is $\text{var}_{T-1}(W_T) = \text{var}_{T-1}[S_{T-1}(\Delta p_T + b_{T-1}\Delta f_T)]$. To minimize $\text{var}_{T-1}(W_T)$, the optimal hedge ratio is

$$B_{T-1} = -\text{cov}_{T-1}(\Delta p_T, \Delta f_T)/\text{var}_{T-1}(\Delta f_T) \tag{2}$$

the conditional covariance between price changes in proportion to the conditional variance of change in futures prices. In fact, B_{T-1} corresponds to the one-period variance minimization solution, which is also called the myopic optimal hedge ratio (see also Heaney and Poitras (1991) for more general cases). In the absence of conditional heteroskedasticity, both $\text{cov}_{T-1}(\Delta p_T, \Delta f_T)$ and $\text{var}_{T-1}(\Delta f_T)$ are independent of the information set and, consequently, B_{T-1} is a constant term. Introducing conditional heteroskedasticity implies that B_{T-1} is a function of the price history $\{p_o, f_o, p_1, f_1, \ldots, p_{T-1}, f_{T-1}\}$.

Proposition 1. *Assume the spot position sequence $\{S_t\}$ is generated by an exogenous process. For a T-period hedging horizon, the optimal hedge ratio at time $k(0 \leq k \leq T-1)$, B_k satisfies the following recursive relation:*

[2] The interest rate factor may be incorporated easily into the framework. Let r_t denote the discount factor from time 0 to time t. That is, a dollar in time t is worth r_t at time 0. Then W_T becomes $W_0 + \sum_{t=0}^{T-1} r_{t+1}S_t(\Delta p_{t+1} + b_t\Delta f_{t+1})$. Denote $r_{t+1}S_t$ by S_t'. The analysis in the text is directly applicable to S_t'. The incorporation of transaction costs is more troublesome if the hedger ever changes from short to long or vice versa. Otherwise, let C_t denote the transaction cost for one futures position at time t. The end-of-period wealth is then $W_T = W_0 + \sum_{t=0}^{T-1} S_t(\Delta p_{t+1} + b_t\Delta f_{t+1} - b_tC_t)$. Upon replacing Δf_{t+1} by $\Delta f_{t+1} - C_t$, the results in the text are applicable.

[3] In the one period framework, it is well known that variance minimization is not generally consistent with expected utility maximization. Nevertheless, variance minimization is well accepted as the objective function of a *bona fide* hedger who attempts to minimize risk. Similarly, for the multiperiod framework, the most general approach is to apply an intertemporal utility function, such as Adler and Detemple (1988) and Heaney and Poitras (1991). The latter acknowledged the possible difficulties when GARCH processes are encountered. It is also unclear how the above work applies to the case of a *bona fide* hedger. To resolve the problem, Howard and D'Antonio (1991) and Mathews and Holthausen (1991) provide a simple characterization of the *bona fide* hedger by supplanting minimization of the variance of end-of-period wealth as the objective. This approach is a direct extension of the one-period framework. While restrictive, it is useful in the consideration of a *bona fide* hedger.

$$B_k = \frac{\mathrm{cov}_k(\Delta p_{k+1}, \Delta f_{k+1})}{\mathrm{var}_k(\Delta f_{k+1})} - \sum_{t=k+1}^{T-1} \frac{E_k(S_t)}{S_k} \frac{\mathrm{cov}_k(\Delta f_{k+1}, \Delta p_{t+1} + B_t \Delta f_{t+1})}{\mathrm{var}_k(\Delta f_{k+1})} \tag{3}$$

Whereas the above equation is derived in a discrete-time framework with variance minimization being the objective function, the result is similar to and a specific expansion of that of Heaney and Poitras (1991) in which a general expected utility function is considered within the continuous-time framework. Note that, in eq. (3), B_t is chosen after Δf_{k+1} is known (given $t \geq k + 1$), so that at time k it is possible that B_t and Δf_{k+1} are correlated. This is particularly true if B_t is a function of Δf_{k+1}. In the absence of conditional heteroskedasticity, it is shown that B_t is independent of Δf_{k+1} through recursive examination [Lien (1992)]. Herein,

$$\mathrm{cov}_k(\Delta f_{k+1}, \Delta p_{t+1} + B_t \Delta f_{t+1}) = \mathrm{cov}_k(\Delta f_{k+1}, \Delta p_{t+1}) + B_t \mathrm{cov}_k(\Delta f_{k+1}, \Delta f_{t+1})$$

Conditional heteroskedasticity generates correlation between B_t and Δf_{k+1}. Consequently, the above equation does not hold. Further consideration requires specification of the conditional heteroskedasticity, such as the GARCH specificaiton. In general, B_k is called the optimal multiperiod hedge ratio. Clearly, the difference between the myopic and multiperiod hedge ratio is measured by the second term in eq. (3). It depends on spot positions over time $\{S_t\}$ and autocovariances of the vector time series (p_t, f_t).

BIVARIATE GARCH PROCESSES

Most researchers agree that both spot and futures prices contain a unit root, but there is a debate concerning the cointegration between the two prices. Lien and Luo (1993) and Quan (1992) support cointegration; but Baillie and Myers (1991) and Schroder and Goodwin (1991) argue against it. This study allows for the possibility of cointegration and propose the following bivariate GARCH(1, 1) model.[4]

$$\Delta p_t = \alpha_1 \theta_{t-1} + \beta_1 \Delta p_{t-1} + \gamma_1 \Delta f_{t-1} + \varepsilon_{1t} \tag{4a}$$

$$\Delta f_t = \alpha_2 \theta_{t-1} + \beta_2 \Delta p_{t-1} + \lambda_2 \Delta f_{t-1} + \varepsilon_{2t} \tag{4b}$$

$$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-1}^2 \\ \varepsilon_{1t-1}\varepsilon_{2t-1} \\ \varepsilon_{2t-1}^2 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} h_{11t-1} \\ h_{12t-1} \\ h_{22t-1} \end{bmatrix} \tag{5}$$

where $\theta_{t-1} = f_{t-1} - p_{t-1}$; $h_{11t} = \mathrm{var}_{t-1}(\varepsilon_{1t})$, $h_{12t} = \mathrm{cov}_{t-1}(\varepsilon_{1t}, \varepsilon_{2t})$, $h_{22t} = \mathrm{var}_{t-1}(\varepsilon_{2t})$, and $E_{t-1}(\varepsilon_{1t}) = E_{t-1}(\varepsilon_{2t}) = 0$. Equations (4a) and (4b) describe changes in means while eq. (5) specifies time variations of conditional second moments. Generally, $(\varepsilon_{1t}, \varepsilon_{2t})$ is assumed to be distributed as bivarietely normally or student-t. The normality assumption is adopted in the empirical estimation portion of this article. On the other hand, the analytical results require only the joint density function to be symmetric. The GARCH(1, 1) specification is found to be useful for asset prices [Engle and Bollerslev (1986)] and commodity spot and futures prices [Baillie and Myers (1991); Kroner and Sultan (1991), Myers, (1991)]. However, the model may be overparametrized as the second moment equation itself contains 21 parameters. Following Bollerslev, Engle, and Wooldridge (1988), a more parsimonious representation can be obtained by assuming that both A and B are diagonal matrices. This is a natural simplification because it implies that each variance and covariance depends only on its own past values and prediction errors. The number of parameters in eq. (5) is reduced to 9. Gains in estimation accuracy are expected.

[4] A drawback of this approach is that the estimated variance–covariance matrix may not be positive definite for all samples. Cecchetti, Cumby, and Figlewski (1988) avoid the problem by assuming two invariant GARCH models for spot and futures prices with a constant correlation coefficient. Nonetheless, the cost of imposing the constancy assumption seems to outweigh the benefits. Baillie and Myers (1991) compare the current approach with an alternative matrix decomposition approach that ensures positive definiteness. They conclude that the current approach has better statistical performance for their sample data.

A two-period model (i.e., $T = 2$) is employed to examined the effects of conditional heteroskedastcity on multiperiod hedge ratios.[5] Two ratios need to be determined: B_1 and B_o. From the previous section:

$$B_1 = \frac{-\operatorname{cov}_1(\Delta p_2, \Delta f_2)}{\operatorname{var}_1(\Delta f_2)} \tag{6}$$

$$B_o = \frac{-\operatorname{cov}_o(\Delta p_1, \Delta f_1)}{\operatorname{var}_o(\Delta f_1)} - \frac{E_o(S_1)}{S_o}\frac{\operatorname{cov}_o(\Delta f_1, \Delta p_2 + B_1\Delta f_2)}{\operatorname{var}_o(\Delta f_1)} \tag{7}$$

Proposition 2: *Suppose the joint density function of $(\varepsilon_{1t}, \varepsilon_{2t})$ is symmetric, within a two-period framework. The optimal hedge ratio at $t = 0$, B_o, can be decomposed into four components:*

$$B_o = B_{o1} + B_{o2} + B_{o3} + B_{o4}, \qquad \text{with} \tag{8}$$

$$B_{o1} = \frac{-\operatorname{cov}_o(\varepsilon_{11}, \varepsilon_{21})}{\operatorname{var}_o(\varepsilon_{21})} \tag{9a}$$

$$B_{o2} = -\frac{E_o(S_1)}{S_o}\frac{(\alpha_1 + \gamma_1)\operatorname{var}_o(\varepsilon_{21}) + (\beta_1 - \alpha_o)\operatorname{cov}_o(\varepsilon_{11}, \varepsilon_{21})}{\operatorname{var}_o(\varepsilon_{21})} \tag{9b}$$

[5] This article concentrates on two period hedging problems because more analytical properties are available. When the hedging horizon exceeds two periods, simulation methods ought to be applied to derive estimated hedge ratios. For example, in a three-period hedging framework, eq. (3) provides optimal hedge ratios as follows.

$$B_2 = -\frac{\operatorname{cov}_2(\Delta f_3, \Delta p_3)}{\operatorname{var}_2(\Delta f_3)}$$

$$B_1 = -\frac{\operatorname{cov}_1(\Delta f_2, \Delta p_2)}{\operatorname{var}_1(\Delta f_2)} - \frac{E_1(S_2)}{S_1}\frac{\operatorname{cov}_1(\Delta f_2, \Delta p_3 + B_2\Delta f_3)}{\operatorname{var}_1(\Delta f_2)}$$

$$B_o = -\frac{\operatorname{cov}_0(\Delta f_1, \Delta p_1)}{\operatorname{var}_0(\Delta f_1)} - \frac{E_o(S_1)}{S_o}\frac{\operatorname{cov}_o(\Delta f_1, \Delta p_2 + B_1\Delta f_2)}{\operatorname{var}_o(\Delta f_1)}$$
$$- \frac{E_o(S_2)}{S_o}\frac{\operatorname{cov}_0(\Delta f_2, \Delta p_3 + B_2\Delta f_3)}{\operatorname{var}_0(\Delta f_2)}$$

To derive estimates of the hedge ratios, one must proceed with estimating all the covariance terms. Note that, $\operatorname{cov}_1(\Delta f_2, \Delta p_3 + B_2\Delta f_3)$ and $\operatorname{cov}_o(\Delta f_1, \Delta p_2 + B_1\Delta f_2)$ are similar to each other (with an one period lag). The latter is analyzed in the text. The only term to be considered is $\operatorname{cov}_o(\Delta f_2, \Delta p_3 + B_2\Delta f_3)$. The following estimation procedure is recommended: suppose the hedging horizon extends from time t to time $t + 2$. Empirical results of GARCH estimation provide variance–covariance matrices of $(\varepsilon_{1t}, \varepsilon_{2t})$ and $(\varepsilon_{1,t+1}, \varepsilon_{2,t+1})$. Now, generate 1000 random samples from $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{1,t+1}, \varepsilon_{2,t+1})$. Successive applications of eq. (5) lead to 1000 random samples of $(h_{11,t+2}, h_{12,t+2}, h_{22,t+2})$, from which 1000 random samples of B_2 are at hand. The covariance term $\operatorname{cov}_o(\Delta f_2, \Delta p_3 + B_2\Delta f_3)$ is then estimated by its sample analogue; whereas 1000 random samples of Δf_2, Δp_3, and Δf_3 are derived from eqs (4a) and (4b) using the samples of $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{1,t+1}, \varepsilon_{2,t+1})$. The above approach can be extended easily to any multiperiod hedging problem.

$$B_{o3} = -\frac{E_o(S_1)}{S_o} E_o(B_1) \frac{(\alpha_2 + \gamma_2)\mathrm{var}_o(\varepsilon_{21}) + (\beta_2 - \alpha_2)\mathrm{cov}_o(\varepsilon_{11}, \varepsilon_{21})}{\mathrm{var}_o(\varepsilon_{21})} \tag{9c}$$

$$B_{o4} = -\frac{E_o(S_1)}{S_o} \frac{(\alpha_2 + \gamma_2)E_o\left(\varepsilon_{21}^2 \delta_o(B_1)\right) + (\beta_2 - \alpha_2)E_o(\varepsilon_{11}\varepsilon_{21}\delta_o(B_1))}{\mathrm{var}_o(\varepsilon_{21})} \tag{9d}$$

where, for any random variable X, $\delta_o(X) = X - E_o(X)$.

In eq. (8), B_{o1} corresponds to the single-period optimal hedge ratio at $t = 0$; whereas, B_{o2}, B_{o3}, and B_{o4} arise from multiperiod considerations. To be specific, B_{o2} is produced by the time series dependence property of changes in spot prices. If spot prices follow a random walk, then $\alpha_1 = \beta_1 = \gamma_1 = 0$ and, consequently, $B_{o2} = 0$. Similarly, B_{o3} mimics the effects of time series dependence of changes in futures prices. In addition to the time series dependence property of futures prices, B_{o4} is nonnegilible only when the optimal hedge ratio B_1 is time dependent. Thus, B_{o4} measures the effects of conditional heteroskedasticity.[6]

AN EMPIRICAL STUDY

The optimal multiperiod hedging policy is examined in five major foreign exchange markets: British pound (BP), Canadian dollar (CD), German mark (GM), Japanese yen (JY), and Swiss franc (SF) using weekly spot prices and futures prices on the nearby contract.[7] Tuesday closing prices are used. Monday or Wednesday closing prices are substituted when data

[6] In previous studies on GARCH hedge ratios, both spot and futures prices follow a random walk (i.e., $\alpha_i = \beta_i = \gamma_i = 0$; $i = 1, 2$). As a consequence, $B_{o1} = B_o$; the optimal multiperiod hedge ratio equates to the myopic optimal hedge ratio. Howard and D'Antonio (1991) allows autocorrelation in spot price series while maintaining the martingale assumption on futures prices. In this case, the multiperiod ratio differs from the myopic ratio by the factor B_{o2}.

[7] The model is developed in terms of price changes. The approach is consistent with Baillie and Myers (1991). Alternatively, one could use returns as the operative variables as in Heaney and Poitras (1991), Howard and D'Antonio (1991), and Kolb and Okunev (1993). Witt, Schroeder, and Havenga (1987) compare simple regression hedge ratios using levels, differences, and percentages of agricultural commodities. They conclude that the price level model is theoretically sound and use the price change model for anticipatory hedges and for highly risk averse hedge. Theoretically, the price difference model would be preferred when dealing with a carrying charge hedge for a storage commodity. If the cash and futures price changes are log-linear, then the percentage change model would be preferred.

are missing. The sample period extends from March 4, 1980 through December 27, 1988 for a total of 455 observations.[8]

The first task is to fit the price data into bivariate GARCH(1, 1) models [i.e., eqs. (4) and (5)]. It is assumed that ε_{1t} and ε_{2t} are jointly normally distributed. A maximum Likelihood Estimation (MLE) method is adopted. Analytical details are provided in Bollerslev (1986). The estimation is carried out using the MGARCH program. Estimation results are displayed in Tables I to V. For each market, the sample data exhibit strong GARCH effects. The asymptotic t-statistics for parameters in the variance equation (i.e., $A_{11}, A_{22}, A_{33}; B_{11}, B_{22}, B_{33}$) are significantat the 1% level in every market except the Canadian dollar.[9] Also, the coefficients of lagged price terms (Δp_{t-1} and Δf_{t-1}) are significant in

TABLE I

Model Estimation: The British Pound Market

Bivariate GARCH(1,1) Model

$$\Delta p_t = -0.1405u_{t-1} - 0.0487\Delta p_{t-1} - 0.0517\Delta f_{t-1}$$
$(-2.82)^{a}$ (-0.84) (-0.95)

$$\Delta f_t = -0.1604u_{t-1} + 0.4934\Delta p_{t-1} - 0.6020\Delta f_{t-1}$$
(-3.40) (10.49) (-12.05)

$$h_{11t} = 0.8976 + 0.1263\varepsilon_{1t-1}^2 + 0.8037h_{11t-1}$$
(13.91) (14.80) (73.67)

$$h_{12t} = 1.6673 + 0.0910\varepsilon_{1t-1}\varepsilon_{2t-1} + 0.7339h_{12t-1}$$
(27.76) (20.06) (200.7)

$$h_{22t} = 0.8874 + 0.0125\varepsilon_{2t-1}^2 + 0.8963h_{22t-1}$$
(18.02) (10.60) (195.8)

Simple Error Correction Model

$$\Delta p_t = -0.2030u_{t-1} - 0.2872\Delta p_{t-1} + 0.2490\Delta f_{t-1}$$
(-1.64) (-2.54) (2.37)

$$\Delta f_t = -0.7625u_{t-1} - 0.0970\Delta p_{t-1} + 0.0511\Delta f_{t-1}$$
(-6.08) (-0.85) (0.48)

[a]The number in the parenthesis is the corresponding asymptotic t-statistics.

[8]The data are provided by A. G. Malliaris at Loyola University of Chicago. In Lien and Luo (1993), it is shown that each of the two price series contains exactly one unit root. Moreover, the two prices are cointegrated. Both properties are necessary for applying the analytical results.

[9]The estimation of the GARCH(1, 1) model generates a negative number for B_{11}, the coefficient for $h_{11,t-1}$ in the equation of $h_{11,t}$. There is a similar case in Baillie and Myers (1991). A statistical test indicates that the coefficient is not significantly different from zero. Therefore, the condition $B_{11} = 0$ is imposed. That is, the spot rate of the Canadian dollar is described by an ARCH(1) process.

TABLE II

Model Estimation: The Canadian Dollar Market

Bivariate GARCH(1,1) Model

$$
\begin{array}{lllll}
\Delta p_t = & -0.3284\theta_{t-1} & +\ 0.0649\Delta p_{t-1} & -\ 0.1512\Delta f_{t-1} \\
 & (-8.98)^a & (1.98) & (-3.27) \\
\Delta f_t = & -0.4394\theta_{t-1} & +\ 0.3239\Delta p_{t-1} & -\ 0.4521\Delta f_{t-1} \\
 & (-12.47) & (14.20) & (-10.70) \\
h_{11t} = & 0.2208 & +\ 0.4478\varepsilon_{1t-1}^2 & \\
 & (18.78) & (17.80) & \\
h_{12t} = & 0.1772 & +\ 0.3132\varepsilon_{1t-1}\varepsilon_{2t-1} & +\ 0.1406h_{12t-1} \\
 & (19.32) & (16.91) & (7.51) \\
h_{22t} = & 0.1385 & +\ 0.4027\varepsilon_{2t-1}^2 & +\ 0.3052h_{22t-1} \\
 & (12.40) & (10.59) & (7.61)
\end{array}
$$

Simple Error Correction Model

$$
\begin{array}{lllll}
\Delta p_t = & 0.5367\theta_{t-1} & -\ 0.2012\Delta p_{t-1} & +\ 0.1200\Delta f_{t-1} \\
 & (6.61) & (-3.14) & (1.48) \\
\Delta f_t = & -0.1473\theta_{t-1} & -\ 0.0258\Delta p_{t-1} & -\ 0.0192\Delta f_{t-1} \\
 & (-2.26) & (-0.50) & (-0.30)
\end{array}
$$

[a] The number in the parenthesis is the corresponding asymptotic t-statistics.

every price series with the exception of the spot rate of the Canadian dollar. Overall, one concludes that bivariate GARCH(1, 1) specifications provide better performance than linear models for the sample data.[10]

To investigate the effects of conditional heteroskedasticity, an error correction model without GARCH error terms is estimated. In this case, the estimators are expected to be consistent but inefficient, provided the GARCH(1, 1) model is the true model. The results indicate that the estimators tend to have larger t-statistics in the GARCH models. This is consistent with the inefficiency argument. For example, while both specifications conclude with significant error correction terms (θ_{t-1}) in futures price equations, lagged price terms are insignificant in every

[10] No attempt is made to investigate the possibility of other nonlinear models for spot and futures prices. For a recent survey of nonlinear time series models, see Gooijer and Kumar (1992). This omission does not suggest that other nonlinear models may not be useful. Indeed, Blank (1991) and Decoster, Labys, and Mitchell (1992) find evidence of chaos in several commodity futures prices. Krager and Kugler (1993) suggest self-exciting autoregressive threshold models for several daily exchange rates. However, all these works deal with univariate time series, whereas any hedging decision requires the consideration of bivariate time series. On the other hand, as second order moments are explicitly specified, GARCH models seem very natural in their applications to hedging problems.

TABLE III
Model Estimation: The German Mark Market

Bivariate GARCH(1,1) Model

$$\Delta p_t = 0.0883\theta_{t-1} + 0.2577\Delta p_{t-1} - 0.2343\Delta f_{t-1}$$
$$(-0.63)^a \qquad (-2.55) \qquad (2.42)$$

$$\Delta f_t = 0.4445\theta_{t-1} - 0.3091\Delta p_{t-1} + 0.3646\Delta f_{t-1}$$
$$(-3.06) \qquad (2.97) \qquad (-3.54)$$

$$h_{11t} = 0.1913 + 0.0412\varepsilon^2_{1t-1} + 0.6863h_{11t-1}$$
$$(39.78) \qquad (21.06) \qquad (109.7)$$

$$h_{12t} = 0.1288 + 0.0250\varepsilon_{1t-1}\varepsilon_{2t-1} + 0.7844h_{12t-1}$$
$$(29.54) \qquad (18.20) \qquad (188.8)$$

$$h_{22t} = 0.0431 + 0.0127\varepsilon^2_{2t-1} + 0.9276h_{22t-1}$$
$$(18.51) \qquad (10.45) \qquad (242.1)$$

Simple Error Correction Model

$$\Delta p_t = 0.1784\theta_{t-1} - 0.1983\Delta p_{t-1} + 0.2389\Delta f_{t-1}$$
$$(1.05) \qquad (-1.26) \qquad (1.52)$$

$$\Delta f_t = 0.3188\theta_{t-1} - 0.0183\Delta p_{t-1} + 0.0565\Delta f_{t-1}$$
$$(-1.88) \qquad (-0.12) \qquad (0.36)$$

[a] The number in the parenthesis is the corresponding asymptotic t statistics.

futures price equation when conditional heteroskedasticity is not taken into account. The coefficient of θ_{t-1} in the futures price equation is negative for both specifications, indicating that the futures price is responsive to the market condition by continuously adjusting itself to eliminate arbitrage opportunities. On the other hand, the spot price does not respond to any deviation from the long-run equilibrium relationship. That is, according to Garbade and Silber (1983), the futures market is more likely to be a satellite. This conclusion is not reliable, however, because simple error correction model is subject to misspecifications. When correcting for the conditional heteroskedasticity, it is found that the spot price does respond to the error correction term; but it acts to promote the price gap rather than the convergence (except for the Japanese yen market).

In general, estimation results for the two specifications differ substantially. For comparison purposes, both specifications are examined when calculating multiperiod hedge ratios.

To utilize the above estimation results, consider the following scenario. In week 1, a short hedger (long in spot and short in futures) assumes a futures position on the nearby contract of a specific foreign

TABLE IV

Model Estimation: The Japanese Yen Market

Bivariate GARCH-ECM Model

$$\Delta p_t = 0.2020 u_{t-1} + 0.5805 \Delta p_{t-1} + 0.4452 \Delta f_{t-1}$$
$$(1.77)^{a} \quad (5.98) \quad (4.62)$$

$$\Delta f_t = 0.4060 u_{t-1} + 0.1982 \Delta p_{t-1} + 0.3011 \Delta f_{t-1}$$
$$(3.13) \quad (1.78) \quad (2.60)$$

$$h_{11t} = 3.4934 + 0.1659 \varepsilon_{1t-1}^2 + 0.8283 h_{11t-1}$$
$$(4.22) \quad (12.70) \quad (95.95)$$

$$h_{12t} = 2.5681 + 0.1454 \varepsilon_{1t-1} \varepsilon_{2t-1} + 0.8426 h_{12t-1}$$
$$(4.52) \quad (16.60) \quad (347.9)$$

$$h_{22t} = 3.6524 + 0.1280 \varepsilon_{2t-1}^2 + 0.8553 h_{22t-1}$$
$$(3.54) \quad (13.43) \quad (94.84)$$

Simple Error Correction Model

$$\Delta p_t = 0.2870 u_{t-1} + 0.2735 \Delta p_{t-1} + 0.1967 \Delta f_{t-1}$$
$$(1.71) \quad (1.97) \quad (1.40)$$

$$\Delta f_t = 0.3170 u_{t-1} + 0.2705 \Delta p_{t-1} + 0.1067 \Delta f_{t-1}$$
$$(1.91) \quad (0.56) \quad (0.18)$$

[a]The number in the parenthesis is the corresponding asymptotic t statistics.

currency. The futures position in adjusted in week 2 to reflect the arrival of new information and change in economic conditions (such as a new spot position). Both spot and futures positions are closed in week 3. If the nearby contract expires during the two week period, the futures position is rolled over to the next nearby contract. Within this setup, eqs. (6) and (9) are applicable in deriving optimal two-week hedge ratios. There are 451 overlapping two-week hedge periods for the entire sample. That is, there are 451 B_o's and B_1's.

As an example, consider the hedging period form week $(j - 1)$ to week $(j + 1)$ where j corresponds to the jth observation. To convert this into the two period framework, time 0 corresponds to week $(j - 1)$, time 1 to week j, and time 2 to week $(j + 1)$. Thus, the second period optimal hedge ratio B_1 is estimated by the ratio of $h_{12,j+1}$ to $h_{22,j+1}$. Both numbers are available from the estimation results of eq. (9). On the other hand, the estimation of B_o is preceded by the calculation of $E_o(B_1)$, $E_o(\varepsilon_{11}\varepsilon_{21}B_1)$, and $E_o(\varepsilon_{21}^2 B_1)$. Simulation methods are used to resolve the problem. To begin, note that $(\varepsilon_{11}, \varepsilon_{21})$ is bivariately normally distributed with zero mean, and $E(\varepsilon_{11}^2) = h_{11,j}$, $E(\varepsilon_{11}\varepsilon_{21}) = h_{12,j}$, $E(\varepsilon_{21}^2) = h_{22,j}$. Estimates of the second moments are, again,

TABLE V
Model Estimation: The Swiss Franc Market

Bivariate GARCH(1,1) Model

$$\begin{array}{llll}
\Delta p_t = & -0.1849u_{t-1} & -0.3371\Delta p_{t-1} & +0.3685\Delta f_{t-1} \\
 & (-2.28)^a & (-2.78) & (3.07) \\
\Delta f_t = & -0.4375u_{t-1} & +0.2202\Delta p_{t-1} & -0.1869\Delta f_{t-1} \\
 & (-4.15) & (1.64) & (-1.34) \\
h_{11t} = & 0.0577 & +0.1250\varepsilon_{1t-1}^2 & +0.7991h_{11t-1} \\
 & (10.78) & (18.28) & (122.3) \\
h_{12t} = & 0.07364 & +0.1419\varepsilon_{1t-1}\varepsilon_{2t-1} & +0.7668h_{12t-1} \\
 & (12.44) & (43.81) & (669.3) \\
h_{22t} = & 0.1061 & +0.1723\varepsilon_{2t-1}^2 & +0.7267h_{22t-1} \\
 & (10.12) & (18.32) & (90.07)
\end{array}$$

Simple Error Correction Model

$$\begin{array}{llll}
\Delta p_t = & -0.0053u_{t-1} & -0.2648\Delta p_{t-1} & +0.2971\Delta f_{t-1} \\
 & (-0.03) & (-1.42) & (1.61) \\
\Delta f_t = & -0.2902u_{t-1} & -0.0347\Delta p_{t-1} & +0.0742\Delta f_{t-1} \\
 & (-1.86) & (-0.19) & (0.40)
\end{array}$$

[a] The number in the parenthesis is the corresponding asymptotic t-statistics.

available from the estimation results of eq. (9). Eight hundred random samples of $(\varepsilon_{11}, \varepsilon_{21})$ are generated from the joint density. For each sample, B_1 can be calculated as $h_{12,j+1}$ and $h_{22,j+1}$ are characterized by $(h_{11,j}, h_{12,j}, h_{22,j})$ and $(\varepsilon_{11}, \varepsilon_{21})$. Given the 800 triplets $(\varepsilon_{11}, \varepsilon_{21}, B_1)$, $E_o(B_1)$, $E_o(\varepsilon_{11}\varepsilon_{21}B_1)$, and $E_o(\varepsilon_{21}^2, B_1)$ are estimated by their sample analogs. Finally, a constant spot position, $S_o = S_1 = S_2$, is assumed.[14] Summary statistics of estimated two-period hedge ratios are presented in Table VI.

With the exception of the Canadian dollar market, hedge ratios fluctuate but remain negative for the whole period. That is, the hedger always sells futures contracts (but in different amounts at different times). In the Canadian dollar market, the cross moment (h_{12t}) varies significantly, resulting in possible Texas hedges (i.e., a hedger is long in both spot and futures positions). In every market, on average the second-period hedge ratio exceeds the first-period hedge ratio in absolute

[14] The assumption is made for illustration purposes. Note that the analysis assumes away the interest income earned by the first period profits at the end of period. Both Howard and D'Antonio (1991) and Mathews and Holthousen (1991) consider this factor. The former article then assumes that the spot position S_t increases over time with a rate equal to the one period interest rate.

TABLE VI
Summary Statistics of Estimated Two-Period Hedge Ratios:
Bivariate GARCH Specification

Hedge Ratio	*BP*	*CD*	*GM*	*JY*	*SF*
B_o					
Maximum	0.0931	11.4780	0.5453	0.6857	0.5494
Minimum	−1.2420	−0.7534	1.1216	0.8650	1.2133
Average	0.6871	0.4078	0.7737	0.7530	0.7877
Std	0.1387	0.5910	0.0682	0.0370	0.0767
B_1					
Maximum	0.1408	2.6202	0.4999	0.3585	0.5160
Minimum	1.8490	2.1599	1.6100	1.1704	1.2579
Average	0.8955	0.8728	0.9068	0.8469	0.8808
Std	0.1551	0.2189	0.1096	0.1243	0.0981

Std: Standard deviation

level. That is, the hedger should sell more futures contracts near the end of the planning horizon. The difference between the two ratios is generally large. While both ratios fluctuate over time, with the exception of the Canadian dollar, the second-period ratio varies more than the first-period ratio. Note that the first-period ratios consists of four elements, one of which is approximately the second-period ratio (i.e., $B_o = B_{o1} + B_{o2} + B_{o3} + B_{o4}$ and $B_{o1} \approx B_1$). The fact that $|B_1| > |B_o|$ on average implies that $B_{o2} + B_{o3} + B_{o4}$ is generally positive. Suppose these three elements change accordingly with the first-period ratio, then one expects $\text{var}(B_o) < \text{var}(B_1)$. Otherwise, $\text{var}(B_o) > \text{var}(B_1)$, as in the case of the Canadian dollar.

Table VII provides summary statistics for each element of the first-period hedge ratio. As stated above, B_{o1} is approximately equal to B_1 in both mean and standard deviation. The time-dependence property of spot prices produce significant effects on the hedge ratio (i.e., significant B_{o2}) in British pound and Swiss franc markets, where significance is determined by the ratio of the average to the standard deviation. In the former market, time-dependence reduces the need to sell futures contracts whereas it promotes futures sales in the latter market. For each market, the time-dependent of futures prices lessens the need to hedge (i.e., $B_{o2} > 0$) but the effect is significant only in three markets: British pound, German mark, and Swiss franc. With the exception of the British pound markets, futures prices are more influential than spot prices in terms of the effects of the time-dependence property on hedge ratios. Finally, although the magnitude itself is rather small, the impacts of conditional heteroskedasticity are significant in the Swiss franc, German

TABLE VII

Summary Statistics for the Decomposition of First-Period Hedge Ratio

Component	*BP*	*CD*	*GM*	*JY*	*SF*
B_{01}					
Maximum	0 1408	2 6202	0 4999	0.3585	0.5161
Minimum	1 3858	2 1599	1.2534	1 1704	1 2579
Average	0.8904	0 8730	0 9051	0.8466	0.8815
Std	0.1370	0 2189	0 1027	0 1239	0.0991
B_{02}					
Maximum	0.1793	1 0094	0.0663	0 2686	0 0079
Minimum	0.0650	0 3245	0.0614	0 3667	-0.1050
Average	0 1105	0 0346	0 0073	0.0153	0 0494
Std	0 0126	0 0611	0 0174	0.0970	0 0151
B_{03}					
Maximum	0 1676	4 8918	0 1648	0.1407	0 1791
Minimum	0 2137	1 2997	0 0013	0.1693	0 0220
Average	0 0859	0 3256	0 1195	0.0504	0 1227
Std	0 0374	0 2573	0 0274	0.0588	0.0217
B_{04}					
Maximum	0.0821	5 5384	0 0089	0.0701	0.0366
Minimum	0.0062	0 0280	0 0005	0 0023	0.0003
Average	0 0159	0 1008	0 0037	0.0284	0.0197
Std	0.0085	0 3287	0.0013	0 0114	0.0055

mark, Japanese yen, and Canadian dollar (to a lesser degree). The result supports the need to consider conditional heteroskedasticity in determining optimal hedge ratios.

For comparison, Table VIII provides optimal two-period hedge ratios (denoted by B_0^* and B_1^*, respectively) when conditional heteroskedasticity is ignored in model specification (i.e., simple error-correction models). Note that both B_0^* and B_1^* remain constant over time under the alternative specification. In general, B_1^* is similar to B_1 in the British pound, Canadian dollar, and German mark markets. The two ratios differ largely in Japanese yen and Swiss franc markets. On the other hand, in every exchange market, the first-period hedge ratios differ drastically

TABLE VIII

Optimal Two-Period Hedge Ratios: Simple Error Correction Specification

Hedge Ratio	*BP*	*CD*	*GM*	*JY*	*SF*
B_0^*	0 8922	0 8608	0 9578	0 9516	0 9633
B_1^*	0 8942	0.8713	0 9635	0.9603	0.9680

between the GARCH and the simple error-correction specification. For example, the average B_o in the Canadian dollar market is -0.4078 while B_o^* equals -0.8608. The difference is about 0.8 standard deviation of B_o. In the Japanese yen market, the average of $B_o^* - B_o$ is -0.1986, more than five times the standard deviation. Overall, differences between B_o and B_o^* are significant in German mark, Japanese yen, Swiss franc, and Canadian dollar (to a lesser degree) markets. These significant differences arise not only from the effects of conditional heteroskedasticity on hedge ratio determination, but also from the effects of conditional heteroskedasticity on parameter estimation for the stochastic process of price changes. When the true model is conditionally heteroskedastic, the simple error-correction model provides consistent but inefficient parameter estimates. In fact, Tables I–V show the two specifications lead to rather different estimates.

PERFORMANCE EVALUATION

While it has been shown that GARCH modeling has better statistical performance in determining hedge ratios, the hedger is, nonetheless, more concerned with the hedging performance of these ratios. In this section, four alternative hedging strategies are compared.

I. No hedge. Herein the hedge ratio is set to zero in both periods.

II. Constant hedge. A constant hedge ratio is applied in both periods. The constant ratio is determined by regressing the change in spot prices on the change in futures prices.[12]

III. Error-Correction hedge. This hedging strategy adopts hedge ratios determined through simple error-correction models. Using notation from the previous section, the hedge ratios are B_o^* and B_1^*, respectively.

IV. GARCH hedge. The two-period hedge ratios are B_o and B_1 respectively.

Both within-sample and post-sample performance are compared and ex-ante and ex-post evaluation methods are applied to generate the performance levels.

The sample is split into two subsamples. The first subsample contains 355 observations. It is used to estimate the joint price processes

[12] Using the within-sample data (i.e., first 355 observations), constant hedge ratios are estimated at 0.8858, 0.8056, 0.9687, −0.9420, and 0.9622 for the British pound, Canadian dollar, German mark, Japanese yen, and Swiss franc markets, respectively.

and to determine the required hedge ratios. Any comparison based on this subsample is a "within-sample" comparison. The remaining 100 observations constitute the second subsample. They are considered in post-sample comparisons. In ex-post evaluation, actual end-of-period wealth levels are calculated for each strategy. The hedging performance is measured by (the mean and) the variance (or standard deviation) of the wealth levels over relevant sample periods.

The ex-ante evaluation method assumes the true price process is described by GARCH(1, 1) models and calculates the first two conditional moments of the end-of-period wealth at $t = 0$ and $t = 1$, respectively. They are denoted by $E_o(W_2)$, $\text{var}_o(W_2)$, $E_1(W_2)$, and $\text{var}_1(W_2)$. Mathematically,

$$\begin{aligned}
\text{var}_o(W_2) &= \text{var}_o(W_o + \Delta p_1 S_o + \Delta f_1 b_o S_o + \Delta p_2 S_1 + \Delta f_2 b_1 S_1) \\
&= b_o^2 \text{var}_o(\Delta f_1 S_o) + 2b_o \text{cov}_o(\Delta f_1 S_o, \Delta p_1 S_o + \Delta p_2 S_1 + \Delta f_2 b_1 S_1) \\
&\qquad + \text{var}_o(\Delta p_1 S_o) \\
&\quad + \text{var}_o(\Delta f_2 S_1 + \Delta f_2 b_1 S_1) + 2\text{cov}_o(\Delta p_1 S_o, \Delta p_2 S_1 + \Delta f_2 b_1 S_1)
\end{aligned} \tag{10}$$

Note that the last term vanishes when b_1 is a constant or a random variable following the GARCH decision rule (i.e., $b_1 = B_1$). Also, using conditional expectation operators, one obtains:

$$\begin{aligned}
\text{var}_o(\Delta p_2 S_1 + \Delta f_2 b_1 S_1) &= E_o(\text{var}_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1)) \\
&\quad + \text{var}_o(E_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1))
\end{aligned} \tag{11}$$

Thus $\text{var}_o(W_2)$ may be expressed as follows:

$$\begin{aligned}
\text{var}_o(W_2) = b_o^2 \text{var}_o(\Delta f_1 S_o) \\
&+ 2b_o \text{cov}_o(\Delta f_1 S_o, \Delta p_1 S_o + \Delta p_2 S_1 + \Delta f_2 b_1 S_1) \\
&+ \text{var}_o(\Delta p_1 S_o) + E_o(\text{var}_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1)) \\
&+ \text{var}_o(E_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1))
\end{aligned} \tag{12}$$

For the cases of no hedge ($b_o = b_1 = 0$) and constant hedge (with $(b_o = b_1 = c)$, eq. (12) is directly applicable. When applying the error-correction hedge, the equation may be simplified because b_o satisfies the following equation: $b_o^2 \text{var}_o(\Delta f_1 S_o) = -b_o \text{cov}_o(\Delta f_1 S_o, \Delta p_1 S_o +$

$\Delta p_2 S_1 + \Delta f_2 b_1 S_1$). Thus,

$$\begin{aligned}\mathrm{var}_o(W_2) = b_o^2\mathrm{var}_o(\Delta f_1 S_o) &+ \mathrm{var}_o(\Delta p_1 S_o)\\ &+ E_o(\mathrm{var}_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1))\\ &+ \mathrm{var}_o(E_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1))\end{aligned} \tag{12a}$$

Finally, GARCH hedge chooses b_1 to minimizes $\mathrm{var}_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1)$, resulting in:

$$\mathrm{var}_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1) = \mathrm{var}_1(\Delta p_2 S_1) - b_1^2\mathrm{var}_1(\Delta f_2 S_1)$$

As a consequence, one obtains:

$$\begin{aligned}\mathrm{var}_o(W_2) = b_o^2\mathrm{var}_o(\Delta f_1 S_o) &- E_o(b_1^2\mathrm{var}_1(\Delta f_2 S_1)) + \mathrm{var}_o(\Delta p_1 S_o)\\ &+ E_o(\mathrm{var}_1(\Delta p_2 S_1)) + \mathrm{var}_o(E_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1))\end{aligned} \tag{12b}$$

In this formulation, the hedger attempts to minimize the conditional variance of the end-of-period wealth by sequentially updating information and choosing the hedge ratio therefrom. The objective function, in spite of all the similarity, is different from $\mathrm{var}_o(W_2)$ because the latter is concerned with the variability of expected second-period profits in addition to other factors. To see this, eq. (12) is examined. By the sequential decision rule, b_1 is chosen to minimize $\mathrm{var}_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1)$. b_o is chosen to minimize $b_o^2\mathrm{var}_o(\Delta f_1 S_o) + 2b_o\mathrm{cov}_o(\Delta f_1 S_o, \Delta p_1 S_o + \Delta p_2 S_1 + \Delta f_2 b_1 S_1)$ given the optimal b_1. The sequential optimal hedge ratios (b_o, b_1) do not minimize $\mathrm{var}_o(W_2)$ because the last term $\mathrm{var}_o(E_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1))$ is not considered. On the other hand, one clearly expects the GARCH hedge to dominate the other strategies in terms of $\mathrm{var}_1(W_2)$, which is simply the one-period hedging performance.

The hedging performance is carried out by assuming that the hedger holds 100 units (10,000 units in the Japanese yen market) of spot exchange in the beginning ($S_o = S_1 = S_2 = 100$). Thus, the actual number of futures contracts purchased is given by the hedge ratio times 100 (10,000 in Japanese yen market). The initial wealth is set at $W_o = 100p_o$. Tables IX–XIII present ex-ante hedging performance evaluation results for five major foreign exchange markets. They are rather similar across all markets qualitatively. When comparing the conditional standard deviation of W_2 at $t = 1$ (i.e., one-period hedging performance), the GARCH hedge dominates the other three hedging strategies for all sample selection. The no hedging scenario is the worst

TABLE IX
Ex-Ante Hedging Performance Comparisons: British Pound

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
$E_1(W_2)$	(I)	215.43	152.56	215.57
	(II)	215.66	152.19	215.68
	(III)	215.67	152.18	215.68
	(IV)	215.57	152.29	215.62
$\sigma_1(W_2)$	(I)	2.987	3.030	2.995
	(II)	1.228	1.324	1.248
	(III)	1.230	1.326	1.250
	(IV)	1.172	1.295	1.198
$E_0(W_2)$	(I)	215.64	152.18	215.66
	(II)	215.71	152.14	215.70
	(III)	215.71	152.14	215.71
	(IV)	215.68	152.17	215.69
$\sigma_0(W_2)$	(I)	4.120	4.174	4.131
	(II)	1.337	1.451	1.361
	(III)	1.325	1.438	1.348
	(IV)	2.219	2.291	2.234

$\sigma_1(W_2)$: the conditional standard deviation of W_2 at $t = 1$
$\sigma_0(W_2)$: the conditional standard deviation of W_2 at $t = 0$

TABLE X
Ex-Ante Hedging Performance Comparisons: Canadian Dollar

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
$E_1(W_2)$	(I)	83.65	75.06	83.68
	(II)	83.77	75.08	83.77
	(III)	83.77	75.08	83.78
	(IV)	83.79	75.13	83.80
$\sigma_1(W_2)$	(I)	0.551	0.589	0.560
	(II)	0.281	0.336	0.294
	(III)	0.279	0.337	0.293
	(IV)	0.266	0.318	0.278
$E_0(W_2)$	(I)	83.62	74.88	83.61
	(II)	83.78	75.08	83.78
	(III)	83.78	75.08	83.79
	(IV)	83.66	75.05	83.68
$\sigma_0(W_2)$	(I)	0.768	0.817	0.778
	(II)	0.387	0.423	0.395
	(III)	0.384	0.420	0.391
	(IV)	0.546	0.586	0.555

$\sigma_1(W_2)$: the conditional standard deviation of W_2 at $t = 1$
$\sigma_0(W_2)$: the conditional standard deviation of W_2 at $t = 0$

TABLE XI

Ex-Ante Hedging Performance Comparisons: German Mark

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
$E_1(W_2)$	(I)	51.17	52.32	51.18
	(II)	51.09	52.25	51.09
	(III)	51.09	52.25	51.09
	(IV)	51.09	52.26	51.09
$\sigma_1(W_2)$	(I)	0.830	0.864	0.841
	(II)	0.247	0.419	0.286
	(III)	0.247	0.421	0.287
	(IV)	0.241	0.376	0.272
$E_0(W_2)$	(I)	51.20	52.30	51.21
	(II)	51.05	52.16	51.04
	(III)	51.05	52.16	51.04
	(IV)	51.07	52.19	51.07
$\sigma_0(W_2)$	(I)	1.168	1.215	1.184
	(II)	0.259	0.429	0.297
	(III)	0.259	0.430	0.297
	(IV)	0.371	0.569	0.415

$\sigma_1(W_2)$: the conditional stardard deviation of W_2 at $t = 1$
$\sigma_0(W_2)$: the conditional standard deviation of W_2 at $t = 0$

TABLE XII

Ex-Ante Hedging Performance Comparisons: Japanese Yen

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
$E_1(W_2)$	(I)	400.58	661.82	400.84
	(II)	399.08	658.28	398.90
	(III)	399.09	658.30	398.91
	(IV)	399.31	658.71	399.19
$\sigma_1(W_2)$	(I)	10.667	16.741	12.140
	(II)	5.312	6.347	5.540
	(III)	5.313	6.348	5.540
	(IV)	5.602	6.109	5.286
$E_0(W_2)$	(I)	399.68	659.50	399.63
	(II)	398.08	656.32	397.66
	(III)	398.09	656.33	397.67
	(IV)	398.31	656.78	397.94
$\sigma_0(W_2)$	(I)	13.888	22.360	15.952
	(II)	5.868	7.015	6.124
	(III)	5.882	7.044	6.141
	(IV)	6.872	10.232	7.701

$\sigma_1(W_2)$: the conditional stardard deviation of W_2 at $t = 1$
$\sigma_0(W_2)$: the conditional standard deviation of W_2 at $t = 0$

TABLE XIII
Ex-Ante Hedging Performance Comparisons: Swiss Franc

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
$E_1(W_2)$	(I)	54.04	65.60	54.04
	(II)	53.90	65.46	53.90
	(III)	53.90	65.46	53.90
	(IV)	53.90	65.46	53.90
$\sigma_1(W_2)$	(I)	1.006	1.100	1.037
	(II)	0.378	0.377	0.378
	(III)	0.380	0.378	0.380
	(IV)	0.356	0.356	0.356
$E_0(W_2)$	(I)	54.08	65.64	54.08
	(II)	53.84	65.39	53.84
	(III)	53.83	65.39	53.83
	(IV)	53.87	65.41	53.87
$\sigma_0(W_2)$	(I)	1.432	1.589	1.484
	(II)	0.395	0.400	0.398
	(III)	0.397	0.400	0.399
	(IV)	0.477	0.472	0.477

$\sigma_1(W_2)$: the conditional stardard deviation of W_2 at $t = 1$.
$\sigma_0(W_2)$: the conditional standard deviation of W_2 at $t = 0$.

case generating a standard deviation more than twice of that which arose from the GARCH hedge. The constant hedge and the error correction hedge are comparable to each other and to the GARCH hedge. These results are similar to Kroner and Sultan (1991) and Myers (1991). That is, unless the hedger is extremely risk averse, the constant or the error-correction hedge performs as well as the GARCH hedge. All intermediate result indicates that is seems unnecessary to include error-correction consideration in determining hedge ratio because the residual risk remains near the same. From comparisons among $E_1(W_2)$, it can be concluded that each of the three hedging strategies reduce the hedger's risk at little or no cost of expected end-of-period wealth.

Based on the conditional variance of the end-of-period wealth at $t = 0$ (i.e., $\text{var}_0(W_2) = \sigma_0^2(W_2)$), the constant hedge and the error-correction hedge both dominate the GARCH hedge, which in turn dominates no hedge. As stated above, the conditional variance deviates from the hedger's objective function by the factor concerning the variability of second-period profits, $\text{var}_0(E_1(\Delta p_2 S_1 + \Delta f_2 b_1 S_1))$. Other things equal, the above term is larger when b_1 is a random variable (instead of a constant). The criterion, therefore, is biased in favor of the constant, error-correction, and no hedge against the GARCH hedge. In sum, using ex-ante evaluation methods it is found that the GARCH hedge results in little or no improvement in hedging performance over the alternative constant or error-correction hedging strategy.

Ex-post hedging performances for each strategy are compared in Tables XIV–XVII. In every foreign exchange market, the sample variance of actual end-of-period wealth is smallest for the constant or error-correction hedge, and largest for no hedge. The GARCH hedge lies in between but is clearly dominated by the former two hedging strategies. These results are, again, similar to Kroner and Sultan (1991) and Myers (1991). The sample mean of actual end-of-period wealth is about the same regardless of hedging strategies.

CONCLUSIONS

This article considers multiperiod hedging decisions when conditional heteroskedasticity characterizes the underlying spot and futures prices. A two-period model is examined in detail to illustrate the influence of conditional heteroskedasticity. It is shown that the optimal first-period hedge ratio may be decomposed into four components. The first component corresponds to the usual one-period optimal hedge ratio; the second and the third component summarize the impacts of autocorrelation in spot and futures prices, respectively. The final component arises from conditional heteroskedasticity. Empirical applications to five major foreign exchange markets are performed.

Strong GARCH effects are found in each market. Parameters estimated for GARCH processes differ much from the corresponding results of simple error-correction models and they are more likely to be statistically significant. For the one-period hedge ratios calculated

TABLE XIV
Ex-Post Hedging Performance Comparisons: British Pound

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
W_2^*	(I)	215.22	152.75	215.45
	(II)	215.63	152.17	215.65
	(III)	215.64	152.16	215.65
	(IV)	215.48	152.29	215.55
$\sigma(W_2)$	(I)	3.692	3.924	3.759
	(II)	1.624	1.664	1.629
	(III)	1.631	1.650	1.631
	(IV)	1.676	1.806	1.699

W_2^*: the sample mean of (actual) W_2.
$\sigma(W_2)$: the sample standard deviation of (actual) W_2.

TABLE XV

Ex-Post Hedging Performance Comparisons: Canadian Dollar

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
W_2^*	(I)	83.69	75.23	83.75
	(II)	83.75	75.09	83.76
	(III)	83.75	75.08	83.76
	(IV)	83.77	75.13	83.79
$\sigma(W_2)$	(I)	0.839	0.976	0.879
	(II)	0.620	0.639	0.624
	(III)	0.621	0.635	0.623
	(IV)	0.605	0.678	0.624

W_2^*: the sample mean of (actual) W_2.
$\sigma_2(W_2)$: the sample standard deviation of (actual) W_2.

TABLE XVI

Ex-Post Hedging Performance Comparisons: German Mark

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
W_2^*	(I)	51.10	52.30	51.12
	(II)	51.15	52.28	51.15
	(III)	51.15	52.28	51.15
	(IV)	51.13	52.28	51.14
$\sigma(W_2)$	(I)	1.180	1.201	1.187
	(II)	0.323	0.500	0.397
	(III)	0.323	0.501	0.397
	(IV)	0.326	0.586	0.426

W_2^*: the sample mean of (actual) W_2.
$\sigma_2(W_2)$: the sample standard deviation of (actual) W_2.

TABLE XVII

Ex-Post Hedging Performance Comparisons: Japanese Yen

Criteria	*Hedging Strategy*	*In-Sample*	*Post-Sample*	*Combined-Sample*
W_2^*	(I)	401.45	663.47	401.91
	(II)	400.05	660.10	400.09
	(III)	400.06	660.59	400.23
	(IV)	400.07	660.59	400.23
$\sigma(W_2)$	(I)	18.656	19.383	18.798
	(II)	5.963	6.314	6.046
	(III)	5.964	6.307	6.045
	(IV)	6.651	7.066	6.766

W_2^*: the sample mean of (actual) W_2.
$\sigma_2(W_2)$: the sample standard deviation of (actual) W_2.

therefrom, the time-dependence of futures prices, the time-dependence of spot prices and conditional heteroskedasticity are all important for the British pound and Swiss franc cases and partially important for the German mark, Japanese yen, and Canadian dollar cases. Notwithstanding the better statistical performance, the hedging performance of the GARCH hedge is not better than that of the constant or error-correction hedges. In sum, these findings are consistent with Kroner and Sultan (1991) and Myers (1991) except when the hedger is extremely risk averse. Ex post criteria are not favorable to GARCH hedges at all.

APPENDIX

Proof of Proposition 1: Using backward induction, consider the hedging problem at time $t = k$ where optimal hedge ratios for future periods $B_{k+1}, B_{k+2}, \ldots, B_{T-1}$ are all derived. The risk for the hedger is

$$\begin{aligned}\mathrm{var}_k(W_T) = \mathrm{var}_k(S_k(\Delta p_{k+1} + b_k \Delta f_{k+1}) \\ + \sum_{t=k+1}^{T-1} S_t(\Delta p_{t+1} + B_t \Delta f_{t+1})) \\ = b_k^2 \mathrm{var}_k(\Delta f_{k+1}) S_k^2 \\ + 2b_k \mathrm{cov}_k(\Delta f_{k+1}, S_k \Delta p_{k+1} \\ + \sum_{t=k+1}^{T-1} S_t(\Delta p_{t+1} + B_t \Delta f_{t+1})) S_k + R_k \qquad \text{(A1)}\end{aligned}$$

where R_k is the residual term, independent of b_k. Using Bohrnstedt and Goldberger (1969), the following obtains:

$$\begin{aligned}\mathrm{cov}_k(\Delta f_{k+1}, S_t(\Delta p_{t+1} + B_t \Delta f_{t+1})) \\ = E_k(S_t)\mathrm{cov}_k(\Delta f_{k+1}, \Delta p_{t+1} + B_t \Delta f_{t+1}) \\ + \mathrm{cov}_k(\Delta f_{k+1}, S_t) E_k(\Delta p_{t+1} + B_t \Delta f_{t+1}) \\ + E_k(\delta_k(\Delta f_{k+1})\delta_k(S_t)\delta_k(\Delta p_{t+1} + B_t \Delta f_{t+1})) \qquad \text{(A2)}\end{aligned}$$

where $E_k(\cdot)$ is the conditional expectation operator with the conditioning set being the information available at time k; also for a given random variable X, $\delta_k(X) = X - E_k(X)$. By assumption, S_t is generated by an exogenous process and henceforth $\mathrm{cov}_k(\Delta f_{k+1}, S_t) = 0$. Moreover,

$$E_k(\delta_k(\Delta f_{k+1})\delta_k(S_t)\delta_k(\Delta p_{t+1} + B_t\Delta f_{t+1})) - E_k(\delta_k(S_t))E_k(\delta_k(\Delta f_{k+1})\delta_k(\Delta p_{t+1} + B_t\Delta f_{t+1})) = 0 \tag{A3}$$

As a consequence, eq. (A2) becomes

$$\mathrm{cov}_k(\Delta f_{k+1}, S_t(\Delta p_{t+1} + B_t\Delta f_{t+1})) = E_k(S_t)\,\mathrm{cov}_k(\Delta f_{k+1}, \Delta p_{t+1} + B_t\Delta f_{t+1}) \tag{A4}$$

Note that the above equations hold for every $t \geq k+1$. Upon substituting eq. (A4) into eq. (A1) and then upon taking the partial derivative with respect to b_k, it is found that the optimal hedge ratio at time k, B_k, satisfies the following recursive relation:

$$B_k = \frac{-\mathrm{cov}_k(\Delta p_{k+1}, \Delta f_{k+1})}{\mathrm{var}_k(\Delta f_{k+1})} - \sum_{t=k+1}^{T-1} \frac{E_k(S_t)}{S_k}\,\frac{\mathrm{cov}_k(\Delta f_{k+1}, \Delta p_{t+1} + B_t\Delta f_{t+1})}{\mathrm{var}_k(\Delta f_{k+1})}$$

Proof of Proposition 2: From eqs. (4) and (5), it can be shown:

$$\begin{aligned}\mathrm{cov}_0(\Delta f_1, \Delta p_2) &= \mathrm{cov}_0(\varepsilon_{21}, \alpha_1\theta_1 + \beta_1\Delta p_1 + \gamma_1\Delta f_1 + \varepsilon_{12}) \\ &= \mathrm{cov}_0(\varepsilon_{21}, \alpha_1(\varepsilon_{21} - \varepsilon_{11}) + \beta_1\varepsilon_{11} + \gamma_1\varepsilon_{21} + \varepsilon_{12}) \\ &= (\alpha_1 + \gamma_1)\mathrm{var}_0(\varepsilon_{21}) + (\beta_1 - \alpha_1)\mathrm{cov}_0(\varepsilon_{21}, \varepsilon_{11})\end{aligned} \tag{A5}$$

Also, by Bohrnstedt and Goldberger (1969),

$$\begin{aligned}\mathrm{cov}_0(\Delta f_1, B_1\Delta f_2) &= E_0(B_1)\mathrm{cov}_0(\Delta f_1, \Delta f_2) + E_0(\Delta f_2)\mathrm{cov}_0(\Delta f_1, B_1) \\ &\quad + E_0(\delta_0(\Delta f_1)\delta_0(B_1)\delta_0(\Delta f_2))\end{aligned} \tag{A6}$$

Herein $\delta_0(X) = X - E_0(X)$ for any given random variable X. More specifically,

$$\delta_0(\Delta f_1) = \varepsilon_{21}; \tag{A7}$$

$$\begin{aligned}\delta_0(\Delta f_2) &= \delta_0(\alpha_2\theta_1 + \beta_2\Delta p_1 + \gamma_2\Delta f_1 + \varepsilon_{22}) \\ &= \alpha_2(\varepsilon_{21} - \varepsilon_{11}) + \beta_2\varepsilon_{11} + \gamma_2\varepsilon_{21} + \varepsilon_{22} \\ &= (\alpha_2 + \gamma_2)\varepsilon_{21} + (\beta_2 - \alpha_2)\varepsilon_{11} + \varepsilon_{22}\end{aligned} \tag{A8}$$

Since ε_{21} and ε_{11} are uncorrelated with ε_{22}, the last term in eq. (A6) may be written as:

$$E_o(\delta_o(\Delta f_1)\delta_o(B_1)\delta_o(\Delta f_2)) = (\alpha_2 + \gamma_2)E_o(\varepsilon_{21}^2\delta_o(B_1))$$
$$+ (\beta_2 - \alpha_2)E_o(\varepsilon_{11}\varepsilon_{21}\delta_o(B_1)) = (\alpha_2 + \gamma_2)[E_o(\epsilon_{21}^2 B_1) - E_o(B_1)\mathrm{var}_o(\varepsilon_{21})]$$
$$+ (\beta_2 - \alpha_2)[E_o(\varepsilon_{11}\varepsilon_{21}B_1) - E_o(B_1)\mathrm{cov}_o(\varepsilon_{11}, \varepsilon_{21})] \quad \text{(A9)}$$

On the other hand, $\mathrm{cov}_o(\Delta f_1, B_1) = \mathrm{cov}_o(\varepsilon_{21}, B_1) = E_o(\varepsilon_{21}B_1)$. From eq. (5) and (6),

$$E_o(\varepsilon_{21}B_1) = E_o\left[\frac{A_{22}\varepsilon_{11}\varepsilon_{21}^2 + B_{22}h_{221}\varepsilon_{21}}{A_{33}\varepsilon_{21}^2 + B_{33}h_{331}}\right] \quad \text{(A10)}$$

Since the joint density function of $(\varepsilon_{11}, \varepsilon_{21})$ is symmetric, $E_o(\varepsilon_{21}B_1) = 0$. Finally, similar to eq. (A5), one can show that

$$\mathrm{cov}_o(\Delta f_1, \Delta f_2) = (\alpha_2 + \gamma_2)\mathrm{var}_o(\varepsilon_{21}) + (\beta_2 - \alpha_2)\mathrm{cov}_o(\varepsilon_{21}, \varepsilon_{11}) \quad \text{(A11)}$$

Upon combining the above results, the proof is completed.

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