COMMON VOLATILITY IN S&P 500 STOCK INDEX AND S&P 500 INDEX FUTURES PRICES DURING OCTOBER 1987

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INTRODUCTION

The S&P 500 index futures market and the underlying cash market fluctuated widely during the week of October 12–19, 1987, dropping nearly 40% in the days following the crash. The Brady Commission (1988) reported that under normal circumstances a strong intraday relationship exists between the index futures prices and the underlying cash market prices. That is, the cash and futures markets, with the exception of October 19 and 26, were essentially highly integrated (i.e., operated as one market). Other studies [Kleidon and Whaley (1992); Stoll and Whaley (1990); Harris (1989); Herbst, McCormack, and West (1987); Kawaller, Koch, and Koch (1987, 1993) among others] show too that there is a strong intraday price relationship in which the index futures

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and the underlying cash market move closely together.[1] It is generally believed, that intraday arbitrage hedging and speculative activity should keep the two markets tied together throughout a typical trading day [see, for example, Harris (1989) and Kawaller, Koch, and Koch (1993), among others]. However, cash and futures prices may diverge over short intervals of time as the arrival of new information in the market place is expected to have a differential impact on market participants' expectations which, in turn, influence their actions. Nonsynchronous trading [see Harris (1989); MacKinlay and Ramaswamy (1988)] may also cause the futures index to be less volatile than the underlying aggregate stock index. Harris (1989), however, shows that even after the effect of nonsynchronous trading is removed the cash market follows the futures market. These considerations strongly suggest that this relationship between cash and futures prices (i.e., the basis) may be evidence of information shocks being disseminated first to the futures, and then to the cash market. Equivalently, this would imply that while the cash and futures index series may have time-varying volatility structures they do not share common volatility process. This study addresses the issue of the common volatility process between index futures and the underlying cash market prices. Studying the nature of the volatility process in each of these two markets and the extent to which they may share a common time-varying volatility factor permits one to measure how well integrated the cash and futures markets are through their second moments rather than through their first moments.

As Engle and Susmel (1993) show, two markets might be uncorrelated (correlated) through their first moments but related (unrelated) through their volatilities. Furthermore, conventional hedge ratios assume that the risk in cash and futures markets is time-invariant, implying that the minimum risk hedge ratio will be the same regardless when the hedging is undertaken. The assumption of intertemporal constant variance is in sharp contrast with the fact that information flows in to the market continuously which, in turn, is likely to change the riskiness of cash and futures index contracts. Therefore, this assumption implies that both cash and futures markets must exhibit time-varying volatility which also implies that the risk-minimizing hedge ratio must be time-varying [see, for instance, Kroner and Sultan (1993)]. A more interesting

[1] Two recent articles by Wahab and Lashgari (1993) and Ghosh (1993) examine the dynamic linkages between the S&P 500 index futures and the S&P 500 stock market index using cointegration and error correction analysis during the post October 1987 period. The results of these studies indicate that the two markets do share a stable long-term equilibrium relationship (i.e., the two markets do act as a cointegrated system).

issue, perhaps, is to investigate whether the risk-minimizing hedge ratio is a function of a common time-varying volatility factor. This would require the presence of a common-ARCH feature between the cash and futures index series. On the other hand, if no common-ARCH feature is discovered it would imply that the hedge ratio should be a function of two independent time-varying volatility factors.

This article uses a recent test developed by Engle and Kozicki (1993) which exploits the time-varying structure of financial time series' variances to test whether the cash and futures markets share a time-varying common volatility process (feature). The objective of this article is two-fold. First, to test for the presence of a time-varying volatility process in intraday futures and stock market index series during the month of October, 1987 which was characterized by high and low volatility trading days. Second, to investigate the issue of common volatility in the index stock and futures markets. This is expected to provide evidence about the degree of integration between these two markets through their second moments and shed light on the number of time-varying risk factors required to model the risk-minimizing hedge ratio between cash and futures index stock markets.

The article consists of three sections. The first presents the common autoregressive conditional heteroskedasticity (ARCH) feature methodology and a description of the data employed. The results are presented and discussed in the second section. The article closes with a brief summary and conclusions.

COMMON ARCH-FEATURE METHODOLOGY

The common ARCH features testing procedure is a fairly new statistical method enunciated in Engle and Kozicki (1993), with a comprehensive financial markets application given in Engle and Susmel (1993). The intuition of this test builds on the statistical idea of cointegration [Granger (1983); Engle and Granger (1987)]. If two time series exhibit autoregressive conditional heteroskedasticity (ARCH), and if a linear combination between them does not display a time-varying variance (i.e., a linear combination of the series that eliminates the ARCH effects) then these two series should have a common feature. Specifically, the common feature methodology is used to examine whether the cash and futures index series share a common volatility process. The volatility feature is said to be common if a linear combination of the series does not have the volatility feature even though each of the series individually has the feature [Engle and Kozicki (1993)]. Therefore, the null hypothesis to be tested is that a linear combination of the two series does not have the volatility feature (i.e., no ARCH). While analysis of first moments has frequently been used to test for linkages between the futures and the underlying cash market its statistical limitations are well documented [Engle and Susmel (1993) among others]. The two markets, for instance, might be correlated in returns which can be interpreted as evidence of integration. They might be independent, however, through their second moments.

Following Engle and Susmel's (1993) definition of common feature, a brief description follows of how the linear combination of two series may not have the feature (i.e., no ARCH or constant variance is obtained). Suppose that the x_t and y_t series can be written without loss of generality as

$$x_t = \delta p_t + e_{xt} \tag{1a}$$

$$y_t = w_t + e_{yt} \tag{1b}$$

where $x_t(y_t)$ represents the intraday observed return, $w_t(p_t)$ is the time-varying risk premium and $e_{xt}(e_{yt})$ is the unexpected component of returns with

$$e_{xt} \mid I_{t-1} \sim D(O, \sigma_x^2) \qquad e_{yt} \mid I_{t-1} \sim D(O, \sigma_y^2)$$
$$p_t \mid I_{t-1} \sim D(O, h_{pt}) \qquad u_t \mid I_{t-1} \sim D(O, h_{ut})$$
$$h_{pt} = \alpha_{po} + \alpha_{p1} p_{t-1}^2 \qquad h_{ut} = \alpha_{uo} + \alpha_{u1} u_{t-1}^2$$

where I_{t-1} is the information set on which agents condition their decisions at time t that includes all past information while $\sigma_x^2(\sigma_y^2)$ is constant variance, $h_{pt}(h_{ut})$ is time-varying variance that follow ARCH (1) processes, and $D(a, b)$ an arbitrary distribution with mean a and variance b.

The variance of x_t and y_t are equal to

$$V(x_t) = \delta^2 h_{pt} + \sigma_x^2 \tag{2a}$$

$$V(y_t) = h_{ut} + \sigma_y^2 \tag{2b}$$

which are both time varying. A feature that is present in each of the two series is said to be common to those series if there exists a non-zero linear combination of the series ($u_t = x_t - \tau y_t$) that does not have the feature. This result is obtained when $w_t = p_t$ and $\tau = \delta$ since u_t has a constant variance[2] (i.e., $V(u_t) = \sigma_x^2 - \tau^2\sigma_y^2$). Alternatively, the null hypothesis (i.e., no ARCH in the linear combination between cash and futures prices) of a common factor can be designed by setting $w_t = p_t$ and $\tau = \delta$.

Testing for common volatility across the futures and cash markets amounts to testing first for ARCH effects in each market. This study is interested in testing for common ARCH features in the S&P 500 index futures and the S&P 500 index series. If ARCH effects are discovered in both series, the next step is to test for common ARCH (factor ARCH), thereby identifying a combination of both series that displays no ARCH. The ARCH test is based on the $T \times R^2$ from the regression of the squared series (T defines the sample size) against own and lagged squared series. That is, minimize over the parameter τ the test statistic $T \times R^2$. The $T \times R^2$ is obtained from the regression of $x_t - \tau y_t$ on lagged y_t and lagged x_t, and the lagged cross product of y_t and x_t. In general terms, the search is for a vector τ that minimizes a test of this form

$$\min_\tau V(u) = T \times R^2(\tau) \tag{3a}$$

$$V(u)(\tau) = (x_t - \tau y_t)^2 \tag{3b}$$

$$R^2 = u_t^2(\delta\tau)'Z_t(Z_t'Z_t)^{-1}Z_t'u_t^2(\delta)/\sigma^2 \tag{3c}$$

where Z_t is the larger information set composed of $y_{t-1}^2, x_{t-1}^2, y_{t-2}^2, x_{t-2}^2, \ldots, y_{t-p}^2, x_{t-p}^2$, and lagged cross-products $y_{t-1}x_{t-1}, y_{t-2}x_{t-2}, \ldots, y_{t-p}x_{t-p}$, y_t is $T \times 1$, x_t is a $T \times K$ vector, τ is $K \times 1$, K is the number of variables and σ^2 is a consistent estimator of the variance of u^2. This test is a general method-of-moments-type test. The objective is to identify a portfolio that is not correlated in the squares with any information included in Z. In other words, if the cash and futures index series are individually characterized by ARCH effects, there might be a portfolio (i.e., a linear combination) between them for which there is no evidence against the null hypothesis of no common volatility feature (i.e., no ARCH). This, then would clearly imply that

[2]Substituting (1a) and (1b) into $u_t = x_t - \tau y_t$ one obtains

$$u_t = \delta p_t + e_{xt} - \tau w_t - \tau e_{yt}$$

Empirically, a value of τ is sought such that $\delta = \tau$ and $p_t = w_t$ which yields.

$$u_t = e_{xt} - \tau e_{yt}$$

The variance of u_t, then, will be $V(u_t) = \sigma_x^2 - \tau\sigma_y^2$ since $\text{cov}(e_{xt}, e_{yt}) = 0$ (i.e., error terms are independent).

the cash and futures markets have a common volatility process. Engle and Kozicki (1993) have shown that, under the assumptions of Hansen (1982), $\min_{\tau} V(u)$ follows a chi-squared distribution with the degrees of freedom given by the number of overidentifying restrictions (i.e., number of instruments included in the information set Z_t minus $K - 1$). To minimize (3) a non-linear simplex grid search method is used.

The stock price sample consists of all primary market[3] trades of each S&P 500 stock from open of trading October 1, 1987, to the close of trading October 30, 1987. The data are from the Futures Industry Institute (Washington D.C.) and include the date, time, and price traded for each transaction on each exchange in the United States. The transaction time is the exact second when a trade was executed in the DOT system or when a record of it was read by an exchange card reader. The futures sample consists of all prices recorded by the CME market recorders in the December 1987 S&P 500 index for the corresponding sample period.[4] Each record includes the exact second that the price was entered by the recorder. Since the tick data is not reported in even intervals, the futures data is matched to the current S&P 500 index value on a minute-by-minute basis. However, it should be noted that futures trade more frequently and the first trading price is recorded within the first few seconds of the minute, while the cash index is updated later in the minute. To account for this lag, the first futures price in the minute is matched with the first value of the index within the minute.

EMPIRICAL EVIDENCE

ARCH Tests

Since all stocks in the S&P 500 are not necessarily traded in any given minute, some will react to the information flow with a time lag leading to the possibility of using stale prices in the calculation of the S&P 500 index. This so-called problem of nonsynchronous trading leads to spurious autocorrelation being incorporated in the prices and return series. To alleviate the problems associated with the nonsynchronous trading effects, five-minute estimated returns are regressed on a constant

[3]The NYSE is the primary market for 462 of the S&P 500 stocks, AMEX for eight, and the NASDAQ National Market System for the remaining 30. The primary markets for all S&P 500 stocks opened at 9:30 AM EST and closed at 4:00 PM except on Friday, October 23, when they all closed at 2:00 PM. Some of the stock traded later on the Pacific Exchange.

[4]The CME market opened at 9:30 AM EST and closed at 4:15 PM EST except on Friday, October 23, when it closed at 2:00 PM EST. On Tuesday, October 20, the exchange closed for about one hour near midday.

and four own lags. The errors obtained from these regressions (ϵ_{t-j}) are used then to test for ARCH and the common-ARCH feature.

To test for common volatility in the intraday S&P 500 index futures and the S&P 500 stock index series, it should be established first that each series displays an ARCH effect (i.e., time-varying volatility). To test for ARCH, the following fourth order ARCH models are estimated

$$h_{xt} = a_{xo} + \sum_{j=1}^{4} a_{xj}\epsilon^2_{xt-j} + v_{xt} \tag{4a}$$

$$h_{yt} = a_{yo} + \sum_{j=1}^{4} a_{yj}\epsilon^2_{yt-j} + v_{yt} \tag{4b}$$

where $a_{xo}(a_{yo})$ represent a constant parameter, $\epsilon_{xt}(\epsilon_{yt})$ are residuals obtained from the autoregressive regression and $v_{xt}(v_{yt})$ are error terms. The null hypothesis tested is that the series exhibit no ARCH. The test statistic is obtained by multiplying the regression R^2 to the sample size (T). Engle (1982) shows that the $T \times R^2$ test has an asymptotic chi-squared distribution with degrees of freedom equal to the number of lagged squared residuals. Table I reports the ARCH results. As can be seen, the null hypothesis of no ARCH is rejected in both the S&P 500 cash and futures index series for the entire month of October 1987, except for the day after the crash (October 20, 1987). The value of the $T \times R^2$ statistic for October 20, 1987 is clearly small (2.97) compared to its critical value 9.43 (13.3) at the 5(1)% level.

Common ARCH-Feature Tests

Since ARCH effects are found for the entire month of October 1987, except for October 20, a test is conducted for common volatility in the cash and futures index markets. The common ARCH-feature test consists of looking for a linear combination of the two series that eliminates the ARCH effects (i.e., a linear combination between the cash and futures index series that is characterized by a constant variance). This linear combination is determined by estimating:

$$\min_{\tau} V(u) = T \times R^2(\tau) \tag{5}$$

The minimum $T \times R^2$ is obtained from the regression of $(\epsilon_{yt} - \tau\epsilon_{xt})^2$ on four lagged squared ϵ_{xt}, four lagged squared ϵ_{yt} and their four lagged cross-product. Engle and Kozicki (1993) show that $\min_{\tau} V(u)$ follows a chi-squared distribution with degrees of freedom given by the

TABLE I
ARCH Tests: October 1–30, 1987

Day	*S&P 500 Index* $T \times R^2$ *Statistics*[a]	*S&P 500 Index Futures* $T \times R^2$ *Statistics*
October 1	43.66	101.34
October 2	48.08	109.04
October 5	73.52	112.75
October 6	69.29	116.69
October 7	92.73	104.86
October 8	53.02	95.36
October 9	28.98	113.27
October 12	96.72	122.84
October 13	76.88	89.55
October 14	59.26	91.55
October 15	80.66	108.92
October 16	95.64	45.64
October 19	47.98	106.75
October 20	2.95	33.10
October 21	63.53	97.05
October 22	67.66	107.85
October 23	36.64	68.34
October 26	36.64	68.12
October 27	41.93	75.33
October 28	73.30	69.02
October 29	32.56	93.00
October 30	85.01	74.79

[a] The five-minute estimated returns are first regressed against a constant and four own lags. The estimated errors are used to obtain the uncentered R^2 by regressing the squared errors against a constant and four own lags. The $T \times R^2$ statistic is obtained by multiplying the uncentered R^2 times the sample size. The distribution of this test is chi-squared with 4 degrees of freedom. Its critical value for 4 df, at the 5(1)% level, is 9.49 (13.3).

number of overidentifying restrictions [number of regressors included in the information set, Z minus $(k - 1)$]. In this study 12 regressors are included and two variables (k) that lead to 11 degrees of freedom. To minimize (5) a non-linear simplex search method is used. In Table II, the results are listed for the entire month of October, except for October 20, which display no ARCH effect. The parameter (τ) that minimizes the linear combination of both series is shown in the second column. The third column reports the minimum $T \times R^2$ statistics. The critical value for 11 degrees of freedom, at the 5(1)% level, is 19.7 (24.7). The results indicate that there is no evidence of common volatility (feature) between the stock index futures and the underlying stock index during the month of October. With the exception of Friday, October 23, when both markets closed at 2:00 PM EST, the $T \times R^2$ statistics decisively reject the no-ARCH effect null hypothesis. That is, the evidence does not support the existence of a linear combination between index stock

TABLE II

Common ARCH Feature Estimates: October 1–30, 1987

Day	*Parameter* (τ) *Values*[a]	*Minimum* $T \times R^2$ *Statistics*[b]
October 1	0.6036	48.74
October 2	0.6369	48.60
October 5	0.6666	43.92
October 6	0.8500	39.35
October 7	1.0425	48.45
October 8	0.7230	25.74
October 9	0.8171	83.76
October 12	0.9503	88.78
October 13	0.7052	26.61
October 14	1.0658	36.01
October 15	1.0617	31.10
October 16	2.5090	98.07
October 19	−6.3030	74.39
October 21	0.7653	60.59
October 22	1.5662	46.84
October 23	1.0162	14.94
October 26	0.6464	41.45
October 27	0.8374	46.88
October 28	0.5294	85.30
October 29	0.2511	32.77
October 30	0.4373	52.79

[a]The parameter that minimizes the linear combination of the intra-day S&P 500 index and the S&P 500 index futures series.
[b]The distribution of this test is chi squared with 11 degrees of freedom ($Z - k - 1$) where Z is the number of instruments used and k is the number of variables used. Its critical value for 11 df, at the 5(1)% level, is 19.7 (24.7).

and futures prices that achieves a constant variance. This result further implies that the two markets do not share a common feature, a common factor that displays a time-varying volatility. The S&P 500 index stock and the S&P 500 index futures series are characterized to possess different time-varying volatility structures. These results also suggest that, while previous studies find the cash and futures markets to be correlated through their first moments and share a long-term equilibrium relationship in returns, the two markets are not related through their second moments. The most interesting implication of the no-common feature results relates to the modeling of the basis (i.e., the difference between the value of the underlying stock index price and the futures index price). Since no support is found for a common ARCH-feature between index stock and futures price series, this implies that a two factor ARCH specification is appropriate in modeling the S&P 500 basis. Furthermore, unlike the conventional hedging models which implicitly assume that risk is time invariant, the evidence of this study suggests that the risk-minimizing hedge ratio is time varying. This is consistent with Kroner and Sultan's (1993) model which derives a time varying risk-minimizing hedge ratio for foreign currency futures contracts. However, it should be noted that the finding of two factors may be associated to the fact that the idiosyncratic noise terms of each series are also time varying. As Engle and Susmel (1993) point out the common ARCH-feature technique can not distinguish between ARCH and idiosyncratic factors.

SUMMARY AND CONCLUSIONS

The purpose of this article is to study, using intraday minute-by-minute data, whether the S&P 500 index futures market and the underlying cash market index have the same volatility process. The sample period begins from the open of trading October 1, 1987, to the close of trading October 30, 1987 — a period uniquely characterized by high and low volatility trading days. The analysis uses the common ARCH-feature testing procedure recently proposed by Engle and Kozicki (1993) to address the issue of a common volatility in these two index markets. First, ARCH effects are found in each series for all trading days in the month of October except for the first trading day after the crash (October 20). Second, Engle and Kozicki's (1993) common ARCH-feature test is applied. The results indicate that the index cash and futures markets do not share the same volatility process. Unlike previous studies that favor the interdependence between these two markets in returns, these results suggest that their second moments are independent. In brief, a common factor that has a time-varying volatility is not supported by the intraday price series of index stock and futures market. Therefore, a two factor ARCH specification appears to be highly attractive in modeling the S&P 500 basis. Finally, the results of this study provide strong evidence in favor of time varying risk-minimizing hedge ratios.

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