FORWARDS OR OPTIONS? NESTING PROCEDURES FOR "FIRE AND FORGET" COMMODITY HEDGING

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INTRODUCTION

Should commodity producers or users use forwards (including futures, swaps) or options to hedge their exposures to fluctuations in the price of the physical? In the absence of basis risks, forward sales or swaps, used to cover the full exposure, reduce that exposure to zero, as the outcome is then certain—sales or purchases of the commodity at the forward price. Put options strategies can be used to limit the producer's exposure on the downward side while preserving a desirable upward exposure, but at the cost of the initial premium or net premium. Another common strategy is to write calls on the producer's own output (in effect) and buying puts to protect the downside, retaining an exposure over an intermediate range ("bull" or "bear" spreads). Theoretically, arbitrage between options and forwards should ensure that it is a matter of indifference which strategy is used. However, the theoretical result relies on the possibility of a continual rebalancing of portfolios to ensure zero net exposure over each infinitesimal interval of time [for which, see most options textbooks, or for very general treatment, Föllmer (1992)]. In practice,

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however, few producers or users behave like this for reasons involving transaction costs, time, expertise, or a variety of other impediments. Instead, most would plan to hold the hedging instrument to maturity, or something close to it. The term of a commodity swap, for instance, might be tailored to the needs of the originating producer, while producers would tend to buy or write options with a maturity equal or close to the planned delivery of their product to the spot market. One might call these "fire and forget" hedging strategies. If such strategies are followed, or at least planned even if not subsequently followed, then the resulting options positions are now fully or partly exposed; and, unlike the no-arbitrage theory, the producer will, in general, prefer one position to the other on the basis of expectations and an expected utility payoff. While many might deprecate the reintroduction of expectations into options theory, it is for many users, an essential element in their hedging decisions. Are there, nonetheless, decision rules based on producer expectations, that are undemanding in their assumptions about the nature of the producer's utility function?

In its full generality this is an intractable problem, because, in effect, one has to form portfolios of instruments some of which have highly nonlinear payoffs. For this reason, existing investigations have been based on simulations of preassigned strategies [see, e.g., Schroeder and Hayenga (1988)]. Because of the lumpiness or discontinuity associated with standard options contracts, it is generally necessary to make some special assumption about the utility function involved, down to a specific choice of form (e.g., quadratic or power). It would be desirable to derive decision rules that are not only analytically based, but enable discrimination between types of hedging instruments or strategies on the basis of very general specifications on utility functions; for instance, if the producer is risk averse or for *any* Von Neumann-Morgenstern utility function.

The present article proposes a general analytical methodology that enables one to form decision rules for hedging choices by exploiting a nesting property, wherein one kind of financial instrument can be parametrically obtained from another. The analogy is with classical statistical hypothesis testing, where to accept or reject a hypothesis is to make a decision one way or another on actions associated with the competing hypotheses. Classical hypothesis testing is always easier if the hypotheses are nested within one another, and one can take advantage of this feature for the present problem. A forward can always be constructed by combining a put option and a call option, each at the same strike price, namely the currently quoted forward price. The option premiums[1] offset exactly, and the payoff is the same as a "sell forward." More generally, a forward can be nested within a bear or bull spread strategy and one can arrange things so that this nesting is in terms of a single parameter (θ) "parametric nesting." An alternative kind of nesting "portfolio nesting," is where one of the strategies can be reproduced as a limiting portfolio, where the portfolio proportions tend to a certain value. A simple put option, for example, can be nested with respect to a forward by the use of such a device. In general, the advantage of instrument nesting, whether parametric or portfolio, is that one can handle the "lumpiness" problem referred to above by means of a continuous deformation from one position to the other, so that the methodology of simple differential calculus becomes applicable.

The nesting methodology is applied in this study to certain basic and common kinds of commodity market hedging [for a practical review see Kaiser (1991), also options textbooks such as Hull (1993)]. These are:

i. Forward sales, swaps or futures (these will be modeled simply as forwards/or swaps, abstracting from mark to market and stochastic interest rate considerations in the case of futures);

ii. Simple put options ("protective puts");

iii. A combination of purchased puts with written calls. Here the producer protects the downside with the put, but takes the opportunity to earn positive premiums by writing calls against (in effect) his or her own output. The put and call strike prices differ, creating a zone of intermediate exposure. The result is either a bull spread or a bear spread (see the third section).

On occasion, the producer may be committed for policy or other reasons to covering an entire output, so that the choice is simply between futures, options or some standard combination. However, it should be remembered that the producer has a third alternative, namely to remain unhedged ("natural," or simply "spot" exposure). The economic literature on production under risk has concentrated almost exclusively on the use of forwards versus the natural exposure [see Holthausen (1979); Feder,

[1]The options prices are described as premiums, to distinguish them from spot and forward physical prices. Note that these are not the "portfolio" risk premiums separating the commodity forwards price from the expected future price, as in Beck (1993) or Bowden (1993b).

Just, and Schmitz (1980); Batlin (1983); Ederington (1979), Anderson and Danthine (1981); Antonovitz and Nelson (1988); Bowden (1993a)]. The classical result is that if the distribution of the unknown product price is normal, then the entire output should be hedged whenever the expected price (μ_s) is less than or equal to the current forward price (F). (More precisely, output should always be fully hedged, and a speculative demand added which is a function of the discrepancy ($F - \mu_s$) depending on agents' risk aversion). This study shows that options availability can affect hedging procedure, even where the possibility of remaining unhedged remains. Rather surprisingly, however, this is limited to the bear spread. Common strategies such as the bull spread or protective put are not superior to forwards/natural combinations, at least when strike prices are chosen at or near the current forward price, as the nesting theory with respect to forwards requires.

The formulation used here suggests a tripartite overall decision rule: (a) if expectations are optimistic relative to the current forward price, the producer should remain unhedged; (b) if expectations are less rosy, increase the forward coverage, becoming total where $\mu_s \leq F$; (c) if producer expectations are very pessimistic, utilize a bear spread strategy at a point determined by solving a certain volatility-dependent equation (Prop. 1B of "Formal Results" section). Volatility is that of the spot price process and is assumed[2] to be a common estimate as between the producer and the (options) market; however, expectations can differ as between producers, or for that matter options price makers, who do not need them for formal pricing purposes.

The following section describes the instrument nesting concept, utilizing the spread strategies and protective puts as examples. A distinction is made between parametric nesting, wherein one position is transformed to another by simply altering a parameter or functional argument (e.g., a strike price) and portfolio nesting, whereby a position is obtained as a limiting case of a portfolio constructed from other instruments or securities. These nesting procedures are utilized in the third and fourth sections to obtain decision regions for the use of spread strategies and protective puts, respectively. The possibility of remaining unhedged is also considered. In all cases, the objective is to display decision regions defined in terms of producer expectations, and in some cases spot price volatilities, within which one kind of hedging strategy is dominant over

[2]A referee points out a possible discrepancy, evidenced by Eales et al. (1990) where producers' estimates of volatility may differ from (generally lower than) the volatility implied in market pricing of options, so the assumption utilized in Proposition 1 is a material one. It would be desirable to explore the effect of misspecified volatility, but this is very difficult to do analytically.

others, for any specification of the utility function involved. The fourth section contains a short concluding discussion. The nesting methodology itself is independent of particular assumptions about spot prices. A basic Ito process with state independent volatility and constant or autonomous drift is employed here for illustrative purposes. Basis risk (temporal or grade) is not considered in the article.

INSTRUMENT NESTING: TYPES AND APPLICATIONS

As indicated in the introduction, instrument nesting refers, in general, to the establishment of one kind of instrument as a special case of another instrument, or a combination of instruments. Indeed, there may be two special cases of interest and it may be possible to turn one into the other by means of a continuous transformation of the parent combination of instruments. The examples in this study differ according to the meaning of the transformation needed to obtain the special cases. Parametric nesting means the creation of special cases by simply altering parameters (or functional arguments, from the pricing point of view) of the instruments making up the base combination, changing the strike price of an option, for example. Portfolio nesting means the creation of special cases in terms of portfolio of the base set of instruments. Choosing portfolio proportions appropriately will give the special cases of interest.

Parametric Nesting

Suppose that a comparison of forwards with bull or bear spreads is the object of study so that one can advise producers which to employ under various hypotheses about expected future price movements. The price of the product to be protected is denoted S and it is assumed that this constitutes the "natural exposure."

Bull or bear spreads entail (i) writing a call to extract some of the benefits from the upside as the premium, while the natural exposure balances out the option exposure; and (ii) buying a put to protect the natural downside exposure. In between the put and call strike prices (X_p, X_c), the producer retains an exposure to S, but this is limited in extent. The usual strategy is to use the *bull spread,* where $X_p < X_c$, i.e., the put strike price is lower than the call. Denote this as case A. Here the net exposure to S is positive between X_p and X_c, zero elsewhere [see Fig. 1(a)]. However, the *bear spread* is also considered by setting

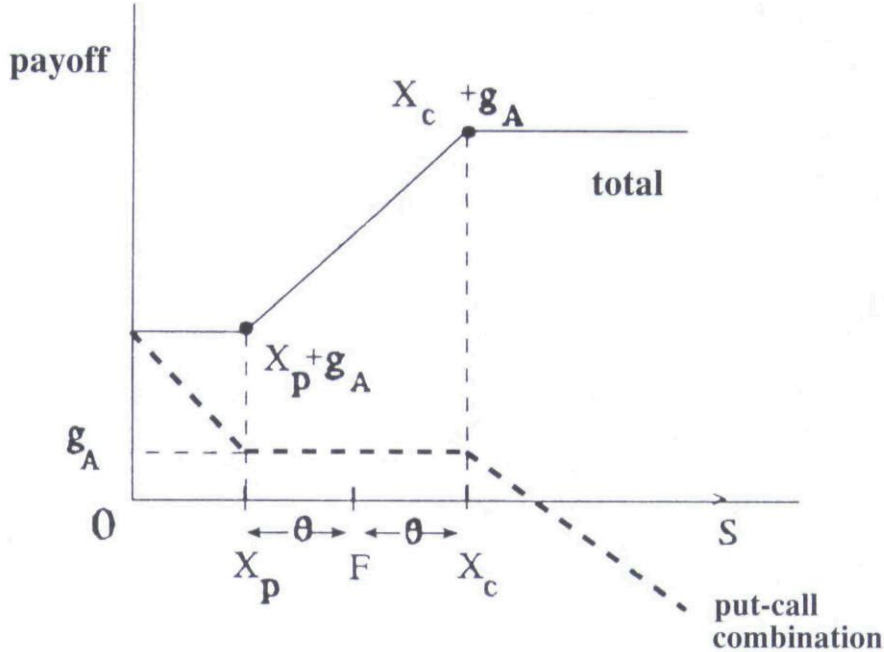

FIGURE 1(a)
Payoffs - $X_p < X_c$ (bull spread).

$X_c < X_p$. Denote this as case B. Here the net exposure to S is negative between X_c and X_p [see Fig. 1(b)]. Contingent payoffs are shown in Tables IA, B, while Figures 1(a), (b) show the payoff profiles from the put–call combination (dotted line) and the net, or total, payoff which encompasses also the natural exposure S (solid line). In each case, options are written with strike price equal to F, the current forward price. The options prices are denoted π_c, π_p for calls and puts, respectively, while the one period interest rate factor $(1 + \rho)$ converts the option premiums paid or received now to the end of the period for incorporation in the contingent payoffs.

Now introduce a parameter[3] $\theta > 0$ such that $X_p = F - \theta$, $X_c = F + \theta$ for case A, and $X_c = F - \theta$, $X_p = F + \theta$ for case B. In other words, parametric nesting can be achieved by simply setting the strike prices, with marginal variations needing no special interpretation. As $\theta \to 0$, the payoff from the put–call combination in either version becomes identical with the payoff from a standard "sell forward" contract:

[3]One could conceivably go further and simply define $X_p = F - \theta$, $X_c = F + \theta$ with θ unrestricted as to sign. Case A would then correspond to $\theta > 0$ and case B to $\theta < 0$. Unfortunately, the partition of the S axis into three regions as in Tables IA, B is not reproduced. The present analysis, therefore, proceeds for the two cases separately, maintaining $\theta > 0$ for each.

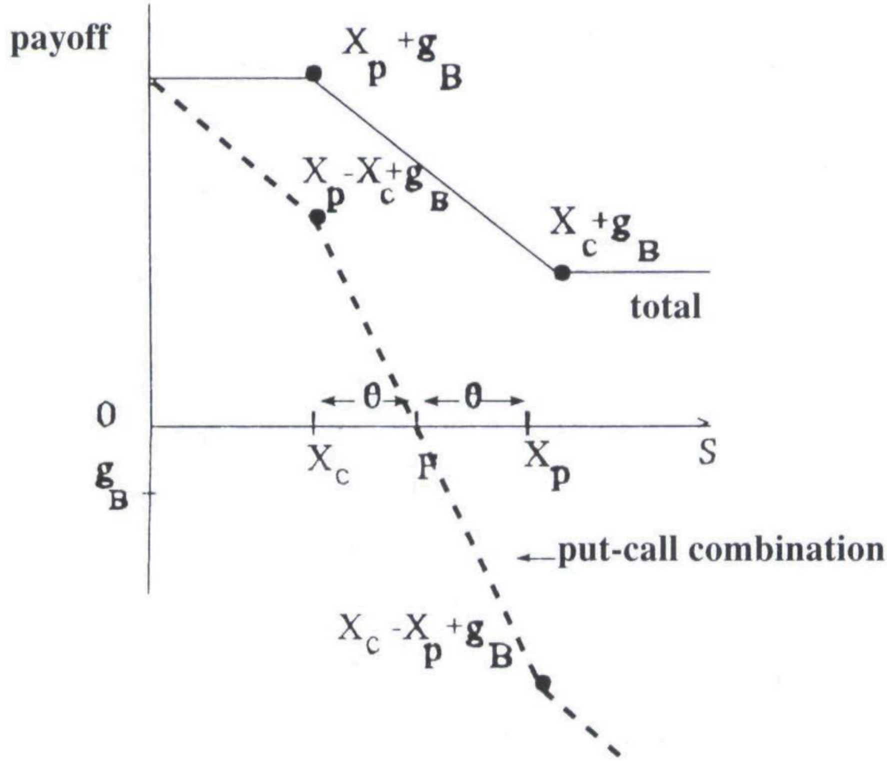

FIGURE 1(b)
Payoffs - $X_c < X_p$ (bear spread).

one has only to verify (see third section) that $\pi_c(F) = \pi_p(F)$, i.e., the net cost of the position is zero, (Fig. 1a,b) illustrate. As $\theta \rightarrow 0$, the strike prices X_c, X_p collapse on to the point F, the current forward price. The dotted line representing the payoff from the put–call combination becomes the 45° downward sloping line $F - S$, which is the payoff from a short forward position (the producer has sold forward).

The deformation from a forward to a spread can therefore be accomplished by simply perturbing the strike price parameter θ away from zero. One can explore the welfare gains of forwards versus symmetric spreads by means of simple differential calculus methods, given an appropriate objective function. The third section continues this line of development.

Portfolio Nesting

Suppose the objective is to compare a forward with a put option; again, this adopts the point of view of a producer. Consider combining a forward with progressively more units of a call option (the operational

TABLE I
Call–Put Strategy Payoffs

Contingency	*Write Call*	*Buy Put* *A: Case* $X_p < X_c$ *(Bull Spread)*	*Natural*	*Total*
$S \leq X_p$	$(1+\rho)\pi_c$	$-(1+\rho)\pi_p + X_p - S$	S	$(1+\rho)(\pi_c - \pi_p) + X_p$
$X_p < S \leq X_c$	$(1+\rho)\pi_c$	$-(1+\rho)\pi_p$	S	$(1+\rho)(\pi_c - \pi_p) + S$
$S > X_c$	$(1+\rho)\pi_c - (S - X_c)$	$-(1+\rho)\pi_p$	S	$(1+\rho)(\pi_c - \pi_p) + X_c$
		B: Case $X_c < X_p$ *(Bear Spread)*		
$S \leq X_c$	$(1+\rho)\pi_c$	$-(1+\rho)\pi_p + X_p - S$	S	$(1+\rho)(\pi_c - \pi_p) + X_p$
$X_c < S \leq X_p$	$(1+\rho)\pi_c - (S - X_c)$	$-(1+\rho)\pi_p + X_p - S$	S	$(1+\rho)(\pi_c - \pi_p) + X_p + X_c - S$
$S > X_p$	$(1+\rho)\pi_c - (S - X_c)$	$-(1+\rho)\pi_p$	S	$(1+\rho)(\pi_c - \pi_p) + X_c$

meaning of this combination is discussed further below). Thus to one forward (protecting one unit of the physical output exposure) add a semipositive fraction θ times a call option with strike price F, the current forward price. Table II shows the contingent end of period payoffs, with all symbols as for Table I. Figure 2 plots these payoffs against the future spot price S, for various values of θ between zero and unity. The lower half depicts the instrument profiles created by combining the forward and the call. The upper half combines this with the natural exposure (S) to give the total payoff. The case $\theta = 0$ is just the pure forward, while the other polar case $\theta = 1$ creates the put option (lower half of Fig. 1), or in fact a call once the natural exposure is incorporated (top half). Values of θ between zero and unity move closer to the put profile as $\theta \rightarrow 1$.

In the context of portfolio nesting, the variable θ represents a portfolio proportion as a decision variable, rather than a parameter, as in parametric nesting. In this context, a variation in θ may be interpreted in two possible ways: (a) the producer might actually be able to combine the forward and call position in an approximately continuous manner, against his or her spot position. Given the lumpiness of traded options contracts, most producers would not be able to do this on their own account, but might be able to do so a part of a pooling arrangement; for example until recently the Australian Wheat Corporation offered a scheme whereby producers could elect to cover any part of their planned output by means of forwards or puts. (b) Alternatively, a small increase in θ might be taken simply as a theoretical device to overcome the discontinuity problem referred to in the first section. If one could show that producer welfare was a monotonic increasing function of θ, then one could assert that a call option on a unit of physical output (the unit chosen in this case to coincide with the option quantity) would be superior to coverage by means of forwards of the same amount of physical output. In other words, one would use the differential behavior for $0 < \theta < 1$ to deduce something about the position value at $\theta = 1$

TABLE II
Contingent Payoffs: Forward Versus Protective Put

Contingency	*Forward*	*Physical*	$\theta \times$ *call*	*Total*
$S \leq F$	$F - S$	S	$-\theta(1 + \rho)\pi_c$	$F - \theta(1 + \rho)\pi_c$
$S > F$	$F - S$	S	$-\theta(1 + \rho)\pi_c + \theta(S - F)$	$-\theta(1 + \rho)\pi_c + (1 - \theta)F + \theta S$

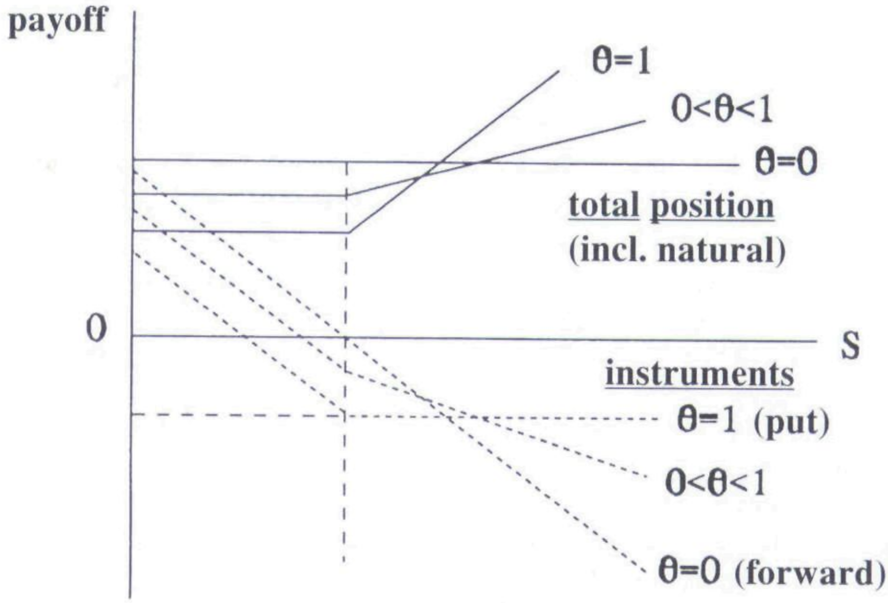

FIGURE 2
Portfolio nesting (forward to put).

versus that at $\theta = 0$. Monotonicity is generally difficult to show, so interpretation (b) remains somewhat conjectural.

Expositional Assumptions

Instrument nesting is used here to explore the relative advantages of forward versus spread strategies (third section) or the protective put (fourth section). It is useful at this point to establish assumptions for the price process involved and to remind the reader of associated Black–Scholes type options pricing results. Of course, the nesting methodology will survive the particular specifications employed for the price process, which is rather basic, but has the virtue of expositional simplicity.

It is assumed that the spot price follows a logarithmic Ito process of the form

$$\frac{dS}{S} = \mu dt + \sigma dz(t) \tag{1}$$

where μ is a constant mean drift (percent change) and σ^2 is the volatility of instantaneous percentage changes in price. The results given are unaffected if the mean drift is an autonomous function of time: $\mu(t)$ as in, say, regular seasonal movements of the physical price. In this case simply replace μ in everything that follows by the integral $\int_0^1 \mu(t)\,dt$, where the unit interval is identical with the holding horizon. More precisely, both the producer and the options market (meaning those who price options) believe that price is generated according to the process (1); producer and market share common estimates of volatility, but may differ in their estimates of the mean drift parameter or function μ, which is indeed not used in pricing options if Black–Scholes theory is employed. It is usually assumed with respect to the forward price that spot-forward arbitrage applies with no storage or convenience costs, so that $F = (1 + \rho)S_0$, where S_0 denotes the current physical (spot) price [see Black (1976)]. Of course this presupposes no inventory shortages, which may interfere with spot-forward parity [for discussions see Wright and Williams (1989) or Bowden (1993)]. If spot-forward arbitrage does apply, however, put–call parity in options theory purports that if π_p is the put premium and X is the common strike price, then

$$\pi_c - \pi_p = S - \frac{X}{1 + \rho} \tag{2}$$

so that if $X = F$, then $\pi_c(F) = \pi_p(F)$.

It is assumed that the option is written on the physical price, S, and that standard Black–Scholes (BS) pricing applies. An alternative is to write an option on the future, F, with Black's formula. If spot-forward parity holds, the two approaches are virtually equivalent, as the spot price differs from the forward only by the interest rate factor, presumed constant. Thus for a general strike price X, with the maturity set at $T = 1$,

$$\pi_c(X) = S_0 N(d_1) - Xe^{-r} N(d_1 - \sigma) \tag{3}$$

gives the call option price, with r defined by $1 + \rho = e^r$, $N(\,)$ the unit normal distribution function, and

$$d_1 = d_1(X) = \frac{1}{\sigma}\left[\log\left(\frac{S_0}{X}\right) + r\right] + \frac{1}{2}\sigma$$

The put option price can be established from put–call parity, i.e., eq. (2) above. In particular, if $X = F$, the current forward price, put–call parity can be used to derive $d_1(F) = (1/2)\sigma$, and

$$\pi_c(F) = S_0(2N(1/2\sigma) - 1) \tag{4}$$

For future use, the derivative with respect to X (more correctly, the partial derivative) is established from expression (3) as

$$\pi_c'(X) = \frac{-S_0}{\sigma X}\, n(d_1) - \frac{1}{1+\rho}\left[N(d_1 - \sigma) - \frac{1}{\sigma} n(d_1 - \sigma) \right]$$

so that under spot forward parity

$$(1+\rho)\ \pi_c'(F) = N\left(\frac{1}{2}\sigma\right) - 1 \tag{5}$$

Finally, from the Ito process (1), the distribution function of terminal price S is given by the lognormal distribution $\Phi(S) = N(\log(S/S_0);\ \mu, \sigma^2)$, where N is the Normal c.d.f. The corresponding density $\phi(S)$ is given by

$$\phi(S) = \frac{1}{S}\, n\left(\log\frac{S}{S_0};\ \mu_1, \sigma^2\right) = \frac{1}{S}\, n\left(\frac{\log\left(\frac{S}{S_0}\right) - \mu}{\sigma}\right)$$

where $n(x) = n(x;\ 0, 1)$ is the unit normal density [similarly $N(x) = N(x;\ 0, 1)$]. It follows from the preceding and spot-forward parity that at $S = F$,

$$\Phi(F) = N\left(\frac{r-\mu}{\sigma}\right) \quad \text{and} \quad \phi(F) = \frac{1}{\sigma F}\, n\left(\frac{r-\mu}{\sigma}\right) \tag{6}$$

SPREAD STRATEGIES VERSUS FORWARDS

Generalities

In this section, the parametric nesting methodology is applied to determine whether circumstances exist where producer welfare can be improved relative to a fully covered forward position by moving toward one or other of the two spread strategies outlined in the "Parametric Nesting" section, the bull spread or the bear spread. For the former, the strike prices are set, for an arbitrary nesting parameter, θ, at $X_c = F + \theta$, $X_p = F - \theta$, while for the bear spread, the strike prices are $X_c = F - \theta$, $X_p = F + \theta$. Setting $\theta = 0$ gives the standard forward and the object is to find circumstances in which setting $\theta > 0$, i.e., moving toward the spread strategies, will improve producer welfare. Subsequently, the alternative of remaining completely unhedged—the natural exposure—is incorporated also.

The analysis establishes bounds or "confidence limits" on price expectations, $\mu_s = ES$, such that if expectations fall in certain ranges, one or other of the three possible strategies [forward, bull spread (A), bear spread (B)] will be followed; or more precisely, incentives will either exist or not exist to depart from the standard forward in one or other of the indicated directions. The bounds are partitions on $\{\mu_s\}$ generated for fixed volatility σ from the drift parameter μ by means of varying x in the form:

$$\mu = r + x\sigma;\ x = \frac{\mu - r}{\sigma}$$

Having established "critical values" x_c, the corresponding critical values for μ and μ_s are generated as $\mu_c = r + x_c\sigma$, and $\mu_s^c = S_0 \exp(\mu_c + (1/2)\sigma^2)$, both monotonic transformations. Here the "critical point" (c) can refer to either case A ($c = A$) or case B ($c = B$) and are taken in turn.

Producer welfare is represented by Von Neumann-Morgenstern utility function $u(\bullet)$; the objective is to choose the course of action that maximizes expected utility of the payoff per unit of physical product. The expected utility from spread strategies are functions $v(\theta)$ of the nesting parameter. Ideally, one would like to optimize with respect to θ, but a more feasible alternative is to examine the behavior of $v'(\theta) = (dv/d\theta)$ at $\theta = 0$. If this is negative or zero, no incentive exists to depart from the forward; if $v'(0) > 0$, the pure forward strategy is dominated by a strategy with a contribution from options. Bounds on μ_s can then be established as earlier outlined.

In some circumstances, these bounds are independent of the precise specification of the utility function. In others, a curvature factor plays a role, defined at the current forward price, F, by

$$\gamma = \frac{Fu'(F)}{u(F)} = \left.\frac{du(S)}{d \log S}\right]_{S=F} \tag{7}$$

The factor γ is rather similar to the standard coefficient of relative risk aversion, but first rather than second derivatives are involved, so that risk properties are not the focus. This said, however, note that γ is constant for a power utility function and in this context has an inverse relationship to the conventional risk parameter, in the sense that decreasing γ corresponds to power utility functions with increasing risk aversion. Its role is not crucial in what follows, but it does help to set regions within which x_A or x_B must lie. For instance, the higher the curvature factor γ, the narrower is the region within which the critical values (boundaries of the decision regions) must lie.

Finally, it is shown in Table I that the payoff to the spread strategies depends not only upon the future spot price, S, in relation to the strike prices, X_p, X_c but also upon the premium income, $(1 + \rho)(\pi_c - \pi_\rho)$. The premium incomes for strategies A, B are in each case functions of the chosen nesting parameter θ:

$$g_A(\theta) = (1 + \rho)[\pi_c(F + \theta) - \pi_p(F - \theta)]$$
$$g_B(\theta) = (1 + \rho)[\pi_c(F - \theta) - \pi_p(F + \theta)]$$

Under case A,

$$\begin{aligned} g_A(\theta) &= (1 + \rho)[\pi_c(F + \theta) - \pi_c(F - \theta)] \\ &\quad + (1 + \rho)[\pi_c(F - \theta) - \pi_p(F - \theta)] \\ &= (1 + \rho)[\pi_c(F + \theta) - \pi_c(F - \theta)] \\ &\quad + (1 + \rho)S_0 - (F - \theta) \qquad \text{(put-call parity)} \\ &= (1 + \rho)[\pi_c(F + \theta) - \pi_c(F - \theta)] + \theta \\ &\qquad \text{(no spot-forward arbitrage)} \end{aligned} \tag{8A}$$

Similarly for case B

$$\begin{aligned} g_B(\theta) &= (1 + \rho)[\pi_c(F - \theta) - \pi_c(F + \theta)] - \theta \\ &= -g_A(\theta) \end{aligned} \tag{8B}$$

Note that $g(0) = 0$, in each case. Figure 3 portray the premium income functions. Note that Figure 1 above assumes $g_A(\theta) > 0$, i.e., $\theta < \theta_0$ in Figure 3.

Formal Results

The following result establishes partitions for cases A (the bull spread) and B (the bear spread), respectively. The results are interpreted more informally in the "Interpretation" section.

Proposition 1.
Case A: $(X_p < X_c)$.
Let $x = x_A$ *satisfy the equation*

$$\sigma\gamma\left[N\left(\frac{1}{2}\sigma\right) - N(-x_A)\right] = n(x_A);\ (n(x) = n(-x)) \tag{9A}$$

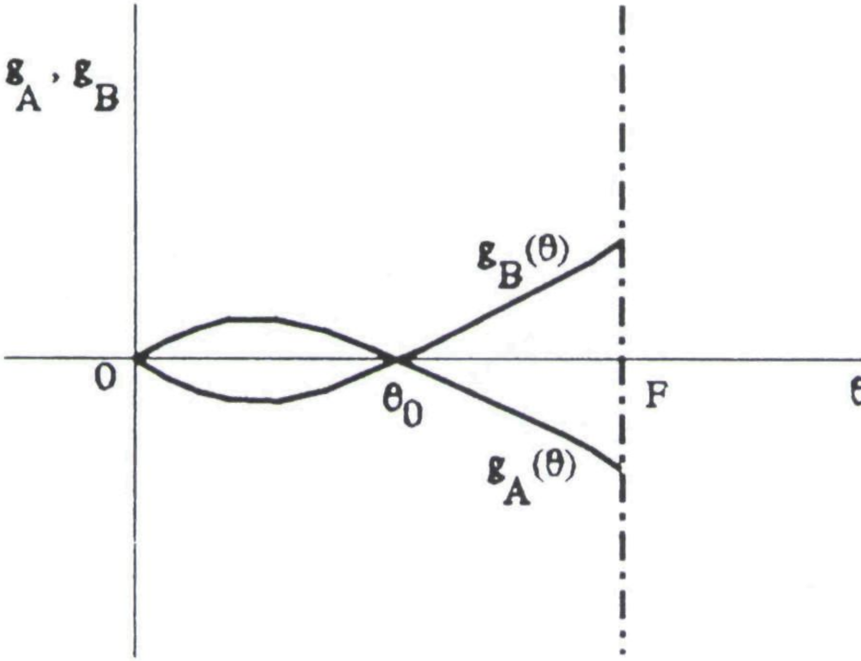

FIGURE 3
The premium income functions.

Then:

$$x_A > 0 \text{ and } \mu_S^A > F. \text{ If in addition } \gamma < \frac{2}{\sigma^2}, \quad \text{then } \mu_A > r \text{ and } \mu_s^A > \mathrm{Fe}^{(1/2)\sigma^2} \tag{a}$$

$$\text{If } \mu > \mu_A \text{ (or } \mu_s > \mu_s^A\text{) it pays to utilize the bull-spread options strategy A. Otherwise, the pure forward should be used.} \tag{b}$$

Case B: $(X_c < X_p)$.

Let $x = x_B$ *satisfy the equation*

$$\sigma\gamma\left[\mathrm{N}\left(\frac{1}{2}\sigma\right) - \mathrm{N}(-x_B)\right] = -n(x_B) \tag{9B}$$

Then:

$$-\frac{1}{\gamma\sigma} - \frac{1}{2}\sigma < x_B < -\frac{1}{2}\sigma \tag{a}$$

$$\textit{If } x < x_{\mathrm{B}}\ (\mu < \mu_{\mathrm{B}},\ \textit{etc.})\ \textit{it pays to reject the standard forward in favor of option strategy B; otherwise, B should not be utilized.} \tag{b}$$

Proof:
Case A ($X_p < X_c$). From Table IA with $g = g_A$,

$$\begin{aligned} v(\theta) = Eu(\theta) &= u(g(\theta) + F - \theta)\Phi(F - \theta) \\ &\quad + \int_{F-\theta}^{F+\theta} u(g(\theta) + s)\phi(s)\,ds \\ &\quad + u(g(\theta) + F + \theta)[1 - \Phi(F + \theta)] \end{aligned} \tag{10}$$

whence [recall $g(0) = 0$],

$$v'(0) = E\left.\frac{dv}{d\theta}\right]_{\theta=0} = u'(F)(1 + g'(0) - 2\Phi(F)) - 2u(F)\phi(F)$$

From eq. (8A), $g'(0) = 2(1 + \rho)\pi_c'(F) + 1$. Hence

$$\begin{aligned} v'(0) &= 2u'(F)[1 - \Phi(F) + (1 + \rho)\pi_c'(F)] - 2u(F)\phi(F) \\ &= 2u'(F)\left[\mathrm{N}\left(\frac{1}{2}\sigma\right) - \mathrm{N}\left(\frac{r - \mu}{\sigma}\right)\right] - 2\frac{u(F)}{\sigma F}n\left(\frac{r - \mu}{\sigma}\right) \end{aligned} \tag{11}$$

by utilizing (5) and (6). Recalling the definition (7) of the (positive) curvature parameter γ, eq. (11) indicates that welfare can be improved by the put–call strategy, relative to the "null" forward strategy, if and only if

$$\sigma\gamma\left[\mathrm{N}\left(\frac{1}{2}\sigma\right) - \mathrm{N}\left(\frac{r - \mu}{\sigma}\right)\right] \geq n\left(\frac{r - \mu}{\sigma}\right) \tag{12A}$$

Equation (9A) is (12A) expressed as an equality and with $x = (\mu - r)/\sigma$. Turning to the two parts of the proposition:

(a) It is clear from (12A) that $x_A > -(1/2)\sigma$, but one can improve on this bound to show that, in fact, $x_A > 0$. Suppose to the contrary that $x_A \leq 0$. Utilizing the concavity of $N(x)$ for $x \geq 0$:

$$\mathrm{N}\left(\frac{1}{2}\sigma\right) \leq \mathrm{N}(-x_A) + \left(\frac{1}{2}\sigma + x_A\right)n(-x_A)$$

Substituting into (9A)

$$n(x_A) \le \sigma\gamma\left(\frac{1}{2}\sigma + x_A\right)n(x_A)$$

whence, $x_A \ge \dfrac{1}{\sigma\gamma} - \dfrac{1}{2}\sigma$.

But if $\gamma < (2/\sigma^2)$ then $(1/\sigma\gamma) > (1/2)\sigma$ and it can only be true that $x_A > 0$, is a contradiction. Thus, $x_A > 0$, whence $\mu_A > r$ and $\mu_S^A > \mathrm{F}e^{(1/2)\sigma^2}$ under spot-forward parity.

(b) It suffices to show for part (b) that for $x > x_a$, $f(x) = \sigma\gamma[N((1/2)\sigma) - N(-x)] - n(x)$ is increasing in x, for then $f(x) > 0$ whenever $x > x_A$ and condition (21A) is satisfied. Now, $f'(x) = n(x)(\sigma\gamma - ((n'(x))/(n(x))) = n(x)(\sigma\gamma + x) > 0$, if $x > 0$, as has been shown. The result for case A follows.

Case B ($X_c < X_p$). The same procedures can be followed, with changes of sign as appropriate, to give $v_B'(0) > 0$, if and only if,

$$\sigma\gamma\left[N\left(\frac{1}{2}\sigma\right) - N\left(\frac{r-\mu}{\sigma}\right)\right] \le -n\left(\frac{r-\mu}{\sigma}\right) \tag{12B}$$

Turning to the two parts of the proposition:

(a) From (9B), it is clear that $x_B < -(1/2)\sigma$. As $-x_B > 0$ one can again utilize the concavity of $N(x)$ for $x > 0$ to derive

$$-n(x_B) < \sigma\gamma\left[N(-x_B) + \left(\frac{1}{2}\sigma + x_B\right)n(x_B) - N(-x_B)\right]$$

the strict inequality applying because x_B cannot be equal to $(1/2)\sigma$. Hence,

$$-1 < \sigma\gamma\left(\frac{1}{2}\sigma + x_B\right),\ \text{i.e.,}\ x_B > -\frac{1}{\sigma\gamma} - \frac{1}{2}\sigma.$$

(b) Let $f(x) = \sigma\gamma[N((1/2)/\sigma) - N(-x)] + n(x)$. As with Proposition 1A, it suffices to show that for $x < x_B$, $f'(x) > 0$. But $f'(x) = n(x)(\sigma\gamma - x)$, so for any $x < 0$, $f'(x) > 0$. □

Interpretation

Figure 4 partitions the μ-line and μ_s-line into the three regions bounded by the critical values μ_B, μ_A and μ_B^s, μ_A^s generated by x_B, x_A as

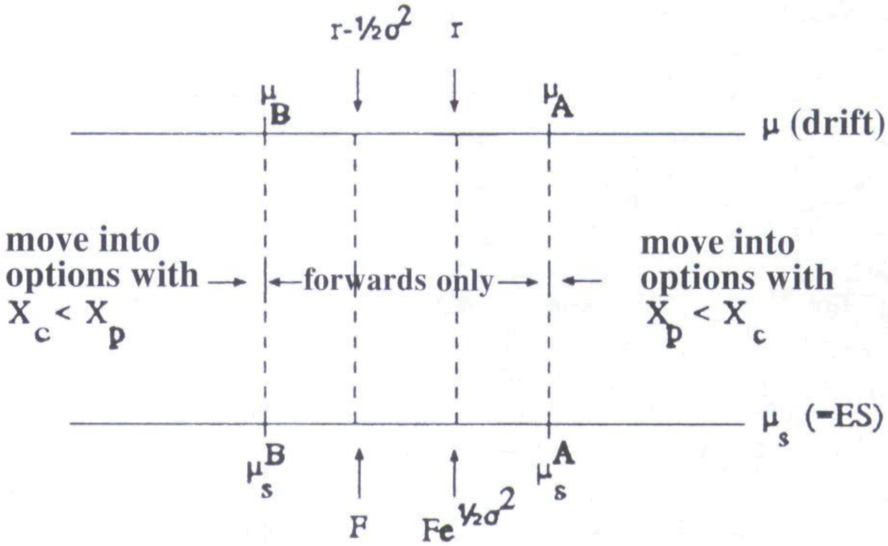

FIGURE 4
Trading bands: forward versus bull/bear spreads.

in Propositions 1A,B, respectively. The precise boundaries may be obtained if desired by solving eq. (9A,B) for x_A, x_B and utilizing $\mu_S = Fe^{x\sigma+(1/2)\sigma^2}$. The region between $\mu_S = \mu_S^B, \mu_S^A$ is evidently a "dead zone" or the zone of inaction—if expectations fall in here, then it pays to take the pure forward or swap and not utilize either spread strategy. A fairly good rule of thumb is not to utilize a bull spread strategy unless the user's personal expectations are such that

$$ES > Fe^{(1/2)\sigma^2} \cong F\left(1 + \frac{1}{2}\sigma^2\right)$$

where σ^2 can either be estimated from the implied volatility of quoted options, or subjectively imputed as $\mathrm{Var}(\log(S/S_0))$, i.e., the expected variance of the rate of commodity price inflation. It should be noted, however, that this sort of rule arises only in comparing forwards with options, i.e., where a prior decision has been taken to fully protect output. The discussion that follows considers a third alternative, namely the unhedged position.

The Spot Alternative

A simple alternative to either forward or spread strategies is to not hedge at all. If $\mu_s < F$, it is well known from the production planning literature surveyed in the first section, that forwards dominate spot, so that the zone of interest relates to $\mu_s > F$. Here one can ask whether moving towards spot (from the assumed initial short forward position) would dominate moving toward the bull spread position.

Although it is hard to give a fully rigorous account, the following argument suggests that spot does in fact dominate the bull spread strategy over $\mu_s > F$. Start with a fully covered long forward position and add θ times a *long* forward position, so that the payoff per unit of the physical is $F + \theta(S - F)$. (The nesting involved here is actually portfolio nesting). Expected utility is

$$v_1(\theta) = \int_{-\infty}^{\infty} u(f + \theta(S - f))\,\phi(S)\,dS$$

and

$$v_1'(0) = u'(F)(\mu_s - F) \tag{13}$$

It is not possible to directly compare $v_1'(0)$ with $v'(0)$ as in Proposition 1 above, for the variation refers to two quite different objects,[4] a long forward and a bull spread, respectively. If not in magnitude, however, it is possible to compare as to sign. For μ_s in the range $F < \mu_s \le \mu_s^A$, eq. (13) indicates an increase in welfare in moving to spot, whereas the corresponding marginal effect is negative for the bull spread. This is in spite of the exposure of the natural position being created to a full downside risk not present with the bull spread [see Fig. 1A]. So where $\mu_s > \mu_s^A$, where the exposure of the natural position to low values of S is even less, one would expect the spot alternative to be even more dominant over the bull spread, especially in view of the truncated (flat) side of the latter. It is therefore a fairly firm conjecture that the spot alternative is uniformly dominant over the bull spread for all values of subjective expectations in excess of the current forward rate.

On the other hand, this sort of argument cannot be applied to the bear spread where $\mu_s < F$, for here the forward position always dominates spot as an alternative. This leaves a hedging role for the put–call strategy in the form of a bear spread where $X_c < X_p$ and where expectations are pessimistic. Figure 5 portrays the suggested trading bands where the natural exposure is included as an alternative.

PROTECTIVE PUTS AND PORTFOLIO NESTING

A similar conceptual procedure can be applied to the protective put versus forward strategies outlined in "Portfolio Nesting" section as an

[4]Thanks go to a referee for this point.

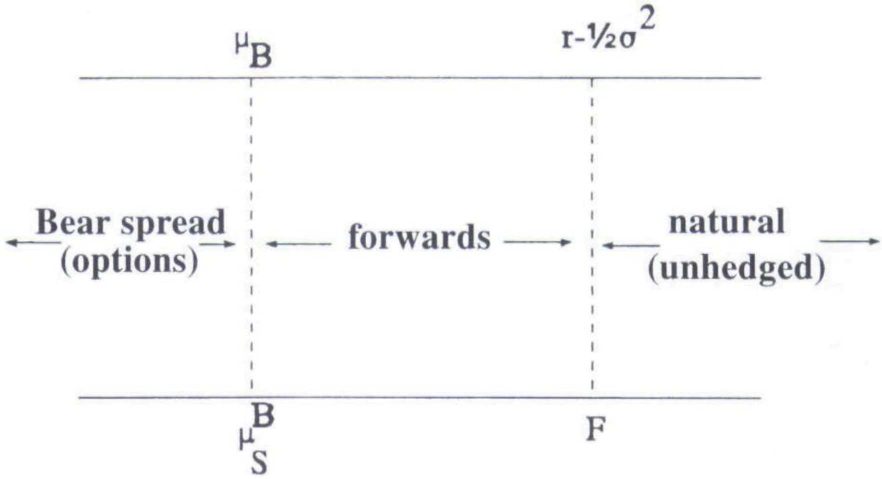

FIGURE 5
Trading bands: including natural exposure.

application of portfolio nesting. Again one starts with $\theta = 0$, which corresponds with the pure forward and investigates whether $\theta > 0$ improves producer welfare. The following is a formal statement of the main result.

Proposition 2. *Suppose: (a) The spot or physical process is assumed by all parties to follow a logarithmic Ito process with constant or autonomous drift $\mu(t)$ and volatility σ^2. Options are priced according to the Black–Scholes formula, and producers and pricemakers agree on the volatility σ^2.* (b) *Spot-forward parity $F = S_0e^r$ holds.* (c) *The producer is constrained to hold all positions to maturity.*

Then the producer will commence substituting put options at strike price F for the forward if and only if $\mu_s > F$.

The proof of this proposition is straightforward but rather long, and is available on request from the author. The starting point is the expected utility as derived from Table II:

$$v(\theta) = E(u) = u(F - \theta(1 + \rho)\pi_c)\Phi(F) + \int_F^\infty u[F - \theta(1 + \rho)\pi_c + \theta(S - F)]\phi(S)\,dS$$

where $\pi_c = \pi_c(F)$ and ϕ, Φ are defined by eq. (6) above. Differentiating with respect to θ and integrating by parts yields

$$v'(o) = u'(F)\left[-\pi_c(1 + \rho) + \mu_s - F + \int_{-\infty}^F \phi(S)\,dS\right]$$

and the remainder of the proof is directed at showing that $v'(o)$ is positive, if and only if, $\mu_s > F$.

The producer will therefore not choose the protective put strategy if he is more pessimistic about future prices than current forward prices would indicate; but will move in that direction (as against a forward), otherwise. This result is independent of any particular specification of the nature of the utility function, which could be risk averse, risk neutral, or risk loving. On the other hand, attitudes to risk will naturally affect the speed with which protective puts (or natural spot–see below) are substituted for the forward, where expectations are favorable.

A similar cut-off point, again independent of volatility, exists for the choice between forwards and spot, i.e., the unhedged position. It is not possible to use a sign-based comparison of marginal utilities to fully compare the relative expected utilities of moving to spot as against the protective put where $\mu_s > F$. One would expect spot to dominate the protective put for very optimistic expectations, which would attach progressively less probability to the lower S values protected by the put, but whether this result holds true for all $\mu_s > F$ remains conjectural.

CONCLUDING REMARKS

Decisions by producers as to whether to use forwards or options acquire some substance when it is planned to hold the relevant instruments to maturity, for in this case things do depend upon producer expectations as to future physical prices. Of course, they then become rather hard decisions because one has to balance expectations against the cost of premiums, and also to incorporate highly nonlinear payoffs, if options strategies are to be employed. The nesting methodology provides a way of reducing the problem to analytically more tractable terms and deriving reasonably simple decision rules as a result. These solutions may be regarded as analogous to confidence intervals, with the forward price determining the decision boundaries.

The decision rules depend upon the alternatives being considered, just as they do in statistical decision theory. For instance, if a prior decision has been made to protect the producer's entire output, with the choice to be made between forwards and put options, then it is optimal to use puts if the expected future price exceeds the current futures price. This result may not, however, hold if one alternative is simply to remain unhedged. One can use the nesting methodology to show that bull spread strategies, where the strike price of the written call is above that of the put, are dominated by forwards/natural exposure mixtures. On the other hand, a role for options continues to exist for bear spread strategies, effective where producer expectations are very pessimistic relative to the current futures price.

One of the most interesting aspects of the decision rules is that they hold regardless of the character of the utility function; in particular, whether agents are risk averse, risk neutral or risk loving—they arise simply because agents prefer more money to less. Insofar as the rules advocate regimes in which one mode of coverage is swapped for another—e.g., forwards to spot where $\mu_S > F$, the speed or completeness with which this is done does depend upon risk aversion; in the forwards to spot example, a risk neutral or risk loving agent would instantly transfer the whole between forwards and spot and perhaps go beyond that.

The decision rules are local, in that only small portfolio variations are considered. As remarked in the first section, global solutions involving discontinuous or lumpy portfolio readjustments will in general require specific assumptions or even the precise functional form of the utility function involved. It would be of interest to check whether the local rules are in fact global for utility functions of designated types or classes. Likewise, further research could investigate the extent to which the choice of strike price validates or invalidates the decision rules established by utilizing the current futures price as the chosen strike price, a necessity forced here by mathematical tractability.

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