SHOULD ACTIVELY TRADED FUTURES CONTRACTS COME UNDER THE DUAL-TRADING BAN?

SUGATO CHAKRAVARTY

INTRODUCTION

In recent years, among many other controversies facing the futures exchanges, a debate has raged as to whether floor traders should enjoy dual-trading facilities.[1] While proponents of dual trading claim that the practice fosters market liquidity, critics argue that such a privilege leads to a conflict of interest between brokers and their customers. This is because a broker[2] can front-run on his customer's order for personal profit. Front-running is illegal in the United States, but as

I would like to thank Mark Bagnoli, Richard Feinberg, Eric Rasmusen, Anjan V. Thakor, Craig Holden, and especially David P. Brown for comments on earlier drafts. Thanks are also due to two anonymous referees and the editor, Mark J. Powers, for suggestions that led to considerable improvement in the exposition of the article. Unfortunately, they cannot be blamed for any errors.

[1]Dual-trading facilities enable certain traders to trade both for their customers as brokers and trade on their own accounts as dealers.

[2]For this article, the term "broker" is used to mean dual-capacity traders. They can both execute orders for their clients as well as trade for their personal accounts.

■ *Sugato Chakravarty is an Assistant Professor, Family and Consumer Economics, Purdue University.*

The Journal of Futures Markets, Vol. 14, No. 6, 661–684 (1994)

CCC 0270-7314/94/060661-24

most traders and officials of the futures exchanges privately admit, it is almost impossible to eradicate.[3]

This article asks: Given that front-running cannot be eradicated by making the mere practice of it illegal, should the futures exchanges ban dual-trading practices on its most actively traded contracts to reduce front-running? This question is particularly important in the light of a May 1991 ruling (Rule 552) by which the Chicago Mercantile Exchange suspended dual trading privileges in contracts with an average daily trading volume of 10,000 contracts or more over the previous six months.

Here, the broker–customer relationship is modeled as a Stackelberg game, as opposed to a Cournot game used by Admati and Pfleiderer (1988), Holden and Subrahmanyam (1992), and others. Specifically, the informed trader (the Stackelberg leader) commits first by submitting orders to the broker. The broker (the Stackelberg follower), in turn, gets to add a personal order which may be based on the informed trader's order. Finally, the market maker sees the broker's order (which may include the broker's personal order as well) along with orders from noise traders, but none of the components separately. The market maker then sets a single market clearing price, such that he makes zero expected profits, conditional on his beliefs about the informativeness of the combined order that he sees.

In the current single-period model, any non-zero quantity traded by the broker after observing his customer's order is interpreted as front-running. This is because the broker chooses his own order after observing his customer's order and before executing the same.

The trading intensity of the customer and broker are inversely related. With multiple brokers, competition drives the expected front-running profit per broker exponentially to zero with only about ten unethical brokers in the market. Thus, banning dual trading in the most actively traded contracts is pointless, since front-running

[3]Three common methods are generally used for a broker to front-run on customers. The first, and the easiest to detect, is when a broker trades a small quantity and then immediately afterwards trades a large quantity. Since extensive trading records by brokers are maintained by nearly all futures exchanges and most certainly by both the Chicago Board of Trade and the Chicago Mercantile Exchange, this obvious attempt at front-running will almost certainly be spotted by the officials of the exchange. A second method involves a broker who, having seen his customer's orders, leans over to a colleague and asks him/her to trade a certain quantity on behalf of the first broker. This practice is hard to discover from a mere examination of the trading records of each broker. A third method, a variation of the second, involves the colleague broker trading a certain quantity on his/her own personal account, after which the trader with the news and the one who does the actual front-running based on this information distribute the profits between themselves. These collusive practices are difficult to detect, without a direct confession by the parties concerned.

profits in these contracts are insignificant to begin with. Additionally, the brokers have no fundamental information of their own and only skim the information of their informed client for personal profit. Thus, their presence forces informed trader to scale down his trading activity, with the brokers picking up the slack, such that in equilibrium, the brokers do not impact the informativeness of prices.

Related literature includes Sarker (1993) who considers a broker who receives orders from multiple informed and noise traders. Later, the model allows for rational behavior by the uninformed traders. In Roell (1990), a broker observes the trades of some uninformed traders. In Fishman and Longstaff (1992), a broker has more information about customer's trading motives than the market maker. Finally, Smith and Whaley (1991) examine the "top-step rule" imposed in June, 1987 in the S&P 500 futures trading pit at the Chicago Mercantile Exchange. This rule prohibits dual trading by floor traders who stand on the top step of the S&P 500 futures pit. They can act only as pure brokers. Smith and Whaley find evidence that the imposition of this rule leads to a widening of the effective bid–ask spread. In contrast, the Fishman and Longstaff empirical work focuses on floor traders' and their customers' profits conditional on whether the floor traders choose to dual trade. They find that dual traders earn higher profits than non-dual traders, and that customers of dual-trading brokers do better than customers of non-dual-trading brokers. While these studies focus on the liquidity-providing aspect of dual traders, the current model focuses on the competition among dual traders, and its effects on front-running profits and informativeness of prices.

The rest of the article is organized as follows. The next section presents the basic model and the model that includes multiple brokers. The third section concludes. All proofs are in the appendix.

THE BASIC MODEL

In this section we consider a single-period, rational expectations model similar to Kyle (1985). This model has three sources of uncertainty: the uncertainty regarding the future spot price of a single risky commodity or asset ($\tilde{v}$), the broker's uncertainty regarding the informativeness of his customer's order ($\tilde{k}$) and, finally, the informational advantage that the broker enjoys over the market maker ($\tilde{u}$). The random variables $\{\tilde{v}, \tilde{k}, \tilde{u}\}$

are assumed to be jointly normally and independently distributed with means $\{0, 0, 0\}$[4] and variances $\{\Sigma_0, \sigma_k^2, \sigma_u^2\}$.

The players in the model are: one informed trader, one broker, many noise traders, and a market maker who deals with the risky asset in question, and prices competitively. The quantity u is assumed to be traded by noise traders, who are price takers and may or may not be exchange members. The non-member noise traders are assumed to reach the market maker directly through another exchange member, like a pure discount broker, by paying him an insignificant sum to just walk the order to the pit. Whatever the exact mechanism, it is assumed that these orders and the broker's total order reach the market maker simultaneously, such that the market maker only see the total order flow.

Trading is structured in the following steps: in stage 1, the informed trader observes a perfect signal about the future spot price $\tilde{v} = v$ and submits a market order $x(v)$ to the broker. It is assumed that the broker has better, though imperfect, information about the quality of his customer's information, than other market participants, specifically the market maker. This is consistent with actual practice, where the broker may be the only person who knows the identity of his customer. The broker may also have information about the past profitability of the customer's trade, which may give him an informational edge over the market maker.

In stage 2, the broker having observed $y_0 = x + k$[5], now chooses $z(y_0)$ for his own account. He then submits $x + k + z$ for execution to the market maker. In stage 3, the market maker, engaged in price competition, observes the total order flow $y_1 = x + k + z + u$, but none of the components separately. Thus, as discussed above, the broker has an informational advantage over the market maker. The market maker sets price $p(y_1)$ to clear the market. The price $p(y_1)$ set by the market maker best reflects the information regarding v contained in the order flow y_1. In this sense, markets can be said to be semi-strong form efficient. Finally, in stage 4, the liquidation value v is publicly announced and both the informed trader as well as the broker realize their respective profits, if any.

In the single-period set up, an equilibrium is defined as a triplet $\{x, z, p\}$, such that the following two conditions hold:

[4]The results of the article are not changed qualitatively if the mean value of the payoff v is a positive constant p_0.

[5]Thus, $\tilde{k}$ is simply a measure denoting the quality of information that the broker gleans from seeing the customer's order. If k is large, the broker's belief about the informativeness of his customer's order is low, and vice versa.

1. Profit maximization for the informed trader and the broker, given by,

$$\underset{x}{\text{Max}}\ E[\{\tilde{v} - p(y_1)\}x|\tilde{v} = v]$$
$$\underset{z}{\text{Max}}\ E[\{\tilde{v} - p(y_1)\}z|x + k]$$

respectively, and

2. Market efficiency condition, given by

$$\tilde{p}(y_1) = E[\tilde{v} \mid y_1]$$

The following can now be stated:

Proposition 1. *A unique linear equilibrium exists with*

$$x(v) = \beta_1 v$$
$$z(y_0) = \gamma(x + k)$$
$$p(y_1) = \lambda(x + k + z + u)$$

where β_1 and γ are the trading intensity parameters of the informed trader and the broker, respectively, and λ[6] is the market maker's pricing coefficient. The coefficient ξ,[7] below, is the broker's personal value coefficient, where

$$E[v \mid y_0] = \xi(y_0)$$
$$\xi = \frac{\beta_1 \Sigma_0}{\beta_1^2 \Sigma_0 + \sigma_k^2}$$
$$\lambda = \frac{1}{\beta_1} - \xi$$
$$\gamma = \frac{\xi - \lambda}{2\lambda}$$

and β_1 is computed as the unique positive root to the cubic equation given by

$$\Sigma_0 \overline{\beta}^3 + \sigma_k^2 \overline{\beta}^2 - \frac{\sigma_k^4}{\Sigma_0}\overline{\beta} - \left[\frac{\sigma_k^6 + 4\sigma_u^6}{(\Sigma_0)^2}\right] = 0$$

where $\overline{\beta} \equiv \beta^2$. Note that β is only a general representation for the informed trading intensity parameter, while β_1 is the unique equilibrium value for a given set of $\{\Sigma_0, \sigma_k^2, \sigma_u^2\}$.

[6]Note that $1/\lambda$ measures the market depth, which is the order flow necessary to change prices by one dollar.

[7]$1/\xi$ measures the order flow to the broker, necessary to increase the value of the asset (to the broker) by one dollar.

Finally, the ex-ante profits of the informed trader and the broker[8] (unconditional on $\tilde{v}$) are, respectively,

$$E\,\tilde{\pi}_i = \frac{\Sigma_0 \beta_1}{2} \quad \text{and} \quad E\,\tilde{\pi}_b = \left(\frac{\xi - \lambda}{2\lambda}\right)\frac{\beta_1}{2}\left[\Sigma_0 - \frac{\sigma_k^2}{\beta_1^2}\right]$$

Proof: See Appendix.

Figures 1 and 2 illustrate the equilibrium graphically. In Figure 1, the trading intensity parameters of the broker and the informed trader, γ and β_1 respectively, are plotted against σ_k^2, while holding σ_u^2 and Σ_0 constant at 10 and 5, respectively. Note that σ_k^2 is allowed to vary from 0.001–8.5. First note the inverse nature of trading between the informed trader and the broker. A low (high) σ_k^2 implies that the customer order seen by the broker is relatively informative (uninformative). Recall that it is the informational advantage of the informed trader over the broker, that derives the trading intensity of the informed trader. As $\sigma_k^2 \rightarrow 0$, this

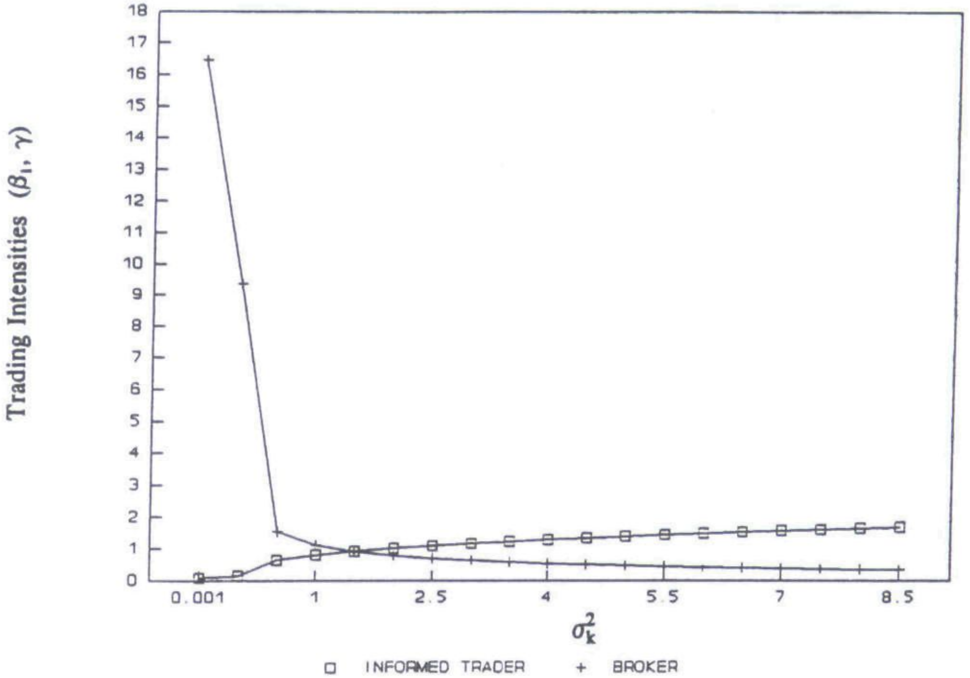

FIGURE 1
Trading of broker and informed trader.

[8]The broker profits computed here, and in proposition 4 are expected front-running profits only. Additionally, the broker earns commissions from executing his customer's order. This commission rate for the broker is switched to zero, since the goal here is to focus on front-running profits.

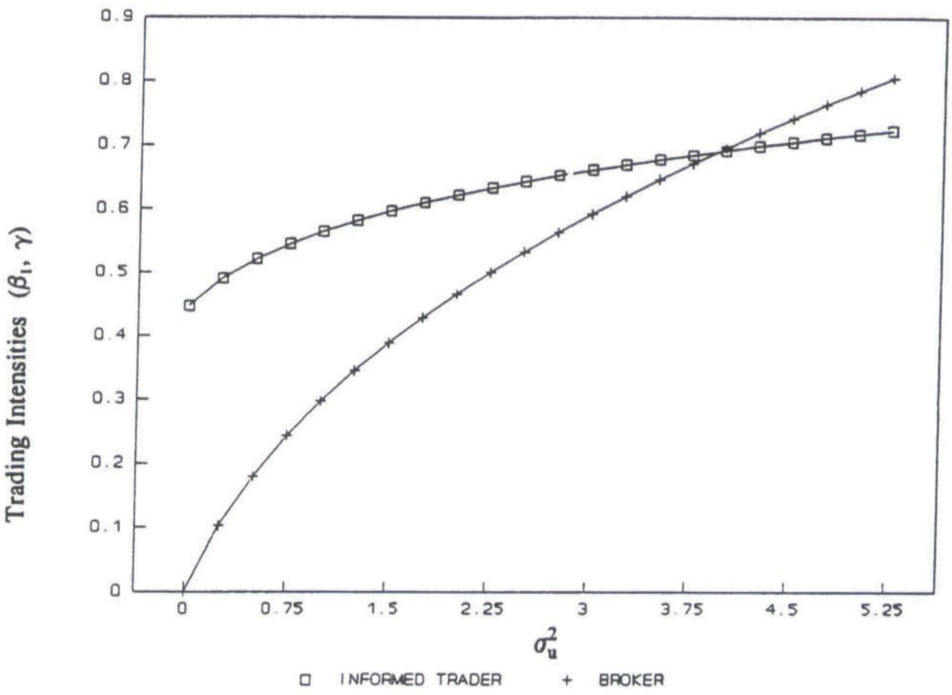

FIGURE 2
Trading of broker and informed trader.

advantage disappears, and $\beta_1 \to 0$. Simultaneously, the inverse nature of trading intensities between the informed trader and broker, described above, dictates a large value of γ.

In Figure 2, the broker and the informed trader's trading intensity parameters γ and β_1, respectively, are plotted against σ_u^2 while holding σ_k^2 and Σ_0 constant at 1 and 5, respectively. A low (high) σ_u^2 implies a relatively small (large) informational advantage of the broker over the market maker. From Figure 2, for relatively small values of σ_u^2 (compared to σ_k^2), the broker trades with low intensity while the informed trader has a correspondingly high trading intensity. At the other extreme, as σ_u^2 becomes large (in comparison to σ_k^2), the trading intensity of the broker becomes large. The informed trader rationally anticipates this and scales down his own trading intensity.

Finally, the expected payoff of the informed trader (unconditional on $\tilde{v}$) predictably increases with the intensity of his trade (β_1). The unconditional expected payoff of the broker increases with β_1 since the broker has more fundamental information, for a given quantity of orders that he receives for execution. Additionally, his expected payoff decreases with σ_k^2 since the broker is able to see his informed customer's order with progressively less noise.

Now, some interesting limiting cases are presented.

Proposition 2.

a. *When* $\sigma_k^2 = 0$, *the informed trader does not trade* (i.e., $\beta_1 = 0$).

b. *As* $\sigma_k^2 \to \infty$, *the unique linear equilibrium goes to*

$$x(v) = \beta_1 v$$
$$z(y_0) = \gamma(x + k)$$
$$p(y_1) = \lambda(x + k + z + u)$$

where

$$\beta_1 \to \infty$$
$$\gamma \to -\frac{1}{2}$$
$$\lambda \to 0$$

c. *When* $\sigma_u^2 = 0$, *the broker does not trade on his own account* (*i.e.*, $z = 0$). *The equilibrium market depth and the trading intensity parameter* (β_1) *of the informed trader are identical to Kyle (1985), and given by*

$$\lambda = \frac{1}{2}\sqrt{\frac{\Sigma_0}{\sigma_k^2}}$$
$$\beta_1 = \frac{1}{2\lambda}$$

Proof: See Appendix.

The informational advantage that the informed trader enjoys over the broker is what drives his intensity of trade. When $\sigma_k^2 = 0$, this advantage vanishes. Thus, the informed trader does not trade.

The magnitude of the broker's trade on his personal account is a function of the informational advantage that he enjoys over the market. He trades in the same direction as the informed trader when he expects the market maker to underestimate the informativeness of the customer's order; and trades in the opposite direction, when he expects the market to overestimate this informativeness. It is the latter that happens when $\sigma_k^2 \to \infty$. If $k > 0$, then on average, the market clearing price p will be greater than the unconditional expected value of v $E(\tilde{v})$. Hence, the broker personally short-sells (i.e., $z < 0$). Conversely, when $k < 0$, then on average, p is less than $E(\tilde{v})$, and the broker buys in his personal account (i.e., $z > 0$). There is a trade-off between quantity and price-impact that prevents the broker from taking a position too large against the total orders that he sees. As he trades more on his personal account, he effectively reduces the difference between p and $E(\tilde{v})$. This trade-off implies a unique interior optimum, since profits are concave in the quantity traded.

Finally, when $\sigma_u^2 = 0$, the broker's informational advantage over the market maker disappears. Thus, he is unable to make positive expected profits, and does not trade.

Notice that the above results are different from the standard multiple informed trader non-cooperative games examined by Admati and Pfleiderer (1988) and Holden and Subrahmanyam (1992), among others. There, the multiple informed traders, who are identical in all respects, simultaneously compute the (identical) equilibrium quantity to trade, based on the identical signal that they all see. Thus, they each end up trading identical quantities, less than that in the monopolist case but a combined quantity greater than that in the monopolist case. In this study's model, even though the broker and the informed trader can be looked upon as multiple informed traders, in the sense that the broker after seeing the informed trader's order is also partially informed about the terminal value of the risky asset, they essentially solve different maximization programs. The Holden and Subrahmanyam, Admati and Pfleiderer, and other models of this class are essentially Cournot games where decisions are made simultaneously. In contrast, this study's model is a Stackelberg game where the informed trader is the first mover (the Stackelberg leader), solving his maximization problem conditional on observing the terminal value of the asset and taking into consideration the effect of his trade on the personal trade of the broker (the Stackelberg follower), which in turn affects the market clearing price charged by the market maker. The Broker, in turn, conditions on his customer's trade, when choosing his personal order, taking into consideration the effect that the combined trade has on the market clearing price.

Many Brokers

The basic model is extended now to allow for many dual traders (**m**) to exist at the trading pit where a single futures contract (with terminal value v) is being traded. Of these m dual traders, m_e choose not to trade on their personal accounts. They are labeled *ethical brokers.* Then there are m_u who trade on their personal accounts after observing their customer's order. These are labeled the *unethical brokers.* Thus,

$m = m_e + m_u$. The quantity m can also be thought of as a measure of the number of brokers per information trader present in a particular contract.

The model assumes a single, large, informed trader. As usual, many noise traders are trading for liquidity reasons. Thus, the informed trader after observing v (at time zero) chooses his optimal trading quantity **x**. He/she then comes to the market and breaks up his order and distributes it among the **m** available brokers. Each of the brokers is unsure about the informativeness of the order that he receives. For the ethical brokers, it does not matter, since they do not trade personal quantities. But for the unethical ones, their beliefs about order informativeness is important since it enables them to choose their personal order quantities from it. The proportion of unethical brokers in the contract is common knowledge, even though they are indistinguishable from the ethical brokers.

Otherwise, the model set up and the sequence of events are the same as those in the basic model. As usual, **z** denotes the quantity chosen by each of the m brokers for his personal account after having observed his customers' orders, part of which is from the informed customer. Also, p denotes the price function, which is linear in y_1 denoting the total orders observed by the market makers.

The following Proposition characterizes the equilibrium in this set up.

Proposition 3. *For any $m < \infty$, and $m_u \leq m$, a unique linear equilibrium exists with*

$$x(v) = \beta_2 v$$

$$z_u = \gamma(x + k)$$

$$p(y_1) = \lambda(x + m_u z_u + \tilde{u} + \tilde{k})$$

where β_2 and γ are the trading intensity parameters of the informed trader and the broker, respectively, and λ is the market maker's pricing coefficient. Additionally, ξ/m is each broker's personal valuation coefficient, where

$$\frac{\xi}{m} = \frac{\beta_2 \Sigma_0}{[\beta_2^2 \Sigma_0 + \sigma_k^2]}$$

$$\gamma = \frac{\dfrac{\xi}{m} - \lambda}{\lambda(m_u + 1)}$$

$$\lambda = \frac{(m_u + 1)}{2\beta_2} - m_u\left(\frac{\xi}{m}\right)$$

and β_2 is computed as the unique root of the cubic equation given by

$$\overline{\beta}^3 + \overline{\beta}^2\left[\frac{\sigma_k^2 - \sigma_u^2(m_u - 1)^2}{\Sigma_0}\right] + \overline{\beta}\left[\frac{2\sigma_k^2\sigma_u^2(m_u^2 - 1) - \sigma_k^4}{\Sigma_0^2}\right] - \left[\frac{\sigma_k^6 + (m_u + 1)^2\sigma_k^4\sigma_u^2}{\Sigma_0^3}\right] = 0$$

where $\overline{\beta} \equiv \beta^2$. Once again, β is only a general representation for the informed trading intensity parameter, while β_2 is the unique equilibrium value for a given set of $\{\Sigma_0, \sigma_k^2, \sigma_u^2, m_u\}$.
Proof: See Appendix.

When $m = m_u = 1$, Proposition 1 holds. Also, notice that the equilibrium is a function of only the number of unethical brokers (m_u) present in the market, along with the exogenous variances $\{\Sigma_0, \sigma_k^2, \sigma_u^2\}$.[9] Intuitively, the equilibrium trading intensity parameter of the informed trader, β_2, depends only on m_u and the exogenous variances. This is because even though the informed trader, like the market maker, cannot ex ante distinguished between ethical and unethical brokers, he knows that only the m_u unethical brokers will free-ride on his information. This, in turn, determines that the market maker's pricing coefficient λ and each unethical broker's personal trading intensity parameter γ, are also functions of m_u and the exogenous variances.

Consider now the question of informativeness of prices in a market with a single informed trader and m_u unethical brokers. This has been defined by Kyle (1985), as $\Sigma_1 = \text{Var}\{v \mid p\}$.
Corollary 1. $\Sigma_1 = \text{Var}\{v \mid p\} = \Sigma_0/2$
Proof: See Appendix.

This result reinforces the notion that brokerage activity does not make market clearing prices any more or less informative. After all, brokers are only free-riding off the fundamental information acquired by one of their customers whose orders they are able to identify, albeit with noise. All that the presence of brokers does is make the traders with fundamental information trade relatively less aggressively than they would in a Kyle world where no brokers exist, with the broker picking

[9]Specifically, notice that ξ/m is a function of β_2 and exogenous variances, while β_2 is a function of m_u and exogenous variances. This makes ξ/m itself a function of m_u and exogenous variances. Next, λ is a function of m_u, β_2, and (ξ/m). Thus, λ is a function of m_u and exogenous variances. Finally, γ is a function of m_u, λ, and (ξ/m). Again, from the above discussion, γ is clearly a function of m_u and exogenous variances.

up the slack, such that, in equilibrium, prices are exactly as informative as in the Kyle world of a monopolist insider and no brokers.

The final result gives the ex-ante profits of the informed trader and each of the m_u unethical brokers, unconditional on $\tilde{v}$, and drives home the main message of the article.

Proposition 4. *The ex ante profits of the informed trader and each of the m_u unethical brokers (unconditional on $\tilde{v}$) are given by, respectively,*

$$E\pi_i = \frac{1}{2}\beta_2\Sigma_0$$

$$E\pi_b = \frac{\frac{\xi}{m} - \lambda}{\lambda(m_u + 1)}\frac{\beta_2}{2}\left[\Sigma_0 - \frac{\sigma_k^2}{\beta_2^2}\right]$$

Proof: See Appendix.

As expected, the unconditional expected profit of the informed trader is proportional to his trading intensity. The expected profit of each (unethical) broker, however, is more interesting. In Figure 3, the expected profit of each unethical broker is plotted as a function of

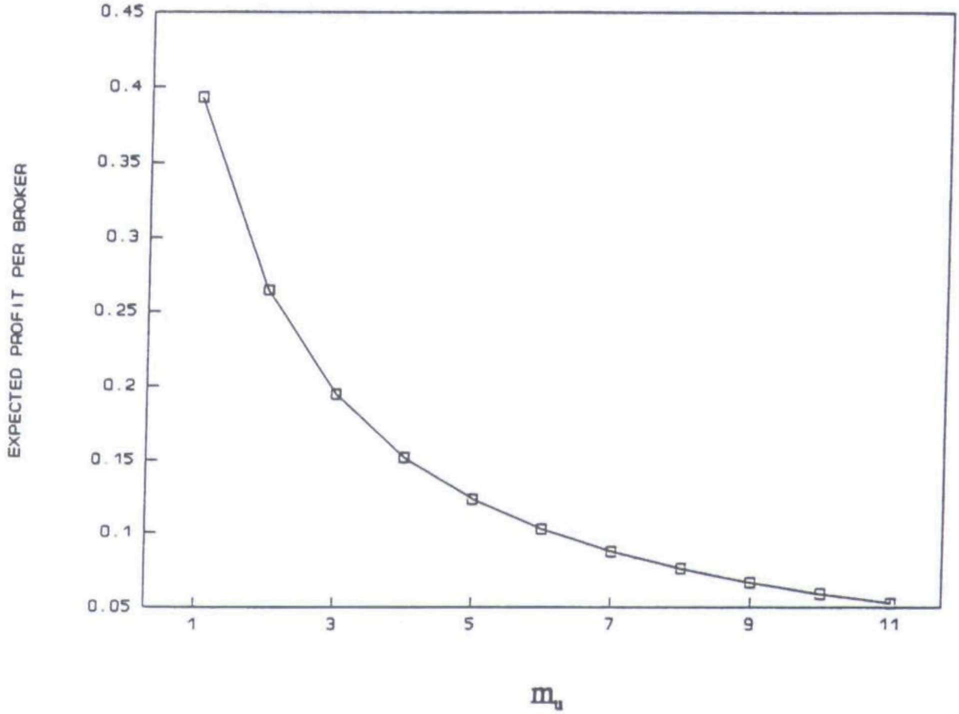

FIGURE 3
Expected profit per broker.

the total number of (unethical) brokers in the market (m_u). Here, $\sigma_u^2 = 5.5$, $\sigma_k^2 = 5$, and $\Sigma_0 = 5$. The expected profit per broker declines exponentially with increasing m_u, and the drop is the steepest for m_u changing from 1 to 2. This indicates that the competition among brokers is most intense when there are just two brokers in the market. Also note that expected profit per broker becomes insignificant with $m_u \approx 10$. The implication of this is obvious. The competition between just ten unethical brokers in a contract ensures that expected front-running profit per broker becomes negligibly small.

CONCLUSION

The current model seeks to demonstrate that competition among dual traders drives expected front-running profits per trader to zero, with only about ten unethical dual traders in the market. An implication of this result for policy makers is that, perhaps the regulation for banning dual trading in the most actively traded contracts needs to be reexamined. Actively traded contracts with their high trading volumes, which include a lot of liquidity trading, attract many dual traders, lured in by the possibility of large commissions, among other things. And it is precisely through this competition, that any possible front-running profits are driven down to zero very quickly.

This result does not change qualitatively, when one introduces multiple informed traders, instead of a single informed trader, as done here. This is because competition between multiple traders sharing common private information causes each of them to trade identical quantities less than that in the monopolist case. But their combined quantity adds up to be greater than that in the monopolist case. This implies that the combined quantity traded by the unethical dual traders adds up to be less than that in the monopolist broker case. Adding multiple unethical dual traders to the mix, competition, once again, drives the expected profit per dual trader to zero with even fewer unethical dual traders, than in the present scenario with single informed trader and multiple unethical dual traders.

In conclusion, it should be mentioned that the author has developed a two-period extension of the current model which is not included in the article. While that model is more complex and includes a two-period, dynamic setup, it does not add any new insights to those captured by the model reported on here. The simple, single-period model tells the story adequately.

APPENDIX

Proof of Proposition 1.
The problem is solved by backward induction, starting with stage 2 and then stage 1. From the market-efficiency condition and the linear pricing rule of the market makers, one can write

$$p(y_1) = E[\tilde{v} \mid y_1] = \lambda(y_1) \tag{1.1}$$

the expected profit equation of the broker can be given as

$$\begin{aligned}
&\operatorname*{Max}_{z} E[\{\tilde{v} - p(x + z + \tilde{u} + \tilde{k})\}z \mid x + k] \\
\text{or, } &\operatorname*{Max}_{z} E[\{\tilde{v} - \lambda(x + z + \tilde{u} + \tilde{k})\}z \mid x + k] \\
\text{or, } &\operatorname*{Max}_{z} E[\tilde{v} \mid x + k]z - \lambda(x + k)z - \lambda z^2
\end{aligned} \tag{1.2}$$

where $E[\tilde{v} \mid x + k]$ denotes the expected value of the contract to the broker, given the order that he receives from his informed customer and his beliefs about the informativeness of the order. With the (correct) conjecture that, in equilibrium, this conditional expected value is a linear function of the order observed by the broker, the conditional expected value can be written at $\xi(x + k)$, where ξ is the broker's personal valuation sensitivity coefficient.

From first order conditions (hereafter, FOC)

$$\begin{aligned}
z &= \frac{\xi - \lambda}{2\lambda}(x + k) \\
&= \gamma(x + k)
\end{aligned} \tag{1.3}$$

where γ, is the trading intensity parameter of the broker. Now in stage 1, the informed trader's problem is solved by:

$$\begin{aligned}
&\operatorname*{Max}_{x} E[\{\tilde{v} - p(x + z + \tilde{u} + \tilde{k})\}x \mid \tilde{v} = v] \\
&\operatorname*{Max}_{x} E[\{\tilde{v} - \lambda(x + \gamma(x + \tilde{k}) + \tilde{u} + \tilde{k})\}x] \\
&\operatorname*{Max}_{x} [Vx - \lambda x^2 - \gamma\lambda x^2]
\end{aligned}$$

Once again from FOC

$$\begin{aligned}
x &= \frac{1}{2\lambda(1 + \gamma)} v \\
&= \beta v
\end{aligned} \tag{1.4}$$

where β is a general trading intensity parameter of the informed trader. Now substituting for γ from (1.3) in (1.4), one can express β more simply as

$$\beta = \frac{1}{\lambda + \xi} \tag{1.5}$$

Additionally, it is known that:

$$E[\tilde{v} \mid y_0 = x + k] \equiv \xi(y_0) \tag{1.6}$$

Hence, from (1.6)

$$\xi = \frac{\mathrm{Cov}(\tilde{v}, \tilde{y}_0)}{\mathrm{Var}(\tilde{y}_0)} \tag{1.7}$$

where y_0 is the total order seen by the broker, which is a noisy realization of the informed order. It can be verified that

$$\begin{aligned}
\tilde{y}_0 &= x + \tilde{k} \\
&= \beta\tilde{v} + \tilde{k} \\
\text{hence, } \mathrm{Var}(\tilde{y}_0) &= \beta^2\Sigma_0 + \sigma_k^2 \\
\text{and, } \mathrm{Cov}(\tilde{v}, \tilde{y}_0) &= \beta\Sigma_0 \\
\text{finally, } \xi &= \frac{\beta\Sigma_0}{\beta^2\Sigma_0 + \sigma_k^2} \qquad (1.8)
\end{aligned}$$

Similarly, from the Market Efficiency condition, the market maker's pricing function can be written as

$$E[\tilde{v} \mid y_1] = \lambda y_1 \tag{1.9}$$

where y_1 denotes the total order seen by the market maker.

$$\begin{aligned}
\tilde{y}_1 &= x + z + \tilde{u} + \tilde{k} \\
&= \beta\tilde{v} + \frac{\xi - \lambda}{2\lambda}(x + \tilde{k}) + \tilde{u} + \tilde{k} \\
&= \beta\tilde{v}\left[\frac{\lambda + \xi}{2\lambda}\right] + \left[\frac{\lambda + \xi}{2\lambda}\right]\tilde{k} + \tilde{u} \qquad (1.10)
\end{aligned}$$

hence,

$$\mathrm{Var}(\tilde{y}_1) = \beta^2\left(\frac{\lambda + \xi}{2\lambda}\right)^2\Sigma_0 + \left(\frac{\lambda + \xi}{2\lambda}\right)^2\sigma_k^2 + \sigma_u^2$$

and,

$$\mathrm{Cov}(\tilde{v}, \tilde{y}_1) = \beta\left(\frac{\lambda + \xi}{2\lambda}\right)\Sigma_0$$

Finally,

$$\lambda = \frac{\beta\left(\frac{\lambda + \xi}{2\lambda}\right)\Sigma_0}{\beta^2\left(\frac{\lambda + \xi}{2\lambda}\right)^2\Sigma_0 + \left(\frac{\lambda + \xi}{2\lambda}\right)^2\sigma_k^2 + \sigma_u^2} \tag{1.11}$$

From (1.11) and (1.5)

$$\lambda^2 = \frac{1}{4\sigma_u^2}\left(\Sigma_0 - \frac{\sigma_k^2}{\beta^2}\right) \tag{1.12}$$

From (1.5), (1.8), and (1.12), after simplification, one obtains

$$\Sigma_0\beta^6 + \sigma_k^2\beta^4 + \left[E_0\left(\frac{\sigma_k^2}{\Sigma_0}\right)^2 - 2\left(\frac{\sigma_k^2}{\Sigma_0}\right)\sigma_k^2\right]\beta^2 - \left[\sigma_k^2\left(\frac{\sigma_k^2}{\Sigma_0}\right)^2 + 4\sigma_u^2\left(\frac{\sigma_u^2}{\Sigma_0}\right)^2\right] = 0 \tag{1.13}$$

Equation (1.13) can be further simplified as

$$\Sigma_0\overline{\beta}^3 + \sigma_k^2\overline{\beta}^2 - \frac{\sigma_k^4}{\Sigma_0}\overline{\beta} - \left[\frac{\sigma_k^6 + 4\sigma_u^6}{(\Sigma_0)^2}\right] = 0 \tag{1.13a}$$

where $\overline{\beta} \equiv \beta^2$.

Now, it is to be shown that the above polynomial has a unique positive root in β. This unique root is defined as β_1. To do this, first express (1.13) as

$$f(\beta^2) = 0 \tag{1.14}$$

It is obvious that $f(-\infty) = -\infty$ and $f(\infty) = \infty$. Also, $f(0) < 0$. Therefore, one can conclude that a positive root exists in β^2. To check for the uniqueness of the positive root, one can rewrite (1.13) as

$$\overline{\beta}^3 + p\overline{\beta}^2 + q\overline{\beta} + r = 0 \tag{1.15}$$

where

$$
\begin{aligned}
p &= \frac{\sigma_k^2}{\Sigma_0} \\
q &= -\left(\frac{\sigma_k^2}{\Sigma_0}\right)^2 \\
r &= -\left[\left(\frac{\sigma_k^2}{\Sigma_0}\right)^3 + 4\left(\frac{\sigma_u^2}{\Sigma_0}\right)^3\right]
\end{aligned} \tag{1.16}
$$

One can further define

$$
\begin{aligned}
a &= \frac{1}{3}(3q - p^2) \\
b &= \frac{1}{27}(2p^3 - 9pq + 27r)
\end{aligned} \tag{1.17}
$$

Then, if the inequality condition given by

$$
\frac{b^2}{4} + \frac{a^3}{27} > 0 \tag{1.18}
$$

is met, one can conclude that one real root and two imaginary roots exist in β^2. It can be verified that condition (1.18) holds, thereby implying that only one real root exists in β^2. That this real root is positive can also be verified. To show this, one needs to compute the expected payoff of the informed trader and see that the expected payoff is maximized for a positive β. The unique positive root in β is labeled β_1.

The expected profit of the insider (unconditional on $\tilde{v}$) is

$$
\begin{aligned}
E\,\tilde{\pi}_i &= E[(\tilde{v} - \tilde{p})x] \\
&= E[\{\tilde{v} - \lambda(x + z + \tilde{u} + \tilde{k})\}\beta_1\tilde{v}] \\
&= E[\beta_1\tilde{v}^2 - \lambda\beta_1^2\tilde{v}^2 - \lambda\gamma\beta_1^2\tilde{v}^2]
\end{aligned} \tag{1.19}
$$

After simplification, and substituting the expression for γ above

$$
E\,\tilde{\pi}_i = \frac{\Sigma_0\beta_1}{2} \tag{1.20}
$$

Similarly, the expected profit of the broker is

$$
\begin{aligned}
E\tilde{\pi}_b &= E[\{\tilde{v} - \lambda(x + z + \tilde{u} + \tilde{k})\}z] \\
&= E[\{\tilde{v} - \lambda\beta_1\tilde{v} - \lambda\gamma\beta_1\tilde{v} - \lambda\gamma\tilde{k} - \lambda\tilde{u} - \lambda\tilde{k}\}\{\gamma\beta_1\tilde{v} + \gamma\tilde{k}\}] \\
&= (\gamma\beta_1 - \lambda\gamma\beta_1^2 - \lambda\gamma^2\beta_1^2)\Sigma_0 - (\lambda\gamma^2 + \lambda\gamma)\sigma_k^2 \qquad (1.21)
\end{aligned}
$$

simplifying (1.21), one obtains

$$E\tilde{\pi}_b = \left(\frac{\xi - \lambda}{4\lambda}\right)\beta_1\left[\Sigma_0 - \frac{\sigma_k^2}{\beta_1^2}\right] \qquad (1.22)$$

Proof of Proposition 2.

(a) when $\sigma_k^2 = 0$, then redoing the computations of Proposition 1 one obtains

$$z = \frac{\xi - \lambda}{2\lambda}x = \gamma x \qquad (2.1)$$

and

$$x = \frac{1}{2\lambda(1 + \gamma)} = \frac{1}{\xi + \lambda} = \beta_1 v \qquad (2.2)$$

Furthermore, from definition

$$\xi = \frac{\mathrm{Cov}(\tilde{v}, \tilde{x})}{\mathrm{Var}(\tilde{x})} = \frac{1}{\beta_1} \qquad (2.3)$$

Finally, from the Market efficiency condition

$$\lambda = \frac{1}{2}\sqrt{\frac{\Sigma_0}{\sigma_u^2}} \qquad (2.4)$$

From (2.2) and (2.3) one obtains

$$\beta_1\lambda = 0 \qquad (2.5)$$

This implies that either β_1 or λ have to be zero. But, from (2.4), λ cannot be zero. Therefore, $\beta_1 = 0$.

(b) When $\sigma_k^2 \to \infty$, then from (1.8), $\xi \to 0$. From (1.5), this implies $\beta_1 \to 1/\lambda$. Furthermore, from (1.11), one finds that when $\sigma_k^2 \to \infty$, $\lambda \to 0$, which implies that $1/\lambda \to \infty$. From (1.3), $\gamma = (\xi - \lambda)/(2\lambda)$. When $\xi \to 0$, this reduces to $\gamma \to -1/2$.

(c) When $\sigma_u^2 = 0$, then from (1.11) and (1.8), $\lambda = \xi$. Now substituting this in the broker's trading intensity parameter γ defined by (1.3), one obtains $\gamma = 0$.

Proof of Corollary 1.

In general, it is known that if $x \sim N(\mu_x, \sigma_x^2)$ and $y \sim N(\mu_y, \sigma_y^2)$, then

$$(x \mid y) \sim N\left\{\mu_x + \rho \frac{\sigma_x}{\sigma_y}(y - \mu_y); \sigma_x^2(1 - \rho^2)\right\}$$

where

$$\rho^2 = \frac{[\mathrm{Cov}(\tilde{x}, \tilde{y})]^2}{\mathrm{Var}(\tilde{x})\,\mathrm{Var}(\tilde{y})}$$

In this case, by definition, $\Sigma_1 = \mathrm{Var}\{\tilde{v} \mid p\}$,
where

$$\mathrm{Var}\{\tilde{v} \mid p\} = \mathrm{Var}\{\tilde{v}\}\left[1 - \frac{\{\mathrm{Cov}(\tilde{v}, \tilde{p})\}^2}{\mathrm{Var}(\tilde{v})\,\mathrm{Var}(\tilde{p})}\right] \tag{c1.1}$$

and,

$$\begin{aligned}\tilde{p} &= \lambda(x + m_u z + \tilde{u} + \tilde{k}) \\ &= \lambda\beta_2(1 + m_u\gamma)\tilde{v} + \lambda(1 + m_u\gamma)\tilde{k} + \lambda\tilde{u}\end{aligned} \tag{c1.2}$$

Hence,

$$\begin{aligned}\mathrm{Var}(\tilde{p}) &= \lambda^2\beta_2^2(1 + m_u\gamma)^2\Sigma_0 + \lambda^2(1 + m_u\gamma)^2\sigma_k^2 + \lambda^2\sigma_u^2 \\ \mathrm{Cov}(\tilde{v}, \tilde{p}) &= \lambda\beta_2(1 + m_u\gamma)\Sigma_0\end{aligned} \tag{c1.3}$$

Substituting (c1.3) in (c1.1) and using $(1 + m_u\gamma) = (\xi + \lambda/((m_u + 1)\lambda)$ and, $\beta_2 = (m_u + 1)/(2(\lambda + \xi))$, yields, after simplification

$$\mathrm{Var}(\tilde{v} \mid p) = \frac{1}{2}\Sigma_0$$

Proof of Proposition 3.

Assume a linear pricing rule with $p = \lambda y$, where y is the total trade observed by the market makers. Once again, the broker's problem is solved first, then the informed trader's problem is solved.

A particular broker (among the m_u unethical brokers) with his demand $\hat{z}_u$ takes the demands of the other $(m_u - 1)$ unethical brokers at $\bar{z}_u$ and the total order coming to him as $c(x + k)$, where $0 < c < 1$. The broker now solves the following problem:

$$\begin{aligned}
&\operatorname*{Max}_{\hat{z}_u} \; E[\{\tilde{v} - \tilde{p}\}\hat{z}_u \,|\, c(x + k)] \\
&\text{or, } \operatorname*{Max}_{\hat{z}_u} \; E[\{\tilde{v} - \lambda(mc(x + k) + (m_u - 1)\bar{z} + \hat{z}_u + \tilde{u})\}\hat{z}_u \,|\, c(x + k)] \\
&\text{or, } \operatorname*{Max}_{\hat{z}_u} \; E[\{\tilde{v} \,|\, c(x + k)]\hat{z}_u - \lambda(m_u - 1)\bar{z}\hat{z}_u - \lambda\hat{z}_u^2 \\
&\qquad\qquad - \lambda mc(x + k)\hat{z}_u \qquad (3.1)
\end{aligned}$$

Since all of the m brokers are considered identical, the informed trader does not have any preference among them. So each broker (correctly) hypothesizes that $c = 1/m$ in equilibrium and then computes his equilibrium demand z. It will be shown that the informed trader's demand is just a function of the total number of brokers m and not how he divides his total orders among them. So the informed trader does not have any incentive to deviate from the conjectured equilibrium strategy of equally dividing his order amongst m brokers.

F.O.C., and at equilibrium $\hat{z}_u = \bar{z}_u = z_u$ and $c = 1/m$ give, after simplification,

$$E\left[\tilde{v} \,\middle|\, \frac{x + k}{m}\right] - \lambda(m_u - 1)z_u - 2\lambda z_u - \lambda m\left(\frac{x + k}{m}\right) = 0 \qquad (3.2)$$

where $E[\tilde{v} \,|\, (x + k)/m]$ denotes the expected value of the contract to each broker (ethical or otherwise), given the equal proportion of the order that each receive from the informed customer and their (identical) beliefs about the informativeness of this order. Now with the (correct) conjecture that, in equilibrium, this conditional expected value is a linear function of the order observed by each broker, $(x + k)/m$, it readily follows that the conditional expected value can be written as $\xi\{(x + k)/m\}$, where ξ is each broker's personal valuation sensitivity coefficient. Substituting this in (3.2) and simplifying,

$$z_u = \gamma(x + k) \qquad (3.3)$$

where,

$$\gamma = \frac{\frac{\xi}{m} - \lambda}{\lambda(m_u + 1)}$$

and γ is the trading intensity parameter of each unethical broker and $m = m_u + m_e$. It will soon be clear that γ is a function of m_u and the exogenous variances only.

The informed trader now solves the following problem:

$$\underset{x}{\text{Max}}\ E[\{\tilde{v} - \tilde{p}\}x \mid \tilde{v} = v]$$

$$\underset{x}{\text{Max}}\ E\left[\left\{v - \lambda\left(m\left(\frac{x + k}{m}\right) + m_u z_u + \tilde{u}\right)\right\}x\right]$$

$$\underset{x}{\text{Max}}\ [vx - \lambda x^2 - \lambda m_u \gamma x^2] \tag{3.4}$$

Once again from FOC, after simplification

$$x = \beta\ v,\ \text{where}\ \beta = \frac{1}{2\lambda(1 + m_u\gamma)} \tag{3.5}$$

Note that β is a general trading intensity parameter of the informed trader. From (3.3) and (3.5), one obtains

$$\beta = \frac{m(m_u + 1)}{2(m\lambda + m_u\xi)} \tag{3.6}$$

Also, it is known that

$$E\left[\tilde{v} \mid y_0 = \frac{x + k}{m}\right] = \xi y_0$$

where y_0 is what each broker observes. From definition,

$$\xi = \frac{\text{Cov}(\tilde{v}, \tilde{y}_0)}{\text{Var}(\tilde{y}_0)}$$

where $y_0 = (x + k)/m$. Following similar procedures as before

$$\frac{\xi}{m} = \frac{\beta\Sigma_0}{[\beta^2\Sigma_0 + \sigma_k^2]} \tag{3.7}$$

Furthermore, from Market Efficiency condition,

$$E[\tilde{v} \mid y_1 = x + m_u z_u + \tilde{u} + \tilde{k}] = \lambda y_1 \tag{3.8}$$

where λ is the market maker's pricing coefficient. Following similar steps as before

$$\lambda = \frac{\beta(1 + m_u\gamma)\Sigma_0}{\beta^2(1 + m_u\gamma)^2\Sigma_0 + (1 + m_u\gamma)^2\sigma_k^2 + \sigma_u^2} \tag{3.9}$$

Hence, there is a system of four equations and four unknowns, given by (3.3), (3.5), (3.7), and (3.9). From (3.3), (3.5), and (3.9), after simplification,

$$\lambda = \frac{1}{2}\sqrt{\frac{1}{\sigma_u^2}\left[\Sigma_0 - \frac{\sigma_k^2}{\beta^2}\right]} \tag{3.10}$$

From (3.3), (3.5), and (3.7), after simplification

$$\lambda = \frac{(m_u + 1)}{2\beta} - m_u\left(\frac{\xi}{m}\right) \tag{3.11}$$

Equating (3.10) and (3.11) and substituting for ξ/m from (3.7), one obtains the cubic given by

$$\begin{aligned}\overline{\beta}^3\Sigma_0^3 + \overline{\beta}^2\left[\sigma_k^2\Sigma_0^2 - (m_u - 1)^2\sigma_u^2\Sigma_0^2\right] + \\ \overline{\beta}\left[2(m_u^2 - 1)\sigma_k^2\sigma_u^2\Sigma_0 - \sigma_k^4\Sigma_0\right] - \\ \left[\sigma_k^6 + (m_u + 1)^2\sigma_k^4\sigma_u^2\right] = 0\end{aligned} \tag{3.12}$$

where $\overline{\beta} \equiv \beta^2$. Now, as in Proposition 1, one first must determine that the above cubic can only have positive real roots in $\overline{\beta}$. To do this, first rewrite (3.12) in the following way.

$$f(\overline{\beta}) \equiv \overline{\beta}^3 + p\overline{\beta}^2 + q\overline{\beta} + r = 0 \tag{3.13}$$

where

$$\begin{aligned} p &= \frac{\sigma_k^2 - (m_u - 1)^2\sigma_u^2}{\Sigma_0} \\ q &= \frac{2(m_u^2 - 1)\sigma_k^2\sigma_u^2 - \sigma_k^4}{\Sigma_0^2} \\ r &= -\left[\frac{\sigma_k^6 + (m_u + 1)^2\sigma_k^4\sigma_u^2}{\Sigma_0^3}\right] \end{aligned}$$

First notice that $f(-\infty) = -\infty$, $f(\infty) = \infty$ and $f(0) < 0$ which shows that there exists a positive real root in β^2. Further, **a** and **b** are defined the same way as in Proposition 1. Then, if

$$\frac{b^2}{4} + \frac{a^3}{27} > 0$$

then there exists one real root and two imaginary roots in β^2. Only the positive root of β^2, called β_2 is considered. But, if

$$\frac{b^2}{4} + \frac{a^3}{27} < 0$$

then all three roots in β^2 are real and positive. In this case, one can say only that it is the positive root of β^2 that gives a monotonic solution. Once again, this is denoted as β_2.

Finally, in Figure 3 the unique root of the cubic (β_2) is numerically solved for, using the numerical computation package GAUSS, for different values of m_u.

Proof of Proposition 4.

The ex ante profit of the informed trader (unconditional on $\tilde{v}$) is given by

$$\begin{aligned} E\pi_i &= E[(\tilde{v} - \tilde{p})x] \\ &= E[\{\tilde{v} - \lambda(x + m_u z + \tilde{u} + \tilde{k})\}x] \\ &= E[\{\tilde{v} - \lambda[\beta_2(1 + m_u\gamma)\tilde{v} \\ &\qquad + (1 + m_u\gamma)\tilde{k} + \tilde{u}]\}\beta_2\tilde{v}] \\ &= \beta_2[1 - \lambda\beta_2(1 + m_u\gamma)]\Sigma_0 \end{aligned} \tag{4.1}$$

Now it is known that

$$1 + m_u\gamma = \frac{\xi + \lambda}{(m_u + 1)\lambda} \quad \text{and} \quad \beta_2 = \frac{(m_u + 1)}{2(\lambda + \xi)}$$

Substituting in (4.1), and simplifying, one obtains

$$E\pi_i = \frac{1}{2}\beta_2\Sigma_0 \tag{4.2}$$

The expected profit of each broker (unconditional on $\tilde{v}$) is

$$\begin{aligned} E\pi_b &= E[(\tilde{v} - \tilde{p})z] \\ &= E[\{\tilde{v} - \lambda(x + m_u z + \tilde{u} + \tilde{k})\}z] \\ &= E[\{\tilde{v} - \lambda[\beta_2(1 + m_u\gamma)\tilde{v} \\ &\qquad + (1 + m_u\gamma)\tilde{k} + \tilde{u}]\}\{\gamma\beta_2\tilde{v} + \gamma\tilde{k}\}] \\ &= \gamma\beta_2\Sigma_0 - \lambda\gamma(1 + m_u\gamma)[\beta_2^2\Sigma_0 + \sigma_k^2] \end{aligned} \tag{4.3}$$

Substituting for $(1 + m_u \gamma)$ from above and for γ from (3.3), one obtains

$$E\pi_b = \frac{\dfrac{\xi}{m} - \lambda}{\lambda(m_u + 1)} \frac{\beta_2}{2}\left[\Sigma_0 - \frac{\sigma_k^2}{\beta_2^2}\right] \tag{4.4}$$

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