NONCONSTANT OPTIMAL HEDGE RATIO ESTIMATION AND NESTED HYPOTHESES TESTS

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INTRODUCTION

There exists a large body of empirical work on the estimation of optimal hedge ratios in futures markets. The hedge ratio measures the risk minimizing proportion of futures relative to a firm's physical position. It is equal to the ratio of the covariance of the cash and futures prices to the variance of the futures price, conditioned upon information available at the time of the hedging decision. A hedge ratio is based, in principle, on a decision maker's subjective beliefs about the moments of the cash and futures joint distribution. In practice, however, hedge ratios are usually estimated using historical price data.

Early studies [Peck (1975); Heifner (1972)] estimate the hedge ratio by regressing the cash price on the futures price where the coefficient estimate on the futures price gives the empirical hedge ratio. Myers and Thompson (1989) show that this method results in an unconditional hedge ratio estimate in that the covariance and variances are implicitly calculated relative to the sample means of cash and futures prices

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instead of conditional means. They modify the regression approach by including a set of regressors that influence the mean of the cash and futures prices. This set of regressors serves as a proxy for the information available to a firm and the regression coefficient can thereby be interpreted as the conditional hedge ratio.

Although the methodology of Myers and Thompson is simple to apply, it may suffer from the assumption that the conditional hedge ratio is constant over an entire sample. This is of particular concern given the evidence that commodity prices exhibit time-varying volatility. One obvious source of variation in crop price volatility is the seasonal variation in the amount of price-relevant information. For example, considerable uncertainty about harvest size is resolved during the summer months, resulting in far greater price volatility than in winter months [Anderson (1985)]. Thus, clumpy information flows may result in nonconstancy in the second moments and in the hedge ratio. The possibility of nonconstant variances has prompted applied researchers to model futures and cash prices with the autoregressive conditional heteroskedasticity (ARCH) and the generalized ARCH (GARCH) methods developed by Engle (1982) and Bollerslev (1986), respectively.

Unfortunately, there are several features of the GARCH methodology that may make it a less than ideal technique for estimating time-varying hedge ratios. From a practical standpoint, estimation of GARCH models requires the use of nonlinear optimization algorithms and possibly the imposition of inequality restrictions on model parameters. The methodology can, therefore, be taxing to the practitioner. More fundamentally, the only evidence of the need for this modeling effort comes from simulations that compare the risk reduction benefits of an estimated nonconstant hedge ratio to those of an estimated constant hedge ratio. Statistical evidence that the hedge ratios vary over time has yet to be provided.

This article presents an econometric methodolgy that facilitates statistical tests on the hedge ratio by nesting the hypothesis of a constant hedge ratio. There are three main features of this framework. First, it directly estimates the conditional hedge ratio as a function of information available to the firm. Second, it models how the unhedgeable risk depends on observable information. Third, it utilizes an iterative generalized least squares (GLS) procedure, making it fairly simple computationally.

An empirical application of this procedure in the corn and live cattle markets indicates that the constant hedge ratio hypothesis cannot be rejected in favor of a nonconstant hedging strategy. Also, the results indicate that firms operating in these markets are exposed to seasonally varying risk levels even when they hedge optimally.

The next section discusses previous work on optimal hedge ratio estimation and is followed by a section that addresses the use of time-varying techniques for estimating hedge ratios. The fourth section presents the econometric modeling procedure used to test the hypothesis of a constant hedge ratio. This methodology is then applied to weekly corn and live cattle cash and futures prices for the period 1976–1992. A description of the results from these hypotheses tests is given in the fifth section.

ESTIMATING HEDGE RATIOS

The most common treatment of hedging assumes that a firm must make a commitment in the current period concerning the quantity of a good to be purchased or sold in the future at an unknown price. With a fixed commitment in the physical position, profit risk occurs only through price risk. To reduce the price risk, the firm can take an offsetting futures position in the current period that will be lifted at the same time as the physical position.

At the time of the hedging decision, the firm uses available information to form the conditional joint distribution of the cash price (P) and the futures price (F). The first two moments of this distribution are denoted by μ and Σ; in general, both μ and Σ are functions of the information available to the firm at the time the hedge is placed. Furthermore, assume that F is a martingale ($E[\Delta F] = 0$), an assumption that is consistent with most studies of futures markets [Kamara (1982)]. The first two moments of the joint distribution of P and ΔF are denoted by

$$\begin{bmatrix} P \\ \Delta F \end{bmatrix} \sim \left[\begin{pmatrix} \mu_p \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_p & \sigma_{pf} \\ \sigma_{pf} & \sigma_f \end{pmatrix} \right]$$

It is well known that in this framework the minimum variance hedge ratio, h, is σ_{pf}/σ_f [Kahl (1983)].

In the work of Myers and Thompson, it is implicitly assumed that Σ is constant. In this case, the conditional hedge ratio, h, can be estimated by the following OLS regression:

$$P_t = h\,\Delta F_t + x_t'\beta + e_t \tag{1}$$

where x_t is a vector of variables which are observed when the hedging decision is made and e_t is a random shock. The variance of the shock,

σ_e, which is assumed to be constant over time, can be expressed as

$$\sigma_e = \sigma_p - \frac{\sigma_{pf}^2}{\sigma_f} = \sigma_p - h\sigma_{pf}$$

which is the variance of the cash price, conditioned on ΔF_t and x_t. From an economic perspective, σ_e represents the unhedgeable risk, i.e., the minimal risk still present given optimal hedging [Fackler and McNew (1993)].

TIME-VARYING HEDGE RATIOS

This model may be inappropriate when futures and cash prices exhibit time variation in volatility. In this case a simple OLS model is in error for two reasons. First, the regression error may be heteroskedastic (e.g., σ_e is nonconstant). Second, h, the regression parameter on ΔF_t, may exhibit systematic variation.

A popular method for modeling time-varying volatility in prices uses the GARCH framework [(Yang and Brorsen (1992); Najand and Yung (1991); and Baillie and Bollerslev (1991)]. The multivariate GARCH(p, q) model is constructed by taking an n-vector of errors (ϵ_t) from a system of regression equations and expressing the covariance matrix of these errors (Σ_t) as an autoregressive moving-average model. This representation is given as:

$$\text{vech}(\Sigma_t) = A + \sum_{i=1}^{p} B_i \, \text{vech}(\Sigma_{t-i}) + \sum_{i=1}^{q} C_i \, \text{vech}(\epsilon_{t-i}\epsilon_{t-i}')$$

where vech() denotes the operator that stacks the lower triangular portion of an $n \times n$ matrix as an $n(n + 1)/2$-vector. The number of unique parameters in this equation equals $(1/2)n(n + 1)[1 + n(n + 1)(p + q)/2]$. In practice, for moderately sized n, some simplifying assumptions must be imposed.

This methodology has also been used to calculate conditional hedge ratios based on an estimate of the time-varying covariance matrix of cash and futures prices. Cecchetti, Cumby, and Figlewski (1988) use univariate GARCH(3, 0) specifications to separately model cash and futures prices in the Treasury bond market. They assume that the correlation between cash and futures is constant, which implies that the hedge ratio, h_t, can be written as

$$h_t = \rho\sqrt{\frac{\sigma_{pt}}{\sigma_{ft}}}$$

where ρ is the correlation coefficient between cash and futures. The constant correlation assumption ensures that the estimated covariance matrix will be positive definite as long as the variance estimates are positive.

Myers (1991) and Baillie and Myers (1991) estimated optimal hedge ratios for a model with GARCH(1, 1) errors. They used a bivariate model for cash and futures prices in which the variances and covariances were estimated simultaneously. In Myers, this framework was used to estimate hedge ratios for wheat, while Baillie and Myers applied it to beef, coffee, corn, cotton, gold, and soybeans.

In all three of these studies, the optimal hedge ratio exhibited considerable variability. For example, in Baillie and Myers the GARCH-based hedge ratios ranged between −0.1 and 0.5 for beef and 0.1 and 1.5 for corn. In comparison, the constant hedge ratios estimated by ordinary least squares (OLS) were 0.07 for beef and 0.61 for corn. Furthermore, they estimated that the time-varying hedge reduced the variance of returns by 1% for beef and 6.1% for corn in the sample relative to the constant hedge position. Baillie and Myers also performed an out-of-sample hedging experiment that reinforced this conclusion. No tests were performed, however, to evaluate whether the results were statistically significant.

One way to perform such a test is to nest the hypothesis of hedge ratio constancy within the estimated statistical model. Inferences can then be made about the risk reduction potential of competing hedging strategies based on in-sample hypotheses tests on parameter estimates. Such tests would be difficult within the GARCH framework. For example, the variance and covariance terms of a GARCH(1, 1) model have the form:

$$\begin{pmatrix} \sigma_{ft} \\ \sigma_{pt} \\ \sigma_{pft} \end{pmatrix} = A + B \begin{pmatrix} \sigma_{ft-1} \\ \sigma_{pt-1} \\ \sigma_{pft-1} \end{pmatrix} + C \begin{pmatrix} e^2_{ft-1} \\ e^2_{pt-1} \\ e_{pt-1}e_{ft-1} \end{pmatrix}$$

A constant hedge ratio requires that rows 1 and 3 of A, B, and C are proportional to one another. This ensures that σ_{pft}/σ_{ft} is constant for any set of random shocks and is equivalent to the following set of six nonlinear restrictions:

$$\frac{A_3}{A_1} = \frac{B_{31}}{B_{11}} = \frac{B_{32}}{B_{12}} = \frac{B_{33}}{B_{13}} = \frac{C_{31}}{C_{11}} = \frac{C_{32}}{C_{12}} = \frac{C_{33}}{C_{13}}$$

Although such nonlinear restrictions can be tested, albeit with some difficulty, it is far more difficult to estimate the 21 parameters of this model. Modelers using the GARCH framework generally place a number of restrictions on the general model to reduce the number of parameters. For example, Myers and Baillie and Myers restrict the matrices B and C to be diagonal. Although this leads to a more parsimonious parameterization, it also eliminates the ability to perform nested tests of the constant hedge ratio hypothesis. Similarly, the restrictions used by Cecchetti, Cumby, and Figlewski do not nest the constant hedge ratio.

AN ALTERNATIVE ESTIMATION FRAMEWORK

If a hedge ratio is computed by taking the ratio of the estimated cash/futures covariance to the estimated futures variance, the constant hedge ratio hypothesis is nested only if the same factors determine both terms in a proportional fashion. Suppose the vectors x_t and z_t represent the information available at time period t used in forming the expectations $\mu(x_t)$ and $\Sigma(z_t)$ (z_t and x_t may contain the same information). Furthermore, suppose σ_p, σ_f, and σ_{pf} depend linearly on z_t, i.e., $\sigma_{pt} = z_t'\alpha_p$, $\sigma_{ft} = z_t'\alpha_f$, and $\sigma_{pft} = z_t'\alpha_{pf}$. The unhedgeable risk then is $\sigma_{et} = z_t'\alpha_p - h_t(z_t'\alpha_{pf})$, where $h_t = z_t'\alpha_{pf}/z_t'\alpha_f$. In this case it will generally be true that σ_{et} varies over time but that h_t is constant if $\alpha_{pf} \propto \alpha_f$.

Unfortunately such a model contains a large number of parameters and can be difficult to test, as with the general GARCH model. An alternative is to extend the single-equation framework to account for the systematic variation in h_t directly. This can be accomplished by modifying eq. (1) to include interaction terms between ΔF_t and the variables in z_t:

$$P_t = \Delta F_t h_0 + (\Delta F_t z_t)'\underline{\text{h}} + x_t'\beta + e_t \qquad (1')$$

The conditional hedge ratio at time t is given by $h_t = h_0 + z_t'\underline{\text{h}}$. The constancy of the hedge ratio is equivalent to the hypothesis that $\underline{\text{h}} = 0$.

Because σ_e is likely to be influenced by z_t, this model should not be estimated by OLS if one is concerned with hypothesis tests unless a heteroskedasticity consistent covariance estimator is employed [Davidson and MacKinnon (1993), pp. 552–554)]. More efficient estimates can be obtained, however, if the systematic variation in σ_e is explicitly modeled. Here it is assumed that e_t is normally distributed with mean 0 and variance that is a linear combination of z_t ($\sigma_{et} = z_t'\alpha$). The parameters α, β, h_0, and $\underline{\text{h}}$ can be estimated by regressing squared OLS residuals from (1′) on z_t and using predicted variances to compute a GLS estimate of β and $\underline{h}$. This gives a new set of residuals and the process is repeated until a convergence criterion is met. It can be shown that this iterative procedure yields maximum likelihood (ML) parameter estimates. This facilitates hypothesis testing using standard F (Wald), likelihood ratio, or score tests that are common in econometrics. The methodology for the estimation and hypothesis tests used in this article is outlined in the appendix for the interested reader.[1]

EMPIRICAL RESULTS

The method for estimating and testing time-varying hedge ratios is applied to the corn and live cattle futures contracts. The futures prices are nearby contract prices, with roll-forward occurring on the first day of the delivery month. The contract months for corn are March, May, July, September, and December, while live cattle has contracts every other month beginning in February and ending in December. The cash price for corn is the central Illinois elevator bid price and the cash price for live cattle is the Oklahoma–Texas steer price. All prices are weekly averages. These data coincide with a firm that faces weekly planning horizons and always uses the nearby contract, except in the expiration month, for hedging purposes.[2] The sample covers the period from February 1976–August 1992 (865 observations).

To estimate the conditional mean of the cash price, x_t is constructed with lags of weekly average cash and futures prices.[3] The conditioning variables should also include terms to account for seasonal patterns in the level of cash prices and its relationship to futures prices. Generally, the cash and futures prices tend toward convergence as the maturity date of the futures approaches. Therefore, the seasonality is modeled as a piecewise linear function of time, with breakpoints at the roll-forward dates. This is accomplished by including a set of dummy variables for each contract, a time-to-roll-forward variable, and a set of interaction

[1]If futures and cash prices exhibit unit root nonstationarity, the use of standard test statistics may be a questionable practice. In the problem discussed here, however, the limiting distribution of the parameters of interest (i.e., h_0 and $\underline{h}$) have a standard distribution and are unaffected by the unit root nonstationarities. Sims, Stock, and Watson (1990) and Stock and Watson (1988) discuss situations of this type and the conditions under which estimators in regressions involving unit root processes have standard distributions.

[2]There are many other possible planning horizons and contract choices that could be used. In contrast, Baillie and Myers use daily data for only one contract expiration period, while Myers performs his analysis on weekly data for two expiration periods. Although there may be suitable justification for very short-term planning horizons, it is unclear why a firm facing such a short horizon would choose a deferred as opposed to a nearby contract for hedging.

[3]The lags of futures prices are from the same contract as the current futures price. This is done to ensure that a hedging decision is based on lagged prices of the contract used in the hedge.

terms between the dummies and the time-to-roll-forward variable. The time-to-roll-forward variable is defined to be 1 in the week prior to the roll-forward date, 2 in the preceding week, etc.

It is hypothesized that Σ depends on the futures contract used for hedging and the time-to-roll-forward. Thus, the same set of contract dummy variables, the time-to-roll-forward variable and the interaction terms are used in the variance equation. Using only these variables, however, may not account for the autoregressive (ARCH) effects that have been detected in price volatility in past studies. It is well known that price volatility and trading volume are highly correlated in speculative markets.[4] Both are related to the amount of information being processed by the market. Hence, conditioning current volatility on lagged volume (V_{t-1}) provides a way to account for ARCH effects that avoids the unobserved variables problem inherent in the ARCH methodology. Specifically used is the one-week lag of average daily futures volume (in 1000s of contracts) for the nearby contract.

The procedure of Said and Dickey (1984) is used to determine the number of lagged cash and futures prices appropriate for the cash price mean equation. The maximum likelihood (iterated empirical GLS) procedure described in the last section is used to estimate eq. (1′). This equation is modified by replacing P_t with ΔP_t as the dependent variable and overfitting the equation with lags of first differences of cash and futures prices. Joint F-tests are then performed on the last cash and futures terms to determine their significance. In both corn and live cattle, the appropriate lag length is four weeks. Thus, the full model can be expressed as

$$P_t = \beta_0 + h_0\,\Delta F_t + \sum_i [h_i(\Delta F_t C_{it}) + h_{qi}(\Delta F_t C_{it} q_t)] + h_V\,\Delta F_t V_{t-1}$$

$$+ \beta_q q_t + \sum_i [\beta_i C_{it} + \beta_{qi} C_{it} q_t] + \sum_{j=1}^{4} [\beta_{Pj} P_{t-j} + \beta_{Fj} F_{t-j}] + e_t$$

$$\sigma_e = \alpha_0 + \sum_i [\alpha_i C_{it} + \alpha_{qi} C_{it} q_t] + \alpha_v V_{t-1} \tag{2}$$

where q_t is the time-to-roll-forward, C_{it} represents the contract dummy. The summations over i are taken over all contract months except December, which is dropped to avoid perfect collinearity in the model.

[4]See Karpoff (1987) for a survey of empirical work on this topic. Garcia, Leuthold, and Zapata (1986) empirically examine this relationship in futures markets.

TABLE I

Hedge Ratio Parameter Estimates Corrected for Heteroskedasticity

Live Cattle			*Corn*		
Parameter	*Estimate*	*P-Value*	*Parameter*	*Estimate*	*P-Value*
h_0	0.242	0.033	h_0	1.046	0.001
h_2	0.062	0.604	h_3	−0.016	0.893
h_4	0.302	0.039	h_5	−0.090	0.444
h_6	0.245	0.047	h_7	−0.022	0.784
h_8	0.312	0.019	h_9	0.015	0.878
h_{10}	0.133	0.337			
h_q	0.030	0.182	h_q	0.012	0.236
h_{q2}	−0.016	0.582	h_{q3}	−0.012	0.447
h_{q4}	−0.044	0.198	h_{q5}	−0.011	0.617
h_{q6}	−0.027	0.337	h_{q7}	−0.014	0.360
h_{q8}	−0.060	0.065	h_{q9}	−0.018	0.256
h_{q10}	−0.011	0.733			
h_v	0.005	0.548	h_v	−0.001	0.070

$$P_t = \beta_0 + h_0\,\Delta F_t + \sum_i [h_i(\Delta F_t C_{it}) + h_{qi}(\Delta F_t C_{it} q_t)] + h_V\,\Delta F_t V_{t-1} + \beta_q q_t + \sum_i [\beta_i C_{it} + \beta_{qi} C_{it} q_t] + \sum_{j=1}^{4} [\beta_{Pj} P_{t-j} + \beta_{Fj} F_{t-j}] + e_t$$

The parameter estimates of h_0 and $\underline{h}$ for eq. (2) are presented in Table I. These parameter estimates can be used to construct the conditional hedge ratios at time t (not presented). The hedge ratios are quite volatile, with corn ranging between 0.78 and 1.14 with an average of 0.96 and live cattle between 0.26 and 0.61 with an average of 0.48.

However, F-tests of the hypothesis that $\underline{h} = 0$ indicate that this apparent volatility is due to sampling error. The first line of Table II presents these test results. For both corn and live cattle, the test results are similar. The hypothesis that the hedge ratio is constant cannot be rejected, as indicated by probability values of 0.248 for live cattle and 0.309 for corn. Also presented in Table II for the nonconstant hedge model are tests of homoskedasticity, acceptance of which would

TABLE II

F-Tests of Hedge Ratio Constancy and Homoskedasticity

Null Hypothesis	*Live Cattle*		*Corn*	
Non-Constant Hedge Model				
(1) constant hedge ratio	1.244	(0.248)	1.168	(0.309)
(2) constant unhedgeable risk	2.273	(0.008)	2.101	(0.022)
Constant Hedge Model				
(3) constant unhedgeable risk	2.142	(0.013)	1.958	(0.035)

P-values are shown in parentheses.

indicate that unhedgeable risk is constant. This test is based on the variance equation estimates and can be stated in parametric form as: $\alpha_i = \alpha_{qi} = \alpha_V = 0$ for all $i \neq 0$. For both live cattle and corn, the hypothesis that unhedgeable risk is constant over a year can be rejected at the 1% level. Taken together these results indicate that Σ is nonconstant, but that σ_{pft} and σ_{ft} are proportional to one another. This confirms the presence of time-varying price volatility but also supports the conclusion that hedge ratios are constant for these commodities.

Given this conclusion, the model is reestimated with the constant hedge ratio restriction imposed (Table III). These results indicate that the optimal hedge ratios for corn and live cattle are estimated to be 0.91 and 0.47, respectively. This model also provides estimates of the variation in unhedgeable risk, the constancy of which is again rejected for both corn and live cattle (see the last line of Table II). Figure 1 displays the seasonal pattern in the unhedgeable risk. It is measured as the standard deviation of the unhedgeable risk evaluated at the average volume over the sample.

For corn, conditional risk is lowest on average in the period December through July, while after July conditional risk is higher. In live cattle, conditional risk is lowest in January and highest in June. In general, risk declines the closer the hedging period is to the maturity of the contract used in the hedge. This is reasonable because the convergence of the cash and futures prices increases the covariability of the prices as the futures contract matures. The main exception to this pattern is the July corn contract, which exhibits increasing risk as the contract approaches maturity ($\alpha_q + \alpha_{q7} < 0$). This can be explained by the fact that corn prices become increasingly volatile during the early part of the growing season when the most important information concerning the size of the upcoming harvest is realized. For live cattle, all contracts exhibit decreasing risk as time-to-expiration declines except the April contract. However, this increase is not statistically significant (the hypothesis $\alpha_q + \alpha_{q4} \geq 0$ cannot be rejected).[5]

It is of interest to note that the parameter on lagged trading volume in the corn variance equation is insignificantly different from zero (and, in fact, is negative). This should not be taken to suggest, however, that volume and volatility are unrelated (or negatively related), a result that would be contrary to the conclusions of previous studies. In this study the variance being modeled is the cash price variability remaining after

[5]For corn, the alternative hypothesis presumes that $\alpha_q + \alpha_{q7} < 0$ has a t-statistic of -37.754 while for live cattle the hypothesis that $\alpha_q + \alpha_{q4} < 0$ has a t-statistic of -0.581.

TABLE III

Parameter Estimates for Constant Hedge Ratio Model Corrected for Heteroskedasticity

Live Cattle			*Corn*		
Parameter	*Estimate*	*P-Value*	*Parameter*	*Estimate*	*P-Value*
β_0	0.511	0.003	β_0	−0.767	0.220
h	0.456	0.001	h	0.909	0.001
β_2	−0.263	0.060	β_3	−0.317	0.499
β_4	−0.230	0.036	β_5	−0.691	0.227
β_6	0.176	0.217	β_7	−1.232	0.019
β_8	−0.199	0.293	β_9	−0.654	0.312
β_{10}	−0.335	0.046			
β_q	−0.014	0.512	β_q	−0.302	0.001
β_{q2}	0.136	0.001	β_{q3}	0.241	0.001
β_{q4}	−0.019	0.001	β_{q5}	0.313	0.004
β_{q6}	0.039	0.253	β_{q7}	0.410	0.001
β_{q8}	0.057	0.192	β_{q9}	0.309	0.018
β_{q10}	0.042	0.286			
β_{P1}	0.643	0.001	β_{p1}	1.193	0.001
β_{F1}	0.533	0.001	β_{f1}	−0.251	0.001
β_{P2}	0.293	0.001	β_{p2}	−0.298	0.001
β_{F2}	−0.267	0.001	β_{f2}	0.369	0.001
β_{P3}	−0.041	0.376	β_{p3}	−0.110	0.029
β_{F3}	−0.196	0.001	β_{f3}	0.142	0.010
β_{P4}	−0.031	0.032	β_{p4}	0.094	0.002
β_{F4}	0.056	0.123	β_{f4}	−0.134	0.001
α_0	−0.247	0.031	α_0	10.191	0.012
α_2	0.173	0.246	α_3	−5.815	0.019
α_4	0.279	0.538	α_5	−2.387	0.062
α_6	−0.346	0.041	α_7	−3.770	0.433
α_8	0.208	0.065	α_9	−1.325	0.079
α_{10}	0.131	0.077			
α_q	0.027	0.070	α_q	0.314	0.048
α_{q2}	−0.005	0.196	α_{q3}	−0.096	0.812
α_{q4}	−0.028	0.780	α_{q5}	−0.165	0.103
α_{q6}	0.059	0.055	α_{q7}	−0.515	0.056
α_{q8}	0.063	0.052	α_{q9}	0.770	0.379
α_{q10}	0.033	0.174			
α_v	0.089	0.001	α_v	−0.026	0.332

$$P_t = \beta_0 + h\,\Delta F_t + \sum_i [\beta_i C_{it} + \beta_{qi} C_{it} q_t] + \beta_q q_t + \sum_{j=i}^{4} [\beta_{Pj} P_{t-j} + \beta_{Fj} F_{t-j}] + e_t$$

$$\sigma_{et} = \alpha_0 + \sum_{i=m}^{k} [\alpha_i C_{it} + \alpha_{qi} C_{it} q_t] + \alpha_q q_t + \alpha_v V_{t-1}$$

optimal hedging ($\sigma_p - h\sigma_{pf}$) rather than the step-ahead forecast error variability (σ_p). It is perfectly possible for the latter to vary systematically with trading volume while the former does not.

Given the previous applications of ARCH models to the estimation of cash-futures prices relationships, a natural question that arises in the

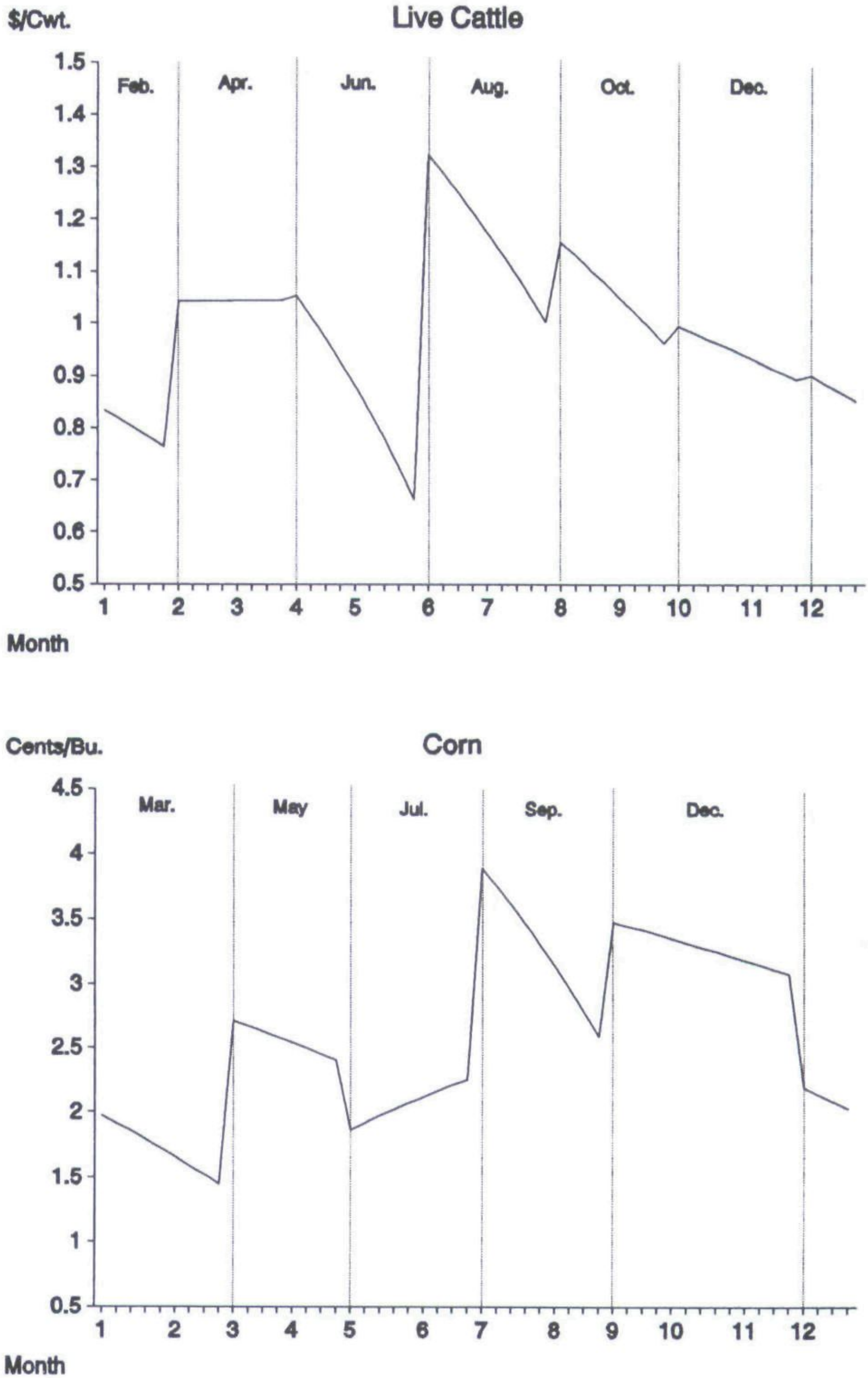

FIGURE 1
Estimated unhedgeable risk by contract month and time of year.

current model is whether any ARCH effects are captured. To address this possibility, Ljung–Box tests on the squared residuals are presented in Table IV. The results provide no evidence that ARCH effects are present. In fact, in the corn model no ARCH effects are present in the squared residuals estimated using OLS (ignoring heteroskedasticity). This may seem to be at variance to the findings of Baillie and Myers. As discussed above, however, the distinction between the unhedgeable risk and the forecasting risk resolves this apparent conflict.

The approach presented here uses a single equation regression of cash price on observable variables to estimate the hedge ratio. Such an equation could be estimated consistently using OLS. It is preferable, however, that the heteroskedasticity in the unhedgeable risk be modeled explicitly for two related reasons. First, if tests are to be performed on the hedge ratio estimates, it is important to correctly represent the error distribution associated with those estimates. It is well known that hypothesis tests must be modified if heteroskedasticity is present. Second, estimates of the hedge ratios are more efficient if the heteroskedasticity is explicitly modeled. However, if one is only concerned with consistent estimates of the hedge ratio, then the approach of Myers and Thompson, which uses OLS to estimate eq. (2), is valid. In the current context, the standard hedge ratio estimates using OLS are 0.96 for corn and 0.48 for live cattle, as opposed to the 0.91 and 0.47 reported above. Thus, the estimates are fairly similar.

CONCLUSIONS

It has become clear that heteroskedasticity in price movements is common and that it should be explicitly modeled. This need not imply, however, that there is nonconstancy in optimal hedge ratios. This article develops an econometric methodology for estimating conditional hedge ratios and for testing hypotheses about them. The method uses an

TABLE IV

Ljung–Box Statistics for Squared Residuals from Constant Hedging Model

Number of Lags	*Live Cattle*		*Corn*	
4	0.589	(0.964)	0.146	(0.997)
8	1.324	(0.995)	1.098	(0.998)
12	4.219	(0.979)	3.430	(0.992)

P-values for the Ljung–Box statistic are given in parentheses. Under the null hypothesis, the Ljung–Box statistic is χ^2 with degrees of freedom equal to the number of lags used in the test.

iterative generalized least squares procedure that produces maximum likelihood estimates and is simpler to apply than the GARCH procedures used in previous studies. It is, nonetheless, able to successfully remove apparent ARCH effects from the data. Furthermore, in contrast to the GARCH approach, it provides a practical method for testing whether the constant hedge ratio is optimal and provides a direct estimate of the risk that remains in a portfolio that is hedged optimally.

These results have two important implications. First, in these two particular examples, a constant hedge ratio appears to be a suitable assumption.[6] Thus, more complicated procedures for estimating time-varying optimal hedge ratios may not be necessary, resulting in hedging strategies that are easier to implement. Also, it appears that the unhedgeable risk varies seasonally and, in the case of cattle, with the level of trading activity in the market. Thus, even with optimal hedging, firms may be exposed to higher risk levels at certain times of the year then at other times.

APPENDIX

Consider a linear model of the form

$$y_t = x_t'\beta + e_t$$

where x_t is a k-vector of regressors, y_t is a scalar, and the error e_t is distributed $N(0, z_t'\alpha)$ and z_t is an s-vector of exogeneous variables. With $\theta = [\beta', \alpha']'$ then the log-likelihood function, given n observations is:

$$L(\theta) = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{n}\ln z_t'\alpha - \frac{1}{2}\sum_{t=1}^{n}\frac{(y_t - x_t'\beta)^2}{z_t'\alpha}$$

The score functions for the likelihood are:

$$\frac{\partial L(\theta)}{\partial \beta} = -\sum_{t=1}^{n}\frac{x_t y_t}{z_t'\alpha} + \sum_{t=1}^{n}\frac{x_t x_t'}{z_t'\alpha}\beta$$

$$\frac{\partial L(\theta)}{\partial \alpha} = -\frac{1}{2}\sum_{t=1}^{n}\frac{z_t}{z_t'\alpha} + \frac{1}{2}\sum_{t=1}^{n}\frac{z_t e_t^2}{(z_t'\alpha)^2}$$

[6]The constant hedge ratio result is specific to this empirical example. Other cash markets may lead to a different conclusion about the constancy of the hedge ratio.

The roots to the score functions give the maximum likelihood estimators of β and α which can be expressed as:

$$\hat{\beta} = \left[\sum \frac{x_t x_t'}{z_t' \hat{\alpha}} \right]^{-1} \left[\sum \frac{x_t y_t}{z_t' \hat{\alpha}} \right]$$

$$\hat{\alpha} = \left[\sum \frac{z_t z_t'}{(z_t' \hat{\alpha})^2} \right]^{-1} \left[\sum \frac{z_t \hat{e}_t^2}{(z_t' \hat{\alpha})^2} \right]$$

where

$$\hat{e}_t = (y_t - x_t' \hat{\beta})$$

Although the $\hat{\alpha}$ equations are non-linear in the parameters, an iterative weighted least squares procedure can be used to obtain a solution. In a more general framework, Jobson and Fuller (1980) show that as long as the preliminary estimator of α is consistent, all estimators of β obtained by this procedure will asymptotically equivalent to the weighted least squares estimator with known weights. They suggest an estimator for α that can be regarded as the result of one iteration and has the same asymptotic properties as the maximum likelihood estimator. Another possible solution strategy is to iterate until some convergence criteria is met so that the maximum likelihood estimators are obtained directly. Consistent estimators for α and β can be found by using the least squares estimators as starting values. These expressions are

$$\hat{\beta}_{(0)} = \left[\sum x_t x_t' \right]^{-1} \left[\sum x_t y_t \right]$$

$$\hat{\alpha}_{(0)} = \left[\sum z_t z_t' \right]^{-1} \left[\sum z_t \hat{e}_t^2 \right]$$

Interation can be performed by replacing the parameter estimates in the current iteration to obtain the new parameter estimates. For example, the ith iteration would be calculated by

$$\hat{\beta}_{(i)} = \left[\sum \frac{x_t x_t'}{z_t' \hat{\alpha}_{(i)}} \right]^{-1} \left[\sum \frac{x_t y_t}{z_t' \hat{\alpha}_{(i)}} \right]$$

$$\hat{\alpha}_{(i)} = \left[\sum \frac{z_t z_t'}{(z_t' \hat{\alpha}_{(i-1)})^2} \right]^{-1} \left[\sum \frac{z_t \hat{e}_{t(i-1)}^2}{(z_t' \hat{\alpha}_{(i-1)})^2} \right]$$

Hypothesis tests can then be carried out using the Wald, likelihood ratio or score statistics. For the Wald and score statistics, one must evaluate the information matrix under the alternative and null hypotheses, respectively. The $(k + s)$-dimension information matrix for this problem is

$$I_n(\theta) = \begin{bmatrix} \sum \frac{x_t x_t'}{z_t'\alpha} & 0_{(kxs)} \\ 0_{(sxk)} & \frac{1}{2} \sum \frac{z_t z_t'}{(z_t'\alpha)^2} \end{bmatrix}$$

Given a hypothesis $R\theta = r$, where R has full row rank q, the three test statistics (Wald, likelihood ratio, and score) are:

$$T_w = (R\hat{\theta} - r)'(RI_n^{-1}(\hat{\theta})R')^{-1}(R\hat{\theta} - r)$$

$$T_{lr} = 2[L(\hat{\theta}) - L(\hat{\theta}_R)]$$

$$T_s = \left[\frac{\partial L(\hat{\theta}_R)}{\partial \theta}\right]' I_n^{-1}(\hat{\theta}_R)\left[\frac{\partial L(\hat{\theta}_R)}{\partial \theta}\right]$$

where θ_R refers to the parameter vector under the null hypothesis. Asymptotically, T_w, T_{lr}, and T_s converge to a chi-squared distribution with q degress of freedom.

In certain applications, computation of an F-statistic for the hypothesis $R\theta = r$ may be more practical. For this study, the F-statistic is used to test hypothesis about θ. This statistic amounts to a degrees of freedom correction to the Wald statistic given above. In particular, the F-statistic has the form:

$$T_F = \frac{T_w}{q} \sim F[q, (n - k - s)]$$

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