A Time Series Approach to Testing for Market Linkage: Unit Root and Cointegration Tests

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INTRODUCTION

The principal economic functions of the futures markets include price discovery and risk management. These economic functions have been examined by numerous studies [Johnson (1960); Stein (1961); Ederington (1978); and Laatsch and Schwarz (1988), to name a few]. It is well known that the futures market can function well only when the futures and cash prices are highly correlated or linked. Despite the fact that markets can be linked without index arbitrage, it is primarily the index arbitrage activities that result in the two markets to be closely linked in both directions of price movements. Thus, market linkage is essential to a successful futures market. The greater the degree of market linkage, the greater the effectiveness of the futures market in terms

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of performing its economic functions. If the markets are not linked, efficient transmission of information from one market to another will be impaired, thus reducing the value of the futures market in price discovery and hedging. The estimation of the degree of market linkage is therefore of paramount importance and interest to practitioners who engage in various hedging and arbitrage programs and to policymakers who set up regulatory policies to protect the well-functioning of the markets.

The purpose of this article is to suggest a cost-effective, statistical methodology that allows one to estimate the level of index arbitrage activities and to make inferences about market linkage between the cash and futures markets using only the readily available intraday market data. The methodology entails the application of the unit root test to the intraday cash-futures bases to test for market linkage. More specifically, based on Garbade and Silber's model of price convergence (1983), it is illustrated that the Dickey–Fuller's (1979) test for a unit root within the Engle and Granger's (1987) cointegration framework can be used to test for market linkage as well as to estimate the level of arbitrage activities between the cash and futures markets. The contribution of this article to extant literature is to show that failing to reject the null hypothesis of the presence of a unit root in the mispricing series within a cointegration framework is equivalent to failing to reject the null hypothesis that the two markets are not linked. Furthermore, the estimated first-order autoregressive coefficient of the mispricing series measures the degree of market linkage between the two markets if it is statistically different from one. More importantly, this article marries a commonly used statistical model—the cointegration test—to an economic model which lends itself to meaningful economic interpretations. As an empirical demonstration, the methodology is applied to the five-minute intraday data of S&P 500 index futures and cash markets surrounding the October 1987 Market Break. The empirical results from this methodology and the inferences made therefrom are consistent with the reported index arbitrage activities from previous studies, and the assertion that markets were not linked during the Break is also statistically supported by this methodology.

The remainder of the article is organized as follows. In the next section, the rationale of how and why the unit root test within the Engle and Granger's cointegration framework can be applied to the mispricing series in the context of Garbade and Silber's model of price covergence to test for market linkage is presented. First, the relationship between the futures and cash prices is explained in terms of the cost-of-carry model.

Since the relationship between the two markets hinges on arbitrage activities, the Garbade and Silber's model which assumes the supply of arbitrage activities as a driving force behind the linkage of markets is introduced next. Then it is shown that when the behavior of intraday observed basis is used to proxy that of mispricing series, the estimated first-order autoregressive coefficient of the mispricing of Garbade and Silber's model can measure the degree of linkage between the two markets. Lastly, the cointegration test is presented and synthesized with the Garbade and Silber's model. In particular, it is shown that the second step of the cointegration model involves the augmented Dickey–Fuller's (ADF) test on the residuals of the cointegrating regression, where market linkage is tested and the elasticity of the supply of arbitrage activities is estimated. The third section reports the empirical results of the application of this methodology on the S&P 500 index futures and cash markets during the October 1987 Market Break.

THE NEW METHODOLOGICAL APPROACH AND ITS ECONOMIC UNDERPINNINGS

Garbade and Silber's Cash-Futures Price Convergence Model and the Unit Root Test

The relationship between futures and cash prices can be summarized in terms of the cost-of-carry. Suppose financial markets are perfect and frictionless[1] and ignoring continuous compounding, the theoretical futures prices, F_t', and theoretical cash prices, C_t', both at time t, can be stated as:

$$F_t' = C_t + (r - d)(T - t) \text{ and} \tag{1}$$

$$C_t' = F_t - (r - d)(T - t) \tag{2}$$

where C_t is the contemporaneous price of the underlying security; F_t is the contemporaneous futures price; r is the risk-free rate; d is the dividend yield; and $(T - t)$ is the time to expiration of the futures contract. When the observed cash price exceeds the theoretical cash price, C_t', at any point in time, one is able to take advantage of the price discrepancy by simultaneously selling the underlying index stocks/portfolios and buying the index futures contracts. Likewise, when the observed cash price

[1] A perfect and frictionless market implies that there are (1) no taxes or transaction costs, (2) no limitation on borrowing, and (3) no limitation on short sales and the use of the proceeds of any short sales.

is below the theoretical cash price, one can buy the cash portfolio and short the futures. The derivation of the cash-futures price relationship is thus based on the argument of arbitrage. Consequently, the applicability of the cost-of-carry to model the futures prices depends on the presence and efficacy of index arbitrage activities which, in turn, insures that futures and cash prices are closely linked.

Garbade and Silber (1983) have developed a model to derive the dynamic price relationship between cash and futures, assuming the presence of arbitrage. They contend that the rate of price convergence depends on the supply elasticity of index arbitrage since the greater the elasticity of supply of arbitrage services, the better the substitutability of futures contracts for cash market positions and improvement of risk transfer function of futures markets, and the quicker the price differential can be arbitraged away. Their model is presented in eqs. (3a) and (3b), where β_c and β_f are elasticities of supply of index arbitrage activities, α_f and α_c are the intercepts, ϵ_{1t} and ϵ_{2t} are the error terms, C_t' is the theoretical cash price and C_t is the observed cash price, and the subscript $t - 1$, denotes the lagged terms of their respective variables.

$$C_t' = \alpha_f + C_{t-1}' - \beta_c(C_{t-1}' - C_{t-1}) + \epsilon_{1t} \tag{3a}$$

$$C_t = \alpha_c + C_{t-1} + \beta_f(C_{t-1}' - C_{t-1}) + \epsilon_{2t} \tag{3b}$$

Equations (3a) and (3b) can be written as a bivariate time series model:

$$\begin{bmatrix} C_t' \\ C_t \end{bmatrix} = \begin{bmatrix} \alpha_f \\ \alpha_c \end{bmatrix} + \begin{bmatrix} (1 - \beta_c) & \beta_c \\ \beta_f & (1 - \beta_f) \end{bmatrix} \begin{bmatrix} C_{t-1}' \\ C_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \tag{3c}$$

From eq. (3c), it is clear that when $\beta_c = \beta_f = 0$, the theoretical cash price, C_t', and the observed cash price, C_t, will have no relationship. The two price series will follow a bivariate random walk. As long as β_c and β_f are not equal to zero, the two price series are linked and will move together to some degree.

Instead of estimating β_c and β_f separately from eq. (3c), eq. (3b) is subtracted from eq. (3a), thus obtaining the univariate autoregressive process of the mispricing series, defined as $C_t' - C_t$, in Garbade and Silber (1983, p. 284, eq. 14):

$$C_t' - C_t = \alpha + \delta(C_{t-1}' - C_{t-1}) + \epsilon_{3t} \tag{3d}$$

where $\alpha = (\alpha_f - \alpha_c)$, $\delta = (1 - \beta_c - \beta_f)$, and ϵ_{3t} is the error term.

Although Garbade and Silber have provided a model to estimate the rate of convergence of cash and futures prices which reflects the corresponding level of index arbitrage activities, they do not furnish a statistical test for the significance of the estimated coefficients, δ of the convergence of the theoretical and observed cash prices from one, which has profound implications in the testing for market linkage. One of the major contributions of this article is to fill this void. The coefficient of the mispricing series, δ, is equal to one when $\beta_c = \beta_f = 0$. This occurs when there is no index arbitrage activity from either the cash or futures markets; hence, the two markets are not linked. On the other hand, when δ is equal to zero, the two markets are fully linked. Therefore, a unit root test for testing the hypothesis that δ equals one is an equivalent test for market linkage. In other words, if there is a unit root in the mispricing series, the two price series have no relationship and the markets are not linked.

One practical difficulty in applying the unit root test directly to the mispricing series is that the intraday interest rates and dividend yields are often unavailable. However, if one is willing to assume that the two variables remain constant during the intraday period,[2] then one can show that Garbade and Silber's model provides a framework for testing for market linkage when the behavior of intraday observed basis $(F_t - C_t)$ instead of the mispricing series $(C_t' - C_t)$ is being examined. To show this, the observed basis is first expressed as follows:

$$F_t - C_t = F_t - C_t' + C_t' - C_t \tag{4a}$$

Substituting eq. (2) into (4a),

$$F_t - C_t = (r - d)(T - t) + C_t' - C_t \tag{4b}$$

Similarly, one can write

$$F_{t-1} - C_{t-1} = (r - d)(T - t - 1) + C_{t-1}' - C_{t-1} \tag{4c}$$

Substituting (4b) and (4c) into (3d), and simplifying,

$$F_t - C_t = \beta_0 + \beta_1(F_{t-1} - C_{t-1}) + \epsilon_{3t} \tag{4d}$$

[2]It is reasonable to assume that the net cost-of-carry, $r_{t,T} = (r - d)(T - t)$, is constant within each trading day. For a longer term period, it is well known that both interest rates and dividend yields are non-stationary [Nelson and Plosser (1982)].

where $\beta_0 = \alpha + (r - d)(T - t) - \delta(r - d)(T - t - 1)$ and $\beta_1 = \delta$ of $(C'_{t-1} - C_{t-1})$ in eq. (3d).

For intraday basis, under the assumption that $(r - d)$ is constant within the day, [i.e., $F_t = (C'_t -$ constant) from eq. (2)], the observed basis, $(F_t - C_t)$, can replace the mispricing variable $(C'_t - C_t)$ in eq. (3d). The intraday mispricing differs from the intraday observed basis only by a constant. Hence, the behavior of intraday observed bases can be considered as a proxy for the behavior of the intraday mispricing series. Subsequently, the coefficient of $(F_{1-t} - C_{t-1})$, β_1 in eq. (5) will have the same interpretation as δ of $(C'_{t-1} - C_{t-1})$ in eq. (3) proposed by Garbade and Silber. Thus, the economic interpretation of a unit root test on the observed basis can be carried over to the mispricing series, i.e., testing for a unit root of the first-order autoregressive process of the mispricing series is equivalent to testing for market linkage between the two markets.

In the following subsection, Engle and Granger's cointegration framework is used to approximate the mispricing series (when the intraday interest rates and dividend yields are not known) because it removes the theoretical basis (i.e., the constant term) from the observed basis, thus providing a better approximation.

The Methodology for Testing for Market Linkage: The Cointegration Test[3]

Engle and Granger (1987) propose that if a linear combination of two difference stationary time series [i.e., each is integrated of order one, denoted I(1)] is stationary, then the two difference time series are cointegrated. For instance, in eq. (6), if the time series F_t is I(1), C_t is I(1), and the residuals, u_t, is I(0) (i.e., a stationary time series), then F_t and C_t are defined to be cointegrated.

$$F_t = \alpha_0 + \alpha_1(C_t) + u_t \tag{6}$$

Engle and Granger (1987) suggest a two-step procedure to test for cointegration between two time series: (1) Run the OLS regression, for example, on eq. 6, which is called the cointegrating regression. The Durbin–Watson statistic of the cointegrating regression residuals (CRDW) is used to test whether the two time series are cointegrated or not. If the CRDW exceeds the critical value, the time series are

[3]Other studies have applied the cointegration models for testing market efficiency [e.g., Hakkio and Rush (1989) and others] and for forecasting [e.g., Ghosh (1993) and others].

cointegrated; (2) Apply the augmented Dickey–Fuller's (ADF) test to the residuals of the cointegrating regression. If the computed test statistic, τ_u, exceeds the critical value, the null hypothesis that the two difference time series are not cointegrated will be rejected.

While Step 1 provides a statistical test for cointegration of two price series, the test per se provides no economic or financial meaning nor does it permit any implications on the supply of index arbitrage services. In contrast, Step 2 of the cointegrating test has a meaningful economic interpretation if the residuals of the cointegrating regression are considered as the proxy for the mispricing behavior of the stock index and index futures.

To show that the residuals (u_t) of eq. (6) can be viewed as a proxy for the mispricing series in eq. (3), the observed basis (i.e., the difference between the observed futures price, F_t, and the cash price, C_t) is first decomposed into two components: the mispricing, $(C_t - C_t')$, denoted by u_t, and the theoretical basis, $(F_t - C_t')$. That is,

$$F_t - C_t = F_t - C_t' + C_t' - C_t, \tag{7}$$

$$= (r - d)(T - t) + u_t. \tag{7a}$$

Second, rearranging eq. (7a), the following is obtained:

$$F_t = (r - d)(T - t) + C_t + u_t; \tag{8}$$

and for estimation purposes, therefore, one obtains:

$$F_t = a + b(C_t) + u_t \text{ [(which is eq. (6)]; or} \tag{8a}$$

$$u_t = F_t - (a + b(C_t)), \tag{8b}$$

where $a = (r - d)(T - t)$, $b = 1$, and $u_t =$ mispricing series.

If the OLS estimation is run on eq. (6) with or without restriction that $\alpha_1 = 1$, then ADF's test on the first-order autoregressive process of the residuals from eq. (6) provides a statistical test for Garbade and Silber's model on linkage between cash and futures markets and activities of index arbitrage. In other words, it is a test for cointegration of the cash and futures prices.

A few words of caution are in order at this juncture. First, since the estimated residual, u_t, depends on the estimated coefficients, α_0 and α_1, the distribution of the standard unit root test proposed by Dickey and Fuller (1979) in a cointegration model is not appropriate. For this reason, the empirical distribution of the ADF test in Engle and Granger's cointegration framework tabulated by MacKinnon (1991) is used.[4] Second, the cointegration test is applied here for statistical reasons, not for the reason of estimating a long-run economic relationship. For further discussion on this point, interested readers are referred to Dickey et al. (1991, pp. 60–61).

AN EMPIRICAL APPLICATION: TESTING FOR MARKET LINKAGE BETWEEN S&P 500 INDEX FUTURES AND CASH MARKETS DURING 1987 MARKET BREAK

As an illustration, the cointegration methodology is applied to the S&P 500 index futures and cash markets during the October 1987 Market Break. Past studies [e.g., MacKinlay and Ramaswamy (1988)] have shown that few profitable arbitrage opportunities existed in the S&P 500 cash and futures markets before the Market Break of October 1987 and concluded that arbitrage kept the cash and futures prices closely linked with each other. However, during the October 1987 Market Break, it has been observed that a large discount of the S&P 500 stock index futures to the S&P 500 index existed in the U.S. Harris (1989) and Moriarty et al. (1990) studied the relationship between the S&P 500 index futures prices and the underlying cash index prices over the period surrounding the Market Break of October 1987. Using different approaches and methods to account for the "nontrading effect" during the Market Break, both studies obtained similar results and conclusions: the nontrading (nonsynchronous trading) effect explains part of the large absolute futures-cash basis observed during the Market Break; the unexplained portion of the large negative basis may be attributed to the breakdown of the linkage mechanism between the two markets.[5] However, neither study offers any statistical method to test the hypotheses.

Another line of research that is closely related to the breakdown of linkage between the markets is continued in studies on arbitrage and program trading surrounding the Market Break. For example, Furbush (1989) showed that, during the Market Break (October 19–20), the normal arbitrage activities were not detected. His analysis was

[4]MacKinnon's table is used as a result of the suggestions of the referees of this journal.

[5]One reason is the increased transaction costs of index arbitrage. Wang et al. (1990) have found that the bid–ask spread of the S&P 500 index increased by seven to eight times during the market break of October 1987. The increase in the bid–ask spread implies a higher cost of index arbitrage. Other reasons for reduced level of index arbitrage services during the market break are discussed by Harris (1987, pp. 84–85).

based on the data on index arbitrage "program trades" supplied by the brokerage firms. Problems with this kind of approach include, among others, data reporting errors and costly collection processes in terms of time and money. The methodology proposed herein, based on sound economic and statistical models, mitigates these problems. Furthermore, the application of the tests requires only the readily available market data.

The Data

The data used in this study to illustrate the application of the methodology is a sample of every five-minute transaction data of the S&P 500 index futures for the period October 14–23, 1987, obtained from the time and sales records of the Chicago Mercantile Exchange (CME) from 8:30 AM to 4:00 PM. The reported S&P 500 cash index used is that captured by the CME from ADP Corp. and disseminated to the trading floor. During the Market Break week, the frequency of capture of the S&P 500 index generally ranged from every 15 to 30 seconds. For each five-minute period the observed basis is calculated by subtracting the S&P 500 index from the December S&P 500 futures price. The futures transaction price selected in each five-minute interval is the one closest in time to the capitalization-weighted average transaction time of the stocks included in that day's proxy portfolio. The tests are applied to the days immediately before and after the Market Break to show the reliability of the procedures/tests and to contrast the findings during the Market Break with those during days before and after the Break.

To illustrate the use of unit root and cointegration tests to test for market linkage and the presence of index arbitrage activities, the "nontrading effect," if present, must be removed from the indexes since they may not indicate the true market condition. Nontrading effect refers to the nonsynchronous trading and reporting of all component securities in the index. In the Appendix, the autocorrelations of rates of returns (measured as the logarithm of the price relatives, P_{t+1}/P_t) of the S&P 500 index, reconstructed index, and index futures for lags up to the twelfth-order and the Ljung-Box Q-statistics are given. Results indicate that taking first difference is adequate to reduce the nonstationary time series to a stationary one. The autocorrelation functions on the cash S&P 500 index reported in the Appendix are consistent with previous empirical results that non-synchronous trading is a problem in using the raw stock index data [Fisher (1966); Atchison et al. (1987), Perry (1985)]. This problem was particularly acute during the period of the Market Break of October 1987. For instance, there are strong autocorrelations observed in the cash S&P 500 index on October 19 (Q-statistic = 74.2) and October 22 (Q-statistic = 65.8), which suggest the presence of a strong "nontrading effect." Hence, the observed cash index may not have been an accurate indicator of the true market condition in those days. Unlike the cash index which is a multiple asset, the S&P 500 index futures does not suffer from the nontrading problem as indicated by the Q-statistics for each individual day. For example, the Q-statistic for the index futures in the Appendix is 13.1 for October 19. Consequently, tests based on the basis computed from the observed cash index and futures prices may be biased by the nontrading problem.

The S&P 500 cash index series is, therefore, first reconstructed to mitigate the "nontrading effect." The Q-statistic for the reconstructed cash index is only 50.4 for October 19 vis-à-vis 74.2 for the S&P 500 cash index. In other words, a substantial portion of the nontrading effect is removed by the reconstruction procedure.[6]

The S&P 500 cash index series reconstruction procedure is described as follows. For each day within the sample period, the trade-by-trade transaction records taken from a NYSE consolidated audit tape, of 480 of the 500 stocks in the S&P 500 index are examined. Those stocks that traded at least once in 90% of the five-minute intervals contained in a given trading day are included in the day's trading proxy portfolio. The number of stocks in the trading proxy portfolio from October 14 through October 23 are 108, 127, 140, 110, 123, 127, 54, and 93, respectively. The transaction price used for each stock in a given five-minute interval is the one closest to the end of the interval. The trading proxy index is calculated on a capitalization-weighted basis. The trading proxy index for a given day is normalized to the S&P 500 index by setting the proxy equal to the S&P 500 during a five-minute interval that contains relatively little price change (and is preceded and followed by such a period) in both the S&P 500 index and the Chicago Mercantile Exchange's December S&P 500 futures. The normalizing interval is also chosen with

[6]A word of caution is in order here. Atchison et al. (1987) show theoretically and empirically that nonsynchronous trading by itself cannot account for all the observed autocorrelations in daily index returns. Their empirical analysis indicates that portfolio returns are autocorrelated even after the effect of nonsynchronous trading is explicitly removed. In addition, using a regression model approach to removing nontrading effect, Harris (1987, pp. 88–89) demonstrates that reconstructed index series often remain autocorrelated (e.g., on October 19) for reasons other than nontrading effects. Therefore, he warns that serious problems would result if nonsynchronous trading problems were modeled using time series methods which assume that underlying values follow a random walk. The series used in this study has the desired time series properties suggested by Harris and Atchison et al. The reconstructed cash index shows less autocorrelations than the original series but it is still often autocorrelated (for example, on October 19).

the consideration that a large proportion of the stocks in the S&P 500 index and the proxy index are traded during that interval. A five-minute time interval is chosen for the analysis as a compromise between a shorter interval, which would decrease the staleness of any transaction prices contained in the interval, and a longer period, which would allow the inclusion of more stocks in the proxy portfolios. Furthermore, a five-minute interval makes it less likely to include a long-term cause of the positive serial correlation such as information lumpiness and the inventory rebalancing of the market makers.

Results

Unit Root Tests for Stationarity

As a prelude to the cointegration test, the two price series are first tested for difference stationarity. In Table I, results are reported on the unit root tests of the index futures (F) and the reconstructed S&P 500 index (C_1) for the period from October 14–October 23, 1987, in the following forms:

$$\Delta F_t = \beta_0 + (\beta - 1)F_{t-1} + \sum_{i=1}^{h} \theta_i \Delta F_{t-i} + u_{1t} \tag{9}$$

where $\Delta F_t = F_t - F_{t-i}$ and $\Delta F_{t-i} = F_{t-i} - F_{t-i-1}$; and

$$\Delta C_{1,t} = \beta_0 + (\beta - 1)C_{1,t-1} + \sum_{i=1}^{h} \theta_i \Delta C_{1,t-i} + u_{2t} \tag{10}$$

where $\Delta C_{1,t} = C_{1,t} - C_{1,t-i}$ and $\Delta C_{1,t-i} = C_{1,t-i} - C_{1,t-i-1}$.

The coefficients of F_{t-1} and $C_{1,t-1}$ are tested by the ADF's τ_u statistics. The τ_u statistics are reported in Table I in parentheses under the estimated coefficients of F_{t-1} and $C_{1,t-1}$. None of the τ_u statistics is significant at the 1% level for both periods, the days before the Market Break (October 14–16) and the days during and after the Market Break (October 19–23). Thus, the null hypothesis that a unit root exists in the time series cannot be rejected for both the reconstructed S&P 500 index and the index futures.[7]

[7]It is usually agreed that the mean of the first difference intraday price data is approximately zero. The empirical results indicate that the intercept terms are statistically insignificant from zero except for October 22 and 23. For those two days, the tests for unit roots include a linear trend term and results are qualitatively the same as those without the trend term. The unit root hypothesis is not rejected. For consistency, results without the trend term are reported in the tables.

The coefficients of the lagged first difference terms (θ_i) are not reported here in the interest of saving space.[8] However, the number of lagged first difference terms and the number of significant θ_i are reported in parentheses in the rows of ΔF_{t-i} and $\Delta C_{1,t-i}$ in Table I. The first element in the parentheses denotes the number of lag terms, and the second element denotes the number of significant θ_i. For example, there are four lagged first difference terms and one of the θ of these terms is significant (4, 1) at 5% level for the futures for October 15.

Cointegration Tests

Since the F and C_1 series have been tested to be difference stationary, the Engle and Granger's cointegration test is appropriate. In Table II, results on the cointegration tests are reported. Results from Step 1 based on the OLS regression of the futures price (F_t) and the reconstructed S&P 500 index ($C_{1,t}$) are reported in column 3 for each day for the period studied. Results from Step 2 are reported in columns 4 and 5. Column 4 provides estimates of the first-order autoregressive coefficient

[8]Results on the lagged first difference terms are available from the authors upon request.

TABLE I

Tests for Unit Roots (Augmented Dickey–Fuller Test) in the S&P 500 Index Futures and the Reconstructed S&P 500 Index: Oct. 14–Oct. 23, 1987 (5-minute interval)[a]

	Oct. 14	*Oct. 15*	*Oct. 16*	*Oct. 19*	*Oct. 20*	*Oct. 21*	*Oct. 22*	*Oct. 23*
(1) Futures								
Constant	−0.0758	−0.0753	−0.0736	−0.1485	0.5668	0.1324	0.2513	0.6631
	(−0.74)	(−0.61)	(−0.46)	(−1.34)	(1.87)	(0.94)	(2.98)	(2.52)
F_{t-1}	0.0131	0.0132	0.0131	0.0265	−0.1061	−0.0239	−0.095	−0.1206
	(0.732)	(0.562)	(0.460)	(1.38)	(−1.88)	(−1.65)	(−2.98)	(−2.52)
ΔF_{t-i}	(4, 0)	(4, 1)	(4, 1)	(4. 0)	(4, 0)	(4, 0)	(4, 1)	(4, 1)
Q-Test(12)	9.2	6.4	3.4	7.2	3.4	5.6	4.7	6.6
(2) Cash(1)								
Constant	−0.0741	−0.0738	−0.0735	−0.0699	0.5171	0.5217	0.2793	0.2838
	(−0.87)	(−0.66)	(−0.69)	(−0.90)	(1.99)	(0.87)	(4.78)	(2.27)
$C_{1,t-1}$	0.0129	0.0129	0.0128	0.0125	−0.0952	−0.0942	−0.0410	−0.0498
	(0.861)	(0.645)	(0.676)	(0.887)	(−2.000)	(−0.877)	(−3.47)	(−2.27)
$\Delta C_{1,t-i}$	(4, 0)	(4, 1)	(4, 1)	(4, 1)	(4, 0)	(4, 1)	(4, 1)	(4, 1)
Q-Test(12)	5.3	4.3	10.7	3.9	3.4	5.6	7.9	5.9

[a]*(1)* The dependent variables are ΔF_t and ΔC_t, respectively.
(2) The critical values of τ_u at $\alpha = 0.01, 0.05$, and 0.10 are 3.51, 2.88, and 2.58, respectively [see Fuller (1976, p. 373)]. The critical values from MacKinnon (1991) for sample size (n) = 49 are 3.57, 2.92, and 2.60 for $\alpha = 0.01, 0.05$, and 0.10, respectively; for $n = 65$, 3.53, 2.91, and 2.59; for $n = 77$, 3.52, 2.90 and 2.59.
(3) Cash(1) denotes the reconstructed S&P 500 cash index.

TABLE II

Cointegration Tests and Estimated Mispricing Behavior of the S&P 500 Index Futures and S&P 500 Index: October 14 to October 23, 1987

(1) Dates	(2) T	(3) CRDW	(4) Coeff. of B_{t-1}	(5) ADF	(6) Coeff. of ΔB_{t-j}	(7) Critical Value 5% (1%)
Oct. 14	77	1.72[a]	0.0707 (0.34)	−4.41[a]	(3, 0)	−3.41 (−4.04)
Oct. 15	75	0.54[a]	0.7635 (8.24)	−2.55	(3, 1)	−3.41 (−4.04)
Oct. 16	78	1.72[a]	0.2218 (0.99)	−3.48[b]	(3, 0)	−3.41 (−4.04)
Oct. 19	73	0.36[b]	0.8414 (10.96)	−2.07	(3, 0)	−3.41 (−4.04)
Oct. 20	65	0.32[b]	0.8597 (11.89)	−1.94	(3, 1)	−3.43 (−4.07)
Oct. 21	76	0.58[a]	0.7539 (7.82)	−2.55	(3, 0)	−3.41 (−4.04)
Oct. 22	74	0.76[a]	0.4743 (4.25)	−4.71[a]	(3, 1)	−3.41 (−4.04)
Oct. 23	49	0.22	0.9117 (12.14)	−1.18	(3, 0)	−3.46 (−4.12)

Notes:
1. T denotes sample size; CRDW denotes the Durbin–Watson statistics of cointegrating regressions.
2. $B_{t-1} = F_{t-1} - (\beta_0 + \beta_1 C_{1,t-1})$
3. ADF is the augmented Dickey–Fuller test.
4. The t-statistics for H_0: $\beta = 0$ for B_{t-1} in column 4 are in parentheses.
5. The critical values of CRDW at $\alpha = 0.01$ and 0.05 are 0.455 and 0.282, respectively [see Engle and Granger (1987, p. 70)]. The critical values for the ADF test reported in the last column are computed from Table I of MacKinnon (1991, p. 275).

[a] Denotes significance at the 1% level.
[b] Denotes significance at the 5% level.

of the proxy for the mispricing variable lagged one period (i.e., B_{t-1}). The proxy for the mispricing, B_t is the residual obtained from the cointegrating regression in Step 1. The estimated equation is specified as follows:

$$B_t = \beta_0 + \beta_1 \beta_{t-1} + \sum_{i=1}^{h} \theta_i \Delta B_{t-i} + u_{3t} \tag{11}$$

where $\Delta B_{t-i} = B_{t-i} - B_{t-i-1}$.

Results in Table II are consistent with the casual observation in the markets and previous empirical findings regarding the relationship between the cash and futures markets surrounding the Market Break. First, the cash and futures markets are shown to be closely linked on October 14 and October 16. This conclusion is robustly supported by tests in both steps of the cointegration test. For instance, the CRDW statistics in column 3 for October 14 and

16 are both 1.72 and they are significant at the 1% level. During the October 19–20 Market Break period, the CRDW are 0.36 and 0.32, although the significance dropped to the 5% level. The CRDW statistics become significant at 1% again on October 21. These results lend support to the hypothesis that there was linkage of the two markets on October 14 and 16 but not on October 19 or 21.

As mentioned before, although the CRDW statistics provide statistical evidence of market linkage between the futures and cash prices, they supply little information about the level of index arbitrage activities. The missing information can be retrieved from the results from Step 2 of the cointegration test. The estimated coefficient of B_{t-1}, β_1, indicates the level of index arbitrage activities which are presumed to be the driving force behind the linkage of the markets. When active index arbitrage activities are in place, the estimated coefficient, β_1, will be approaching zero, which signifies that the markets are fully linked. Conversely, when the level of index arbitrage activities declines, the estimated coefficient will be close to one, revealing that the markets are disengaged. In column 4 of Table II, for October 14, β_1 is close to zero (.0707); whereas, for October 19, it is quite close to one (.8414). And, these figures are not due to chance. The ADF test statistics (in a cointegration framework) on the significant difference of the coefficients of B_{t-1} from one are reported in column 5.[9] The coefficients are significant at 1% for October 14 and 5% for October 16, but they are not significant at the conventional levels for October 15, 19–21, and 23. Column 6 reports the number of the lagged first difference in terms of ΔB_{t-j} in eq. (11) and the number of those which are significant.[10] Results in Table II suggest that there had been active index arbitrage activities which fostered the linkage between the cash and futures markets on October 14, 16, and 22, but not on the other days surrounding the Market Break. These results are consistent with the volume of index arbitrage surrounding the Market Break reported by the Commodity Futures Trading Commission (CFTC) in its *Final Report* and Furbush (1989). The CFTC (1988) estimated that the index arbitrage activity accounted for 14.4%

[9]In some studies [e.g., MacKinnon (1991)], the ADF test is also called the Engle–Granger (EG) test.

[10]The coefficients of the lagged first difference terms (ΔB_{t-j}) are not reported here in the interest of saving space. The first element in the parentheses denotes the number of lag terms, and the second element denotes the number of significant ΔB_{t-j}. For example, there are three lagged first difference terms of ΔB_{t-j} and one of them is significant (3, 1) at the 5% level for October 15.

of the NYSE volume on October 14, 9.1 percent on October 19, 0.6 percent on October 20, and 1.3 percent on October 21. Furbush (1989) estimated that the dollar percentages of index arbitrage of the NYSE dollar volume were 12.68% on October 14, 7.49% on October 15, 9.56% on October 16, 7.18% on October 19, and 0.89% on October 20.

CONCLUSIONS

Garbade and Silber (1983) have provided a model to estimate the rate of convergence of cash and futures prices and level of index arbitrage activities. However, they do not furnish a statistical test for the estimated coefficient of the first-order autoregressive process of the mispricing price series which has profound implications in the testing for market linkage. The contribution of this article is to fill this void. It is shown that failing to reject the null hypothesis of the presence of a unit root in the mispricing series is equivalent to failing to reject the null hypothesis that the two markets are not linked. It is shown also that the estimated first-order autoregressive coefficient of the mispricing series can measure the degree of market linkage between two markets if it is statistically different from one.

This is illustrated empirically by examining the intraday S&P 500 index futures and cash market data surrounding the October 1987 Market Break. It is found that: (1) The two markets were strongly linked on October 14, 16, and 22, and (2) the price changes of the two markets were nearly independent during October 15, 19–21, and 23. It is shown that the cash and futures markets were disengaged and index arbitrage was not active during October 19–21. These findings, consistent with previous results, suggest that the volume of index arbitrage activities during October 19–21 was probably too low to cause a substantial discount on the S&P 500 index futures.

The cointegration methodology used in this study is a cost-effective alternative of making inferences on market linkage and the level of index arbitrage activities based on readily available market price data. The test does not require the collection of firm-level arbitrage volume data, which is time-consuming and costly. The empirical results from this study demonstrate that the methodology can also provide a cross-validation of the estimated index arbitrage activities based on other techniques.

APPENDIX

Autocorrelations of Rates of Returns of the S&P 500 Index Index Futures and S&P 500 Indexes (5-minute interval) October 14 to October 23, 1987

Lags	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*12*	*Q-test (12)*
(October 14)											
Futures	−0.02	−0.15	−0.30	−0.08	0.23[a]	−0.07	−0.17	−0.01	0.01	−0.12	13.5
Cash	0.20[a]	−0.10	−0.06	0.01	0.06	−0.09	−0.14	−0.07	0.03	−0.01	7.0
Cash(1)	0.12	−0.14	−0.10	−0.08	0.11	−0.05	−0.19	0.05	0.05	−0.08	9.2
(October 15)											
Futures	0.28[a]	0.16	0.17	0.17	−0.01	−0.04	−0.13	−0.06	0.02	−0.06	16.3
Cash	0.46[a]	0.31[a]	0.02	0.17	−0.02	−0.04	−0.15	−0.12	−0.14	0.09	33.2
Cash(1)	0.36[a]	0.24	0.12	0.18	0.04	−0.05	−0.11	−0.09	−0.09	0.07	22.5
(October 16)											
Futures	0.05	−0.26	−0.16	0.04	−0.01	−0.10	0.04	0.11	−0.07	−0.09	11.8
Cash	0.49[a]	−0.07	−0.21	−0.25	−0.21	−0.11	0.03	0.10	0.00	−0.04	36.8
Cash(1)	0.34[a]	−0.07	−0.17	−0.20	−0.15	−0.11	0.06	0.17	0.06	−0.03	27.8
(October 19)											
Futures	0.30[a]	0.09	0.70	0.06	−0.03	−0.04	0.00	−0.14	−0.12	0.13	13.1
Cash	0.65[a]	0.49[a]	0.27[a]	0.07	−0.00	−0.07	−0.11	−0.17	−0.22	−0.20	74.2
Cash(1)	0.45[a]	0.37[a]	0.16	0.11	0.04	−0.03	−0.10	−0.15	−0.10	−0.08	50.4
(October 20)											
Futures	0.11	−0.25	−0.06	0.01	−0.12	−0.11	0.23	0.10	−0.11	−0.04	21.7
Cash	0.27[a]	0.16	0.07	−0.01	−0.00	0.10	0.15	0.06	0.03	−0.16	15.0
Cash(1)	−0.19	0.15	0.10	−0.15	0.10	−0.00	−0.00	0.05	−0.05	−0.03	12.4
(October 21)											
Futures	−0.17	−0.03	−0.01	−0.12	0.00	0.00	0.01	−0.03	−0.04	0.07	4.3
Cash	0.38[a]	0.11	−0.02	−0.00	0.03	0.01	0.01	−0.00	0.05	0.03	13.4
Cash(1)	0.08	−0.01	−0.09	−0.04	−0.02	−0.01	−0.02	−0.00	0.03	−0.04	2.3
(October 22)											
Futures	0.10	−0.07	−0.08	−0.10	0.07	−0.08	−0.01	0.08	−0.15	0.07	7.3
Cash	0.52[a]	0.38[a]	0.14	−0.07	−0.23	−0.33	−0.35	−0.26	−0.12	0.12	65.8
Cash(1)	0.48[a]	0.23	0.17	0.02	0.01	−0.13	−0.26	−0.17	0.01	0.09	34.1
(October 23)											
Futures	0.43[a]	0.18	−0.04	−0.04	0.02	−0.08	−0.21	−0.24	−0.21	0.06	19.0
Cash	0.52[a]	0.27	0.20	0.04	−0.00	−0.15	−0.26	−0.26	−0.19	−0.09	31.9
Cash(1)	0.45[a]	0.30	0.16	0.02	−0.04	−0.13	−0.26	−0.08	−0.01	0.01	24.5

Note: 1 Futures, Cash, and Cash(1) denotes the S&P 500 December 1987 index futures, S&P 500 cash index, and the reconstructed S&P 500 cash index, respectively.
2 The cut-off points of chi-square with 12 d.f. at $\alpha = 0.01, 0.05$ are 26.2 and 21.0, respectively.

[a]Denotes significance at the 5% level.

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