EXAMINING THE DEPENDENCY IN INTRA-DAY STOCK INDEX FUTURES

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INTRODUCTION

The behavior of speculative prices has been the subject of extensive research. One key issue is whether price changes are independent over time. This issue is important because it relates to the efficient market hypothesis which suggests that asset prices are not predictable [LeRoy (1989); Fama (1976)]. Financial economists are interested in the existence of dependency in speculative prices for two reasons. First, its existence is inconsistent with rational expectations [Grabbe (1991, chapter 7)]. Second, its existence would make a strong case for technical forecasting.

Empirical investigations on memory in speculative prices have followed three alternative routes. First, variance ratios (VR) analysis

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has been used. A recent study by Peterson, Ma, and Ritchey (1992) uses VR to examine the random walk hypothesis in the cash prices of 17 commodities. They obtain results supporting positive dependency in price changes. VR has been widely used in foreign exchange and stock market analysis [Liu and He (1991); Lo and MacKinlay (1988)]. Although VR can also be used to detect mean conversion, it is less powerful in detecting long-memory than the rescaled range method discussed below [Mandelbrot and Taqqu (1979); Mandelbrot (1972, 1975); Mandelbrot and Wallis (1969)].

Second, the long-range memory in financial assets has been analyzed using the rescaled range (RR) method [e.g., Mandelbrot (1971)]. Because the RR analysis is robust enough to accommodate different distribution assumptions of the stochastic process, it is particularly useful in analyzing the behavior of futures prices, which are not necessarily normally distributed [e.g., see Junkus (1991)].

Using the RR method, many studies [Helms, Kaen, and Rosenman (1984); Booth, Kaen, and Koveos (1982); Greene and Fielitz (1977)] have found evidence of long-range dependency in many financial assets such as futures, exchange rates, and stock prices. A recent study by Lo (1991) demonstrated that the RR analysis is sensitive to short-memory of the stochastic process. Thus, rejection of the RR's null hypothesis (i.e., the process has only short-memory) which validates long-memory may not be justified. Using the modified rescaled range (MRR) in analyzing stock market returns, he did not find the long-range memory found in earlier studies employing RR [e.g., see Greene and Fielitz (1977)]. Using MRR, Fund and Lo (1993) and Kao and Ma (1992) did not find long-memory in interest rate and currency futures prices changes.

Third, the autocorrelation function of the speculative prices has been examined [e.g., Chan (1992); Stoll and Whaley (1990); Goldenberg (1989); Alexander (1961); Kendall (1953)]. The use of autocorrelation analysis does not intend to distinguish the short-term dependency from the long-term dependency. However, an autoregressive fractionally integrated moving-average (ARFIMA) model is designed to capture the long-memory as well as the short-memory component. The long-memory parameter of fractional integration may be estimated by a variety of methods, including a two-stage semiparametric procedure [Cheung and Lai (1991); Diebold and Rudebusch (1989); Geweke and Porter-Hudak (1983)], approximate frequency domain maximum likelihood [Diebold, Husted, and Rush (1991); Fox and Taqqu (1986)] and exact time domain maximum likelihood [Sowell (1992)].

Using VR, MRR, and Geweke and Porter-Hudak's fractional integrated model, this study examines the dependency in the intra-day S&P 500 Index Futures. Since these three methods complement each other, they allow a comparison of the robustness of the results. In addition, this study examines whether dependency in these futures price changes, if it exists, is related to the maturity of the contract. The use of intra-day data for the S&P 500 Index Futures is of particular interest for two reasons. First, this data set represents real transaction prices on a minute-to-minute basis. These transaction data enable analysis of the microstructure of the futures price behavior. Second, the use of these data avoids the issue of using daily observations that involve break points due to overnight non-trading. Many traders, including scalpers and day traders, usually take advantage of the temporal futures price movement and close out their positions before the end of the day.

RESEARCH METHODOLOGY

Variance Ratio Test

The variance ratio test investigates the relation that the variance of the q-differences of an uncorrelated series is q-times the variance of its first difference. For example, given a time series, $X_1, X_2, \ldots X_T$, the variance of $X_t - X_{t-2}$ is twice the variance of $X_t - X_{t-1}$. Because of this proportional relationship, the uncorrelated increments of a time series can be tested by estimating the ratio of the variance of q-difference to the variance of the first-difference. The null hypothesis underlying the variance ratio (VR) for a random walk is equal to one. In turn, this study tests the hypothesis that increments of the time series are uncorrelated.

Lo and MacKinlay (1988) derive an estimator for the asymptotic variance of the variance ratio, which allows for heteroscedasticity including autoregressive conditional heteroscedastic (ARCH) process. Thus, a standard normal Z-statistic can be calculated for various q-intervals. The allowance of heteroscedasticity in the estimation is important and can accommodate the possible Samuelson's (1965) maturity effect (i.e., volatility of futures price changes per unit of time increases as the time to maturity decreases).

Using a 1-minute interval as the base, the Z-statistic can be computed for VR, which is defined as:

$$\mathrm{VR}(q) = \sigma_c^2(q)/\sigma_a^2(q) \tag{1}$$

$\sigma_c^2(q)$ is an unbiased estimator of $1/q$ of the variance of the q-difference and $\sigma_a^2(q)$ is an unbiased estimator of the variance of the first difference. In addition, $\sigma_c^2(q)$ and $\sigma_a^2(q)$ are defined as:

$$\sigma_c^2(q) = \left(\sum_{t=q \text{ to } nq} (X_t - X_{t-q} - qu)^2 \right) \Big/ m \tag{2}$$

$$\sigma_a^2(q) = \left(\sum_{t=1 \text{ to } nq} (X_t - X_{t-q} - u)^2 \right) \Big/ (nq - 1) \tag{3}$$

where $m = q(nq - q + 1)(1 - q/nq)$; $u = (X_{nq} - X_0)/nq$. $X = \log(F_t)$, where F_t is the S&P 500 Index Futures price at time t.

The estimated asymptotic variance of the variance ratio, VAR(q) is:

$$\mathrm{VAR}(q) = \sum_{j=1}^{q-1} [2(q - j)/q]^2 \delta(j) \tag{4}$$

where

$$\delta(j) = \sum_{t=j+1}^{nq} (X_t - X_{t-1} - u)^2 (X_{t-j} - X_{t-j-1} - u)^2 \div \left(\sum_{t=1}^{nq} (X_t - X_{t-1} - u)^2 \right)^2$$

The heteroscedsticity-consistent Z-statistic, Z(q), an asymptotic standard normal statistic, is:

$$Z(q) = [\mathrm{VR}(q) - 1]/[\mathrm{VAR}(q)]^{0.5} \tag{5}$$

The Z-statistic can be used to test the random walk hypothesis. If the Z-statistic is greater than 1.96 at a 5% level, it would indicate that VR is significantly different from one, suggesting that the time series is not a random walk or its increments are correlated.

The Rescaled Range Statistic

The traditional method of detecting long-range dependence is the rescaled range statistic (RR), denoted by V_n^* (where n is the number of observations). RR is the range of partial sums of deviations of a time series or process, X, from its mean, rescaled by its standard deviation, s_n [Mandelbrot (1971)]. In this analysis, $X_t = \log(F_t) - \log(F_{t-1})$, where F_t is the futures price at time t; thus, X_t represents the price change or return of the S&P 500 Index Futures. The traditional rescaled range statistic (RR) is computed as follows:

$$V_n^* = (1/s_n)\left[\underset{1\le k\le n}{\text{Max}}\sum_{j=1}^{k}(X_j - \overline{X}_n) - \underset{1\le k\le n}{\text{Min}}\sum_{j=1}^{k}(X_j - \overline{X}_n)\right] \quad (6)$$

where

$$s_n^2 = (1/n)\sum_{j=1}^{n}(X_j - \overline{X}_n)^2$$

A stochastic process, X, is strong mixing if the maximal dependence of two stochastic events approaches zero when the time span between them increases. Since short-range/long-range dependence is a matter of degree, strong mixing is a desirable operational definition to draw a line between them. Also, hypothesis testing involving strong mixing could include all Gaussian finite-order stationary autoregressive and moving average (ARMA) models and heterogeneously distributed sequences in the analysis. The null hypothesis is that the process of the S&P 500 Index Futures changes is strong mixing (i.e., short-range dependence). The process is assumed to have two components: an arbitrary, fixed constant and a random variable, ϵ, which has a zero mean, and bounded second and certain higher moments at all times, t. ϵ is not required to be independently, identically, distributed (i.i.d.). There is ample evidence of skewness and changing variance such as the generalized autoregressive conditional heteroscedastic (GARCH) model in futures contracts [Junkus (1991); Myers (1991)]. Because ϵ_t allows unconditional leptokurtosis via time-varying conditional moments such as conditional heteroscedasticity, RR is appropriate for analyzing long-memory in futures contracts. Thus, RR allows for the possible Samuelson's maturity effect.

The modified rescaled range (MRR) statistic, $V_n(q)$ is similarly defined as in the traditional RR model except that s_n is replaced by $\sigma_n(q)$, where q is the lag length. $\sigma_n(q)$ includes the autocorrelations of the time series up to lag q. In calculating $\sigma_n(q)$, the selected number of lags (q) cannot be too large or too small because the sample distribution of the estimator changes. The MRR statistic is computed as:

$$V_n(q) = (1/\hat{\sigma}_n(q))\left[\underset{1\le k\le n}{\text{Max}}\sum_{j=1}^{k}(X_j - \overline{X}_n) - \underset{1\le k\le n}{\text{Min}}\sum_{j=1}^{k}(X_j - \overline{X}_n)\right] \quad (7)$$

where

$$\hat{\sigma}_n^2(q) = (1/n)\sum_{j=1}^{n}(X_j - \overline{X}_n)^2 + (2/n)\sum_{j=1}^{q} w_j(q)\left\{\sum_{i=j+1}^{k}(X_i - \overline{X}_n)(X_{i-j} - \overline{X}_n)\right\}$$

and

$$w_j(q) = 1 - j/(q + 1), \qquad q < n$$

The RR statistic is a useful tool in detecting long-range dependence in highly non-Gaussian series with large skewness and kurtosis [Mandelbrot and Wallis (1969)]. In addition, it can detect nonperiodic cycles, cycles with periods equal to or greater than the sample period [Mandelbrot (1972)]. However, the classical RR statistic is sensitive to short-range dependence. Rejection of the null hypothesis may merely be a symptom of short-term memory. For example, the classical RR statistic may be biased if the random component of the underlying process exhibits first-order autocorrelation [Lo (1991)]. The MRR statistic is robust to the short-range memory.

The null hypothesis of no long-range dependency in the time series can be tested using the rescaled range method. Critical values for testing significance of rescaled range are tabulated in Lo (1991). When $q = 0$, MRR will be the same as RR. At a 5% level, the critical value for testing the null hypothesis of no memory is contained in the interval [0.0809, 1.862].

Fractional Integrating Model

Long-memory can be found in the class of autoregressive fractionally integrative moving-average (ARFIMA) model. A time series $X = \{X_1, \ldots X_T\}$ follows an ARFIMA (p, d, q) process if

$$\Phi(L)(1 - L)^d X_t = \Theta(L)\epsilon_t \tag{8}$$

where ϵ_t is iid $(0, \sigma^2)$, L is the backward-shift (lag) operator, $\Phi(L) = 1 - \phi_1 - \ldots \phi_p L^p$, $\Theta(L) = 1 + \theta_1 \ldots \theta_q L^q$, $(1 - L)^d$ is the fractional differencing operator defined by $(1 - L)^d = \sum_{k=0 \text{ to } \infty} \Gamma(k - d)L^k/\Gamma(k + 1)\Gamma(-d)$ where $\Gamma(.)$ is the Gamma function. X is a stationary process with $d < |0.5|$. X_t denotes the price change or return of the S&P 500 Index Futures at time t (i.e., $X_t = \log(F_t) - \log(F_{t-1})$).

Geweke and Porter-Hudak (GPH) demonstrate that the differencing parameter d can be consistently estimated from the least squares regression at frequencies near zero, i.e.,

$$\ln(I(\omega_j)) = c - d \ln(4 \sin^2(\omega_j/2)) + u_t, \qquad j = 1, \ldots n \tag{9}$$

where $\omega_j = 2\pi j/T$, $j = 1, \ldots T - 1$, $n = g(T) \ll T$, and $I(\omega_j)$ is the periodogram of X at frequency ω_j which is defined by

$$I(\omega) = \left| \sum_{t=1}^{T} e^{it\omega}(X_t - \overline{X}) \right| \Big/ (2\pi T) \tag{10}$$

If the least squares estimate of d is significantly greater than zero, this result suggests long-memory. With a proper choice of n, the asymptotic distribution of the estimated d does not depend on the order of the ARMA component or the distribution of the error term of the ARFIMA process [Yajima (1989)]. In empirical analysis, it has been suggested that $n = T^{0.5}$.

DATA

The time series data of the S&P 500 Index Futures prices are from the Chicago Mercantile Exchange. The futures data include all trades when the price is different from the previous trade. When multiple transactions are recorded during a given minute, only the last recorded transaction is used. When no trade is recorded during a minute, the previous minute's price is retained. Using this minute-to-minute price data series for each day, one can generate the futures price return (X_t) which is defined as $\log(F_t/F_{t-1})$, where F_t is the futures price at time t. Over the course of a trading day, there are 405 minute-to-minute observations.

Two contracts are selected for this analysis: September 1987 and March 1988. For each contract, returns are generated for six trading days (i.e., 6/17, 7/22, 8/19, 9/02, 9/12, and 9/17 for September 87 contract, and 12/08, 1/12, 2/18, 3/03, 3/10, 3/17 for the March 88 contract). Thus, there are a total of 12 trading days. Two of the days are the expiration day at the nearby futures contract and the day the futures contract becomes nearby (12 weeks before the expiration day). In addition, the days one, two, four, and eight weeks before the expiration day are included in the sample.

An interesting feature of the data design is that the last two trading days of the contract represent a less liquid contract because the "soon-to-be-nearby" contract becomes the main pit contract (instead of the existing nearby contract) approximately two weeks before expiration date of the nearby contract. As a result, of the six trading days selected for each contract, four trading days are very actively traded while two are much less liquid.[1] The results of the less liquid contract allows an examination of the impact of liquidity on memory. Thus, the results can be compared to the findings of Helms, Kaen, and Rosenman (1984) who use less liquid agricultural futures to examine long-term memory.

These two futures contracts also represent the main pit contract before and after the October 1987 crash which allows an analysis of the impact, if any, of the crash on the futures price behavior.

EMPIRICAL RESULTS

Table I reports the empirical results analyzing the random walk behavior of the S&P 500 Index Futures return using the variance ratio analysis, which is adjusted for heteroscedasticity. Two contracts are used: one with maturity date on September 1987 and the other with maturity date on March 1988. The results shown in both September 1987 and March 1988 contracts clearly reject random walk in the price changes.

As shown in Lo and MacKinlay (1988), the variance-ratio minus one is approximately $q - 1$ times the first $q - 1$ autocorrelation coefficients. Thus, if the variance ratios are less (more) than one, they suggest negative (positive) serial correlations in price changes. For the September 1987 contract, the estimated variance ratios, VR(q) on June 17 and September 10 trading days are significantly less than one at a 5% level. For the March 1988 contract, four out of the six trading days indicate that variance ratios are significantly less than one at a 5% level. Two variance ratios (i.e., on July 22 and January 12 trading days) are statistically greater than one, implying positive correlations; however, other days indicate negative serial correlations in futures price changes or returns. These results are, in general, consistent with Chan (1992) and Stoll and Whaley (1990). Because the bid–ask spread is small in the S&P 500 Index Futures, the significant correlations detected here may reflect the market impact of futures trading.

[1]There are approximately 2500 recorded daily trades in a time stamped tape when a contract trades in the main pit. However, recorded daily trades decrease to approximately 500 trades when the contract moves out of the main pit.

TABLE I
Results of Estimating the Variance Ratio Statistics
Adjusted for Heteroscedasticity for the S&P 500 Futures

Trading Day	*q* (lags in minutes)					
	2	4	6	8	10	12
Contract maturity: Sept 87						
6/17	0.922	0.912	0.884	0.824	0.763	0.726
	(−1.43)	(−1.17)	(−1.26)	(−1.67)	(−2.03)[a]	(−2.19)[a]
7/22	1.122	1.169	1.162	1.121	1.116	1.088
	(1.78)	(2.01)[a]	(1.63)	(1.05)	(0.87)	(0.87)
8/19	1.048	1.064	1.009	0.997	1.008	0.970
	(0.83)	(0.76)	(0.09)	(−0.03)	(0.06)	(−0.22)
9/02	0.973	1.061	1.151	1.170	1.134	1.098
	(−0.47)	(0.74)	(1.54)	(1.49)	(1.07)	(0.72)
9/10	0.719	0.606	0.567	0.555	0.545	0.509
	(−4.74)[a]	(−4.86)[a]	(−4.20)[a]	(−3.77)[a]	(−3.69)[a]	(−3.54)[a]
9/17	1.017	0.999	0.919	0.890	0.855	0.831
	(0.41)	(−0.02)	(−0.85)	(−1.03)	(−1.14)	(−1.21)
Contract maturity: March 88						
12/18	0.992	0.976	0.882	0.795	0.776	0.719
	(−0.11)	(−0.27)	(−1.09)	(−1.69)	(−1.69)	(−1.96)[a]
1/12	1.034	1.052	1.103	1.193	1.235	1.295
	(0.63)	(0.67)	(1.06)	(1.70)	(1.85)	(2.12)[a]
2/18	0.904	0.824	0.754	0.691	0.666	0.616
	(−1.66)	(−2.08)[a]	(−2.45)[a]	(−2.53)[a]	(−2.48)[a]	(−2.55)[a]
3/03	0.877	0.770	0.673	0.662	0.638	0.614
	(−2.31)[a]	(−2.88)[a]	(−3.30)[a]	(−2.99)[a]	(−2.89)[a]	(−2.85)[a]
3/10	0.981	1.107	1.166	1.192	1.198	1.262
	(−0.24)	(0.99)	(1.34)	(1.39)	(1.32)	(1.61)
3/17	0.911	0.925	0.855	0.792	0.718	0.671
	(−1.48)	(−0.97)	(−1.51)	(−1.77)	(−1.92)	(−2.15)[a]

[a]Denotes significant at a 5% level. *Z*-values are in parentheses.

Futures prices may behave differently toward maturity because of (1) liquidity effect and (2) volatility effect. For example, the futures prices are more volatile near maturity as traders attempt to unwind their positions as the contract expires. The results from the variance ratios here indicate that the pattern of dependency in price changes on and before the maturity day is similar to other trading days.

Stoll and Whaley (1990) argue that opening prices of the S&P 500 Index may be inaccurate (or stale) because some of the underlying stocks of the index may not have been traded. The start of the futures trading day has information and trade orders built up from the nontrading period that must be incorporated. In addition, trading activity is typically heavy toward the end of the trading day as many speculators close out their open positions before the market closes. Admati and Pfleiderer (1988) have shown theoretically that the pricing behavior in financial assets may be different in a concentrated-trading period than in other periods. Ekman (1992) documents high return volatility for the S&P 500 Index Futures near the open and the close. Thus, the beginning (and end) of the trading day has trading pressures that are not present during the remaining trading hours. Hence, price behavior during these intervals may not be representative. To minimize any potential bias associated with the opening and closing time period prices, the first 30-minute and last 15-minute observations of the trading day are deleted.

Table II reports the results on dependency using the reduced sample size. The null hypothesis of random walk behavior in the price

TABLE II

Results of Estimating the Variance Ratio Statistics Adjusted for Heteroscedasticity for the S&P 500 Futures After Deleting First 30-Minute and Last 15-Minute Observations

Trading Day	*q (lags in minutes)* 2	4	6	8	10	12
Contract maturity: Sept 87						
6/17	0.888	0.834	0.785	0.725	0.699	0.690
	(−1.98)[a]	(−2.13)[a]	(−2.25)[a]	(−2.56)[a]	(−2.51)[a]	(−2.43)[a]
7/22	1.166	1.236	1.147	1.143	1.053	1.033
	(2.27)[a]	(2.67)[a]	(1.43)	(1.22)	(0.39)	(0.22)
8/19	1.062	1.080	1.019	1.001	1.011	0.992
	(1.04)	(0.91)	(0.18)	(0.01)	(0.08)	(−0.05)
9/02	0.991	1.079	1.172	1.197	1.128	1.063
	(−0.15)	(0.92)	(1.68)	(1.68)	(0.99)	(0.45)
9/10	0.808	0.737	0.725	0.695	0.650	0.636
	(−3.31)[a]	(−3.26)[a]	(−2.80)[a]	(−2.58)[a]	(−2.68)[a]	(−2.63)[a]
9/17	1.031	0.992	0.892	0.858	0.830	0.820
	(0.68)	(−0.10)	(−1.08)	(−1.28)	(−1.27)	(−1.26)
Contract maturity: March 88						
12/18	0.973	0.919	0.818	0.754	0.794	0.761
	(−0.35)	(−0.86)	(−1.55)	(−1.90)	(−1.45)	(−1.59)
1/12	1.043	1.110	1.184	1.287	1.317	1.361
	(0.73)	(1.38)	(1.78)	(2.39)[a]	(2.33)[a]	(2.43)[a]
2/18	0.897	0.821	0.760	0.687	0.664	0.625
	(−1.70)	(−2.02)[a]	(−2.27)[a]	(−2.50)[a]	(−2.42)[a]	(−2.42)[a]
3/03	0.883	0.785	0.687	0.681	0.648	0.634
	(−2.23)[a]	(−2.52)[a]	(−2.97)[a]	(−2.66)[a]	(−2.62)[a]	(−2.52)[a]
3/10	1.049	1.229	1.316	1.344	1.357	1.337
	(0.65)	(2.04)[a]	(2.42)[a]	(2.39)[a]	(2.21)[a]	(1.93)
3/17	0.924	0.938	0.869	0.792	0.742	0.689
	(−1.20)	(−0.76)	(−1.35)	(−1.69)	(−1.73)	(−1.98)[a]

[a]Denotes significant at a 5% level. *Z*-values are in parentheses.

changes is rejected at a 5% level, a result consistent with Table I. Rejection of the random walk is not explained by the opening and closing prices movements.

Because the variance ratio analysis indicates statistically significant correlations in the S&P 500 Index Futures data, it is of interest to find out whether these correlations are of relatively short-term nature (i.e., lasting for several minutes) or whether they persist throughput the day. To this end, the dependency issue is examined by using the rescaled range and fractional differencing methods.

Table III reports the results using the classical and modified rescaled range analyses for the full sample size (i.e., without deleting the first 30-minute or last 15-minute observations). When $q = 0$, the modified rescaled range (MRR) is the classical rescaled range (RR) method. The results indicate that for the two contracts with different trading days toward maturity, the RR and MRR statistics are less than the critical value of 1.862, a finding indicating no long memory. The result of no long memory is consistent with two recent studies using MRR procedure in interest rate and currency futures markets [Fung and Lo (1993); Kao and Ma (1992)]. However, Helms, Kaen and Rosenman (1984) found long-memory in the agricultural futures prices.

TABLE III

Results of Estimating the Classical and the Modified Rescaled Range Statistics for the S&P 500 Futures for a Full Sample Size

Trading Day	*q (lags in minutes)*					
	0	2	4	6	10	50
Contract maturity: Sept 87						
6/17	0.903	0.937	0.940	0.970	1.009	1.291
7/22	1.075	0.985	0.986	1.009	1.026	1.270
8/19	1.464	1.428	1.457	1.479	1.473	1.319
9/02	1.245	1.232	1.184	1.149	1.172	1.509
9/10	0.831	0.970	1.001	1.021	1.040	1.317
9/17	0.909	0.900	0.911	0.932	0.959	1.058
Contract maturity: March 88						
12/18	0.946	0.949	0.951	0.974	1.024	1.213
1/12	1.305	1.283	1.277	1.275	1.286	1.204
2/18	1.139	1.235	1.298	1.347	1.419	1.351
3/03	1.085	1.187	1.292	1.320	1.359	1.198
3/10	1.699	1.639	1.591	1.570	1.578	1.449
3/17	0.905	0.937	0.950	1.003	1.092	1.411

The critical value for a 5% level of significance lies between (0.0809, 1.862) [see Lo (1991)]. When $q = 0$, the modified rescaled range (MRR) is the same as the classical rescaled range statistic.

The difference in results may be due to the fact that agricultural futures may have seasonality that may indicate long memory.

Using Geweke and Porter-Hudak's (GPH) method, the long-memory in S&P 500 Index Futures return for a 1-minute interval is analyzed. Table IV reports the results of the test for the full sample size (i.e., without deleting the first 30-minute or last 15-minute observations). For different sample sizes used, no pattern is found that consistently indicates long memory. Only during the last trading day (i.e., March 17) on the March 1988 contract, is a weak long-term dependency detected. However, after deleting the first 30-minute and last 15-minute observations (results not reported here), the finding

TABLE IV

Results of Estimating the Fractional Differencing Parameter, d Using GPH for the S&P 500 Futures for a Full Sample Size

Trading Day	$\mu = 0.5$	$\mu = 0.525$	$\mu = 0.55$
Contract maturity on Sept 87			
6/17	0.034	−0.036	−0.039
	(0.469)	(0.827)	(0.795)
7/22	−0.080	−0.182	−0.091
	(0.663)	(0.284)	(0.551)
8/19	0.231	0.151	0.217
	(0.218)	(0.369)	(0.159)
9/02	0.042	0.021	−0.083
	(0.817)	(0.897)	(0.582)
9/10	−0.209	−0.223	−0.249
	(0.262)	(0.193)	(0.108)
9/17	−0.016	0.079	−0.031
	(0.930)	(0.640)	(0.840)
Contract maturity on March 88			
12/18	−0.074	−0.147	−0.200
	(0.687)	(0.385)	(0.192)
1/12	0.065	0.178	0.177
	(0.723)	(0.293)	(0.247)
2/18	0.239	0.092	−0.039
	(0.203)	(0.583)	(0.794)
3/03	0.093	0.166	0.064
	(0.613)	(0.158)	(0.674)
3/10	0.387	0.173	0.234
	(0.047)	(0.309)	(0.130)
3/17	−0.387	−0.322	−0.319
	(0.059)	(0.065)	(0.043)

p-values are in parentheses. n (sample size) T^{μ}, where T is the number of observations in the time series data. The null hypothesis $d = 0$ (stationary) is tested against the alternative hypothesis of long-memory.

of long-memory on the March 17 trading day disappears. This result suggests that long-memory found may be due to the opening or closing prices. In general, this analysis using MRR and GPH implies that there is no consistent pattern indicating long-term memory in the intra-day price changes.

Because these two contracts (September 1987 and March 1988) represent contracts before and after the October 1987 crash and their behavior is similar, it appears that the crash did not have a significant impact on the futures prices. In addition, it appears that less liquid contracts (as indicated in 9/10 and 9/10 trading days for Sept 87 contract, and 3/10 and 3/17 for March 88 contract) do not necessarily have long-term memory.

CONCLUSION

Using intra-day data of S&P 500 Index Futures, price dependency in its price changes is investigated. Two contracts (September 1987 and March 1988) are used for the dependency analysis. For each contract, different trading days approaching maturity are selected to investigate the likely impact of the liquidity or maturity's effect on price movements.

The results using the variance ratio analysis suggest that the price changes are not a random walk. In fact, there exist both positive and negative serial correlations in these price changes. Further investigation of dependency in these price changes using modified rescaled range and fractional differencing methods indicate that the dependency is only short-term. The short-term dependency in price changes may reflect market impact of the futures trading.

It appears that there is no consistent pattern of long memory to be observed throughout the trading day. In addition, there is no significant impact of liquidity or maturity effect on price movements. This result has an important implication for forecasting using technical analysis because it is unlikely that the long cycle would be exploited for profit taking in the day. In addition, the analysis indicates that the October 1987 crash does not appear to have had a significant impact on the memory in futures prices.

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