
ASSESSING THE INTRADAY RELATIONSHIP BETWEEN IMPLIED AND HISTORICAL VOLATILITY

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INTRODUCTION

Historical volatility of the price of an asset or security is typically computed as the standard deviation of daily returns over some recent period (for example, the past 20–60 days). An option's price, however, reflects expected volatility yet to be realized, over the life of the option. If an instrument's return is perceived to be more volatile, the option on that instrument should be worth more, and vice versa. Given a value for this perceived volatility and the other parameters in an option pricing model, the theoretical price of the option can be determined. On the other hand, given the current price of a specific option contract along with the model's other parameters, the model can be solved backwards for the value of the volatility parameter implied by the

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current price of the option. This outcome is called "implied volatility" of the option.

A market's perception of future volatility conditions can change instantaneously. Such changes in the market's mood about an underlying instrument should be reflected in option prices and, therefore, in implied volatility. Implied volatility is thus a forward-looking measure of likely future volatility conditions. On the other hand, historical volatility is a backward-looking measure of recent volatility conditions. Furthermore, since the traditional measure of historical volatility incorporates return behavior over the past 20–60 days, such a measure displays substantial inertia. That is, as volatility conditions change from day to day, traditional measures of historical volatility are sluggish to respond because current conditions are diluted by the market's behavior over this past horizon. These characteristics of traditional volatility measures suggest a relationship in which implied volatility might offer predictive information regarding subsequent historical volatility behavior.

Previous work investigates the relationship between implied volatility of option prices and historical volatility of the underlying asset.¹ Some studies question the hypothesis that option prices offer substantial information content regarding expected volatility conditions in the underlying market [Beckers (1981); Wilson and Fung (1990)]. However, other work suggests that implied volatility may be useful in predicting subsequent movements in historical volatility of the underlying instrument's return [Heaton (1986); Latane and Rendleman (1976)].

Throughout this literature little attention has been given to whether, over a shorter time frame, some measure of historical volatility offers any predictive information regarding subsequent option prices. Intuition suggests that option traders may very well react to changes in short-term volatility by adjusting their volatility expectation upward or downward, to provide timely inputs to their option pricing models. With this in mind, this study investigates whether such a process can be measured objectively, by using measures of historical volatility computed over shorter time intervals. Also tests are used to determine whether implied volatility serves as a useful predictor of historical volatility.

Time intervals of two different lengths are used to measure implied and historical volatility: one interval per day and ten intervals per day.

¹See, for example, Ball and Torous (1986); Beckers (1981); Brenner, Courtadon, and Subrahmanyam (1985); Chiras and Manaster (1978); Franks and Schwartz (1988); Han and Misra (1990); Heaton (1986); Latane and Rendleman (1976); Manaster and Rendleman (1982); Park and Sears (1985); Poterba and Summers (1986); and Wilson and Fung (1990).

The measures of historical volatility used here employ transactions data throughout the course of a single trading day, rather than aggregating daily data across the 20- to 60-day time frame traditionally used by other researchers and market participants. Moreover, the time series of intraday volatility measures are computed over non-overlapping time intervals, so that price information during one interval is not carried over to the next. Options on the following four futures contracts are examined: (i) the S&P 500 index; (ii) deutsche marks; (iii) Eurodollars; and (iv) live cattle.

Any such investigation must take into account the well-documented association between volume and volatility in futures markets [Ekman (1992); Kawaller, Koch, and Koch (1990); Merrick (1987)]. This relationship is controlled for by incorporating the volume of both the underlying futures market and the option market into the model.

For all contracts and time intervals investigated, historical volatility is found to be strongly associated with movements in futures volume. After accounting for the relationship with volume, Granger test results are mixed across these contracts and time intervals. For the two time intervals investigated, the analysis often suggests no substantive lead/lag relationship in either direction. When significant relationships do appear, however, the market involved and the nature of the relationship depend upon the time interval of measurement used.

DATA AND METHODOLOGY

Data

Transactions data are from the Chicago Mercantile Exchange for the four futures contracts listed above, for the fourth quarter, 1988, nearby contract period.² These data include all quotes when the price is different from the previous trade. In addition, all price quotes for at-the-money call and put options on each contract are obtained for the same period.³

²For each market the time series begins on the day the futures contract becomes nearby, and ends one week prior to expiration. The sample periods follow: (i) S&P 500 futures, Sept. 16–Dec. 8, 1988; (ii) deutsche mark futures, Sept. 20–Dec. 9, 1988; (iii) Eurodollar futures, Sept. 20–Dec. 12, 1988; and (iv) live cattle futures, Sept. 1–Nov. 17, 1988.

³The Black (1976a) and the Black/Scholes (1972 and 1973) models have a tendency to missprice options that are further out-of-the-money. Hence, the at-the-money option is selected here as the option with a strike price closest to the actual price on the underlying contract throughout each day. Within the day it is not unusual to have a change in the designated at-the-money instrument.

- i. F_{1T} = price of S&P 500 stock index futures contract at time T ,
 c_{1T} = price of call option on S&P 500 stock index futures at time T ,
 p_{1T} = price of put option on S&P 500 stock index futures at time T ;
- ii. F_{2T} = price of deutsche mark futures contract at time T ,
 c_{2T} = price of call option on deutsche mark futures at time T ,
 p_{2T} = price of put option on deutsche mark futures at time T ;
- iii. F_{3T} = price of Eurodollar futures contract at time T ,
 c_{3T} = price of call option on Eurodollar futures at time T ,
 p_{3T} = price of put option on Eurodollar futures at time T ;
- iv. F_{4T} = price of live cattle futures at time T ,
 c_{4T} = price of call option on live cattle futures at time T ,
 p_{4T} = price of put option on live cattle futures at time T ,

where T indexes the intraday price quotes on each futures and option. These transactions price data are used to construct time series reflecting intraday movements in *historical volatility* of the underlying returns (σ_{ht}), to be matched with *implied volatility* in option prices (σ_{it}).

Constructing Measures of Historical Volatility

The computation of historical volatility for each futures contract is complicated by the fact that the frequency of trading in each underlying contract varies throughout any trading day. To measure intraday historical volatility for each futures contract, one must first have a time series in each futures price itself, taken at equispaced time intervals. Therefore, the last price quote every minute on each underlying futures contract is used to generate four minute-to-minute price series $\{F_{jT}; j = 1 - 4\}$.⁴ Then the natural log of the price relative is computed between successive prices to generate a series of minute-to-minute returns $[r_{jT} = \ln(F_{jT}/F_{jT-1}); j = 1 - 4]$. Thus, over the course of a trading day (e.g., 6 hours and 40 minutes for S&P 500 futures), the sample includes as many as 400 minute-to-minute observations on the instrument's return.

Movements in intraday historical volatility (σ_{ht}) are measured using each underlying contract's minute-to-minute return during non-overlapping time intervals of two different lengths. First daily volatility measures are constructed using all return observations throughout each

⁴If no price quote appears during one or more minutes for any underlying contract, the most recent quote is retained as the relevant price. The absence of a futures trade is reflective of a period of zero volatility. By retaining the most recent futures quote for all minutes when no quote appears, a futures price series is generated which displays no variation across all such minutes.

day. Second, the trading day is divided into ten equal time intervals for each underlying futures contract.⁵

Over each time interval, t (whether the day-long interval or the 1/10th day interval), the sample standard deviation of each return series is recorded as that interval's observation on historical volatility (σ_{ht}):

$$\sigma_{ht} = [(N - 1)^{-1} \sum_{T=1}^N (r_T - \bar{r})^2]^{1/2} \quad (1)$$

where N = the number of minute-to-minute returns available within each interval (e.g., $N = 400$ for each 6-hour and 40-minute daily time interval, or $N = 40$ for each 40-minute interval within that day); $\bar{r}$ equals the sample mean for r_T in each interval; T indexes the minute-to-minute observations on r_T within each interval; and t indexes the time intervals chosen to measure historical volatility.

Constructing Measures of Implied Volatility

For each instrument, every time interval's observation on *historical volatility* (σ_{ht}) is matched with the average *implied volatility* of all at-the-money call and put option quotes within that same time interval:

$$\sigma_{it} = \left[\sum_{c=1}^{N_{ct}} \sigma_{ic} + \sum_{p=1}^{N_{pt}} \sigma_{ip} \right] \div (N_{ct} + N_{pt}) \quad (2)$$

where σ_{it} = denotes the measure of implied volatility during interval t ; σ_{ic} refers to the implied volatility of the c th call quote during interval t ; σ_{ip} refers to the implied volatility of the p th put quote during interval t ; and N_{ct} and N_{pt} are the number of call and put quotes, respectively, during interval t .⁶ This procedure yields the matched time

⁵Traditional measures of historical volatility reflect variation in daily returns over the previous 20–60 days [e.g., Latane and Rendleman (1976)]. Since the intraday measures used here reveal changes in intraday volatility over much shorter nonoverlapping time intervals, the measures may be perceived as containing more “noise” than traditional measures. An alternative view would regard these measures as containing more timely “information” that might be germane to options pricing.

⁶The Black (1976a) model is used to compute implied volatility for options on the four futures contracts. It should be noted that the reliability of these implied volatility measures rests on the validity of this option pricing model. For each implied volatility computation, this model requires data on the fraction of a year to expiration, the risk-free rate, the strike price, and the actual price quote of the option. The annualized return on the Treasury Bill maturing nearest the expiration of each option contract investigated is used as a proxy for the risk free rate. This relationship is investigated also using the average implied of calls and puts separately in the analysis. Results are generally robust and available upon request.

Numerous diagnostic checks and robustness checks are used throughout the analysis. To conserve space they are not presented here. These diagnostics uniformly point to robust results, and they are available upon request.

series, $(\sigma_{ht}, \sigma_{it})$, over both daily and 40-minute time intervals that are necessary to investigate any predictive relationships between intraday movements in implied and historical volatility.

Methodology

Granger tests [Granger (1969)] are used to investigate the potential predictive relationships between each pair of matched time series on historical and implied volatility. In this approach, each volatility measure is hypothesized to depend potentially upon its own past movements and past movements in the other volatility measure:

$$\sigma_{ht} = \text{int}_1 + \sum_{k=1}^{M_1} a_k \sigma_{ht-k} + \sum_{k=1}^{M_2} b_k \sigma_{it-k} + e_{1t} \quad (3)$$

$$\sigma_{it} = \text{int}_2 + \sum_{k=1}^{M_2} c_k \sigma_{ht-k} + \sum_{k=1}^{M_1} d_k \sigma_{it-k} + e_{2t} \quad (4)$$

where e_{1t} and e_{2t} are disturbances that reflect variation in each volatility measure over time, beyond that attributable to past movements in both volatility measures.⁷ This two-equation model can be rewritten using the lag operator, L ($L^k x_t = x_{t-k}$), as follows:

$$\begin{bmatrix} \sigma_{ht} \\ \sigma_{it} \end{bmatrix} = \begin{bmatrix} A(L) & B(L) \\ C(L) & D(L) \end{bmatrix} \begin{bmatrix} \sigma_{ht} \\ \sigma_{it} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \quad (5)$$

where the intercepts are inherent; where each distributed lag from eqs. (3) and (4) appears as a polynomial in the lag operator; and where the disturbances are assumed to be normally distributed and not autocorrelated, but may be contemporaneously correlated across equations.⁸

In the context of this model, the null hypothesis that implied volatility does not lead (or "Granger-cause") historical volatility can

⁷An expanded version of this model in which each volatility measure depends on its own past and *current* and past movements in the other volatility measure is estimated also. The addition of these contemporaneous effects transforms the model in (3) and (4) into a system of two simultaneous equations. When this expanded model is estimated using Three Stage Least Squares, the contemporaneous coefficients are almost always statistically insignificant. Furthermore, the remaining results on the lead/lag relationships are robust. This analysis provides empirical justification for the Granger test specification in eqs. (3) and (4).

⁸Theoretically, if enough lagged values of the left-hand side variable are included in each equation, the residuals will be reduced to white noise [Geweke (1978); Granger (1969)]. This assumption of no autocorrelation in each disturbance is important for the validity of the F tests on Granger-causality, and is tested formally with the Ljung-Box (1978) test on the autocorrelation function of the residuals of each equation. Results indicate that the residuals are white noise in all cases.

be tested with zero-restrictions on the lagged coefficients of implied volatility in the first equation of (5):

$$H_1: \sigma_{it} \not\rightarrow \sigma_{ht} [B(L) = 0]$$

Likewise, the null hypothesis of no Granger-causality in the opposite direction can be tested with zero-restrictions in the second equation of (5):

$$H_2: \sigma_{ht} \not\rightarrow \sigma_{it} [C(L) = 0]$$

Results will reveal whether implied volatility contains statistically significant predictive information regarding subsequent movements in historical volatility, and/or vice versa. It is noteworthy that the notion of Granger-causal priority merely reflects the predictive information in one variable regarding another. This notion has little to do with the philosophical concept of causality.

Nonprice Factors that May Affect Volatility

Both equations in (5) are expanded to include the following five variables that may also influence historical and/or implied volatility. Each potential influence is then discussed in turn.

Trend = time trend;

FVOL = trading volume in the underlying futures market;

OVOL = trading volume in the options market;

Return = lagged return in the underlying futures market;

Dk = dummy variables to reflect different days of the week or different intraday time intervals.

First, the argument has been made that the variance of returns on each futures contract should be expected to increase systematically toward the end of the nearby contract period [Samuelson (1965)]. If futures prices are unbiased estimates of subsequent spot prices, and if spot prices follow a stationary first-order autoregressive process, the impact of new information on futures prices will be greater near expiration [Anderson and Danthine (1983)]. Since market information thus becomes more important over time, the variance could rise. This possibility is addressed by including a time trend in each equation of system (5). Lack of a robust positive coefficient for this variable may create some doubt regarding either the assumptions above or the process hypothesized in Samuelson (1965).

Second, trading volume in the futures contract may affect volatility in the underlying futures prices [Ekman (1992); Kawaller, Koch, and

Koch (1990); Merrick (1987)]. If futures volume affects futures volatility, it may also affect implied volatility in option prices on the futures contract. Third, a similar argument can be made to suggest that total volume on all call and put options may affect both implied and historical volatility for a given contract. Therefore, when daily time intervals are analyzed, daily volume on the underlying futures contract and daily volume on all call and put options are included as additional explanatory variables in each equation of (5). For these four futures contracts and their related options, volume data are generally unavailable for time intervals shorter than one day. Thus, when analyzing 10 intervals per day, the following proxies are used for volume on futures and options during the t th intraday time interval:⁹

$FVOL = N_{ft}$ = number of futures quotes during interval t ;

$OVOL = N_{ct} + N_{pt}$ = total number of call and put quotes in interval t .

Fourth, market volatility may vary consistently with general market movements. For example, Black (1976b) demonstrates that stock market volatility tends to increase as stock prices fall, and decrease as stock prices rise. In this model, general market movements are captured by including lagged returns in the underlying futures market. Specifically, this market return is measured as the natural log of the ratio of the last futures quote to the first futures quote in each time interval. In this analysis, one lagged return is included in each equation of (5). If market volatility consistently rises when futures prices fall and falls when futures prices rise, negative coefficients for these lagged returns should be observed. The model including several lagged values of the return in each equation is also estimated with generally robust results.

Finally, it is also possible that implied and historical volatility vary systematically across different days of the week, or across different intraday intervals. Hence, when daily intervals are analyzed, five day-of-the-week dummy variables are added to each equation of (5). These five dummies replace the intercept term in each equation. Then, for each equation a test is conducted of the joint hypothesis that the Monday intercept is identical to that on the other four days of the week. When analyzing ten intervals per day, a separate dummy variable is added for

⁹Harris (1987) suggests that the number of transactions should be closely related to actual trading volume throughout any trading day. For NYSE data, Harris shows that the number of transactions is highly correlated with volume throughout the trading day. For futures data, Ekman (1992) uses the number of recorded transactions to approximate trading volume on a contract.

each intraday interval. Again, these ten dummies replace the intercept in each equation of (5), and the joint hypothesis that the intercept of the day's first interval is identical to that of the other intraday intervals is tested. The coefficients of these intraday dummies will display any systematic intraday pattern in σ_{ht} or σ_{it} inherent in the data, and the joint test will reveal whether such a pattern is statistically significant.

Econometric Considerations

Note that system (5) is specified in terms of the levels of volatility. Consistent with the relationships analyzed by Wiggins (1987) and Kawaller, Koch, and Koch (1990), a natural log transformation of σ_{ht} and σ_{it} is employed in the regression analysis. This corrects for possible heteroskedasticity in the error terms.¹⁰

Before the system can be estimated, a finite lag parameterization must be selected. While longer lag lengths lessen the chance of mis-

¹⁰Kawaller, Koch, and Koch (1990, p. 384) point out that the sample variance has the following asymptotic distribution, if the underlying futures price series is normally distributed:

$$\hat{\sigma}_{ht}^2 \stackrel{a}{\sim} N\{\sigma_{ht}^2, 2\sigma_{ht}^4/(N-1)\}$$

where N is the sample size, and σ_{ht}^2 is the true volatility within the time interval t . This distribution demonstrates that the *precision* (variance) of each observation on the measure of historical volatility depends upon the true σ_{ht}^2 during that interval. Thus, the *precision* of the data on $\hat{\sigma}_{ht}^2$ varies as the true volatility does. This phenomenon may translate into heteroskedasticity in the error term of the first equation in the model, which determines historical volatility. Given the normality assumption underlying the futures price series, this heteroskedasticity disappears with the natural log transformation:

$$\ln(\hat{\sigma}_{ht}^2) \longrightarrow N\{\ln(\sigma_{ht}^2), 2/N\}$$

The practice of applying the natural log transformation follows the rule provided in Judge et al. (1985, p. 160, eq. 5.3.20), which suggests that one consider functions of the volatility measures that will stabilize the error variances. The natural log transformation is such a function to correct for heteroskedasticity that might be transferred to the regression error terms.

The error term (e_{2t}) for the second equation in the model, which determines implied volatility, may exhibit further heteroskedasticity due to the effect of grouping [Theil (1971, p. 249)]. Each observation on implied volatility is the average implied of all option quotes during that time interval (that day or that 40-minute period). The *precision* of this sample mean depends on the number of option quotes during each interval. As a result, for example, the sample mean from a time interval that experiences ten option quotes would have a smaller variance than the sample mean from a time interval with one quote.

To resolve this heteroskedasticity problem, the following weighted least squares transformation can be applied to all variables in the implied volatility equation, (4): let transformed $\sigma_{it} = \sigma_{it} * (N_{ct} + N_{pt})^{0.5}$. After applying this weighted least squares scheme to all variables in eq. (4), generally robust results are obtained. However, only the results without this transformation of the second equation are used in the presented model, because while this procedure results in an error variance for e_{2t} that is homoskedastic, it may also introduce spurious correlation into the model. This problem is due to the fact that the variable, $(N_{ct} + N_{pt})$, already appears on the right-hand side of the model as a proxy for options volume, when ten intervals per day are analyzed.

specification, they also consume more degrees of freedom. Hence one should choose the minimum lag length that specifies the relationships accurately. Geweke (1978) argues that, for each equation, the number of lags on the dependent variable (M_1) should be kept generous to minimize the chance of serially correlated errors, while the number of lags on the other variables (M_2) should be set lower to retain power in the hypothesis tests. With this in mind, $M_1 = 10$ lags and $M_2 = 5$ lags when employing daily time intervals, and $M_1 = 15$ lags and $M_2 = 5$ lags when using the higher frequency intervals of ten per day. The model is also estimated using several other sets of lag lengths, with generally robust results.

The system of seemingly unrelated regressions in (5) is estimated using the standard Zellner–Aitken technique [Johnston (1984, pp. 337–341)]. If the information embodied in each error term affects both volatility measures, this would imply contemporaneous correlation between the errors, and Ordinary Least Squares will yield inefficient estimates.

EMPIRICAL RESULTS

Data Characteristics

Table I presents the sample means of all variables for each futures contract investigated, along with the average frequencies of option quotes in each market. The average market return is negative for Eurodollar futures, and positive for the other three markets. This result indicates that the fourth quarter 1988 nearby contract period represents a time of generally falling futures prices in the Eurodollar market, and generally rising prices in the other three markets.

The last three columns of Table I indicate reasonable liquidity for these four futures markets during the sample period. The option markets also appear to be fairly liquid. The only possible exception is for options on Eurodollar futures, which experience an average of 18 option quotes per day (eight calls and ten puts). While this average liquidity varies substantially across markets, these averages appear to represent consistent activity on most days, as there are very few intraday intervals which experience no futures and/or option trades.

Daily Time Intervals

Initially consider the influence of the five nonprice factors on historical and implied volatility. Coefficient estimates for three of these five nonprice factors (futures volume, options volume, and lagged

TABLE I
Mean Values of Variables for Daily and Intraday Time Intervals

Market	Variable Means					
	Historical Volatility (σ_{ht})	Implied Volatility (σ_{it})	Market Return (r_t)	Futures Volume (FVOL)	Option Volume (OVOL)	Frequency of Option Quotes Nt: Nct, Npt ^a
One INTERVAL Per Day						
S&P 500 (N = 59 days)	0.114	0.176	0.00148	39,100	758	58: 30, 29
Deutsche mark (N = 58 days)	0.070	0.101	0.00153	23,984	3,947	75: 47, 28
Eurodollar (N = 59 days)	0.017	0.013	-0.00009	43,136	2,989	18: 8, 10
Live cattle (N = 55 days)	0.125	0.147	0.00092	8920	740	47: 25, 22
10 INTERVALS Per Day						
S&P 500 (N = 590 intervals)	0.112	0.176	0.00015	196.6 ^b (0)	—	6.1: 3.3, (0) (0)
Deutsche mark (N = 570 intervals) ^d	0.066	0.101	0.00016	81.2 (6)	—	7.8: 5.0, 3.3 (0) (0)
Eurodollar (N = 590 intervals)	0.015	0.013	-0.00001	15.0 (18)	—	2.6: 1.7, 1.9 (0) (0)
Live cattle (N = 550 intervals)	0.120	0.147	0.00006	55.1 (0)	—	(0): 2.7, 2.9, (0) (2)

^aNt represents the average number of total option quotes within each time interval. Nct represents the average number of call quotes, and Npt represents the average number of put quotes (Nt = Nct + Npt).

^bThere are no data available on actual futures volume for ten time intervals per day. The actual number of futures quotes for which the futures price changes during each interval is used as a proxy in the regression analysis. Hence, this column presents the average number of futures quotes per interval, over the entire sample period. In addition, the frequency of time intervals during which there are no futures transactions is presented in parentheses beneath the average number of futures transactions for each market.

^cThere are no data available on actual option volume for ten time intervals per day. As a proxy, the actual number of option quotes during each interval (Nt) is used. These proxies are presented in the last column of this table, along with the frequencies of intervals during which there are no option transactions (in parentheses beneath Nt, Nct, and Npt).

^dNote that there are 58 days in the nearby contract period for Deutsche mark futures. However, when analyzing 10 intervals per day, there are only 570 total intervals included in this sample period. This apparent inconsistency is due to a change in trading hours in the deutsche mark futures pit during the sample period. After the first ten business days of this sample period, trading hours were extended another 30 minutes. As a result, the first ten trading days of this sample period are shorter than the last 48. With this in mind, these first ten trading days are partitioned into just nine intraday time intervals. The remaining 48 days are then partitioned into ten intervals. This procedure results in 570 time intervals of comparable length over the entire 58-day sample period.

TABLE II
Influence of Futures Volume, Options Volume, and
Lagged Market Returns One Interval Per Day^a

<i>Dependent Variable</i>	<i>Historical Vol. (σ_{ht})</i>		<i>Implied Vol. (σ_{it})</i>	
S&P 500 futures				
Futures volume (std err) ^b	1.51	(0.23) ^c	0.05	(0.09)
Options volume (std err) ^b	-6.52	(7.42)	-0.92	(2.72)
Lagged mkt return (std err)	-0.88	(3.28)	-1.99	(1.10) ^d
Deutsche mark futures				
Futures volume (std err) ^b	2.68	(0.57) ^c	0.20	(0.14)
Options volume (std err) ^b	0.002	(1.68)	-0.46	(0.43)
Lagged mkt return (std err)	0.05	(8.22)	0.15	(1.93)
Eurodollar futures				
Futures volume (std err) ^b	0.78	(0.14) ^c	-0.08	(0.05)
Options volume (std err) ^b	1.05	(1.80)	-0.21	(0.61)
Lagged mkt return (std err)	15.1	(31.27)	-5.4	(11.55)
Live cattle futures				
Futures volume (std err) ^b	2.96	(1.74) ^d	0.24	(0.35)
Options volume (std err) ^b	8.48	(9.86)	-0.38	(1.91)
Lagged mkt return (std err)	-1.86	(5.11)	-0.01	(1.03)

^aParameter estimates are presented for the influence of futures volume, options volume, and lagged market returns on historical volatility [from eq. (3)] and on futures volatility [from eq. (4)]. Standard errors are provided in parentheses.

^bThis coefficient and standard error are multiplied by 10^5 for exposition.

^cSignificance at the 0.05 level.

^dSignificance at the 0.10 level.

market returns) are presented in Table II. Coefficients of the other two nonprice factors (the time trend and day-of-the-week dummies) are not important statistically or economically, and are therefore only discussed here briefly. First, the time trend is rarely significant across all contracts and lag lengths investigated. This result suggests that no systematic increase (or decrease) in historical or implied volatility develops throughout the life of these nearby futures and options contracts. Second, coefficients of the day-of-the-week dummies in each equation are similar in magnitude for each contract investigated. Moreover, tests of the joint hypothesis of identical day-of-the-week intercepts are never rejected for any futures or options contract investigated. This result indicates no systematic pattern in either volatility measure across different days of the week, for any market.

Consider the coefficients of the other three nonprice factors, appearing in Table II for each market investigated. First, daily volume on the underlying futures contract always displays a significant direct relationship with historical volatility, but never displays a significant impact on implied volatility, for each contract investigated. Second, total volume on all call and put options never displays a significant

relationship with either implied or historical volatility. Finally, the lagged market return is significantly negative (at the 0.10 level) only for implied volatility in the S&P 500 market. This result suggests that market declines (increases) tend to be followed by increases (declines) in implied volatility for this market.

Granger test results are presented in Table III for each contract investigated. The first hypothesis, $H_1(\sigma_{it} \neq \sigma_{ht})$, is rejected at the 0.05 level for the Eurodollar and live cattle futures markets, but is not rejected for the other two markets. The second hypothesis, $H_2(\sigma_{ht} \neq \sigma_{it})$, is also rejected at the 0.05 level for the Eurodollar and live cattle futures markets, indicating a bidirectional lead/lag relationship between implied and historical volatility for these two markets. The second hypothesis is also rejected at the 0.10 level for the S&P 500 futures market. No lead/lag relationship is indicated for deutsche mark futures.

Ten Intervals Per Day

Before discussing the Granger test results over these shorter time intervals, consider the nonprice factors influencing historical and implied volatility. The time trend is not presented here but it reveals a significant positive coefficient for the S&P 500 and Eurodollar futures and options markets. On the other hand, a significant negative trend appears for

TABLE III
Granger Tests: Relationship between Implied
and Historical Volatility One Interval per Day^a

	DF ^b	$H_1: \sigma_{it} \neq \sigma_{ht}^c$	$H_2: \sigma_{ht} \neq \sigma_{it}^c$
S&P 500 futures	(5, 50)	0.42 (0.833)	2.14 ^d (0.076)
Deutsche mark futures	(5, 48)	1.00 (0.426)	1.46 (0.222)
Eurodollar futures	(5, 50)	3.35 ^e (0.011)	2.42 ^e (0.049)
Live cattle futures	(5, 42)	2.67 ^e (0.035)	3.06 ^e (0.019)

^aIn each case, Granger tests are conducted by estimating eqs. (3) and (4) as a system of seemingly unrelated regressions, using ten daily lags on the dependent variable and five daily lags on the right-hand side variable in each equation. Results are generally robust when other sets of lag lengths are used, and when OLS estimation is employed.

^bDF provides the degrees of freedom for each asymptotic F statistic.

^c H_1 and H_2 test the hypotheses of no Granger causality in each direction, between historical volatility in the underlying futures contract and implied volatility in options on that futures contract. Approximate marginal significance levels are provided in parentheses.

^dSignificance at the 0.10 level.

^eSignificance at the 0.05 level.

historical volatility in deutsche mark futures, and for both historical and implied volatility in the live cattle market. This result apparently reflects a notable decline in volatility for these markets during the sample period under scrutiny. These contrasting results for the trend call into question the validity of the hypothesis that volatility consistently rises nearer expiration [Samuelson (1965)].

The coefficients of futures volume, options volume, and lagged market returns are presented in Table IV for each market analyzed. Futures volume displays a significant positive association (at the 0.05 level) with historical volatility in all four markets, and with implied volatility in the live cattle market. This result is consistent with previous work [Ekman (1992); Kawaller, Koch, and Koch (1990); Merrick (1987)] and underscores the importance of controlling for futures volume. It is, however, the only result that is robust across markets and time intervals investigated.

Options volume reveals a significant positive relationship (at the 0.05 level) with implied volatility for deutsche mark futures and with historical volatility for S&P 500 futures; but no statistically significant relationship is found in the remaining tests.

TABLE IV
Influence of Futures Volume, Options Volume, and
Lagged Market Returns Ten Intervals Per Day^a

<i>Dependent Variable</i>	<i>Historical Vol. (σ_{hi})</i>		<i>Implied Vol. (σ_{ii})</i>	
S&P 500 futures				
Futures volume (std err) ^b	2.87	(0.15) ^c	0.01	(0.02)
Options volume (std err) ^b	4.50	(2.11) ^c	-3.75	(2.40)
Lagged mkt return (std err)	-10.26	(2.99) ^c	-1.65	(0.34) ^c
Deutsche mark futures				
Futures volume (std err) ^b	6.97	(0.30) ^c	-0.03	(0.53)
Options volume (std err) ^b	0.71	(2.41)	0.86	(0.42) ^c
Lagged mkt return (std err)	4.26	(6.04)	3.17	(1.07) ^c
Eurodollar futures				
Futures volume (std err) ^b	19.42	(0.98) ^c	-0.31	(0.16) ^d
Options volume (std err) ^b	9.57	(6.42)	0.70	(1.07)
Lagged mkt return (std err)	0.24	(44.82)	15.55	(7.42) ^c
Live cattle futures				
Futures volume (std err) ^b	10.32	(0.58) ^c	0.16	(0.08) ^c
Options volume (std err) ^b	2.52	(3.86)	0.15	(0.52)
Lagged mkt return (std err)	-1.87	(4.56)	-1.89	(0.61) ^c

^aParameter estimates are presented for the influence of futures volume, options volume, and lagged market returns on historical volatility [from eq. (3)] and on futures volatility [from eq. (4)]. Standard errors are provided in parentheses.

^bThis coefficient and standard error are multiplied by 10^3 for exposition.

^cSignificance at the 0.05 level.

^dSignificance at the 0.10 level.

Similarly, lagged returns display a significant negative association with both historical and implied volatility in the S&P 500 market, as well as with implied volatility in the live cattle market. At the same time, significant positive coefficients appear for implied volatility in both the deutsche mark and Eurodollar options markets.

The Granger tests are presented in Table V for each contract. Results indicate that, when the time series cover ten intervals per day, implied volatility never leads historical volatility (H_1 is never rejected). This result contrasts dramatically with the Table III results over daily time intervals, and with previous work which uses different data and methodology and employs longer time intervals to measure volatility [Heaton (1986); Latane and Rendleman (1976)]. On the other hand, historical volatility now leads implied volatility (H_2 is rejected at the 0.05 level) for deutsche marks and live cattle. Thus, whenever a substantive lead/lag relationship exists across these shorter time intervals, historical volatility always leads implied volatility.

The contrasting Granger test results for one interval per day versus ten intervals per day suggest that the behavior of option traders is dependent upon the time interval of analysis. In some markets, option traders display an ability to *predict* forthcoming movements in historical volatility *across days*. However, when shorter time intervals are examined *within the trading day*, option traders are more likely

TABLE V
Granger Tests: Relationship between Implied
and Historical Volatility Ten Intervals per Day^a

	DF^b	$H_1: \sigma_{ii} \nrightarrow \sigma_{hi}^c$	$H_2: \sigma_{hi} \nrightarrow \sigma_{ii}^c$
S&P 500 futures	(5, 1082)	1.03 (0.399)	1.18 (0.318)
Deutsche mark futures	(5, 1030)	0.64 (0.668)	2.36 ^d (0.038)
Eurodollar futures	(5, 1048)	1.57 (0.167)	1.34 (0.244)
Live cattle futures	(5, 1002)	1.12 (0.346)	3.47 ^d (0.004)

^aIn each case, Granger tests are conducted by estimating eqs. (3) and (4) as a system of Seemingly Unrelated Regressions, using 15 lags on the dependent variable and five lags on the right-hand side variable in each equation. Results are generally robust when other sets of lag lengths are used, and when OLS estimation is employed.

^bDF provides the degrees of freedom for each asymptotic F statistic.

^c H_1 and H_2 test the hypotheses of no Granger causality in each direction, between historical volatility in the underlying futures contract and implied volatility in options on that futures contract. Approximate marginal significance levels are provided in parentheses.

^dSignificance at the 0.05 level.

to *respond* to previous intraday short-term volatility conditions in the underlying market.

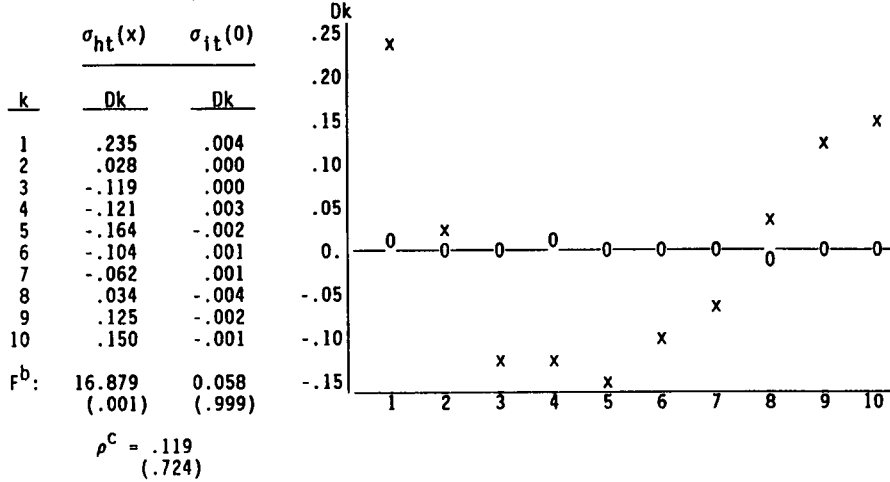
Intraday Patterns in σ_{ht} and σ_{it}

When analyzing ten intervals per day, larger volatility is anticipated during the first and last intervals each day. Although trading in these markets is closed overnight during the sample period, new information continuously accumulates and may be expected to influence trading more heavily at the market's open and close [Ahimud and Mendelson (1987); Ekman (1992); Kawaller, Koch, and Koch (1990); Wood, McInish, and Ord (1985)].

A preliminary investigation of this possibility is conducted for all four markets, by estimating an ordinary least squares regression of each volatility measure on a time trend and ten intraday dummy variables. Results are presented in Panel A of Figures 1–4, for the four markets examined. These results represent the systematic intraday patterns in historical and implied volatility inherent in each market, *before* accounting for the intraday variation in each volatility measure due to the other influences in the model.

Scrutiny of Panel A in Figures 1, 3, and 4 reveals a distinct “U”-shaped pattern in historical volatility for these three markets and significant *F*-tests indicating that this pattern is statistically profound. This pattern is consistent with previous work [Ahimud and Mendelson (1987); Ekman (1992); Kawaller, Koch, and Koch (1990); and Wood, McInish, and Ord (1985)] and reflects significantly greater volatility at the open and close of each day for these three markets. The intraday pattern for historical volatility of deutsche mark futures, in Panel A of Figure 2, deviates somewhat from this “U” shape. This market reveals significantly greater volatility at the beginning and middle of the typical trading day, followed by declining volatility toward the day's close. This pattern likely reflects the importance of trading out of London, which tends to “close” between 11:00 AM and 12:00 noon, Chicago time. Also, note that for Eurodollars (in Fig. 3) some of the “London effect” still is apparent, although the effect seems less prominent here. On the other hand, for live cattle and S&P markets (Figs. 1 and 4), London is much less important and the London effect is not in evidence.

Scrutiny of the results for implied volatility in Panel A of Figures 1–4 indicates no detectable patterns in this volatility measure for any market. This result supports the intuition that options traders

Panel A: Intraday Patterns estimated without the other influences in the Model^a

Panel B: Intraday Patterns after controlling for other influences in the Model

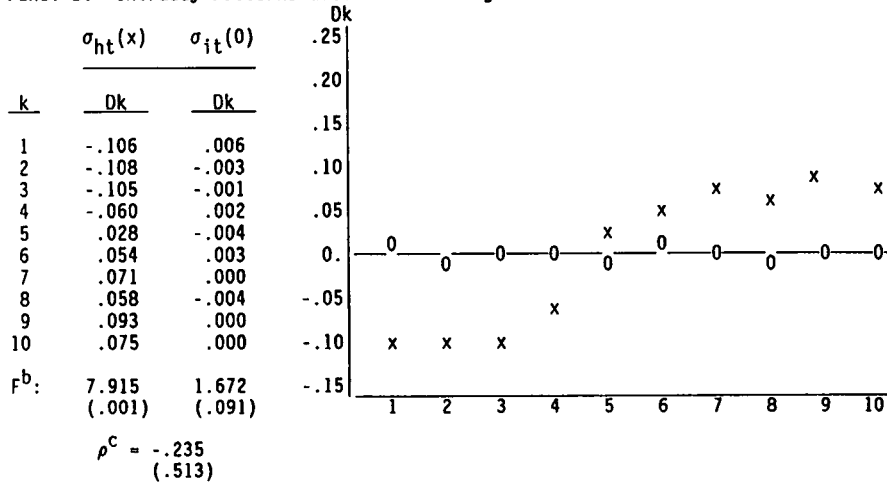
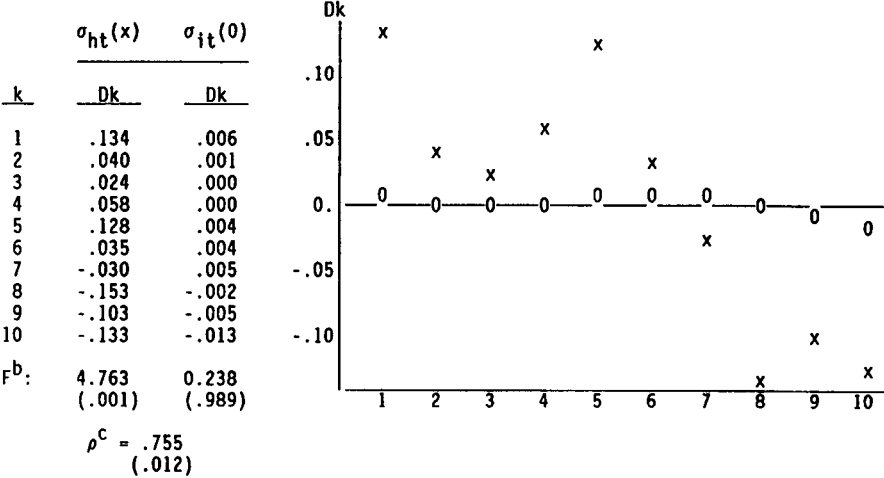


FIGURE 1

Coefficients of intraday dummies (Dk: $k = 1, 10$), intraday patterns in historical and implied volatility of S&P 500 futures. ^aPanel A presents the intraday patterns for historical volatility (σ_{ht}) and implied volatility (σ_{it}), estimated using ordinary least squares regression of each volatility measure on a time trend and ten intraday dummy variables. Results are presented as deviations from the mean dummy coefficient. Note that these coefficients reflect the typical intraday patterns in σ_{ht} and σ_{it} , before taking into account the other influences incorporated in the model in eqs. (3) and (4). Panel

B then presents the intraday patterns remaining in σ_{ht} and σ_{it} after taking into account these other influences in the model. ^bThe F statistic presented for each set of intraday dummy coefficients tests the null hypothesis that the last nine coefficients are identical to the first coefficient on a given day. The approximate marginal significance level is presented in parenthesis under each F statistic. ^cThe correlation between the set of intraday dummies for σ_{ht} and for σ_{it} is presented, along with the approximate marginal significance level associated with the null hypothesis that $\rho = 0$.

Panel A: Intraday Patterns estimated without the other influences in the Model^a



Panel B: Intraday Patterns after controlling for other influences in the Model

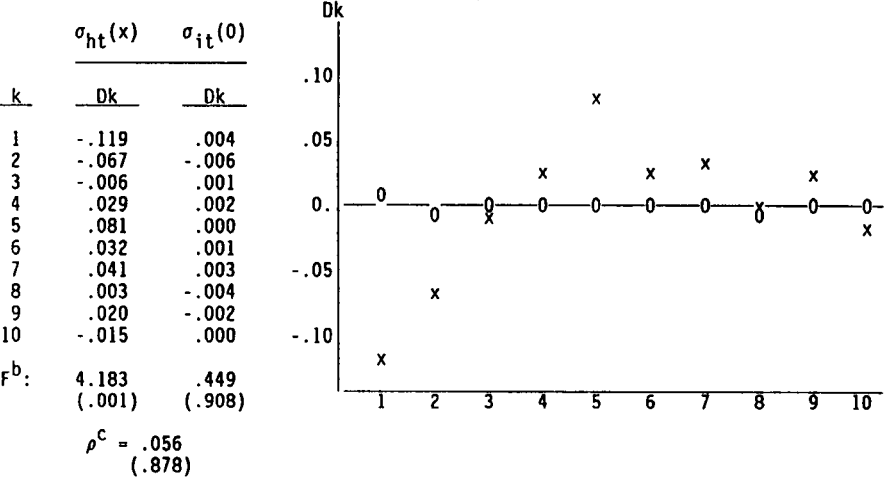
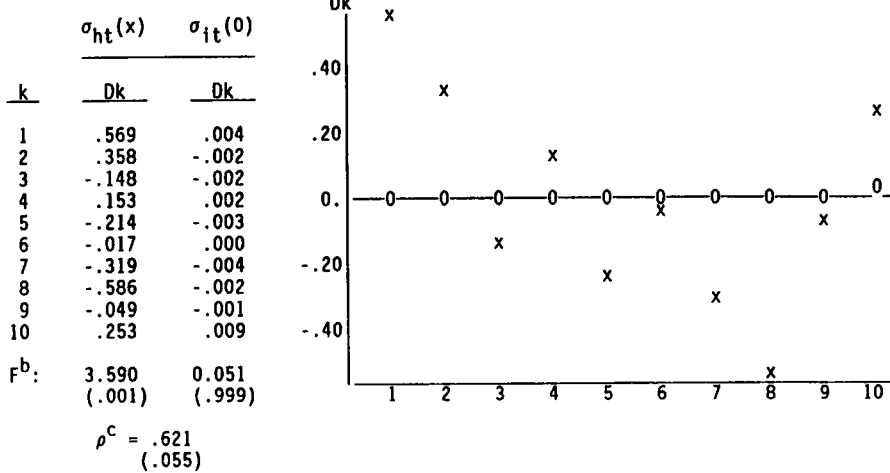


FIGURE 2

Coefficients of intraday dummies (Dk: $k = 1, 10$), intraday patterns in historical and implied volatility of Deutsche mark futures. ^aPanel A presents the intraday patterns for historical volatility (σ_{ht}) and implied volatility (σ_{it}), estimated using ordinary least squares regression of each volatility measure on a time trend and ten intraday dummy variables. Results are presented as deviations from the mean dummy coefficient. Note that these coefficients reflect the typical intraday patterns in σ_{ht} and σ_{it} , before taking into account the other influences incorporated in the model in eqs. (3) and (4). Panel B then presents the intraday patterns remaining in σ_{ht} and σ_{it} after taking into account these other influences in the model. ^bThe F statistic presented for each set of intraday dummy coefficients tests the null hypothesis that the last nine coefficients are identical to the first coefficient on a given day. The approximate marginal significance level is presented in parenthesis under each F statistic. ^cThe correlation between the set of intraday dummies for σ_{ht} and for σ_{it} is presented, along with the approximate marginal significance level associated with the null hypothesis that $\rho = 0$.

Panel A: Intraday Patterns estimated without the other influences in the Model^a

Panel B: Intraday Patterns after controlling for other influences in the Model

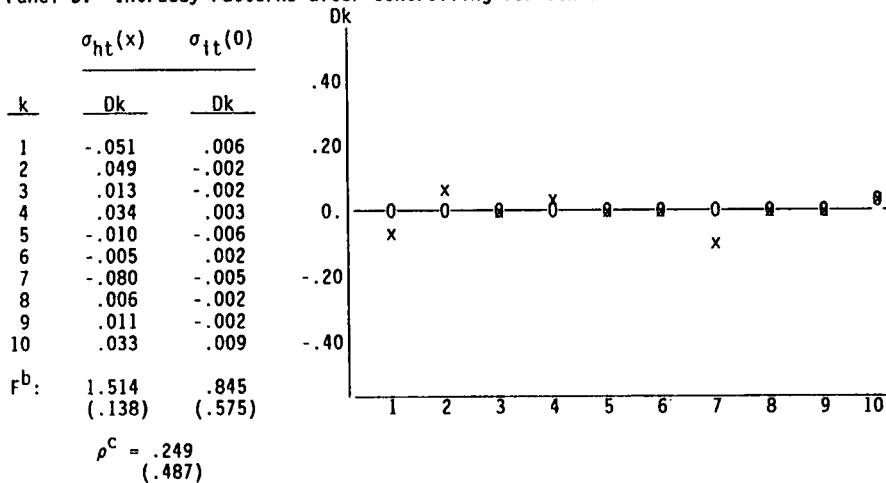
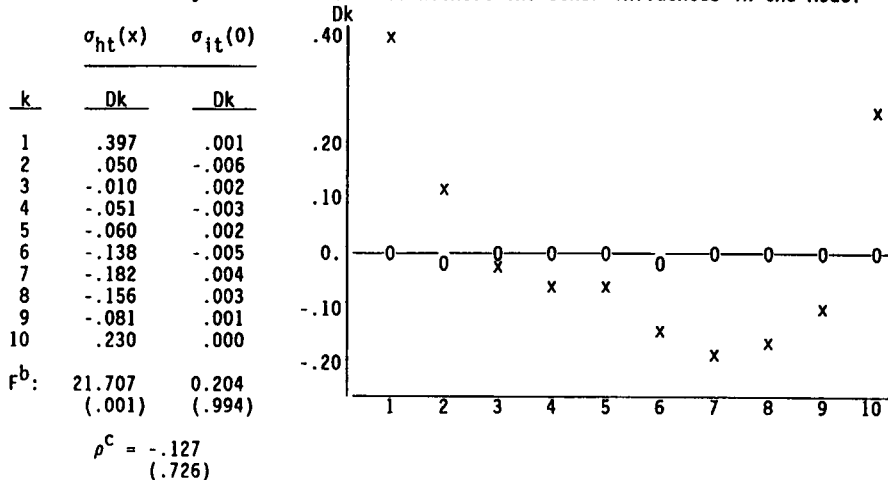


FIGURE 3

Coefficients of intraday dummies (Dk : $k = 1, 10$), intraday patterns in historical and implied volatility of Eurodollar futures. ^aPanel A presents the intraday patterns for historical volatility (σ_{ht}) and implied volatility (σ_{it}), estimated using ordinary least squares regression of each volatility measure on a time trend and ten intraday dummy variables. Results are presented as deviations from the mean dummy coefficient. Note that these coefficients reflect the typical intraday patterns in σ_{ht} and σ_{it} , before taking into account the other influences incorporated in the model in eqs. (3) and (4). Panel

B then presents the intraday patterns remaining in σ_{ht} and σ_{it} after taking into account these other influences in the model. ^bThe F statistic presented for each set of intraday dummy coefficients tests the null hypothesis that the last nine coefficients are identical to the first coefficient on a given day. The approximate marginal significance level is presented in parenthesis under each F statistic. ^cThe correlation between the set of intraday dummies for σ_{ht} and for σ_{it} is presented, along with the approximate marginal significance level associated with the null hypothesis that $\rho = 0$.

Panel A: Intraday Patterns estimated without the other influences in the Model^a

Panel B: Intraday Patterns after controlling for other influences in the Model

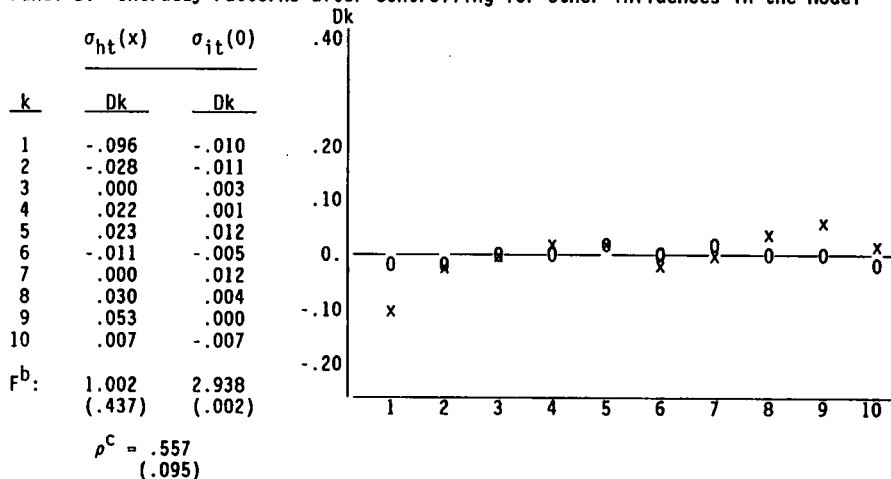


FIGURE 4

Coefficients of intraday dummies (Dk: $k = 1, 10$), intraday patterns in historical and implied volatility of live cattle futures. ^aPanel A presents the intraday patterns for historical volatility (σ_{ht}) and implied volatility (σ_{it}), estimated using ordinary least squares regression of each volatility measure on a time trend and ten intraday dummy variables. Results are presented as deviations from the mean dummy coefficient. Note that these coefficients reflect the typical intraday patterns in σ_{ht} and σ_{it} , before taking into account the other influences incorporated in the model in eqs. (3) and (4). Panel

B then presents the intraday patterns remaining in σ_{ht} and σ_{it} after taking into account these other influences in the model. ^bThe F statistic presented for each set of intraday dummy coefficients tests the null hypothesis that the last nine coefficients are identical to the first coefficient on a given day. The approximate marginal significance level is presented in parenthesis under each F statistic. ^cThe correlation between the set of intraday dummies for σ_{ht} and for σ_{it} is presented, along with the approximate marginal significance level associated with the null hypothesis that $\rho = 0$.

do not price the intraday patterns inherent in historical volatility for these four futures markets.

The systematic daily patterns for historical volatility indicated in Panel A of Figures 1–4 reflect behavior in each underlying market that must be accounted for in this model. For example, if futures *volume* is greater at the beginning and close of the typical trading day, inclusion of futures *volume* in the historical volatility equation may explain a substantial portion of the “U”-shaped pattern appearing in three of these markets. On the other hand, the right-hand side variables included in this model may not account for all of this intraday pattern.

This evidence points to the need for employing separate dummy variables for each intraday interval in both equations of the model in (5). The joint hypothesis that each equation’s intercept for the first time interval is identical to that for the other nine intervals on a given day is tested also. The coefficients of these dummy variables should display any intraday patterns remaining in the data, *after* accounting for the intraday variation in σ_{ht} and σ_{it} due to the other influences in the model. The joint *F*-test should then reveal the statistical significance of any such intraday patterns *remaining after* the inclusion of the model’s other parameters. It is crucial to use this dummy variable approach to account for any such potential intraday patterns remaining in σ_{ht} or σ_{it} , so that one can sensibly estimate potential leadlag relationships across days in the model. This problem is analogous to the need to “seasonally adjust” when analyzing any data that display seasonality.

A priori, the dummy coefficients of the model in (5) are expected to display a much less distinct pattern than the patterns presented in Panel A of Figures 1–4. This is born out in scrutiny of Panel B in Figures 3 and 4, which reveal no distinct pattern in historical volatility after controlling for the other influences in the model. Panel B of Figures 1 and 2 similarly reveals a less notable pattern in historical volatility given the model’s other parameters. However, the plots and joint *F* tests indicate that the patterns for these two markets are still distinct. For both S&P 500 futures and deutsche mark futures, this pattern suggests a relatively low level of historical volatility at the beginning of the typical trading day, which increases throughout the day, after accounting for volume changes and other intraday influences in the model.

CONCLUSIONS

This study addresses the question of whether intraday movements in implied volatility lead or follow movements in historical volatility of the

underlying instrument's return, after accounting for changes in trading volume. Put another way, the analysis is designed to assess: (a) whether option traders are able to make reliable predictions about forthcoming movements in the volatility of underlying returns; and/or (b) whether option traders respond to recent changes in historical volatility and adjust option prices accordingly. Futures contracts on the S&P 500 index, deutsche mark, Eurodollars, and live cattle are examined along with their associated options. Matched time series in historical and implied volatility are constructed from transactions data, using time intervals of two different lengths: one interval per day and ten intervals per day.

Results reveal a strong association between trading volume in each futures contract and historical volatility in the futures return, for both time intervals investigated. This result is consistent with previous work [Ekman (1992); Kawaller, Koch, and Koch (1990); Merrick (1987)], and it underscores the need to control for changes in volume when attempting to uncover the lead/lag relationship between historical and implied volatility. No stable analogous relationship is evidenced between volume and implied volatility in option prices. This result may be attributable to the forward-looking nature of implied volatility versus the backward-looking nature of historical volatility, or the possibility of more noise inherent in implied volatilities.

Concerning the issue of leads and lags between implied and historical volatility, the analysis shows one area of consistency. When using data measured over ten intervals per day, implied volatility never leads historical volatility. Thus, on a very short-term basis (within the course of the day), option traders seem unable to forecast impending volatility changes with any degree of reliability. This conclusion, however, does not generally hold when the time interval is expanded to one-day periods, where an improved forecasting capability for option traders is found in some markets.

When considering the possibility that historical volatility leads implied volatility, the results are inconsistent across markets. In this case, however, mixed results are shown for both time intervals. Thus, option traders may respond to recent changes in historical volatility; but if they do so, the mechanism by which they respond does not appear to be systematic across all markets.

The inconsistencies shown in this study suggest it may be inappropriate to generalize about these phenomena across markets; and that conclusions based on time intervals of one length may not apply for alternative intervals based on different lengths of time. It is also

important to emphasize that findings in this study may be reflective of conditions pertaining to the fourth quarter of 1988 that may not be robust over time.

The final contribution of the analysis deals with intraday patterns in volatility. Significant intraday patterns of historical volatility are observed, but no analogous robust patterns are detected with implied volatility. Moreover, the pattern associated with historical volatility reveals some variation across markets. While opening and closing times are consistently associated with higher historical volatility, it should be noted that a midday (Chicago time) historical volatility "spike" occurs in markets where substantial trading volume emanates from London.

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