Estimating Multiperiod Hedge Ratios in Cointegrated Markets

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INTRODUCTION

The concept of cointegration was introduced in Granger (1981) and fully discussed in Engle and Granger (1987). Cointegration between two random variables is a direct consequence of some prevailing long-run equilibrium relationship. As such, cointegration tests are useful in validating the efficient markets hypothesis. Most empirical results support cointegration between spot and futures markets. For examples, Lai and Lai (1991) conclude that, in five major foreign exchange markets, 1-month forward rates and spot rates at the maturity date are cointegrated; Chowdhury (1991) finds cointegration between 3-month futures prices and spot prices at the maturity date for copper, lead, tin, and zinc traded at the London Metal Exchange. Both articles claim cointegration on the basis of convergence between spot and futures prices at the maturity date.

A different approach is taken in Quan (1992). Using price data for the same date, Quan finds cointegration between spot and short-term (1-month and 3-month) futures prices in crude oil markets but not between spot and long-term (6-month and 9-month) futures prices. Quan's results are not surprising. Assuming sufficient arbitrage, the difference between spot and futures prices reflects the carrying charge, which is proportional to the number of days to the maturity of the futures contract, abstracting from the impact of risk premia, transaction costs, and so forth. For short-term contracts and assuming low interest rates, the above time-varying relationship may be well captured by an alternative time-invariant relationship.[1] In the case of long-term contracts, with the

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[1]The approximation argument may not hold for every possible case. For example. Schroeder and Goodwin (1991) investigate the relationship between spot and futures prices for nearby contracts in live hog markets. They conclude the two price series are not cointegrated. Similarly, if the maturity of the futures contract is fixed, the approximation argument holds for short periods. When extending to long periods, cointegration is not expected between spot and futures prices. Baillie and Myer (1991) examined and validated no cointegration for several commodity markets. On the other hand, when the carrying charge is near zero, due to the tradeoff between convenience yield and storage cost, cointegration is implied by the no arbitrage condition and holds for any length of sample period.

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number of days to maturity being more variable, the approximation is not expected to perform well.

In this article, optimal multiperiod hedge ratios for major foreign exchange and stock indices are derived. Since the ratios are conditioned by the stochastic processes for spot and futures prices, specifications of such processes are necessary for determining the ratios. Only 3-month ahead futures contracts are considered. Therefore, the approximation procedure is employed in an attempt to search for cointegration. Similar results to Quan (1992) are obtained from which estimates of multiperiod hedge ratios are derived.

Hedge ratios are usually analyzed within the conventional single time-period framework. With daily settlement it is natural to set the period (i.e., unit) of time equal to one day. Realism suggests the representative hedger's planning horizon covers multiple periods. Thus, to more fully consider both the usual hedging practice and the influence of daily resettlement, a multiperiod arrangement seems more appropriate. Following Howard and D'Antonio (1991), the decision of a hedger with a given planning horizon is examined. Solutions are characterized by a recursive equation with parameters embodied in the stochastic processes of spot and futures prices. When changes in spot and futures prices remain constant (in addition to disturbance terms), multiperiod hedge ratios are identical to the corresponding single-period hedge ratios [Lien (1992b)]. However, since spot and futures prices for both foreign currency and stock index markets appear to be cointegrated, an error correction model is more appropriate. In such cases, changes in either price are affected by previous spot and futures prices, and multiperiod consideration becomes necessary. The model is estimated with one autoregressive lag. From the estimation results, multiperiod hedge ratios for each market are generated. Finally, the optimal multiperiod spreading strategy is considered. In terms of risk reduction, the analysis indicates spreading outperforms hedging.

The remaining sections of this article are organized as follows. The next section formulates the multiperiod decision problem and characterizes the solution. Data description and cointegration tests for each market are then provided and estimation of multiperiod hedge ratios is performed. By introducing additional assumptions, we illustrate multiperiod spread ratios and, in the final section, offer conclusions.

OPTIMAL HEDGE RATIOS

Consider a hedger who has a planning horizon of T periods. At time t, $t = 0, 1, 2, \ldots, T$, the hedger takes a spot position S_t and, subsequently, lifts at time $t + 1$. It is assumed S_t is exogenously given and known at $t = 0$. This assumption of perfect foresight on the future spot positions S_t at the initial period can be relaxed without changing the conclusions.[2] For a given spot position, the hedger may choose to participate in the futures market upon taking the futures position, $F_t = b_t S_t$, at time t, where b_t denotes the t-period hedge ratio. The hedging decisions are made sequentially so that b_t is chosen with all of the information available at time t, of which price histories in both markets are basic elements. At the end of the planning horizon, the hedger's wealth is

$$W_T = W_o + \sum_{t=0}^{T-1} S_t[P_{t+1} - P_t + b_t(f_{t+1} - f_t)]$$
$$= W_o + \sum_{t=0}^{T-1} S_t[\Delta P_{t+1} + b_t \Delta f_{t+1}]$$

[2]In this case, any future spot position, S_t, is replaced by its expectation, $E(S_t)$. This substitution does not affect any of the results.

where W_o denotes the initial wealth; P_t and f_t are spot and futures prices at time t, and Δ denotes the difference operator, respectively. Assuming the case of a pure hedger, the appropriate objective is to minimize the variance of the end-period wealth, var(W_T), by sequentially choosing optimal hedge ratios. This problem may be solved backwards using the dynamic programming technique.

Specifically, at time $T - 1$, $\mathrm{var}(W_T) = \mathrm{var}[S_{T-1}(\Delta P_T + b_{T-1}\Delta f_T)]$. To minimize var($W_T$), the optimal hedge ratio is

$$B_{T-1} = -\mathrm{cov}(\Delta P_T, \Delta f_T)/\mathrm{var}(\Delta f_T) \tag{1}$$

where cov($\cdot,\cdot$) and var($\cdot$) are, respectively, covariance and variance operators. The solution corresponds to the traditional one-period solution [see, for example, Ederington (1979)] which is also called the myopic hedge ratio. Consider time period, $t = k$, at which optimal hedge ratios for subsequent periods, $B_{k+1}, B_{k+2}, \ldots, B_{T-1}$, are available. The hedger's problem at $t = k$ is to minimize:

$$\begin{aligned}\mathrm{var}(W_T) &= \mathrm{var}\left(S_k(\Delta P_{k+1} + b_k \Delta f_{k+1}) + \sum_{t=k+1}^{T-1} S_t(\Delta P_{t+1} + B_t \Delta f_{t+1})\right)\\ &= b_k^2 S_k^2 \,\mathrm{var}(\Delta f_{k+1}) + 2b_k S_k^2 \,\mathrm{cov}(\Delta P_{k+1}, \Delta f_{k+1})\\ &\quad + 2b_k S_k \sum_{t=k+1}^{T-1} S_t \,\mathrm{cov}(\Delta f_{k+1}, \Delta P_{t+1} + B_t \Delta f_{t+1}) + R_k\end{aligned}$$

where R_k is the residual term, independent of b_k. Upon taking the partial derivative with respect to b_k, the following recursive relationship is derived:

$$B_k = -\frac{\mathrm{cov}(\Delta P_{k+1}, \Delta f_{k+1})}{\mathrm{var}(\Delta f_{k+1})} - \sum_{t=k+1}^{T-1} \left(\frac{S_t}{S_k}\right) \frac{\mathrm{cov}(\Delta f_{k+1}, \Delta P_{t+1} + B_t \Delta f_{t+1})}{\mathrm{var}(\Delta f_{k+1})} \tag{2}$$

Note that B_t is chosen after Δf_{k+1} is known when $t \geq k + 1$. Clearly, it is possible that B_t and Δf_{k+1} are correlated, as in the case when spot and futures prices follow a bivariate ARCH (Autoregressive Conditional Heteroscedastic) process.[3] In this article, it is assumed cash and futures price series are cointegrated such that the second-order moments of the two price series are invariant to price histories. As a result, B_t is a constant term independent of the past prices.

As shown in the following section, cointegration prevails in major foreign currency markets and stock index markets. In this situation, assuming the cointegrating coefficient equal to -1, it is possible to characterize the stochastic processes of spot and futures prices as follows.

$$\Delta P_t = \alpha_1 \theta_{t-1} + \beta_1 \Delta P_{t-1} + \gamma_1 \Delta f_{t-1} + \varepsilon_{1t} \tag{3a}$$

$$\Delta f_t = \alpha_2 \theta_{t-1} + \beta_2 \Delta P_{t-1} + \gamma_2 \Delta f_{t-1} + \varepsilon_{2t} \tag{3b}$$

The difference between spot and futures prices is given by $\theta_t = f_t - P_t, \varepsilon_{1t}$ and ε_{2t} denote joint white noise. From eqs. (3a) and (3b):

$$\theta_t = (1 + \alpha_2 - \alpha_1)\theta_{t-1} + (\beta_2 - \beta_1)\Delta P_{t-1} + (\gamma_2 - \gamma_1)\Delta f_{t-1} + \varepsilon_{3t} \tag{3c}$$

[3]When the error terms are modeled by a bivariate GARCH process, the determination of multiperiod hedge ratios becomes complicated. Lien (1992b) provides some preliminary results.

where $\varepsilon_{3t} = \varepsilon_{2t} - \varepsilon_{1t}$. Upon adopting a vector representation: $X_t = (\Delta P_t \Delta f_t \theta_t)'$ and $\varepsilon_t = (\varepsilon_{1t}\varepsilon_{2t}\varepsilon_{3t})'$, then

$$X_t = AX_{t-1} + \varepsilon_t; \tag{4a}$$

$$\text{where } A = \begin{pmatrix} \beta_1 & \gamma_1 & \alpha_1 \\ \beta_2 & \gamma_2 & \alpha_2 \\ \beta_2 - \beta_1 & \gamma_2 - \gamma_1 & 1 + \alpha_2 - \alpha_1 \end{pmatrix} \tag{4b}$$

Consequently, when $t \geq k + 1$,

$$\begin{aligned} \text{cov}(\Delta f_{k+1}, \Delta P_{t+1} + B_t \Delta f_{t+1}) &= (0\ 1\ 0)\ \text{cov}(X_{k+1}, X_{t+1})\,(1\ B_t\ 0)' \\ &= (0\ 1\ 0)\ \text{var}(\varepsilon)\,(A^{t-k})'(1\ B_t\ 0)' \end{aligned}$$

By substituting the relationship into eq. (2), it holds that

$$B_k = -\sigma - \sum_{t=k+1}^{T-1} \left(\frac{S_t}{S_k}\right)(0\ 1\ 0)\ \text{var}(\varepsilon)\,(A')^{t-k}(1\ B_t\ 0)'\ \text{var}^{-1}(\varepsilon_2) \tag{5}$$

with $\sigma = \text{cov}(\varepsilon_1, \varepsilon_2)\ \text{var}^{-1}(\varepsilon_2)$ when $k < T - 1$, and $B_{T-1} = -\sigma$ at $k = T - 1$. For the empirical studies undertaken in this study, it is assumed $S_t/S_k = 1$ for every t and k. The assumption is imposed for illustration. A more general approach is to choose $S_t/S_k = (1 + \theta)^{t-k}$ for some θ whenever $t \geq k$. The specific case considered here follows Howard and D'Antonio (1991) by assuming $\theta = 0$. This assumption implies the hedger's spot position remains constant over the planning horizon. When the horizon is short, it may be well justified. Another example that fits this assumption is anticipatory hedging. In such cases, the hedger will undertake a spot transaction at the end of the planning horizon in a certain amount. Thus, $S_t = S_k$ for every t, $k \leq T$. From the sample data, estimates of var(ε) and A are obtained and the multiperiod hedge ratios are calculated accordingly.

DATA DESCRIPTION AND PRELIMINARY TESTS

The major foreign currency and stock index futures markets are considered in this article. The former consists of the British pound, Canadian dollar, German mark, Japanese yen, and Swiss franc while the latter includes the MMI (Major Market Index), NYSE (New York Stock Exchange), and S&P 500 (Standard and Poor 500). Weekly spot prices and futures prices for the nearby contract are employed. More specifically, Tuesday closing prices are used; in case of missing data, Monday or Wednesday closing prices are substituted. For the currency futures, the sample period extends from March 4, 1980, through December 27, 1988, for a total of 459 observations. The stock index futures have a shorter sample period, from January 1, 1984, to December 27, 1988, for a total of 260 observations.

For each contract, statistical tests are conducted to examine cointegration. The procedure for establishing the existence of cointegration begins with the Augmented Dickey–Fuller (ADF) test for a unit root in spot and futures prices. The ADF test determines whether a time series, y_t, contains a unit root. This test hinges on the realization of the ratio of c to its standard deviation, or the so-called τ-statistics in the following regression:

$$\Delta y_t = d_o + cy_t + \sum_{j=1}^{q} d_j\,\Delta y_{t-j} + e_t \tag{6}$$

The number of lagged terms, q, is first chosen to be 15, and is gradually reduced until the t-statistics for d_q is significant. The ADF test is applied to each of the price series, being 16 in total, and results show each of them contains a unit root. A similar procedure is applied to the first-difference-series. For each, the stationarity hypothesis is not rejected. In other words, each of the spot or futures price series considered in this article contains exactly one unit root. Table I shows the test statistics.

For each pair of spot and futures prices, the following cointegration test is performed. First, the spot price is regressed on the futures price. The ADF test is then applied to the resulting regression residuals to examine the stationarity property. Mathematically, consider the τ-statistics of c from the regression model:

$$P_t = m_o + m_1 f_t + e_t \tag{7a}$$

$$\Delta e_t = d_o + c e_{t-1} + \sum_{j=1}^{q} d_j \Delta e_{t-j} + u_t \tag{7b}$$

Table I
RESULTS OF TESTS FOR UNIT ROOT

Market	ADF Statistics for the Series[a]	ADF Statistics for the Differenced Series[a]
British pound		
spot	−1.51	−22.15[b]
futures	−1.52	−22.70[b]
Canadian dollar		
spot	−1.22	−10.31[b]
futures	−1.36	−22.47[b]
German mark		
spot	−1.18	−6.02[b]
futures	−1.06	−11.18[b]
Japanese yen		
spot	−0.54	−8.07[b]
futures	0.15	−10.74[b]
Swiss franc		
spot	−1.03	−6.20[b]
futures	−1.02	−6.17[b]
MMI		
spot	−1.31	−5.84[b]
futures	−1.33	−5.86[b]
NYSE		
spot	−1.39	−6.31[b]
futures	−1.38	−6.06[b]
S&P 500		
spot	−1.36	−6.12[b]
futures	−1.32	−6.61[b]

[a]Fuller (1976, p. 373) provides the following critical values: −3.46, −2.88, and −2.57 for 1%, 5%, and 10% significance levels, respectively.

[b]The null hypothesis of a unit root is rejected at 1% level.

The results are shown in Table II. Except for the Swiss franc, the cointegration hypothesis cannot be rejected at 10% significance level. In fact, strong evidence of cointegration is established for six out of eight contracts, i.e., six out of eight pairs of markets.

An alternative test for cointegration relies upon Durbin–Watson (DW) statistic from eq. (7a). More precisely, given that each price series contains a unit root resulting zero DW statistic. Table II presents the empirical results. In each case, the null hypothesis of no cointegration is rejected at 1% significance level.

Besides the above 2 two-step test procedures, cointegration can be tested directly from the corresponding error correction representation. Kremers, Ericsson, and Dolado (1992) show that the test statistic based upon the estimation of an error correction model is asymptotically normally distributed under the null hypothesis of no cointegration. This approach is generally more powerful than the ADF test; however, it requires an assumption regarding the value of the cointegrating coefficient. Applying arbitrage

Table II
RESULTS OF COINTEGRATION TESTS

Market	Cointegration Regression	DW[a]	ADF Statistics[b]
British pound	$P_t = 0.013 + 0.993 f_t$ (4.06)[d] (509.7)	1.43[c]	−8.59[c]
Canadian dollar	$P_t = 0.090 + 0.990 f_t$ (2.44) (202.6)	1.84[c]	−13.22[c]
German mark	$P_t = 0.000 + 0.996 f_t$ (−0.33) (619.9)	1.68[c]	−4.16[c]
Japanese yen	$P_t = 0.000 + 0.995 f_t$ (0.32) (632.0)	1.74[c]	−3.84[c]
Swiss franc	$P_t = -0.01 + 0.999 f_t$ (−1.49) (624.6)	1.21[c]	−2.77
MMI	$P_t = -1.508 + 1.000 f_t$ (−1.24) (877.7)	1.24[c]	−3.95[c]
NYSE	$P_t = -1.006 + 1.003 f_t$ (−4.32) (572.6)	1.44[c]	−2.90[e]
S&P 500	$P_t = -1.419 + 1.001 f_t$ (−2.39) (393.0)	1.41[c]	−3.70[e]

[a]Engle and Granger (1987, Table II) provide the following critical values: 0.511, 0.386, and 0.322 for 1%, 5%, and 10% significance levels, respectively.

[b]Engle and Granger (1987, Table II) provide the following critical values: −3.77, −3.17, and −2.84 for 1%, 5%, and 10% significance levels, respectively.

[c]The null hypothesis of no cointegration is rejected at 1% significance level.

[d]The number in the parenthesis is the corresponding t-value.

[e]The null hypothesis of no cointegration is rejected at 10% significance level.

conditions, the error correction model is constructed as[4]:

$$\Delta P_t = \alpha_1 \theta_{t-1} + \text{lagged}(\Delta P_t, \Delta f_t) + d(B)\varepsilon_{1t}; \tag{8a}$$

$$\Delta f_t = \alpha_2 \theta_{t-1} + \text{lagged}(\Delta P_t, \Delta f_t) + d(B)\varepsilon_{2t} \tag{8b}$$

Results from empirical studies indicate only a single-period lag for the variables, ΔP_t and Δf_t, i.e., ΔP_{t-1} and Δf_{t-1} need to be considered. Also, it is assumed $d(B) = 1$ so that there is no moving average disturbance term included in the estimation. The final equations estimated are reduced to eqs. (3a) and (3b). Estimation results are summarized in Tables III and IV.

Table III
ESTIMATION OF THE COINTEGRATION SYSTEM: FOREIGN CURRENCY MARKETS

			Coefficients for	
Markets		$f_{t-1} - P_{t-1}$	ΔP_{t-1}	Δf_{t-1}
British pound	ΔP_t	−0.2030	−0.2872[a]	0.2490[a]
		(1.6360)[b]	(2.5470)	(2.3738)
	Δf_t	−0.7625[c]	−0.0970	0.0511
		(6.0765)	(0.8507)	(0.4821)
Canadian dollar	ΔP_t	0.5367[c]	−0.2012[c]	0.1200
		(6.6111)	(3.1443)	(1.4882)
	Δf_t	−0.1473[a]	−0.0258	−0.0192
		(2.2613)	(0.5029)	(0.2965)
German mark	ΔP_t	0.1784	−0.1983	0.2389
		(1.0514)	(1.2663)	(1.5234)
	Δf_t	−0.3188[d]	−0.0183	0.0565
		(1.8822)	(0.1168)	(0.3611)
Japanese yen	ΔP_t	0.2870[d]	−0.2735[d]	0.1967
		(1.7143)	(1.9711)	(1.3985)
	Δf_t	−0.3170[d]	−0.2705	0.1967
		(1.9118)	(0.5618)	(0.1835)
Swiss franc	ΔP_t	−0.0053	−0.2648	0.2971
		(0.0341)	(1.4222)	(1.6103)
	Δf_t	−0.2902[d]	−0.0347	0.0742
		(1.8638)	(0.1851)	(0.3999)

[a]The coefficient differs from zero at 5% significance level.
[b]The number in the parenthesis is the corresponding t-statistics.
[c]The coefficient differs from zero at 1% significance level.
[d]The coefficient differs from zero at 10% significance level.

[4]In other words, the cointegrating coefficient is assumed to be −1. An alternative method makes use of the coefficient derived from the two step test procedure, i.e., the coefficient of f_t in the cointegration regression equation. Results in Table II show support for choosing $\theta_t = f_t - P_t$ as the cointegrating variable.

Table IV
ESTIMATION OF THE COINTEGRATED SYSTEM: STOCK INDEX MARKETS

Markets		Coefficients for $f_{t-1} - P_{t-1}$	ΔP_{t-1}	Δf_{t-1}
MMI	ΔP_t	−0.9323	0.0059	0.0003
		(1.2467)[a]	(0.0081)	(0.0005)
	Δf_t	−1.5250[b]	0.1466	−0.0959
		(1.9985)	(0.1981)	(0.1325)
NYSE	ΔP_t	−0.3277	−0.1795	0.1223
		(0.9235)	(0.4430)	(0.3205)
	Δf_t	−0.6355[c]	0.2104	0.2227
		(1.7366)	(0.5058)	(0.5572)
S&P 500	ΔP_t	−0.4418[c]	−0.5742[b]	0.5581[b]
		(1.6544)	(1.9853)	(2.2143)
	Δf_t	−1.0313[d]	−0.5127	0.5214[c]
		(3.2641)	(1.4983)	(1.7486)

[a]The number in the parenthesis is the corresponding t-statistics.
[b]The coefficient differs from zero at 5% significance level.
[c]The coefficient differs from zero at 10% significance level.
[d]The coefficient differs from zero at 1% significance level.

With the exception of S&P 500 contract, the lagged term is not significant in the futures price equations for every market. They are somewhat significant in the spot price equation for the British pound, Canadian dollar, Japanese yen, and S&P 500 markets. The coefficient associated with the deviation from long-run equilibrium, θ_{t-1}, is negative and significant in the futures price equation for each of the markets, supporting cointegration between spot and futures prices. This implies the futures price is responsive to market conditions by continuously adjusting to eliminate arbitrage opportunities. On the other hand, with the exception of the Canadian dollar, Japanese yen, and S&P 500 markets, the spot price does not respond to the deviation from the long-run equilibrium. In the Canadian dollar and Japanese yen markets, the spot price adjusts to eliminate arbitrage opportunities.[5] As suggested by Garbade and Silber (1983), the futures market is more likely to be a "pure satellite."

In sum, the results of the three different test procedures provide evidence for cointegration between spot and futures prices in stock index markets and foreign exchange markets, albeit the support for Swiss franc is a bit weak. In the latter market where the other two tests do support cointegration, the ADF test fails to detect cointegration. The result may be explained by the low power of the ADF test.

ESTIMATING MULTIPERIOD HEDGE RATIOS

It is now possible to apply the estimation results of the error correction model to derive the multiperiod hedge ratios for each of the contracts. Assume that the spot position is

[5]A joint test for $\alpha_1 = \alpha_2 = 0$ is conducted on each pair of markets. In each case, the null hypothesis is rejected at 1% significance level.

Table V
MULTIPERIOD HEDGE RATIOS: FOREIGN CURRENCY MARKETS

Time	British Pound	Canadian Dollar	German Mark	Japanese Yen	Swiss Franc
0	−0.8448	−0.8806	−0.9639	−0.9770	−0.9755
1	−1.0254	−0.9822	−0.9998	−0.9809	−0.9881
2	−0.8644	−0.8854	−0.9626	−0.9767	−0.9738
3	−1.0092	−0.9762	−1.0003	−0.9800	−0.9870
4	−0.8761	−0.8882	−0.9607	−0.9755	−0.9712
5	−0.9933	−0.9676	−0.9998	−0.9776	−0.9848
6	−0.8832	−0.8850	−0.9579	−0.9706	−0.9675
7	−0.9703	−0.9467	−0.9954	−0.9713	−0.9806
8	−0.8922	−0.8609	−0.9578	−0.9516	−0.9633
9	−0.8942	−0.8714	−0.9635	−0.9603	−0.9680

fixed: $S_t = S_k$ for every t and k. For illustration, assume $T = 10$. Multiperiod hedge ratios are calculated and shown in Tables V and VI. The last row of both tables (i.e., $t = 9$) corresponds to the myopic hedge ratio. In general, multiperiod hedge ratios (in absolute value) exhibit a common cyclic pattern in all markets: it first increases, then decreases, and then repeats the up and down cycles.[6] For example, the hedge ratio for British pound over a planning horizon of 10 weeks is 0.8448 for the first week. It then increases to 1.0254, followed by a decrease to 0.8644, and then raises to 1.0092. The pattern continues to the end of the planning horizon. This result can be explained by the existence of complex roots for eq. (4b). If the single lagged terms are discarded from eqs. (3a) and (3b) (which seems reasonable for MMI, NYSE, German mark and Swiss franc markets) the multiperiod hedge ratios decrease over time. A similar result is established in Howard and D'Antonio (1991). For the present case, since the myopic hedge ratio pertains to the odd time period ($t = 9$), consideration of the ratios for the odd time periods ($t = 1, 3, 5, 7, 9$) leads to the same conclusion. While applying the myopic ratio uniformly to multiperiod problems induces hedging ineffectiveness, the extent of ineffectiveness varies across markets. It is relatively more ineffective in the British pound, Canadian dollar, and S&P 500 markets. The solution obtained via the single-period method is comparable to the multiperiod solution for the other markets.

OPTIMAL SPREAD RATIOS: SOME EXAMPLES

When the transaction costs are rather small, for risk reduction purposes, it is always beneficial for the hedger to participate in as many futures markets as possible. In fact, simultaneous trading in multiple futures markets, i.e., spreading, generally incurs less cost. In this section, it is shown that optimal spreading, measured by risk reduction, may be of greater advantage than optimal hedging. Consider the following model:

$$\Delta f_{1t} = \alpha_1 \theta_{t-1} + \alpha_2 \Phi_{t-1} + \varepsilon_{1t} \tag{9a}$$

$$\Delta f_{2t} = \beta_1 \theta_{t-1} + \beta_2 \Phi_{t-1} + \varepsilon_{2t} \tag{9b}$$

$$\Delta P_t = \gamma_1 \theta_{t-1} + \gamma_2 \Phi_{t-1} + \varepsilon_{3t} \tag{9c}$$

where f_{1t} and f_{2t} are the prices for two different futures contracts at time t; $\theta_{t-1} = f_{1,t-1} - P_{t-1}$, and $\Phi_{t-1} = f_{2,t-1} - P_{t-1}$ are the deviations from the long-run equilibrium in markets 1 and 2, respectively. The spot position is given. An underlying assumption is that the two futures prices follow a long-run equilibrium relationship.

Since the equation system (9a)–(9c) does not allow for lagged price terms, consider the combination of German mark and Swiss franc for the case of foreign currency markets, and the combination of MMI and NYSE for stock index markets.[7] For the former combination, assume the hedger holds Swiss francs only. The optimal spreading strategy is derived, assuming the hedger trades in both German mark and Swiss franc futures markets. The solution for the cointegration system described by eqs. (3a)–(3c) is characterized in Lien (1992a). Specifically, let H_{ik} denote the optimal spread ratio in the ith futures market at time k, where $i = 1, 2, \ldots, k \leq T - 1$. Then

$$\begin{pmatrix} H_{1k} \\ H_{2k} \end{pmatrix} = D^{-1}(R^{T-k-1} - I)\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} + \sum_{i=0}^{T-k-2} D^{-1}R^{i}H + D^{-1}R^{T-k-1}D\begin{pmatrix} H_{1k-1} \\ H_{2k-1} \end{pmatrix} \tag{10}$$

where

$$D = \begin{pmatrix} \alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} \mathrm{cov}(\varepsilon_1, \varepsilon_1 - \varepsilon_3) & \mathrm{cov}(\varepsilon_1, \varepsilon_2 - \varepsilon_3) \\ \mathrm{cov}(\varepsilon_2, \varepsilon_1 - \varepsilon_3) & \mathrm{cov}(\varepsilon_1, \varepsilon_2 - \varepsilon_3) \end{pmatrix}$$

and

$$R = (S_{k+1}/S_k) \cdot D \cdot \Sigma_{12}^{-1}\Omega \cdot (A' \cdot \Omega^{-1} \cdot \Sigma_{12} \cdot D^{-1} - I)$$

$$H = (I - (S_{k+1}/S_k) \cdot D \cdot \Sigma_{12}^{-1}\Omega \cdot A' \cdot \Omega^{-1} \cdot \Sigma_{12}^{-1}D) \cdot (\gamma_1, \gamma_2)'$$
$$- D \cdot \Sigma_{12}^{-1}(I - \Omega \cdot A' \cdot \Omega^{-1}) \cdot (\mathrm{cov}(\varepsilon_1, \varepsilon_3), \mathrm{cov}(\varepsilon_2, \varepsilon_3))'$$

with

$$\Sigma_{12} = \begin{pmatrix} \mathrm{var}(\varepsilon_1) & \mathrm{cov}(\varepsilon_1, \varepsilon_2) \\ \mathrm{cov}(\varepsilon_2, \varepsilon_1) & \mathrm{var}(\varepsilon_2) \end{pmatrix}$$

$$A = \begin{pmatrix} 1 + \alpha_1 - \gamma_1 & \alpha_2 - \gamma_2 \\ \beta_1 - \gamma_1 & 1 + \beta_2 - \gamma_2 \end{pmatrix}$$

[7]From Table III and IV, all of the lagged terms are statistically insignificant in these four markets.

Table VI
MULTIPERIOD HEDGE RATIOS: STOCK INDEX MARKETS

Time	MMI	NYSE	S&P 500
1	−1.0015	−0.9823	−0.8721
2	−0.9677	−0.9515	−0.9340
3	−1.0041	−0.9789	−0.8704
4	−1.0061	−0.9736	−0.8666
5	−0.9580	−0.9459	−0.9273
6	−1.0037	−0.9645	−0.9273
7	−0.9491	−0.9426	−0.9099
8	−0.9719	−0.9449	−0.8320
9	−0.9729	−0.9485	−0.8379

[6]Two alternative assumptions on the spot positions are considered: (i) $S_t = (1 + 0.05)^t$; and (ii) $S_t = (1 - 0.005)^t$ for $t = 0, 1, \ldots, T$. In both cases, the estimated multiperiod hedge ratios (in absolute value) exhibit a common cyclic pattern in all markets.

Estimators for the foreign currency futures combination are given by

$$\Delta f_{1t} = \underset{(1.4862)}{-0.0105\theta_{t-1}} - \underset{(2.2144)}{0.3036\Phi_{t-1}}$$

$$\Delta f_{2t} = \underset{(1.6839)}{-0.0150\theta_{t-1}} - \underset{(2.6115)}{0.4499\Phi_{t-1}}$$

$$\Delta P_t = \underset{(0.2759)}{0.0025\theta_{t-1}} + \underset{(0.6150)}{0.1059\Phi_{t-1}}$$

where f_{1t} and f_{2t} denote futures prices for German mark and Swiss franc, respectively; P_t is the spot rate of Swiss franc. The multiperiod spread ratios, calculated from the above model, are summarized in Table VII. Contrary to the case of hedging, the ratio for German mark and the ratio for Swiss franc both increase until the end of the planning horizon. The myopic solution (the ratios correspond to $t = 9$) differs considerably from the dynamic solution.

Estimators for the stock index futures combinations are given by the following equations:

$$\Delta f_{1t} = \underset{(1.4897)}{-0.0074\theta_{t-1}} - \underset{(1.9491)}{2.2055\Phi_{t-1}}$$

$$\Delta f_{2t} = \underset{(1.8116)}{-0.0034\theta_{t-1}} - \underset{(2.7911)}{1.1969\Phi_{t-1}}$$

$$\Delta P_t = \underset{(1.0114)}{0.0018\theta_{t-1}} - \underset{(1.3078)}{0.5386\Phi_{t-1}}$$

where f_{1t} and f_{2t} denote the MMI and NYSE futures prices, respectively; P_t is the spot price corresponding to the portfolio of the NYSE index. The resulting multiperiod spread ratios are shown in Table VII. Similar conclusions to the foreign currency futures are derived. Nonetheless, the myopic ratio is much smaller than the dynamic ratio for the NYSE futures market. The hedger should buy MMI futures in the early periods instead of selling as suggested by the myopic ratio.

Table VII
OPTIMAL SPREAD RATIOS

	Foreign Currency Markets		Stock Index Markets	
Time	**German Mark**	**Swiss Franc**	**MMI**	**NYSE**
0	−0.0306	−0.9694	0.0024	−1.0023
1	−0.0307	−0.9693	0.0023	−1.0021
2	−0.0309	−0.9692	0.0022	−1.0016
3	−0.0313	−0.9687	0.0019	−1.0007
4	−0.0323	−0.9678	0.0012	−0.9985
5	−0.0345	−0.9657	−0.0002	−0.9937
6	−0.0395	−0.9610	−0.0034	−0.9832
7	−0.0507	−0.9503	−0.0105	−0.9600
8	−0.0758	−0.9264	−0.0260	−0.9089
9	−0.1325	−0.8725	−0.0603	−0.7963

CONCLUSION

In this article, multiperiod hedge ratios for major foreign currency and stock index markets are provided. A general recursive equation is derived. For estimation purposes, stochastic processes to characterize spot and futures prices need to be established. While alternative approaches [such as Garbade and Silber (1983)] are possible, a cointegration process using an approximation argument coupled with an efficient market hypothesis is assumed. For most markets considered herein, the existence of cointegration between spot and futures prices is confirmed. An error correction model is then applied in the estimation procedure. In general, the model presented in this article may be considered as an extension of the well-known GS model, extended by the addition of lagged price terms. Although the results obtained here are based on price levels, similar conclusions are derived when considering the logarithms of spot and futures prices.

The optimal multiperiod hedge ratios exhibit a cyclic pattern with a tendency for the amplitude of the cycles to decrease. Applying the myopic ratio uniformly to each period is clearly ineffective, but the loss of the effectiveness varies in different markets. Finally, two examples are used to illustrate the possibility of spreading among markets. It is shown that hedging in a single market may be much less effective than the optimal spreading strategy.

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