A Transactions Data Analysis of Arbitrage between Index Options and Index Futures

Jae Ha Lee
Nandkumar Nayar

INTRODUCTION

Stock index options and futures rank among the most remarkable financial innovations the securities markets have witnessed. In particular, a futures contract based on the Standard and Poors (S&P) 500 Index was launched on April 21, 1982, by the Chicago Mercantile Exchange (CME). This instrument has evolved into the most successful and viable stock index futures contract. Due to the success of this contract, the CME introduced options on the S&P 500 futures on January 28, 1983. The popularity of the S&P 500 index soon spawned the birth of another derivative product. On July 1, 1983, the Chicago Board Options Exchange (CBOE) started trading in S&P 500 (SPX) index option contracts. The SPX options are European options and employ a cash settlement procedure at expiration.[1]

Noting that these derivative securities are all based on the same underlying asset, the S&P 500 stock index, it is to be expected that their prices be interrelated with one another and the underlying stock index itself. If prices of the different instruments and/or the index itself do not satisfy the relevant intermarket relationships, the relative mispricing should be instantaneously corrected given efficient markets and the high degree of sophistication of market participants. The price adjustments could be accomplished by a variety of cross-market strategies. These include arbitrage between index options and index futures, index futures and options on the index futures, index futures and the stocks comprising the index, index options and the index stocks, and so on. Research in finance has examined some of these arbitrages. For example, see Klemkosky and Lee (1991) for index futures

We are grateful to Louis Ederington, Scott Linn, and Lee Willinger for helpful discussions. This article was presented at the 1992 F.M.A. meetings and we thank Gary Gastineau and Tom Miller for their comments. The Center for Financial Studies at the University of Oklahoma provided financial support to acquire data used in this study.

[1]The SPX option contract is not the most heavily traded stock index option contract. The OEX option contract, based on the S&P 100 Index, trades more heavily.

Jae Ha Lee is an Assistant Professor of Finance at the University of Oklahoma.

Nandkumar Nayar is an Assistant Professor of Finance at the University of Oklahoma.

The Journal of Futures Markets, Vol. 13, No. 8, 889–902 (1993)

CCC 0270-7314/93/080889–14

and index stocks, and Evnine and Rudd (1985) for index options and index stocks.[2] Generally, buying (selling) pressure will drive the price of the underpriced (overpriced) instrument(s) up (down), thus wiping out any arbitrage possibilities. These arguments all imply that the stock index and the associated derivative products all trade in the same "market".

To address this issue of the existence of an integrated market, it is important to examine *other arbitrage combinations* employing the different instruments. It is in this area that this article makes a contribution. Specifically, the relationship between SPX options and S&P 500 index futures contracts is examined to determine the existence of profitable arbitrage opportunities.

This article is partially motivated by the concept that pricing relationships between the different derivative products should hold with greater strength if market makers themselves set prices taking into account the various interrelationships. This could apply particularly in the interrelationship between prices for the S&P 500 futures contract and SPX options. This is because most market makers in SPX option contracts are continually hedging their positions with the companion S&P 500 futures contract. In fact, market makers in the SPX options pit at the CBOE are known to be in contact on a real time basis with traders in the S&P 500 futures pit at the CME. Thus, this study also examines a trading strategy that is actually practiced by market makers.

In this article, the following questions are addressed: (a) From the standpoint of institutional investors who face relatively larger transactions costs than member firms of the CBOE and CME (including market makers for the SPX options), is arbitrage between SPX options and S&P 500 futures profitable? (b) If such profitable arbitrage opportunities exist, what is the frequency and the magnitude of profits? (c) How do the magnitude of profits and frequency of such arbitrage opportunities vary with different transactions costs? This sensitivity to transactions costs is important because different trader types face different levels of such costs.

The theoretical framework for the arbitrage strategies is first developed. Specifically, SPX call and put options with the same exercise price and S&P 500 futures are employed to establish an arbitrage position that yields a zero net cash flow position at expiration.[3] To prevent arbitrage profits, the arbitrage position should produce a nonpositive initial net cash flow. This framework is employed to develop no-arbitrage bounds on the futures price in terms of index option prices and transactions costs. In the empirical tests, violations of these no-arbitrage bounds are examined. To eliminate problems of nonsynchronous prices, intraday data on index options and index futures instruments are used. Employing intraday observations on the derivative securities lends greater credibility to the empirical tests than would end-of-day observations. "Ex-ante" tests for arbitrage are also conducted.[4] Specifically, after a mispricing occurs, it is assumed that transactions are undertaken to exploit the mispricing at the next possible set of prices. This procedure helps to further determine whether it is possible to realize profits from the proposed arbitrage strategies.

[2]Also see Brennan and Schwartz (1990); Brenner, Subrahmanyam, and Uno (1989); Kawaller (1987); Kawaller, Koch, and Koch (1987); MacKinlay and Ramaswamy (1988); Miller, Muthuswamy, and Whaley (1991); and Stoll and Whaley (1990).

[3]Since SPX options are European options, early exercise is not of any concern in this study's framework.

[4]See Klemkosky and Resnick (1980) for a further description and the logic underlying ex-ante tests. Rhee and Chang (1992) also use ex-ante tests in their study of derivative instruments on foreign currencies.

ARBITRAGE STRATEGIES

Strategies involving transactions in both the stock index futures and stock index options markets are developed. These strategies first assume the complete absence of transactions costs and that effects of marking-to-market in the futures markets are of no consequence. The analysis does, however, control for differential borrowing and lending rates. Following this, strategies to account for the effects of transactions costs are proposed. Two points in time are defined to be time t, the initial point, and time T, the expiration of the derivative securities. In this temporal framework, the following transaction sets are considered:

> **Transaction Set 1.** *Long position in stock index futures contract versus buying a futures contract indirectly via stock index options and borrowing/lending*

The relevant transactions are presented in Table I. In this framework, it is essential that the strike price, X represent the strike price of both the SPX call and put options. It is obvious that the cash flows from buying the futures contract can be replicated using index options.

Transaction sets 1.1 and 1.2 are expected at time t to produce the same cash flows of $(S_T - F_t) \times \$500$ with certainty, at expiration, T. Consequently, to prevent arbitrage, it must be the case that the initial time t cash flows associated with the said transactions must be the same. Specifically, the following condition must hold:

$$\left(-C_t + P_t + \frac{F_t - X}{(1 + r_{o1})^{T-t}}\right) \times \$500 = 0$$

Table I

DETAILS OF CASH FLOWS FOR TRANSACTION SET 1: LONG POSITION IN STOCK INDEX FUTURES CONTRACT VERSUS BUYING A FUTURES CONTRACT INDIRECTLY VIA STOCK INDEX OPTIONS AND TRANSACTING IN THE BORROWING/LENDING MARKETS

Cash Flow at t	**Cash Flow at T**	
Transaction set 1.1: Buy a futures contract at time t, at a known futures price of F_t which expires at time T.		
0	$(S_T - F_t) \times \$500$	
Transaction set 1.2: (1) buy 5 index call options at C_t, (2) sell 5 index put options at P_t, and (3) transact in the borrowing/lending markets such that an amount $(F_t - X) \times \$500$ is paid out at time T.[a]		
	if $S_T < X$	if $S_T > X$
$-5\ C_t \times \$100$	0	$5(S_T - X) \times \$100$
$+5\ P_t \times \$100$	$-5(X - S_T) \times \$100$	0
$\frac{(F_t - X) \times \$500}{(1+r_{o1})^{T-t}}$	$-(F_t - X) \times \$500$	$-(F_t - X) \times \$500$
$\left\{-C_t + P_t + \frac{(F_t - X)}{(1+r_{o1})^{T-t}}\right\} \times \$500$	$(S_T - F_t) \times \$500$	$(S_T - F_t) \times \$500$

[a]In what follows, if $F_t > X$, then r_{o1} becomes the borrowing rate, r^l. Else, it is the lending rate, r.

where: if $F_t > X$, then r_{o1} becomes the borrowing rate, r'. Else, it is the lending rate, r. This condition implies:

$$F_t = \{C_t - P_t\}(1 + r_{o1})^{T-t} + X \tag{1}$$

Next, consider a strategy that appears to be the reverse of the one above.

Transaction Set 2. *Short position in stock index futures contract versus selling a futures contract indirectly via stock index options and borrowing/lending*

The relevant transaction details appear in Table II.

To prevent arbitrage, the cash flows at time t for each of Transaction sets 2.1 and 2.2 must be identical which implies:

$$\left(C_t - P_t - \frac{F_t - X}{(1 + r_{o2})^{T-t}}\right) \times \$500 = 0$$

where: if $F_t > X$, then r_{o2} becomes the lending rate, r. Else, it is the borrowing rate, r'. This condition implies:

$$F_t = \{C_t - P_t\}(1 + r_{o2})^{T-t} + X \tag{2}$$

Arbitrage strategies are now developed where transactions costs are explicitly accounted for. The notation employed for transactions costs is:

Transactions costs = Q_{ai} for transaction a,
involving instrument i

Table II
DETAILS OF CASH FLOWS FOR TRANSACTION SET 2: SHORT POSITION IN STOCK INDEX FUTURES CONTRACT VERSUS SELLING A FUTURES CONTRACT INDIRECTLY VIA STOCK INDEX OPTIONS AND TRANSACTING IN THE BORROWING/LENDING MARKETS

Cash Flow at t	Cash Flow at T	
Transaction set 2.1: Sell a futures contract at time t, at a known futures price of F_t which expires at time T.		
0	$(F_t - S_T) \times \$500$	
Transaction set 2.2: (1) sell 5 index call options at C_t, (2) buy 5 index put options at P_t, and (3) transact in the borrowing/lending markets such that an amount $(F_t - X) \times \$500$ is received at time T.[a]		
	if $S_T < X$	if $S_T > X$
$+5\ C_t \times \$100$	0	$-5(S_T - X) \times \$100$
$-5\ P_t \times \$100$	$+5(X - S_T) \times \$100$	0
$-\frac{(F_t - X) \times \$500}{(1 + r_{o2})^{T-t}}$	$(F_t - X) \times \$500$	$(F_t - X) \times \$500$
$\left\{C_t - P_t - \frac{(F_t - X)}{(1 + r_{o2})^{T-t}}\right\} \times \$500$	$(F_t - S_T) \times \$500$	$(F_t - S_T) \times \$500$

[a]In what follows, if $F_t > X$, then r_{o2} becomes the lending rate, r. Else, it is the borrowing rate, r'.

where: $a = [b$ (for buy), s (for sell)] and $i = [f$ (for transacting in one futures contract with an amount equal to \$500× futures price), c (for five call option contracts) and p (for five put option contracts)][5].

Any transactions costs for borrowing and lending funds are ignored. The magnitude of the actual futures price relative to that suggested by the critical values developed in eqs. (1) and (2) are of primary interest. Two propositions are advanced to define bounds within which the actual futures price should lie to prevent profitable arbitrage.

Proposition 1. *To eliminate the existence of arbitrage profits, the following condition establishes an upper bound for the actual index futures price,* F_t:

$$F_t \leq \{C_t - P_t + 0.002(Q_{sf} + Q_{bc} + Q_{sp})\}(1 + r_{o1})^{T-t} + X \tag{3}$$

Proof: If the actual futures price is overpriced, this calls for a short hedged position in index futures to be undertaken. Specifically consider undertaking (a) Transaction set 2.1 and (b) Transaction set 1.2 to neutralize net cash flows at time T to zero (see cash flows at time T for these two sets in Tables I and II). After including transactions costs, to prevent arbitrage profits, it must be true that this initial cash flow must be nonpositive. Specifically, the following condition must hold:

$$\left(-C_t + P_t + \frac{F_t - X}{(1 + r_{o1})^{T-t}}\right) \times \$500 - [Q_{sf} + Q_{bc} + Q_{sp}] \leq 0 \tag{4}$$

This condition simplifies to the expression in Proposition 1.

Proposition 2. *To eliminate the existence of arbitrage profits, the following condition establishes a lower bound for the actual index futures price,* F_t:

$$F_t \geq \{C_t - P_t - 0.002(Q_{bf} + Q_{sc} + Q_{bp})\}(1 + r_{o2})^{T-t} + X \tag{5}$$

Proof: If the actual futures price is underpriced, this calls for a long hedged position in the index futures contract by implementing (a) Transaction set 1.1 and (b) Transaction set 2.2. To prevent arbitrage profits, the following condition must hold:

$$\left(C_t - P_t - \frac{F_t - X}{(1 + r_{o2})^{T-t}}\right) \times \$500 - [Q_{bf} + Q_{sc} + Q_{bp}] \leq 0 \tag{6}$$

This condition simplifies to the expression in Proposition 2.

THE DATA

In this study, actual transactions prices of all securities concerned are employed. Intraday transactions prices for the SPX index call and put options are from the Chicago Board Options Exchange for the period from November 1, 1989, to June 20, 1991. Intraday transactions prices for the S&P 500 index futures are for the same time period from the Chicago Mercantile Exchange. Daily Treasury bill yield data as well as call money rates are from the Wall Street Journal. The yield on the Treasury bill whose maturity is closest to the maturity of the futures contract is used as a proxy for the lending rate of interest. The call money rate plus 1% is used as the borrowing rate of interest. This is typical for institutional investors based on conversations with representatives from Merrill Lynch, Goldman Sachs, and Morgan Stanley.

[5] In what follows in the rest of this article, it is assumed that trading occurs in five call option and five put option contracts. This is because of the difference in multipliers in SPX options and futures markets which are \$100 and \$500, respectively.

For transactions costs in the futures market, a round trip commission of $20 per contract for institutional investors is assumed.[6] The one-way futures market impact cost (half of the bid–ask spread) associated with buying at the asked and selling at the bid price is estimated at $12.50. This assumes a bid–ask spread of $25, the minimum price change per trade. With respect to the notation used, this implies that: $Q_{bf} = Q_{sf} = \$32.50$ for trading in futures contracts. Arbitrageurs can submit Treasury bills for the initial margin requirement when trading in futures contracts. Thus, the possibility of forgone interest on initial margin does not arise as a concern here.

For index options, transactions costs are not fixed but vary with the price of the option. To be conservative, the costs charged by a typical leading full service (not discount) commission brokerage firm are used. While it is assumed here that index futures require a round trip commission up front, in the index options market investors pay commissions twice; at the time of initial trade and then to close out the position (unless the position is allowed to expire unexercised). In general, one-way commission and bid/ask spreads are all combined and are charged to institutional investors. In the empirical tests, it is assumed that double the one-way transactions costs (one-way commissions plus the bid/ask spread) are incurred even if the option is allowed to expire unexercised.[7] The estimates of transactions costs, Q_{bc}, Q_{bp}, Q_{sc}, and Q_{sp}, that are employed are consistent with those charged by Merrill Lynch and assume that five option contracts are being traded to accommodate the difference in multipliers in SPX options and S&P futures. These costs take the following form:

Base commission = (\$14 + 0.016 × option price × \$100) × 5 contracts
subject to a maximum of 17% of principal value.
less: 0.03× Base commission × 5 contracts
With the following exceptions:
If the commission calculated is less than \$30, then there is a minimum charge of: \$30.00 on orders of principal value of \$187.50 and greater or 16% of principal value on orders less than \$187.50. The maximum charge per contract can never exceed \$92.00.

The data for index put and call option prices are matched by exercise price, expiration date, and time of trade. This is then merged with the futures price data such that prices of all three instruments are matched in time for the same expiration date.[8]

The requirement that transaction prices of the three instruments match up in time constraints the sample to a total of 1900 observations. The frequency distribution of the sample by contract expiration month and by exercise price of the index options is provided in panels A and B, respectively, of Table III. The sample spans the contract expiration months quite evenly except for the first one, December 1989. The sample also includes a wide range of exercise prices for index options.

[6]This is also based on conversations with representatives from Merrill Lynch, Goldman Sachs, and Morgan Stanley. Member firms are generally charged a round trip commission of about $10 per contract.

[7]Thus, an overestimate of transactions costs is used. Violations that arise despite this indicate strong evidence of existence of arbitrage opportunities.

[8]Technically, the S&P 500 futures expire on the third Thursday of March, June, September, and December. The SPX options used to hedge these futures contracts expire on the third Friday of those months. Theoretically, the one day difference may lead to nonzero cash flows at expiration for the arbitrage positions that are proposed. However, practitioners have indicated that they mostly unwind their positions any time prior to expiration if such early unwinding guarantees a sizable profit. Consequently, they rarely wait until expiration to unwind; thus the one day difference in expiration times is unlikely to be of material concern to arbitrageurs.

Table III
DISTRIBUTION BY CONTRACT EXPIRATION MONTH AND EXERCISE PRICE OF INTRADAY OBSERVATIONS FOR S&P 500 INDEX PUT AND CALL OPTION AND FUTURES PRICES MATCHED IN TIME

Panel A. Distribution of Observations by Contract Expiration Month

Contract expiration month	Number of observations
December 1989	95
March 1990	222
June 1990	230
September 1990	366
December 1990	396
March 1991	287
June 1991	304
Total	1900

Panel B. Distribution of Observations by Index Option Exercise Price

Index option exercise price	Number of observations
300	3
305	3
310	20
315	126
320	197
325	180
330	244
335	144
340	139
345	73
355	72
360	137
365	158
370	162
380	156
385	69
390	17
Total	1900

The sample covers the period from November 1, 1989 to June 20, 1991. Data for index options are obtained from the Chicago Board Options Exchange and the futures data from the Chicago Mercantile Exchange.

EMPIRICAL RESULTS

Using the 1900 time-matched observations, the following test is conducted to determine whether (on average) conditions (3) and (5) are violated. Rearranging (3) and (5) leads to the test equations below:

$$\left(-C_t + P_t + \frac{F_t - X}{(1 + r_{o1})^{T-t}}\right) - 0.002 \times \{Q_{sf} + Q_{bc} + Q_{sp}\} \leq 0 \qquad (7)$$

$$\left(C_t - P_t - \frac{F_t - X}{(1 + r_{o2})^{T-t}}\right) - 0.002 \times \{Q_{bf} + Q_{sc} + Q_{bp}\} \leq 0 \quad (8)$$

If the null hypothesis of nonexistence of arbitrage opportunities applies, then conditions (7) and (8) above must be satisfied. This null hypothesis is tested by computing the terms on the left-hand side of (7) and (8) for all the observations in the sample. The means of the resulting sampling distributions are used to test the null hypothesis. Two statistical tests are conducted on the means for this purpose, a parametric t test and a nonparametric signed rank test. If the mean is significantly positive, then the null hypothesis does not apply.

Table IV
DETAILS OF SAMPLING DISTRIBUTIONS FOR NO-ARBITRAGE CONDITIONS BETWEEN INDEX OPTIONS AND INDEX FUTURES

Transactions Costs	Mean[a]	t-Statistic[b]	Signed Rank[b]	Median[a]	Standard Deviation[a]
Panel A: Location test for the mean of the sampling distribution of the left-hand term in condition (7): $\left\{-C_t + P_t + \frac{F_t - X}{(1 + r_{o1})^{T-t}}\right\} - 0.002 \times (Q_{sf} + Q_{bc} + Q_{sp}) \leq 0$					
None assumed	0.219	11.79	445226	0.204	0.809
0.1% of futures price	−0.128	−6.87	−311943	−0.141	0.809
0.3% of futures price	−0.820	−44.04	−877712	−0.835	0.811
0.5% of futures price	−1.512	−80.72	−898745	−1.532	0.817
1.0% of futures price	−3.243	−168.12	−901073	−3.241	0.841
Actual assumption about Q_{sf}, Q_{bc}, Q_{sp}	−0.645	−34.24	−852638	−0.637	0.821
Panel B: Location test for the mean of the sampling distribution of the left-hand term in condition (8): $\left\{C_t - P_t - \frac{F_t - X}{(1 + r_{o2})^{T-t}}\right\} - 0.002 \times (Q_{bf} + Q_{sc} + Q_{bp}) \leq 0$					
None assumed	−0.227	−12.20	−457370	−0.212	0.811
0.1% of futures price	−0.573	−30.79	−807859	−0.561	0.811
0.3% of futures price	−1.265	−67.77	−894944	−1.254	0.814
0.5% of futures price	−1.958	−104.14	−902796	−1.948	0.819
1.0% of futures price	−3.689	−190.40	−902975	−3.671	0.844
Actual assumption about Q_{bf}, Q_{sc}, Q_{bp}	−1.090	−57.29	−890577	−1.062	0.829

The sampling distributions of the terms on the left-hand side of conditions (7) and (8) are analyzed for the 1900 time-matched observations. Under the null hypothesis of nonexistence of arbitrage opportunities between the index options and futures markets, the means of the distributions should be nonpositive.

[a]Mean, median, and standard deviation of the distributions are expressed in index points.

[b]The t-statistics as well as the signed ranks are all significant at 0.001 level.

Panels A and B of Table IV report the results. When the assumed actual transactions costs are used (last row in each panel), the sample means for the terms on the left-hand side of (7) and (8) are -0.645 and -1.090 index points, respectively. The parametric t test as well as the nonparametric signed rank test indicate that these means are negative and significant at the 0.001 level. This supports, on average, the null hypothesis of nonexistence of arbitrage opportunities between index options and futures markets.

To examine the sensitivity of the results to the magnitude of transactions costs, various estimates that are a fraction of the futures price are used. Evidence on the sensitivity of the results to transactions costs is important because it facilitates determination of arbitrage opportunities for different trader types. For example, members of the CME and CBOE (including market makers for SPX options on the CBOE) may face much smaller costs than the transactions costs estimates for institutional investors. Furthermore, individual investors may face even higher transactions costs than institutional investors (although individual investors may put their money with member firms who perform arbitrage on their behalf for a commission).

Only when transaction costs are assumed to be nonexistent (see first row of panel A of Table IV) does the mean appear to be positive and significant for condition (7), i.e., for the majority of the observations, arbitrage opportunities exist with respect to overpriced futures contracts. Otherwise, the results are consistent with those for the assumed estimates of actual transactions costs.

The above analysis is refined further by testing the arbitrage bounds on the futures price that are developed in Propositions 1 and 2. For those observations which violate the arbitrage bounds, the consequent average profits (in index points) are computed. This is done for the overall sample of 1900 time-matched observations, as well as over various time periods that lie within the sample period. The results are summarized in panels A and B of Table V, respectively.

From the last row of panel A where the assumed actual transactions costs are used, the majority of the sample (90.5%), appears to comply with the no-arbitrage bounds developed earlier. However, there are 141 (7.4% of the sample) upper bound violations and 40 (2.1% of the sample) lower bound violations implying more overpriced futures contracts than underpriced ones. The average arbitrage profits from these opportunities amount to 0.474 and 0.452 index points, respectively. These profits are defined as ex-post profits because it assumes that trades can be consummated at those very prices that are identified as profitable opportunities by the arbitrage model. For the purpose of comparison, results are also provided for transactions costs assumed to be a percentage of the futures price. With respect to this sensitivity to transactions costs, the frequency of violations of the no-arbitrage bounds decreases with increased transactions costs. When these costs are 1% of the futures price, 99.9% of the observations conform to the no-arbitrage bounds. Thus, while violations of the no-arbitrage bounds do occur, the vast majority of the observations seem to be efficiently priced. In panel B, the results are categorized by contract expiration month using the assumed actual transactions costs. It appears that the pattern within any contract expiration month is consistent with that for the overall sample; specifically, the vast majority of observations lie within the no-arbitrage bounds. There is also no distinct pattern in the frequency of mispricings across subperiods. This implies that the overall results are not due to any particular subperiod's results.

Ex-ante tests of arbitrage [see Klemkosky and Resnick (1980)] are also conducted. Specifically, whenever a violation of the no-arbitrage bounds occurs, this acts as a signal to initiate transactions to exploit it. It is assumed that transactions cannot be conducted

Table V

FREQUENCY OF VIOLATIONS AND COMPLIANCE WITH ARBITRAGE BOUNDS IN INDEX OPTIONS AND INDEX FUTURES

Transaction Costs	Contract Expiration Month	Total Number of Observations	Number of Efficiently Priced Observations[a]	Number of Upper Bound Violations[a]	Ex Post Profits (in Index Points)	Number of Lower Bound Violations[a]	Ex Post Profits (in Index Points)
Panel A: Results for all 1900 time-matched observations							
None assumed	all	1900	15 (0.8%)	1315 (69.2%)	0.464	570 (30.0%)	0.334
0.1% of futures price	all	1900	1029 (54.2%)	687 (36.2%)	0.380	184 (9.7%)	0.366
0.3% of futures price	all	1900	1782 (93.8%)	87 (4.6%)	0.631	31 (1.6%)	0.475
0.5% of futures price	all	1900	1882 (99.1%)	9 (0.5%)	3.472	9 (0.5%)	0.483
1.0% of futures price	all	1900	1898 (99.9%)	2 (0.1%)	12.355	—	—
Assumed actual costs	all	1900	1719 (90.5%)	141 (7.4%)	0.474	40 (2.1%)	0.452

Panel B: Results categorized by contract expiration month

(These results reflect the use of assumed transactions costs that we elaborate on in the second section)

Assumed actual costs	December 1989	95	93 (97.9%)	1 (1.1%)	0.134	1 (1.1%)	0.144
	March 1990	222	206 (92.8%)	13 (5.9%)	0.288	3 (1.4%)	0.203
	June 1990	230	208 (90.4%)	16 (7.0%)	0.290	6 (2.6%)	0.175
	September 1990	366	324 (88.5%)	30 (8.2%)	0.335	12 (3.3%)	0.439
	December 1990	396	366 (92.4%)	26 (6.6%)	0.345	4 (1.0%)	0.332
	March 1991	287	231 (80.5%)	49 (17.1%)	0.796	7 (2.4%)	0.875
	June 1991	304	291 (95.7%)	6 (2.0%)	0.047	7 (2.3%)	0.510

[a]The number in parenthesis is the frequency expressed as a percentage of overall number of observations examined in that row.

at the prices that violate the no-arbitrage bounds but instead at the next time-matched set of prices for index options and index futures contracts. The number of observations for these ex-ante tests is not necessarily the same as the number that violate the no-arbitrage bounds. This is because subsequent to the violations, a time-matched set of index option and futures prices may not exist for each violation. These trades are examined over three time intervals subsequent to the actual mispricing. The arbitrage profits from these "after-the-violation" trades are termed ex-ante profits. The results for these ex-ante tests appear in Table VI.

If trades could be consummated at the actual prices that violate the no-arbitrage bounds, arbitrage opportunities appear to be positive and significant. However, the ex-ante profits are negative as indicated by the mean and the median. This is true for even the shortest time interval of 1 to 10 min after the actual violation. Furthermore, the ex-ante profits become more negative as time elapses after the violation. The parametric and nonparametric test statistics confirm this. This evidence implies that any mispricing in index futures and/or options is corrected soon after the mispricing so as to preclude profitable (ex-ante) arbitrage opportunities between the two markets.

SUMMARY

Prices of stock index options, stock index futures, options on stock index futures, and the stocks comprising the index itself should be interrelated since the "underlying asset" is the same. The efficient markets paradigm would imply that the market for these instruments must be an integrated one. Specifically, if significant deviations existed from prices dictated by the relevant interrelationships, then arbitrageurs would exploit such situations. Consequently, such deviations or mispricings would not be longlived.

To fully investigate the issue of an integrated market, it is useful to explore the different arbitrage positions possible between the instruments. Some of these positions have been formally investigated in the finance literature but the one involving index options and futures has not been addressed. This article investigates this particular arbitrage strategy and thus provides incremental evidence on the issue of an integrated market for the different instruments.[9]

The investigation of this arbitrage strategy includes both a theoretical as well as an empirical examination. Specifically, arbitrage strategies using stock index options and index futures are developed to exploit any mispricing in the markets for these instruments. These lead to the propositions for the upper and lower bounds on the actual stock index futures price to eliminate the possibility of arbitrage. Following this, tests are conducted for violations of the no-arbitrage bounds to determine whether the derivative security markets are efficient from the viewpoint of arbitrage opportunities. The results indicate that, after accounting for transactions costs, the futures price lies between the no-arbitrage bounds for the majority of the observations in the sample.

If it is assumed that arbitrage trades can be executed at the time of the violations of the no-arbitrage bounds, it appears that economically significant profits can be obtained. For the sample, 9.5% of the observations violate the bounds. These violations yield ex-post profits of 0.463 index points on average. However, on an ex-ante basis, where ex-post violations are used as a signal to form arbitrage positions at subsequent points in time, the profits disappear. Furthermore, the transactions produce increasing losses as time elapses after the initial violation. This implies that although relative mispricing does exist between

[9]It should be noted that although the strategy investigated has received little or no formal attention, it is actually practiced. Consequently, the results should be interesting to actual traders and investment professionals.

Table VI
TESTS OF PROFITABILITY FROM TRANSACTIONS SUGGESTED BY VIOLATIONS OF NO-ARBITRAGE BOUNDS AND ASSUMED ACTUAL TRANSACTIONS COSTS

	Actual Mispricing	Time Interval Subsequent to the Mispricing		
		1–10 min	11–25 min	26–50 min
	Panel A. Upper bound violation suggested trades			
No. of observations[a]	105	41	40	38
Mean time difference[b]	0	7 min (2)	20 min (4)	39 min (7)
		Arbitrage Profit[c]		
Mean	0.295	−0.421	−0.562	−0.684
Median	0.175	−0.519	−0.535	−0.695
Standard deviation	0.399	0.476	0.565	0.776
		Test statistics[d]		
t-statistic	7.58	−5.67	−6.29	−5.44
Signed rank	2783	−331	−357	−322
	Panel B. Lower bound violation suggested trades			
No. of observations[a]	29	11	11	11
Mean time difference[b]	0	7 min (3)	19 min (5)	39 min (9)
		Arbitrage Profit[c]		
Mean	0.460	−0.896	−1.070	−1.333
Median	0.252	−0.971	−0.989	−1.422
Standard deviation	0.499	0.453	0.496	0.675
		Test statistics[d]		
t-statistic	4.96	−6.56	−7.15	−6.55
Signed rank	218	−33	−33	−33

[a]The number of observations here differs from that in Table V. This is because for some violations, subsequent time-matched prices for index options and index futures may not exist within the same day.

[b]This refers to the time that elapses between the violation of the no-arbitrage bound and the relevant time-matched set of index option and futures prices. The standard deviation of the time difference appears in parenthesis.

[c]All entries are in terms of index points.

[d]All the statistics are significant at the 0.001 level.

index options and futures contracts, it does not persist. In sum, the evidence suggests that the SPX options market and the S&P 500 futures market constitute an integrated market.

Bibliography

Brennan, M. J., and Schwartz, E. S. (1990): "Arbitrage in Stock Index Futures," *Journal of Business,* 63:S7–S31.

Brenner, M., Subrahmanyam, M. G., and Uno, J. (1989): "The Behavior of Prices in the Nikkei Spot and Futures Market," *Journal of Financial Economics,* 23:363–383.

Evnine, J., and Rudd, A. (1985): "Index Options: The Early Evidence," *Journal of Finance,* 40:743–758.

Kawaller, I. G. (1987): "A Note: Debunking the Myth of a Risk-Free Return," *The Journal of Futures Markets,* 7:327–331.

Kawaller, I. G., Koch, P. D., and Koch, T. W. (1987): "The Temporal Price Relationship between S&P Futures and the S&P 500 Index," *Journal of Finance,* 42:1309–1329.

Klemkosky, R. C., and Lee, J. H. (1991): "The Intraday Ex Post and Ex Ante Profitability of Index Arbitrage," *The Journal of Futures Markets,* 11:291–311.

Klemkosky, R. C., and Resnick, B. G. (1980): "An Ex Ante Analysis of Put Call Parity," *Journal of Financial Economics,* 8:363–378.

MacKinlay, A. C., and Ramaswamy, K. (1988): "Index-Futures Arbitrage and the Behavior of Stock Index Futures Prices," *Review of Financial Studies,* 1:137–158.

Miller, M. H., Muthuswamy, J., and Whaley, R. E. (1991): "Predictability of S&P 500 Index Basis Changes: Arbitrage Induced or Statistical Illusion?" Working Paper, University of Chicago and Duke University.

Rhee, S. G., and Chang, R. P. (1992): "Intra-Day Arbitrage Opportunities in Foreign Exchange and Eurocurrency Markets," *Journal of Finance,* 47:363–379.

Stoll, H. R., and Whaley, R. E. (1990): "The Dynamics of Stock Index and Stock Index Futures Returns," *Journal of Financial and Quantitative Analysis,* 25:441–468.