Impacts of Shifts in Uncertainty on Spot and Futures Price Change Serial Correlation and Standardized Covariation Measures

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INTRODUCTION

Leistikow (1990) developed a model and used it to determine the relative variability of spot and futures prices. That model is used here to examine how changes in the mix of uncertainty types affect spot and futures price change serial correlation and standardized covariation measures. The standardized covariation measures considered are the minimum-risk hedge ratio for a futures contract, the hedging-effectiveness of a futures contract, and the parameters estimated in a regression in which the independent variable is a futures contract price while the dependent variable is the futures contract's subsequent maturity date spot price. An analysis is included on how the minimum-risk hedge ratio changes dynamically through time and how the hedge should be allocated across futures contract maturities. Finally, the relative sizes of near and far futures market risk premia are investigated. The importance of these issues and the empirical findings regarding each follow.

Ederington's "minimum-risk hedge ratio" and "hedging-effectiveness" concepts [Ederington (1979)] are central to futures markets. Ederington's minimum-risk hedge ratio is given by the covariance between the spot and futures price change divided by the futures price change variance. Hedging-effectiveness is defined as the square of the correlation between the spot and futures price change. The hedger hedges with the futures contract that has the highest hedging-effectiveness. The size of the hedger's futures position is given by the product of their spot position and the minimum-risk hedge ratio. Ederington (1979), Hill and Schneeweis (1982), Grammatikos and Saunders (1983), and others have empirically estimated the minimum-risk hedge ratio for and hedging-effectiveness of various futures contracts. Hill and Schneeweis (1982) and Grammatikos and Saunders (1983) noted that the empirical estimates of these variables have been nonstationary across time for debt and foreign exchange markets, respectively.

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Agents frequently need a point estimate of future spot prices to plan their operating activities. Rather than building a sophisticated model or subscribing to a forecasting service, the agent may choose to use the existing futures price as an unbiased estimator of the subsequent maturity date spot price. However, Stein (1981) shows that futures market risk premia cause avoidable social losses when the futures price is used in this way. Futures market risk premia have been investigated by many authors in the context of many different models.[1] Of the many empirical studies focussing on risk premia, a few have focused on the relative size of near and far futures market risk premia. In particular, Capozza and Cornell (1979), Stein (1981), and Rzepczynski (1987) empirically find that risk premia are greater in far than in near futures contracts. This suggests that avoidable social losses are greater using far futures than near futures contract prices. This study provides a theoretical foundation for the empirical finding that far futures market risk premia exceed near futures market risk premia.

Alternatively, agents forecasting future spot prices may use the existing futures price and the historic linear relation between the futures price and its subsequent maturity date spot price to derive their forecast. Tests examining the linear relation between the futures price and its subsequent maturity date spot price have been performed by Levich (1978), Frenkel (1981), and Agmon and Amihud (1981), among others. These authors assume there is no basis[2] risk[3] and they estimate the relationship by regressing the maturity date futures price on its pre-maturity date price. The test is performed on a non-overlapping matched series of futures prices and their subsequent maturity date futures prices. Specifically, the following regression is estimated:

$$F_{TT} = \alpha + \beta F_{TT-K} + \epsilon_T$$

where F_{TI} is the time I price of a futures contract that matures at time T, ϵ_T is an error term, and time T is K periods later than time $T{-}K$. Another version of the test is sometimes performed. Under the null hypothesis that $\alpha = 0$ and $\beta = 1$, the above regression is equivalent to the following regression.

$$F_{TT} - P_{T-K} = \alpha + \beta[F_{TT-K} - P_{T-K}] + \epsilon_T$$

where P_I is the time I spot price. The conclusions derived from these tests are mixed. Frenkel (1981) does not find that α and $\beta - 1$ are significantly different from zero, while Levich (1978) and Agmon and Amihud (1981) find some cases where they are significantly different from zero.

Some trading strategies are founded on the assumption that serial correlation exists between price changes. If the serial correlation between price changes is zero, these trading strategies can be rejected. Tests for dependence between price changes have been performed by Smidt (1965), Larson (1960), and Stevenson and Bear (1970), among others. Those tests included serial correlation tests of varying lags, runs tests, filter rule tests, and an "index of continuity" test. The tests are applied to both intra-day and inter-day prices. The findings with respect to dependence between price changes were not uniform.

[1]Though the models and authors are too numerous to list here, the models include hedging-pressure, CAPM, consumption-based CAPM, and multi-good equilibrium models. The authors studying futures market risk premia include Keynes (1930), Dusak (1973), and Richard and Sundaresan (1981).

[2]The maturity T, time I basis is defined as the price of the futures contract maturing at time T minus the spot price, where both prices are observed at time I.

[3]There is no basis risk if, at all times before the maturity date, it is known that the spot and expiring futures prices will be equal. Sufficient conditions for no basis risk are that the spot and futures delivery grades and locations match and there are no transactions costs.

The lack of uniformity in the empirical findings regarding the spot and futures price change serial correlation and standardized covariation measures may be due to changes in the mix of uncertainty types. The Leistikow (1990) model is now reviewed and used to demonstrate these results.

THE MODEL

Introduction

Trading occurs at times one, two, and three. At time one, the "spot" asset, "near" futures contract, and "far" futures contract trade. Each futures contract calls for delivery of one unit of the spot asset. The spot market requires immediate delivery. The "near" and the "far" futures contracts call for time two and time three delivery of the spot asset, respectively. At time two, there is trading in the spot asset and the far futures contract. At time three, there is trading only in the spot asset. The expiring futures price and spot price are equal (see footnote 3). Therefore, the time two spot and near futures markets are indistinguishable as are the time three spot and far futures markets. Time three is the terminal date.

The participants in the markets are of four types: inventory-carriers, futures market speculators, producers, and consumers. There are n_I identical inventory-carriers and n_S identical futures market speculators. Each is a perfect competitor. Each maximizes a linear function of the mean and variance of time three wealth. The production supply and consumption demand functions are exogenously given in this short-run model.

The inventory-carrier's time three wealth is given by

$$W_{3I} = W_{1I}R^2 + RS_1\left[P_2 - RP_1 - \frac{c}{2}S_1\right] + S_2\left[P_3 - RP_2 - \frac{c}{2}S_2\right] - RX_{21I}[P_2 - F_{21}] - RX_{31I}[F_{32} - F_{31}] - X_{32I}[P_3 - F_{32}] \quad (1)$$

In (1), W_{1I} represents the inventory-carrier's time one wealth. R denotes one plus the one period riskless interest rate. R is constant and greater than one. F_{ji} is the time i price of the futures contract which requires time j delivery. P_i is the time i spot price. $P_i = F_{ii}$ from the no basis risk assumption. S_i is the time i carry-out chosen by the individual inventory-carrier for $i = 1, 2$. c is a positive constant that reflects increasing marginal costs of carry. $S_i[P_{i+1} - RP_i - \frac{c}{2}S_i]$ is the profit on inventory bought at time i and carried to time $i + 1$. X_{jiI} is the inventory-carrier's short time i position in futures contracts which require time j delivery. $X_{jiI}[F_{j(i+1)} - F_{ji}]$ is the loss from the short position in futures contract j taken at time i and held to time $i + 1$.

The Optimization Problems

The inventory-carrier chooses S_1, X_{21I}, and X_{31I} at time one and S_2 and X_{32I} at time two. The time two and time one choices are formally represented as eqs. (2) and (3), respectively.

$$\max_{X_{32I}, S_2} E_2W_{3I} - \frac{\alpha_I}{2} Var_2(W_{3I}) \quad (2)$$

given X_{21I}, X_{31I}, and S_1, where: E_i is the expectations operator conditional on time i information, α_I is the constant absolute risk-aversion of the inventory-carrier, and $Var_i(.)$

is the variance of (.) conditional on time i information.

$$\max_{X_{21I}, X_{31I}, S_1} E_1 W_{3I} - \frac{\alpha_I}{2} Var_1(W_{3I}) \tag{3}$$

given that X_{32I} and S_2 solve (2). The inventory-carrier begins by solving the time two maximization. Because the market-participants are perfect competitors, current time prices are taken as deterministic in the maximization. Future time prices and positions are taken as stochastic.

The time two first order conditions are given by (4) and (5).[4]

$$E_2P_3 - RP_2 - cS_2 = \alpha_I[S_2 - X_{32I}]Var_2(P_3) \tag{4}$$

$$-E_2P_3 + F_{32} = \alpha_I[X_{32I} - S_2]Var_2(P_3) \tag{5}$$

In (4), the expected marginal revenue for carrying stock from time two is equated with the marginal cost of purchasing and carrying the stock out of time two plus an endogenous risk premium. In (5), the marginal futures market profit is equated with an endogenous risk premium.

Because the futures market speculator is assumed not to participate in the spot market, the speculator's wealth and optimization problems are the same as the inventory-carrier's, except, for the speculator the "I" subscripts are replaced by "S" and $S_S = 0$. Similar to (5), the futures market speculator's first order condition at time two is given as (6),

$$-E_2P_3 + F_{32} - \alpha_S X_{32S} Var_2(P_3) = 0 \tag{6}$$

For the speculator, the expected marginal profit is equated with the endogenous risk premium.

Specification of the Rest of the Model and Derivation of the Results

The time three equilibrium has consumption demanded plus inventory carried-out equal to production supplied plus inventory carried-in. Each of the n_I inventory-carriers brings in stock S_2. The time three market equilibrium is given by (7):

$$A_3 - aP_3 + \phi_{32} + \phi_{33} = n_I S_2 \tag{7}$$

In (7), A_3 and a are positive constants. The ϕ_{3i} are consumption demand plus storage demand less production supply shocks realized at time three and revealed in time i, where $i = 2, 3$. For simplicity, the ϕ_{3i} shocks are referred to as "net consumption demand" shocks. Throughout the article all of the fundamental shock terms are assumed to be independently and symmetrically distributed about zero with finite variances.

The government periodically makes announcements which affect the expected level of future time net consumption demand. For example, the U.S. Department of Agriculture makes monthly projections of the upcoming harvest for various grains. Housing starts announcements can cause revisions in forecasts of future demand for mortgages. Similarly, the Department of Commerce's GNP and unemployment announcements are important to the financial markets to the extent that they change expectations regarding the level of future economic activity.

[4]Because, given time two information, the time two prices are taken as deterministic by the perfect competitors, $Var_2(P_3) = Var_2(P_3 - P_2) = Var_2(P_3 - F_{32}) = \mathrm{Cov}_2(P_3 - P_2, P_3 - F_{32})$. Only the stochastic part of the arguments for $\mathrm{Cov}_i(.,.)$ and $Var_i(.)$ are included in the paper to save space. Because the second order conditions are satisfied, a solution to (4) and (5) will satisfy (3). Conditions are established in the appendix of Leistikow (1990) so an interior solution to (4) and (5) is achieved.

Government announcements shift the resolution of uncertainty from later dates to earlier dates. As of time one, the uncertainty regarding time three net consumption demand is $Var_1(\phi_{32}) + Var_2(\phi_{33})$. At time two, $Var_1(\phi_{32})$ of the uncertainty regarding time three net consumption demand is eliminated. For subsequent analytical simplicity, $Var_1(\phi_{32}) + Var_2(\phi_{33})$ is defined as $(Y + Z)Var\phi$ and $Var_1(\phi_{32})$ is defined as $(Z + G)Var\phi$. Therefore, $Var_2(\phi_{33})$ equals $(Y - G)Var\phi$, where $Y > G \geq 0$ and $Z > 0$. A comparative static rise in G is used to represent a government announcement at time two concerning time three net consumption demand.

Using (7) and the definition of $Var_2(\phi_{33})$, the variance of the time three spot (or equivalently far futures) price, conditional on time two information, is derived as

$$Var_2(P_3) = \frac{(Y - G)Var\phi}{a^2} \tag{8}$$

From (5), (6), (8), and the futures market clearing condition, (5′) and (6′) are derived. Equations (5′) and (6′) are well-known two-period inventory-hedging model results [see Stein (1985), for example]. These two-period results will be contrasted with the three-period results.

$$E_2(P_3) - F_{32} = \theta n_I S_2(Y - G)Var\phi/a^2, \text{ where: } \theta = \alpha_I\alpha_S/(n_S\alpha_I + n_I\alpha_S) \tag{5′}$$

$$X_{32I} = \frac{S_2 n_S \alpha_I}{n_S\alpha_I + n_I\alpha_S} \tag{6′}$$

The θ term may be interpreted as the "market risk-aversion"; it rises with α_I and α_S, while it declines with n_I and n_S. In (5′), the one period time two risk premium is positive, where the risk premium is defined as the expected futures price minus the current futures price. In (6′), the futures position for the inventory-carrier is positively related to his storage demand and negatively related to the market risk-aversion parameter.

The time two spot market equilibrium has consumption demanded plus storage demanded equal to production supplied plus carry-in. It is given as (9):

$$A_2 - aP_2 + \phi_{22} + n_I S_2 = n_I S_1 \tag{9}$$

In (9), A_2 is constant, ϕ_{22} represents the time two current time net consumption demand (i.e., consumption demand minus production supply) shock, $E_1\phi_{22} = 0$, and $Var_1(\phi_{22}) = Var\phi$.

Therefore, at time two there are shocks concerning time two net consumption demand and expectational revisions concerning time three net consumption demand with variances of $Var\phi$ and $(Z + G)Var\phi$, respectively. The ratio of the variance of expectational shocks concerning time three net consumption demand to the variance of shocks concerning time two net consumption demand, where each is received at time two, is $(Z + G)$. The ratio could be high due to governmental and/or non-governmental factors.

Using (4) through (9), the rational expectations assumption, and the futures market-clearing conditions, the inventory-carrier's carry-out from time two is derived as (10):

$$S_2 = \frac{[A_3 + \phi_{32}] - R[A_2 - n_I S_1 + \phi_{22}]}{\chi}$$

$$\text{where } \chi = (R + 1)n_I + ca + a\gamma n_I \text{ and } \gamma = \frac{\theta(Y - G)Var\phi}{a^2} \tag{10}$$

Substituting (10) into (9) and rearranging terms yields the time two spot price,

$$P_2 = \frac{n_I[A_3 + \phi_{32}] + (n_I + ca + a\gamma n_I)[A_2 - n_I S_1 + \phi_{22}]}{a\chi} \tag{11}$$

Using (4), (5), (10), and (11), the time two far futures price is determined,

$$F_{32} = \frac{(ac + Rn_I)[A_3 + \phi_{32}] + R(n_I + a\gamma n_I)[A_2 - n_I S_1 + \phi_{22}]}{a\chi} \tag{12}$$

The variance of the time two spot (or near futures) price is determined from (11), the definition of $Var_1(\phi_{32})$, the definition of $Var_1(\phi_{22})$, and the independence of ϕ_{22} and ϕ_{32} as (11′),

$$Var_1(P_2) = \frac{[(n_I + ca + a\gamma n_I)^2 + n_I^2(Z + G)]Var\phi}{a^2\chi^2} \tag{11′}$$

Similarly, using (12), the definition of $Var_1(\phi_{32})$, the definition of $Var_1(\phi_{22})$, and the independence of ϕ_{22} and ϕ_{32}, the variance of the time two far futures price is derived as (12′),

$$Var_1(F_{32}) = \frac{[R^2n_I^2(1 + a\gamma)^2 + (Rn_I + ac)^2(Z + G)]Var\phi}{a^2\chi^2} \tag{12′}$$

The Leistikow (1990) analysis focused on (and stopped with) eqs. (11), (12), (11′), and (12′). This analysis begins with the covariance between the time two spot and far futures prices. It is derived from (11), (12), the definition of $Var_1(\phi_{32})$, the definition of $Var_1(\phi_{22})$, and the independence of ϕ_{32} and ϕ_{22} as (13).

$$\mathrm{Cov}_1(P_2, F_{32}) = \frac{[(n_I + ca + a\gamma n_I)Rn_I(1 + a\gamma) + n_I(Z + G)(Rn_I + ca)]Var\phi}{a^2\chi^2} \tag{13}$$

The covariance between the time two spot and far futures prices is positive and depends on the same parameters as the variances of the time two spot and far futures prices. Like the time two price variances, the covariance between the time two spot and far futures prices is positively related to Z. Intuitively, as the uncertainty rises for a factor that has a similar effect on spot and futures prices, their covariability rises.

Ederington's minimum-risk hedge ratio and hedging effectiveness terms are now examined.

Result 1: In a comparative static sense, Ederington's minimum-risk hedge ratio is lower over announcement periods than it is over control periods; it is also lower over periods for which Z is high than it is over control periods.

To illustrate the application of the result, consider two cases. In each case, there is a futures contract with the same time-to-maturity, say two months. More uncertainty, about the net consumption demand in two months, is to be resolved in the next 2 months in the first case than in the second case. The uncertainty may concern a war, securities legislation, GATT talks, or the upcoming crop harvest. Given this scenario, the ex ante minimum risk hedge ratio is lower in the first case than in the second case. Consider soybeans as a specific example. For the most part, soybeans are planted in May and harvested in September. The weather in July and August is a critical determinant of the soybean harvest size. For soybeans, the minimum-risk hedge ratio would be lower in July than it would be in January for a two month-to-maturity futures contract. This is due to the fact that for soybeans greater future time net consumption demand uncertainty is resolved in July and August than in January and February.

The result with respect to Z is generated by comparing $[Cov_1(F_{32}, P_2)/Var_1(F_{32})]|_{z=z_H}$ with $[Cov_1(F_{32}, P_2)/Var_1(F_{32})]|_{z=z_N}$ where H denotes High and N denotes Normal. To show the result with respect to announcements, partially differentiate $Cov_1(F_{32}, P_2)/Var_1(F_{32})$ with respect to G; for this result to hold it is assumed that $\theta Var\phi\{R^2 n_I^2(1 + a\gamma)^2 ac + (Rn_I + ac)^2(Z + G)[(1 + a\gamma)(R - 1) - ca]\} < (Rn_I + ca)(1 + a\gamma)c\chi a^2$. This condition is met when the product of the "market risk aversion" and "market risk" is small (i.e., $\theta Var\phi$ is small). Market risk aversion (i.e., θ) is small when there are many speculators. The impact of G changes on the minimum-risk hedge ratio is the same as for Z changes, except there is a convoluting effect due to the reduced uncertainty to be resolved at time three. This convoluting effect is minimal when there are many speculators.

The intuition for result one is as follows. When Z is high, relatively more of the uncertainty being resolved concerns future time net consumption demand. While positive future time net consumption demand shocks affect both spot and futures prices positively, the impact on the futures price is greater than that on the spot price because the marginal cost of carry rises as well.[5] The situation where the futures price is more responsive than the spot price is the opposite of the Samuelson effect, referring to Samuelson (1965, 1976). Leistikow (1990) theoretically derived this anti-Samuelson effect result for future time net consumption demand shocks and empirically documented it in the frozen concentrated orange juice market. However, while positive current time net consumption demand shocks affect both spot and futures prices positively, the impact on the futures price is less than that on the spot price because the marginal cost of carry declines.[6] The situation where the futures price is less responsive than the spot price is known as the Samuelson effect. Leistikow (1990) theoretically derived this Samuelson effect result for current time net consumption demand shocks and empirically documented it in the frozen concentrated orange juice market. Hence, when Z is high, the anti-Samuelson effect type information is prevalent and the hedge ratio is low. However, when Z is low, the Samuelson-effect type information is prevalent and the hedge ratio is high.

Many empirical studies have estimated the minimum-risk hedge ratio ex post. Some studies find the ratio to be nonstationary across observation periods. Differing amounts of uncertainty regarding future time net consumption demand and announcement considerations may account for the observed nonstationarity. Leistikow (1989, 1991) found empirical support for result one in grain and precious metals markets.

Ederington's ex ante one period minimum-risk hedge ratio may be greater than, less than, or equal to one. $Cov_1(F_{32}, P_2)/Var_1(F_{32}) \gtreqless 1$ is equivalent to $Rn_I(1 + a\gamma)ca \gtreqless Rn_I^2(1 + a\gamma)^2(R - 1) + (Rn_I + ca)[n_I(R - 1) + ca](Z + G)$. When interest rates are low (i.e., $R - 1$ is small), when c is large, and when a low proportion of the uncertainty to be resolved concerns future time net consumption demand (i.e., $Z + G$ is small), the hedge ratio is greater than one. Markets for which c is large will have hedge ratios which are greater than one, where c represents the rate of change in the marginal cost of carry. In the agricultural and industrial markets, c is large because of the high and rapidly diminishing marginal convenience yield given the relatively low level of stocks. Stocks in these markets are low in the sense that they are generally low relative to annual production and consumption. These commodities will have hedge ratios which are greater than one. On the other hand, in the case of the precious metal and equity markets, c is small because of the low and slowly diminishing marginal

[5]From (4) and (5), $F_{32} - P_2 = P_2(R - 1) + cS_2$, where the right side of the equation is the time two marginal cost of carry. The time two marginal cost of carry rises when there are positive future time net consumption demand shocks because both P_2 and S_2 rise. Given that F_{32}, P_2, and the right side of the equation rise, it follows that F_{32} rises more than P_2.

[6]The marginal cost of carry declines because S_2 declines. This assumes that the downward pressure exerted on the marginal cost of carry from the decline in S_2 is greater than the upward pressure exerted by the increase of P_2.

convenience yield given the relatively high level of stocks. Stocks in these markets are high in the sense that they are high relative to annual production and consumption. On the basis of their low c, equities and precious metals will have hedge ratios which are less than one.[7]

Result 2: Ederington's ex ante one period minimum-risk hedge ratio declines, stays the same, or rises as the contract dynamically approaches its maturity date as $Rn_I(1 + a\gamma)ca \gtreqless Rn_I^2(1 + a\gamma)^2(R - 1) + (Rn_I + ca)[n_I(R - 1) + ca](Z + G)$.

Result two follows from the fact that Ederington's ex ante one period minimum-risk hedge ratio at time one, defined as $Cov_1(F_{32}, P_2)/Var_1(F_{32})$, may be greater than, less than, or equal to one and the fact that the ex ante one period minimum-risk hedge ratio at time two, defined as $Cov_2(F_{33}, P_3)/Var_2(F_{33})$, is one due to the absence of basis risk. When $(R - 1)$ is small, when c is large, and when $(Z + G)$ is small, the one period minimum-risk hedge ratio will decline as the contract dynamically approaches its maturity. This might be the case for copper and the grains. On the other hand, markets for which c is small, will have the one period hedge ratio rising as the contract approaches its maturity date. Given the small c for metals and equities, it is expected that precious metals and equities have one period hedge ratios that rise as the futures contract dynamically approaches its maturity.

Result 3: Ederington's hedging-effectiveness varies in a comparative static sense across announcement periods and control periods. It also varies in a comparative static sense with respect to changes in Z.

This result can be demonstrated using the correlation, conditional on time one information, between the time two spot and far futures price changes, denoted as $\rho_1(P_2, F_{32})$. If $R^2n_I^2(1 + a\gamma) \neq (Rn_I + ca)(n_I + ca + a\gamma n_I)$, the correlation squared over a high Z period will exceed the correlation squared over a normal Z period when $R^2(1 + a\gamma)^2(n_I + ca + a\gamma n_I)^2/(Rn_I + ca)^2$ is less than $(Z_H + G)(Z_N + G)$. Similarly, the correlation squared over a high Z period will be less than the correlation squared over a normal Z period when $R^2(1 + a\gamma)^2(n_I + ca + a\gamma n_I)^2/(Rn_I + ca)^2$ is greater than $(Z_H + G)(Z_N + G)$. The intuition is that when Z_N is sufficiently large, the mix of uncertainty to be resolved at time two $[Var_1(\phi_{22}), Var_1(\phi_{32})]$ is dominated by uncertainty concerning future time net consumption demand and a higher Z makes the uncertainty mix more homogeneous. On the other hand, when Z_N is sufficiently small, the mix of uncertainty to be resolved at time two is dominated by uncertainty concerning current time net consumption demand and a higher Z makes the uncertainty mix less homogeneous. In the special case where $R^2n_I^2(1 + a\gamma) = (Rn_I + ca)(n_I + ca + a\gamma n_I)$, the square of the correlation between spot and futures price changes will be stable with respect to comparative static changes in Z, because the ratio of the spot price response to the futures price response is independent of the type of information [see Leistikow (1990)]. The result with respect to announcements is derived similarly, holds under similar conditions, and assumes $\theta Var\phi$ is small.

To illustrate the application of this result, consider soybeans. Assume that soybean future time net consumption demand uncertainty is sufficiently large relative to current time net consumption demand uncertainty. Recall that soybean supply uncertainty resolution is seasonal, where much of the supply uncertainty is resolved in July and August. In this case, the hedging-effectiveness would be higher in July than it would be in January for a two month-to-maturity futures contract. This is due to the fact that

[7]One could also distinguish markets by the size of Z. For example, Z would be high for the precious metals markets as they are commonly referred to as "expectations markets." According to this line of argument, the precious metal hedge ratio is less than one, as it is with regard to the argument concerning the size of c.

future time net consumption demand uncertainty is the dominant type of uncertainty and in July greater than normal soybean future time net consumption demand uncertainty is resolved while less than normal soybean future time net consumption demand uncertainty is resolved in January. The greater than normal future time net consumption demand uncertainty in July makes the mix of uncertainty types to be resolved more homogeneous, while the lower than normal future time net consumption demand resolution in January makes the mix of uncertainty types to be resolved less homogeneous.

Empirical studies have found the ex post square of the correlation between spot and futures prices to be nonstationary across time periods. Announcement considerations and changing uncertainty regarding future time net consumption demand may explain the observed nonstationarity.

The time three spot price is determined now and the serial correlation for spot and futures price changes is examined. Substituting (10) into (7), the time three spot price is derived as (14),[8]

$$P_3 = \frac{(\chi - n_I)[A_3 + \phi_{32}] + Rn_I[A_2 - n_I S_1 + \phi_{22}] + \chi\phi_{33}}{a\chi} \tag{14}$$

Result 4: The sign of the ex ante serial correlation between one-period price changes in spot, as well as futures, prices is dependent upon the dominant type of uncertainty to be resolved.

This result is seen by examining the signs of $\rho_1(P_2 - P_1, P_3 - P_2)$ and $\rho_1(F_{32} - F_{31}, P_3 - F_{32})$. The sign of $\rho_1(P_2 - P_1, P_3 - P_2)$ is the same as the sign of $[\chi - 2n_I]n_I(Z + G) - [n_I + ca + a\gamma n_I](\chi - n_I)$. The sign of $\rho_1(F_{32} - F_{31}, P_3 - F_{32})$ is the same as the sign of $(Rn_I + ca)(Z + G) - n_I^2 R^2(1 + a\gamma)$. For both spot and futures prices, if $(Z + G)$ is large, the one-period price changes are positively serially correlated. Intuitively, when $(Z + G)$ is large, the dominant type of uncertainty to be resolved at time two concerns time three net consumption demand. On average, at time two, when there is a positive shock regarding time three net consumption demand, the price changes between times one and two will be above what was anticipated at time one and more stock will be carried out of time two. The greater P_2 and S_2 imply that the price changes from time two to time three will also be, on average, above what was anticipated at time one. On the other hand, when $(Z + G)$ is small, the one-period price changes are negatively serially correlated. Intuitively, when $(Z + G)$ is small, the dominant type of uncertainty to be resolved at time two concerns time two net consumption demand. On average, at time two, when there is a positive shock regarding time two net consumption demand, the price changes between times one and two will be above what was anticipated at time one and less stock will be carried out of time two. The lower stock carried out of time two implies that the price changes from time two to time three will be, on average, below what was anticipated at time one. In this efficient market model, the consecutive price changes are linked via the storage function, not by the gradual dissemination of information from informed traders to uninformed traders, irrational bandwagon effects, or scalping activity. The serial correlation of price changes for storable commodities may be positive, zero, or negative across different time periods as has been found by empirical studies. This extends the Stein (1979) rational expectations model which shows that spot price changes will be negatively serially correlated.

[8]The time three spot price is positively related to the deterministic component of net consumption demand in periods two and three, upward revisions in the expected time three net consumption demand received at time two, and positive current time net consumption demand shocks received in periods two and three. The time three spot price is negatively related to the stock carry-out from time one.

The time two and time three prices and positions, expressed as functions of the time one carry-out, are now used to solve for the one-period time one risk premia in the near and far futures markets. The time one optimization problems for the inventory-carrier and speculator are solved first. The perfectly competitive inventory-carrier treats the time two positions as exogenous stochastic parameters. The time one prices are taken as deterministic. Hence, from (1), (3), and the assumption of no basis risk, the time one first order conditions for the inventory-carrier are given in (15)–(17).

$$E_1P_2 - RP_1 - cS_1 = \alpha_I\Big\{[S_1 - X_{21I}]RVar_1(P_2) - RX_{31I}Cov_1(P_2,F_{32}) + Cov_1\Big(P_2,S_2\Big(P_3 - RP_2 - \frac{c}{2}S_2\Big)\Big) - Cov_1(P_2,X_{32I}(P_3 - F_{32}))\Big\} \quad (15)$$

$$-E_1P_2 + F_{21} = \alpha_I\Big\{(X_{21I} - S_1)RVar_1(P_2) + RX_{31I}Cov_1(P_2,F_{32}) - Cov_1\Big(P_2,S_2\Big(P_3 - RP_2 - \frac{c}{2}S_2\Big)\Big) + Cov_1(P_2,X_{32I}(P_3 - F_{32}))\Big\} \quad (16)$$

$$-E_1F_{32} + F_{31} = \alpha_I\Big\{RX_{31I}Var_1(F_{32}) + (X_{21I} - S_1)RCov_1(F_{32},P_2) - Cov_1\Big(F_{32},S_2\Big(P_3 - RP_2 - \frac{c}{2}S_2\Big)\Big) + Cov_1(F_{32},X_{32I}(P_3 - F_{32}))\Big\} \quad (17)$$

Anderson and Danthine (1983) derived analogs to (15) and (17) for a producer. Conditions (15) and (16) are the time one analogs of time two conditions (4) and (5), respectively. Whereas the right sides of (15) and (4) represent the one-period storage risk premia, the right sides of (16) and (5) represent the negative of the one-period risk premia in the expiring futures contract.[9]

Analogous to the inventory-carrier's time one first order conditions (16) and (17), the speculator's time one first order conditions are, respectively, (18) and (19).

$$-E_1P_2 + F_{21} = \alpha_S\Big\{RX_{21S}Var_1(P_2) + RX_{31S}Cov_1(P_2,F_{32}) + Cov_1(P_2,X_{32S}(P_3 - F_{32}))\Big\} \quad (18)$$

$$-E_1F_{32} + F_{31} = \alpha_S\Big\{RX_{31S}Var_1(F_{32}) + RX_{21S}Cov_1(F_{32},P_2) + Cov_1(F_{32},X_{32S}(P_3 - F_{32}))\Big\} \quad (19)$$

[9]From (4), the one-period time two storage risk premium depends positively upon the product of the inventory-carrier's risk-aversion, current time storage underhedged in the expiring futures contract, and the variance of the next period spot price; while from (16) the one-period time one storage risk premium depends positively upon the compounded product of the inventory-carrier's risk-aversion, current time storage underhedged in the expiring futures contract, and the variance of the next period spot price. The one-period time one storage risk premium also depends upon factors for which there is no time two analog. In particular, the time one one-period storage risk premium depends positively upon the marginal (with respect to time one storage) covariance of the one-period time one storage profits with the compounded one-period time one short position far futures market profits, the profits on the time two storage, and the profits on the short far futures market position taken at time two. From condition (17), the one-period time one far futures market risk premium depends negatively upon the compounded product of the inventory-carrier's risk-aversion, the inventory-carrier's short position in the far futures market, and the variance of the time two far futures price. It also depends negatively upon the marginal (with respect to the short position in the far futures contract) covariance of the one-period time one profits in the far futures market with the compounded one-period underhedged in the near futures contract time one storage losses, the one-period losses on storage from time two, and the one-period losses on the far futures short position taken at time two.

The second order conditions for a maximum are satisfied. Using first order conditions (16)–(19) and the time one futures market equilibrium conditions, the one-period time one futures market risk premia can be derived.

The futures markets at time one are in equilibrium when the net futures supplied is zero in each market. Using the time one futures market-clearing conditions, (7), (9), (10), (11′), the assumption that the disturbance terms are independently and symmetrically distributed, and (16)–(19), the one-period time one near futures market risk premium (the time one analog of (5′)) are derived as

$$\begin{aligned} E_1P_2 - F_{21} = {} & Rn_IS_1\theta[(n_I + ca + a\gamma n_I)^2 + n_I^2(Z + G)]\,Var\phi/(a^2\chi^2) \\ & + n_I\theta[(Z + G)n_I - R(n_I + ca + a\gamma n_I)] \\ & \times [ca + 2a\gamma n_I]E_1S_2Var\phi/(a^2\chi^2) \end{aligned} \tag{16′}$$

Analogous to (16′), the one-period time one far futures market risk premium is derived using (7), (9), (10), (12), (13), the assumption that the disturbance terms are independently and symmetrically distributed, and (16)–(19) as

$$\begin{aligned} E_1F_{32} - F_{31} = {} & Rn_IS_1\theta[(n_I + ca + a\gamma n_I)Rn_I(1 + a\gamma) \\ & + n_I(Z + G)(Rn_I + ca)]Var\phi/(a^2\chi^2) \\ & + n_I\theta[(Z + G)(Rn_I + ca) - R^2n_I(1 + a\gamma)] \\ & \times (ca + 2a\gamma n_I)E_1S_2Var\phi/(a^2\chi^2) \end{aligned} \tag{17′}$$

Using (6′), (10)–(14), the assumption that the fundamental sources of uncertainty are independently and symmetrically distributed, (16′), (11′), (17′), (16), and (17), the inventory-carrier's optimal time one near and far futures market positions are derived as (20) and (21), respectively.

$$X_{21I} = \frac{n_S\alpha_I(S_1 - E_1S_2)}{(n_S\alpha_I + n_I\alpha_S)} \tag{20}$$

$$X_{31I} = \frac{n_S\alpha_IE_1S_2}{(n_S\alpha_I + n_I\alpha_S)R} \tag{21}$$

Result 5: The inventory-carrier should hedge in the near futures contract that part of current time storage which is expected to be liquidated by the maturity of the near futures contract. Similarly, the inventory-carrier's far futures market position should be the discounted expected level of storage from the maturity of the near futures contract to the maturity of the far futures contract.

This result follows directly from (20) and (21). It is quite different from that derived in the two-period hedging models with one futures market and contrasts sharply with the prescriptions given to the inventory-carriers. The positive implications from the two-period hedging models and the advice given to the inventory-carriers is to go short in the near futures contract by the size of their current storage. Contrarily, in (20) the inventory-carrier carrying positive stocks may even be long in the near futures market when the next period is characterized by high production supply and low consumption demand. The producer analog to (21) and the interest rate discounting phenomenon in (21) were first discussed by Anderson and Danthine (1983). Anderson and Danthine do not have an analog to (20). An artifact of the discounting phenomenon is that the sum of the time one short positions in the near and far futures markets is less than the time one near futures market short position prescribed in the existing literature for the inventory-carrier. Both the near futures and far futures positions should also be discounted by the

$n_S\alpha_I/(n_S\alpha_I + n_I\alpha_S)$ term as well; this term also appeared in the time two far futures hedge term given in (6′) above.

Analogous to (20) and (21), the time one supply of near and far futures contracts from the speculators, given as (20′) and (21′), respectively, are derived from the time one futures market-clearing conditions and (20) and (21), respectively.

$$X_{21S} = -\frac{\theta n_I(S_1 - E_1S_2)}{\alpha_S} \tag{20′}$$

$$X_{31S} = -\frac{\theta n_I E_1 S_2}{\alpha_S R} \tag{21′}$$

In the far futures market, whereas the inventory-carriers are short, the speculators are long.

The model is completed by specifying the consumption demand, production supply, and carry-in at time one. The time one spot market is in equilibrium when the consumption demanded plus storage demanded equal the carry-in plus production supplied and is given as (22),

$$A_1 - aP_1 + n_I S_1 = 0 \tag{22}$$

where A_1 is a constant.

Finally, the time one storage for the inventory-carrier is determined by applying rational expectations to (9) and (10) and using these equations along with (15), (16), (22), (16′), and the futures market clearing conditions to derive (23).

$$\begin{aligned} S_1 = {} & \frac{A_3\{n_I a\chi^2 - n_I\theta Var\phi(ca + 2a\gamma n_I)[n_I(Z + G) - R(n_I + ca + a\gamma n_I)]\}}{\lambda} \\ & + \frac{A_2\{(n_I + ca + a\gamma n_I)a\chi^2 + Rn_I\theta Var\phi(ca + 2a\gamma n_I)[n_I(Z + G) - R(n_I + ca + a\gamma n_I)]\}}{\lambda} \\ & - \frac{A_1 Ra\chi^3}{\lambda} \end{aligned} \tag{23}$$

where:

$$\begin{aligned} \lambda = {} & a\chi^2[n_I(n_I + ca + a\gamma n_I) + (Rn_I + ca)\chi] \\ & + \theta n_I RVar\phi\{(n_I + ca + a\gamma n_I)[Rn_I^2(1 - a\gamma) + (n_I + ca + a\gamma n_I)^2] \\ & + (Z + G)n_I^2\chi\} \end{aligned}$$

The relative sizes of near and far futures market risk premia are examined theoretically now. Capozza and Cornell (1979), Stein (1981), and Rzepczynski (1987) empirically find that the time-to-maturity risk premia are greater in the far than in the near futures contracts. This study theoretically supports this view.

Result 6: The one-period risk premium in the far futures market exceeds that in the near futures market.

This result applies to time one and is derived using (16′), (17′), (10), and (23). The result assumes

$$\begin{aligned} (Z + G)(Rn_I + ca - n_I)(Rn_I S_1 + E_1S_2) + R[S_1(n_I + ca + a\gamma n_I) - E_1S_2] \\ \times[Rn_I(1 + a\gamma) - (n_I + ca + a\gamma n_I)] > 0 \end{aligned}$$

Result 7: The time-to-maturity risk premium is greater in the far than in the near futures market.

This result follows from result six and (5′). As of time one, the far futures market time-to-maturity risk premium (i.e., the two-period risk premium) exceeds the near futures market time-to-maturity risk premium (i.e., the one-period risk premium).

The parameters in the regressions empirically studied by Frenkel (1981), Levich (1978), and Agmon and Amihud (1981) are now theoretically examined. In light of results eight and nine below, the ambiguity of their empirical results is understandable.

Result 8: The coefficient of a futures price in a regression in which the futures price is the independent variable and the futures contract's respective subsequent maturity date spot price is the dependent variable may be greater than, less than, or equal to one. In a comparative static sense, the coefficient is positively related to Z.

To demonstrate the second part of the result, partially differentiate $Cov_1(F_{32}, P_3)/Var_1(F_{32})$ with respect to Z. With respect to the first part of the result $Cov_1(F_{32}, P_3)/Var_1(F_{32}) \gtreqless 1$ is equivalent to $(Z + G) \gtreqless R^2 n_I^2(1 + a\gamma)/[(Rn_I + ca)(\chi - n_I)]$. Under fairly general conditions, using (12), (14), and result eight, it can be shown that the intercept of the same regression is inversely related to Z; $Cov_1(P_3, F_{32})(1 + a\gamma) \geq Var_1(F_{32})$ is a sufficient condition for this result. This information should be internalized by agents using the futures price to forecast the futures contract's respective subsequent maturity date spot price.

Result 9: The coefficient of the basis in a regression in which the basis is the independent variable and the resultant spot price change between the basis observation date and the futures contract's maturity date is the dependent variable may be greater than, less than, or equal to one. In a comparative static sense, the coefficient is positively related to Z.

The result's second part is demonstrated by partially differentiating $Cov_1[F_{32} - P_2, P_3 - P_2]/Var_1(F_{32} - P_2)$ with respect to Z. For the first part of the result, $Cov_1[F_{32} - P_2, P_3 - P_2]/Var_1(F_{32} - P_2) \gtreqless 1$ is equivalent to $(Z + G) \gtreqless R[n_I(1 + a\gamma)(R - 1) - ca]/[(R - 1)n_I + ca]$. This information should be internalized by agents using the basis to forecast the spot price change.

Finally, having determined time one storage in reduced form, one can derive the reduced forms for the one-period time one near and far futures market risk premia. These risk premia may be positive, negative, or zero. Moreover, the other endogenous variables in the model can be unconditionally determined. The appendix of Leistikow (1990) contains a listing of conditions sufficient for prices, expected prices, storage, expected storage, consumption, and expected consumption to be positive in each period.

SUMMARY AND IMPLICATIONS

This study examines the effects of uncertainty shifts on spot and futures price change serial correlation and standardized covariation measures. Optimal futures positions and risk premia in near and far futures markets are determined also. The analysis is conducted in the context of the Leistikow (1990) three-period inventory-hedging model. The model resolves uncertainty for both current time and future time net consumption demand.

The novel findings and implications of the model include the following: (1) The minimum-risk hedge ratio for futures contracts is lower over announcement periods than it is over control periods. Similarly, the minimum-risk hedge ratio is lower when the amount of uncertainty to be resolved regarding future time net consumption demand is higher. The minimum-risk hedge ratio also predictably changes as the contract dynamically approaches its maturity date. (2) The hedging-effectiveness of a futures contract is nonstationary across announcement and nonannouncement periods. It is also nonstationary across periods for which the amount of uncertainty to be resolved regarding future time net consumption demand varies. (3) In efficient spot and futures markets, the sign of the serial correlation of price changes depends upon the mix of the uncertainty types to be resolved. (4) The coefficient of the futures price in a regression, where the futures price is the independent variable and the dependent variable is the futures contract's respective subsequent maturity date spot price, is positively related to the amount of uncertainty to be resolved regarding future period net consumption demand. The intercept of the same regression is inversely related to the amount of uncertainty to be resolved regarding future time net consumption demand. (5) The one-period risk premium in the far futures market is greater than that in the near futures market. Similarly, the risk premium to the futures contract's maturity date is greater for far futures contracts than it is for near futures contracts. (6) The inventory-carrier should hedge in the near futures contract that part of current time storage which is expected to be liquidated by the maturity of the near futures contract. Likewise, the inventory-carrier's far futures market position should be the discounted expected level of storage from the near futures contract's maturity to the far futures contract's maturity.

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