Memory in Interest Rate Futures

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INTRODUCTION

The behavior of futures prices has been researched extensively. In the futures literature, these prices have been widely modeled as a random walk or martingale [Goldenberg (1986); Samuelson (1965); Larson (1960); Working (1958)]. The implication is that futures price changes are not predictable, a result consistent with the efficient market hypothesis [LeRoy (1989); Fama (1976)]. By implication, a random walk model suggests that futures price changes do not have memory, meaning that the current price does not have a systematic pattern with historical prices [Goldenberg (1989)].

Early evidence suggests an interdependence of futures prices, implying that profitable trading is apt to be generated from these contracts [see, e.g., Leuthold (1972)]. In many recent studies, futures prices are found to be cointegrated with spot prices, thus suggesting that futures prices can be predicted from historical prices [Chowdhury (1991); Schroeder and Goodwin (1991); Bessler and Covey (1991)].

Using a rescaled range analysis, Helms, Kaen, and Rosenman (1984) examine the long-term dependence (i.e., the statistical dependence between events separated by successive long spans of time) in commodity futures contracts. Their results support the presence of persistent dependence, or irregular cyclic patterns, in these futures contracts. Using the same technique, other studies that examine the issue of long-term memory in stock market returns and exchange rate returns have also documented long-term dependence in these markets [Mendelbrot (1971); Greene and Fielitz (1977); Booth, Kaen, and Koveos (1981)]. However, using a modified rescaled range method, Lo (1991) does not find long-term memory in stock returns. A recent study by Peterson, Ma, and Ritchey (1992) finds a memory component in the cash prices of 17 commodities using variance ratio analysis. The finding of memory in futures price changes is important because it serves as a basis for the formulation of trading strategies based on the correlation structure of transaction price data.

The purpose of this article is to examine the long-range memory in Eurodollar and Treasury bill futures markets. These futures contracts are selected because they are highly liquid and very actively traded. Using the traditional and modified rescaled range methods,

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both the Eurodollar and Treasury bill futures markets are found to be without long-term memory, a result consistent with the notion that speculative prices are not predictable.

This article is organized as follows. The next section is a discussion of the null hypothesis in the rescaled range analysis, first proposed by Mandelbrot (1971), among others, and later modified by Lo (1991) for detecting long-term memory in financial assets. The third section presents the empirical results of the daily, weekly, and monthly rates for the Eurodollar and Treasury bill futures markets. The final section contains the conclusion.

TESTING NULL HYPOTHESIS

The R/S Statistic

The traditional method of detecting long-range dependence is the rescaled range statistic (R/S), denoted by V_n^* (where n is the number of observations). The R/S is the range of partial sums of deviations of a time series or process, X, from its mean, rescaled by its standard deviation, s_n [Mandelbrot (1971)]. In this case, $X = r_t - r_{t-1}$, where r_t is either the Eurodollar or Treasury bill futures interest rate at time t. Thus, X represents the increment or changes in the interest rates. The traditional rescaled range statistic (R/S) is computed as follows:

$$V_n^* = (1/s_n)\left[\operatorname*{Max}_{1\le k\le n}\sum_{j=1}^{k}(X_j - \overline{X}_n) - \operatorname*{Min}_{1\le k\le n}\sum_{j=1}^{k}(X_j - \overline{X}_n)\right] \tag{1}$$

where

$$s_n^2 = (1/n)\sum_{j=1}^{n}(X_j - \overline{X}_n)^2$$

The term "strong mixing" is adopted as an operational definition of short-range dependence. Heuristically speaking, a stochastic process, X, is strong mixing if the maximal dependence of two stochastic events approaches zero when the time span between them increases. If a process is strong mixing, the spectral density at frequency zero is zero.[1] Since short-range/long-range dependence is a matter of degree, strong mixing is a desirable operational definition to draw a line between them. Also, hypothesis testing involving strong mixing could include all Gaussian finite-order stationary autoregressive and moving average (ARMA) models and heterogeneously distributed sequences in the analysis.

The null hypothesis is that the process of the interest rate changes is strong mixing (i.e., short-range dependence). The process is assumed to have two components: (A1) an arbitrary, fixed constant and a random variable, ϵ, which has a zero mean, and (A2) bounded variances at all times, t. ϵ is not required to be independently, identically, distributed (i.i.d.). However, to develop a sampling theory of the test statistic, a further restriction (A3) is imposed that the maximum of the higher (i.e., greater than 2) absolute moments of the random variable must be finite. Also, (A4) the maximal degree of dependence of the random variable must decline at a rate restricted by the order of the existing absolute moments.

[1]Spectral density (defined on a frequency domain) is the Fourier cosine transform of the autocorrelation function of a time series. Knowledge of the autocovariance function is mathematically equivalent to knowledge of the spectral density and vice versa.

Condition (A1) is standard. Conditions (A2) and (A3) rule out infinite variance marginal distributions of ϵ_t but still allow unconditional leptokurtosis via time-varying conditional moments such as conditional heteroscedasticity. There is ample evidence of skewness and changing variance such as the generalized autoregressive conditional heteroscedastic (GARCH) model in futures contracts [Junkus (1991); Myers (1991)], but unstable volatilities can be a symptom of conditional heteroscedasticity which can manifest itself in stationary time series. Thus, the conditions (A1–A3) are likely to be satisfied in the interest rate series.

Modified R/S Statistic

In the modified R/S statistic, $V_n(q)$ is similarly defined as in the traditional R/S model except s_n is replaced with $\sigma_n(q)$, where q is the lag length. $\sigma_n(q)$ includes the autocorrelations of the time series up to lag q. In calculating $\sigma_n(q)$, the selected number of lags (q) cannot be too large or too small because the sample distribution of the estimator changes. The modified R/S statistic is computed as:

$$V_n(q) = (1/\hat{\sigma}_n(q))\left[\operatorname*{Max}_{1\le k\le n}\sum_{j=1}^{k}(X_j - \overline{X}_n) - \operatorname*{Min}_{1\le k\le n}\sum_{j=1}^{k}(X_j - \overline{X}_n)\right] \qquad (2)$$

where

$$\hat{\sigma}_n^2(q) = (1/n)\sum_{j=1}^{n}(X_j - \overline{X}_n)^2 + (2/n)\sum_{j=1}^{q} w_j(q)\left\{\sum_{i=j+1}^{k}(X_i - \overline{X}_n)(X_{i-j} - \overline{X}_n)\right\}$$

and,

$$w_j(q) = 1 - j/(q+1), \qquad q < n$$

In the null hypothesis for the modified R/S statistic, in addition to the above assumptions (A1–A4), a revised assumption (A3) is used, and an additional assumption (A5) is imposed. The revised condition (A3) is that the maximum of the even higher (i.e., greater than 2) absolute moments of the random variable in the process are finite. Condition (A5) requires that the truncation lag q increases at a lower rate than the sample size, n such that $q \sim o(n^{1/4})$. Conditions (A1), (A2), revised (A3), (A4), and (A5) will guarantee the consistency of the modified R/S statistic. (Under some special cases, the revised conditions (A3) and added condition (A5) could be relaxed to preserve consistency.)

The classical R/S statistic is a useful tool in detecting long-range dependence in highly non-Gaussian series with large skewness and kurtosis [Mandelbrot and Wallis (1969)]. In addition, it can detect nonperiodic cycles, cycles with periods equal to or greater than the sample period [Mandelbrot (1972)]. However, the classical R/S statistic is sensitive to short-range dependence. Rejection of the null hypothesis may merely be a symptom of short-term memory. For example, the classical R/S statistic may be biased if the random component of the underlying process exhibits first-order autocorrelation [Lo (1991)]. However, the modified R/S statistic is robust to the short-range memory.

The null hypothesis of no long-range dependency in the time series can be tested using the rescaled range method. Critical values for testing significance of $V_n(q)$ and V_n^* are tabulated in Lo (1991). At a 5% level, the critical value for testing the null hypothesis of no memory is contained in the interval [0.0809, 1.862].

EMPIRICAL ANALYSIS AND RESULTS

Eurodollar and Treasury bill futures contracts are traded in the International Monetary Market (IMM) of the Chicago Mercantile Exchange. These contracts are based on a 3-month, $1 million Eurodollar time deposit or Treasury bill. Daily observations of interest rate for both Eurodollar and Treasury bill nearby futures contracts are from the *Wall Street Journal.* When a futures contract comes to maturity, the next nearby contract is used.[2] The study covers the period from May 1982 to the end of 1991.

In the empirical analysis, the increment of interest rates are used because using the level of interest rate entails a stationarity problem. The standard augmented Dickey–Fuller (ADF) test is used to test for a unit root in both Eurodollar and Treasury bill interest rates futures as follows:

$$\Delta r_t = b_0 + b_1 r_{t-1} + \Sigma c_k \Delta r_{t-k} + u_t \tag{3}$$

where r_t is the interest rate in either Eurodollar or Treasury bill futures contracts; u_t is the random noise; and k is the number of lags in the autoregressive representation of r_t. If the autoregressive representation of r_t contains a unit root, the t-ratio for the parameter b_1 should be consistent with the hypothesis $b_1 = 0$. Conventional t-tables are inappropriate for this hypothesis test, so the results of Dickey and Fuller (1979) and the tabulated distribution in Fuller (1976) are used to interpret the t-ratio.

Table I reports the unit root tests, which indicate that the Eurodollar and Treasury bill interest rates futures have a unit root while their first difference or their increment is stationary. In this analysis, k is selected to be 20. Results are similar for other lag lengths, not reported here. These results suggest that a first difference of these interest rate data is necessary before conducting the rescaled range analysis. The results of finding a unit root in these interest rates futures are consistent with the findings in the literature that report nonstationarity in futures prices [see e.g., Goldenberg (1989)].

Table II reports results using the traditional rescaled range method and Lo's (1991) modified rescaled range statistics for various lags. The results indicate that both methods show no statistical significance at a conventional 5% level. Regarding the memory issue

Table I
TESTS OF UNIT ROOT USING AUGMENTED DICKEY–FULLER TEST (ADF)

Panel (a): Test For the Level	**Eurodollar** b_1	**Treasury bill** b_1
Daily	−0.003	−0.004
	(−2.29)	(−2.11)
Weekly	−0.013	−0.008
	(−1.66)	(−1.03)
Monthly	−0.038	−0.039
	(−1.06)	(−1.11)

(continued)

[2]The interest rate behavior in either futures market appears to be smooth across contracts. This is most likely due to the liquidity of the contracts.

Table I (continued)

Panel (b): Test for the first difference		
Daily	−1.055	−1.35
	(−10.09)[a]	(−10.07)[a]
Weekly	−0.878	−1.059
	(−4.81)[a]	(−5.30)[a]
Monthly	−1.148	−1.045
	(−3.19)[a]	(−2.82)[b]

ADF-t values are in parentheses.
[a]Significance at a 5% level.
[b]Significance at a 10% level. The unit root test is conducted as follows [eq. (3) in text]:

$$\Delta r_t = b_0 + b_1 r_{t-1} + \Sigma c_k \Delta r_{t-k} + u_t$$

where Δr_t denotes either the change in Eurodollar or Treasury bill futures interest rates.

in futures prices, the conclusion of this study differs from that of Helms, Kaen, and Rosenman (1984). Because the Eurodollar and Treasury bill futures contracts are highly liquid and actively traded as compared to the commodity futures contracts used by Helms, Kaen, and Rosenman, it is expected that these interest rates futures markets are efficient, and not surprising that cyclical patterns are not found in these interest rates data.

Table II
R/S ANALYSIS OF DAILY EURODOLLAR AND T-BILL FUTURES RATES FROM MAY, 1982, TO DECEMBER, 1991, USING THE TRADITIONAL RESCALED RANGE, V_n^* AND THE MODIFIED RESCALED RANGE, $V_n(q)$.

	Eurodollar	Treasury Bill
$V_n(50)$	1.41	1.38
%-bias	−2.1	−22.9
$V_n(100)$	1.35	1.34
%-bias	2.5	−20.7
$V_n(150)$	1.32	1.32
%-bias	4.4	−19.7
$V_n(200)$	1.33	1.32
%-bias	3.8	−19.6
$V_n(250)$	1.34	1.36
%-bias	2.8	−21.8
$V_n(300)$	1.35	1.37
%-bias	1.8	−22.8
$V_n(350)$	1.36	1.37
%-bias	1.1	−22.9
$V_n(400)$	1.38	1.38
%-bias	0.0	−23.2
V_n^*	1.38	1.06

Rates are differenced for stationarity purpose. Numbers in %-bias rows are calculated as $[(V_n^*/V_n(q) - 1] \cdot 100$.

Table III
R/S ANALYSIS OF EURODOLLAR AND T-BILL FUTURES RATES FROM MAY, 1982, TO DECEMBER, 1991, USING THE TRADITIONAL RESCALED RANGE, V_n^* AND THE MODIFIED RESCALED RANGE, $V_n(q)$.

(a) Weekly Rates

	Eurodollar	**Treasury Bill**
$V_n(20)$	1.30	1.21
%-bias	−5.0	−13.4
$V_n(30)$	1.55	1.19
%-bias	−5.2	−14.4
$V_n(40)$	1.28	1.22
%-bias	−7.12	−16.0
$V_n(50)$	1.30	1.25
%-bias	−8.7	−18.2
$V_n(60)$	1.32	1.27
%-bias	−10.0	−19.4
$V_n(70)$	1.33	1.27
%-bias	−10.7	−19.3
V_n^*	1.10	1.02

(b) Monthly rates

	Eurodollar	**Treasury bill**
$V_n(6)$	1.16	1.13
%-bias	8.2	5.0
$V_n(9)$	1.15	1.13
%-bias	9.5	5.1
$V_n(12)$	1.18	1.18
%-bias	6.4	1.1
$V_n(15)$	1.20	1.20
%-bias	4.5	−0.6
V_n^*	1.26	1.19

Numbers in %-bias rows are calculated as $[(V_n^*/V_n(q) - 1] \cdot 100$.

Table III shows the results of Lo's modified rescaled range statistics for the weekly and monthly rates that have been differenced for inducing stationarity. Consistent with the daily results, the traditional and modified rescaled range statistics do not indicate statistical significance at a conventional 5% level. This result supports the null hypothesis of no memory in both interest rate series. This result of no memory in the interest rate futures data in a longer period is consistent with the recent results examining the long-horizon return in the stock market literature [McQueen (1992)].

CONCLUSION

Using both the traditional and modified rescaled range method, this article examines the long-term dependency in the Eurodollar and Treasury bill futures markets. For daily, weekly, and monthly data from 1982 to the end of 1991, the results indicate that these futures markets do not have memory, a result supporting the weak-form efficient market hypothesis.

These results differ from the existing futures literature that supports memory in the commodity futures contracts. If the source of serial correlation is lagged adjustment to new information, the absence of strong dependence in interest rate futures should not be surprising from an economic standpoint because Eurodollar and Treasury bill futures contracts are very highly liquid and actively traded. An important economic implication of the results is that it is unlikely to have profitable trading based on historical rates in these interest rate futures contracts due to lack of a detectable pattern.

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