Reliability of Soybean and Corn Option-Based Probability Assessments

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INTRODUCTION

Individuals who attempt to select production, marketing and/or financial strategies could improve their decisions by using probabilities of future price ranges rather than relying on the use of point forecasts. For example, grain and oilseed producers could determine the probability that the price would be below their cost of production. Such information would be useful in allocating land to corn and soybean production.

The probabilities of future price ranges can be generated by using historical price data, by surveying market participants, or by using the option-based probability assessment method. In general, the option-based probability assessment method is the preferred generating method. The probabilities generated by this method are based on futures prices and option premiums and thus reflect the price expectations of market participants. These probability assessments are relatively cheap to generate and can always be up-to-date, because they can be generated using current price information.

Before using the probability assessments generated from option markets, decision-makers need to know whether these assessments are a reliable representation of the actual stochastic behavior of prices. Option-based probability assessments are considered to be reliable if statistical test results imply no problems in assessing the parameters that describe the distribution function of a given price [Sherrick, Irwin and Forster (1990)]. If probability assessments are reliable, then they can be used without any adjustment (calibration). However, if problems are found in assessing the parameters of the distribution, then option-based probability assessments must be calibrated to make them reliable.

The reliability of option-based probability assessments can be questioned for two basic reasons. First, the probability assessments are derived from option pricing models that make simplifying assumptions (e.g., assuming no taxes, no transaction costs, and

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European rather than American options).[1] However, empirical studies have shown that the option-pricing models perform well for the nearest-to-the-money American options [Plato (1985)]. Second, futures and/or options markets may be inefficient (i.e., prices may be biased because of normal backwardation, low volume, etc.). Therefore, reliability has to be determined empirically for each specific options market [Fackler and King (1990)].

In an empirical study of the soybean, corn, live cattle and hog markets during 1985–1988, Fackler and King (1990) found differences across commodities in the reliability of option-based probability assessments generated assuming lognormality. Fackler and King (1990) hypothesized that the need for calibration of probability assessments for some commodities (soybeans and hogs) may be caused by the immaturity of agricultural options markets. Even though option premiums may have been biased when options first began trading, a more accurate process of determining option prices may have evolved over time. In addition, the increased volume and increased liquidity of option markets as they mature may improve the reliability of the probability assessments over time. The need for calibration in certain markets may be caused also by the method used to derive the probability distribution. The finding of unreliability may be caused by the lognormality assumption if the lognormal distribution does not approximate the true distribution well enough.[2] Possibly, option-based probability assessments derived using an alternative approach may be reliable.

This article represents an extension of Fackler and King's (1987; 1990) research by evaluating the reliability of soybean and corn option-based probability assessments during 1985–1990. The specific objectives of this research are: (1) to determine whether assessments become reliable as option markets mature, by comparing reliability during two sub-periods (1985–1987 and 1988–1990), and (2) to determine whether reliability is dependent on the approach used to derive the probability distribution (in particular, whether reliability is dependent on the specific distributional form assumed).

Soybeans and corn are analyzed for two reasons. First, options on soybean and corn futures contracts are two of the most actively traded agricultural options in the U.S. Second, Fackler and King (1990) found differences in the reliability of soybean and corn probability assessments during 1985–1988.

Two approaches are used to generate probability assessments 60 days before option expiration. In the first approach, the underlying distribution is assumed to be lognormal. In the second approach, no assumption is made regarding the underlying distribution. The second approach seems more general because no restrictions are placed on the distributional form. However, the increase in generality regarding distributional form is possible only by introducing the possibility of large measurement errors. In particular, only a few points on a distribution can be derived using this approach. Thus, the complete distribution can be derived only through interpolation and extrapolation.

In the following section, the two approaches to derive option-based probability assessments are described. The concept of a calibration function is presented next as

[1]European options can be exercised only at the maturity date. American options can be exercised at any time during the life of the option.

[2]In an earlier article, Fackler and King (1987) analyzed the reliability of cattle and soybean probability assessments during the early years of option trading (February, 1985 to February, 1987 for cattle and March, 1985 to January, 1986 for soybeans). Probability assessments were derived assuming lognormality, assuming the Burr-12 distribution, and making no assumption about the distributional form. They found that the probability assessments from the three methods generally differed by only a few percentage points. They also found problems in assessing the probabilities for soybeans but no problems for cattle. However, they indicated that their conclusions were preliminary given their small data set caused by the recent introduction of trading in agricultural options.

well as statistical tests to evaluate the reliability of option-based probability assessments. The article concludes with a discussion of the empirical results and implications of these results.

OPTION-BASED PROBABILITY ASSESSMENTS

Assuming a Log-Normal Distribution

If the expiration date futures price, i.e., the closing or settlement futures price on the options expiration day, is lognormally distributed, then the cumulative distribution function (cdf) is:

$$F(P_T) = N((\ln(P_T) - \mu)/\sigma) \tag{1}$$

where

P_T = futures price at option expiration, which is a random variable;
ln = natural logarithm;
μ = mean of distribution; and
σ = standard deviation of distribution.

Thus, the cdf is completely specified once the mean and variance are known. If the expected expiration date futures price equals the current futures price, the assessed mean of the distribution function is [Fackler and King (1990)]:[3]

$$\mu = \ln(P_o) - \sigma^2/2 \tag{2}$$

where P_o = current futures price. The variance of the distribution can be estimated by substituting the current premium, strike price, futures price, risk-free interest rate, and time to maturity into the Black (1976) option-pricing model.[4]

Assuming No Theoretical Distribution

If no theoretical distribution form is assumed, then King and Fackler (1985) calculate the cumulative probability of the average of any two consecutive strike prices from the call and put premiums, respectively, as

$$F((P_s^1 + P_s^2)/2) = 1 - \{[V_c(P_s^1, t) - V_c(P_s^2, t)]e^{rt}/(P_s^2 - P_s^1)\} \tag{3}$$

and

$$F((P_s^1 + P_s^2)/2) = e^{rt}(V_p(P_s^2, t) - V_p(P_s^1, t))/(P_s^2 - P_s^1) \tag{4}$$

where P_s^1, P_s^2 = strike or exercise prices with $P_s^2 > P_s^1$; $F((P_s^1 + P_s^2)/2)$ = cdf of P_T evaluated at the average of the two strike prices; V_c, V_p = option premium, where c denotes a call and p denotes a put; t = time to option maturity; e = exponential value (i.e., 2.7183); and r = risk-free interest rate.

[3]The expected futures price at any time is equal to the current futures price if the systematic risk of a futures contract is zero [Black (1976)]. Dusak (1973) found the systematic risk of soybean and corn futures contracts to be approximately zero.

[4]An iterative procedure or a direct method can be used to assess the variance by substituting values for the premiums and values for all variables other than the variance into the Black option-pricing model. Curtis and Carriker's (1988) direct method is used here, because it is more expeditious than an iterative procedure and it provides a reasonable estimate of the variance if nearest-to-the-money options are used. Following Curtis and Carriker (1988), the estimate of the standard deviation used is a simple average of the individual values of the standard deviation obtained from the Black (1976) option-pricing formulas for puts and calls.

The cumulative probability, given by a simple average of the results of eqs. (3) and (4), represents one point on the assessed cdf of P_T.[5] Different points on the curve can be identified by repeating the procedure for different strike prices. However, this procedure can identify only a few points on the assessed cdf (i.e., the number of strike prices available for each option contract minus one). The entire assessed cdf can be derived by linear interpolation and extrapolation based on identified points [King and Fackler (1985)].[6]

EVALUATION OF OPTION-BASED PROBABILITY ASSESSMENTS

A calibration function can be used to evaluate the reliability of the option-based probability assessments and to transform the original assessments into calibrated (reliable) assessments [Fackler and King (1990)]. However, a calibration function can be estimated only if a random sample of independent probability assessments exists. This condition is fulfilled in this analysis if one option-based probability assessment is generated for each futures contract and the forecasting and expiration dates do not overlap [Fackler and King (1990)]. Therefore, in this article, the calibration function represents the relationship between the assessed probability of the actual futures prices at option expiration for n option contract months and the "real" probability.

Following Fackler and King (1990), the empirical calibration function, $C(U_T)$, is specified as

$$C(U_T) = T/n \quad \text{for } T = 1, 2, \ldots, n \tag{5}$$

where $U_T = F(P_T^a)(T = 1, 2, \ldots, n)$ = the cumulative probability assessment for P_T^a, the actual futures price at the maturity date T, and the values of U_T have been placed in ascending order (i.e., $U_1 \leq U_2 \leq \ldots \leq U_n$); and $\frac{T}{n}$ = the estimator of the "real" cumulative probability.

Option-based probability assessments are perfectly calibrated if $C(U_T) = U_T$ for all U_T on the unit interval $[0, 1]$. In the plane $(U, C(U))$, perfect calibration is described by the 45° line. The shape of the estimated calibration curve may indicate particular assessment problems. An S-shaped curve shows underconfidence (i.e., the variability of the assessed distribution function is overassessed). A reverse-S-shaped curve indicates overconfidence. A concave curve indicates overestimation (i.e., the measure of central tendency, called the location of the distribution, is overassessed). A convex curve shows underestimation [Curtis et al. (1988)].

EXAMPLE

In this section, an example is given using only five soybean contract months to illustrate the derivation of a calibration curve (Table I). The lognormality assumption is used to generate probability assessments in this example. The standard deviation (given in column two) is estimated using the Black (1976) formula for each option contract month. The estimated standard deviation and the futures price 60 days before option expiration (given in column three) are substituted into eq. (2) to estimate the mean of the distribution (in

[5]Equations (3) and (4) are derived by using the mean value theorem for integrals which is by definition correct for infinitesimally small intervals. The use of this theorem for the interval between strike prices necessarily introduces estimation error.

[6]The first identified point on the cdf is joined with the origin to estimate the first part of the distribution. If the cumulative probability of the last identified point is less than one, then the rest of the cumulative distribution function is derived by extending the line connecting the last two points that were explicitly identified (keeping the slope constant) until the cumulative probability is equal to one.

Table I
EXAMPLE TO DEMONSTRATE ASSESSED CUMULATIVE PROBABILITIES FOR EXPIRATION DATE FUTURES PRICES FOR FIVE SOYBEAN CONTRACT MONTHS

Contract Month	Estimated Standard Deviation at Option Expiration	Futures Price 60 Days Before Expiration	Estimated Mean in Natural Log Form at Option Expiration	Futures Price at Option Expiration	Assessed Cumulative Probability
	%	¢/bu.		¢/bu.	%
Mar. 85	6.38	607.50	6.4073	596.50	39.96
May 85	7.92	602.00	6.3971	598.50	48.64
Jul. 85	6.71	601.50	6.3972	583.00	33.28
Nov. 85	9.03	520.00	6.2498	502.25	36.71
Jan. 86	8.46	513.50	6.2377	523.00	60.22

column four). The assessed cumulative probability of the expiration date futures price (in column six) is determined by substituting the estimated mean and standard deviation of the distribution and the actual futures price at option expiration (in column five) into eq. (1).

Table II gives the actual values that would be used to generate the calibration function for the five soybean contract months used in Table I. First, the assessed cumulative probabilities (given in Table I) are placed in ascending order (Table II). The "real" cumulative probabilities are calculated by dividing the order number of a given observation by the sample size (five in this example). Equation (5) establishes the relationship between the assessed cumulative probabilities and the "real" cumulative probabilities. The assessed and "real" cumulative probabilities can be plotted in $U, C(U)$-space to generate the calibration function.

TESTS OF CALIBRATION

If the calibration curve coincides with the 45° line, then U is uniformly distributed on the unit interval. Thus, one can test whether the calibration curve is significantly different from the 45° line by testing for uniformity on the unit interval [Fackler and King (1990)].

The Cramer–von Mises statistic (W^2), the Watson statistic, (U^2), and the Anderson–Darling statistic (A^2) are considered the best tests for uniformity on the unit interval

Table II
EXAMPLE TO DEMONSTRATE ASSESSED AND "REAL" CUMULATIVE PROBABILITIES FOR EXPIRATION DATE FUTURES PRICES FOR FIVE SOYBEAN CONTRACT MONTHS

Contract Month	Ordered Assessed Cumulative Probability	"Real" Cumulative Probability
	%	%
Jul. 85	33.28	20
Nov. 85	36.71	40
Mar. 85	39.96	60
May 85	48.64	80
Jan. 86	60.22	100

[Stephens (1974); Quesenberry and Miller (1977)]. The Cramer–von Mises and the Anderson–Darling tests are more powerful in detecting problems in assessing location, while the Watson test is more powerful in detecting variability assessment problems [Stephens, (1974)].[7]

Two-tailed sign tests are also conducted in the interval $[0, 0.5]$ and in the interquartile range (IQR) $[0.25, 0.75]$. The null hypothesis states that the percentage of U_T values in each interval follows a binomial distribution with a probability of "success" equal to 50%. Over- or underestimation is detected by the sign test in the interval $[0, 0.5]$. Over- or underconfidence is detected by the sign test in the IQR [Fackler and King (1990)].

DESCRIPTION OF THE DATA

As noted above, option-based probability assessments are generated 60 days before option contract expiration. Soybean futures contracts expire in January, March, May, July, August, September, and November. Corn futures contracts expire in March, May, July, September, and December. An option on a given soybean and corn futures contract expires in the month before the respective futures contract delivery month.

Following Fackler and King (1990), some option contract months are deleted to eliminate the possibility of overlapping time periods (and thus obtain a random sample of independent probability assessments). In addition, those contract months that have fewer than three strike prices with volume in both put and call options are omitted.[8] The decision to delete these contract months is made for two reasons. First, if a put or call option has zero volume, the option does not trade on that day, implying that the reported premium is not a transaction price. Second, an arbitrary decision is made that at least two points on the pdf (or cdf) of a futures price should be identified using the distribution-free approach.[9]

In the distribution-free approach, in a few contract months the calculated cumulative probability of some average strike prices is negative or greater than one.[10] All cumulative probabilities that do not obey the laws of probability are deleted. In addition, in a few contract months the calculated cumulative probability for two consecutive averages of strike prices decreases. In this case, the cumulative probability associated with the second average of strike prices is deleted.

In an attempt to begin the data set as soon as possible after option trading began and to get equal sample sizes for both commodities, the data sets defined for soybeans and corn are March, 1985–May, 1990 and July, 1985–July, 1990, respectively. A global sample size of 23 observations is obtained for each commodity. The data sets are divided into 1985–1987 and 1988–1990 to analyze whether option-based probability assessments become reliable as option markets mature.

Closing (settlement) futures prices and nearest-to-the-money option premiums are obtained from the Statistical Annuals (CBOT) before 1986 and from Chicago Board

[7]The Cramer–von Mises statistic and the Watson statistic have been modified to remove the sample size dependence [Stephens (1974)]. The modified forms of these statistics (denoted by W^{2*} and U^{2*} are used in this analysis.

[8]A given contract month is dropped only if that condition is not satisfied on the calculated forecasting day and on the previous and next two trading days.

[9]The September, 1985 and May, 1987 soybean contracts and the September, 1986 corn contract are omitted from the analysis based on this rule. In each case, these contracts are skipped and the following contract months included.

[10]For example, such results can occur when the time value increases as the option becomes more in-the-money [see eqs. (3) and (4)]. Such findings are rare in this analysis and occur in the early part of the data set.

of Trade data tapes after 1986. The interest rate on the Treasury Bill that matures as close as possible to the option maturity date, obtained from the Wall Street Journal, is the risk-free interest rate. The interest rate on Treasury Bills is a simple average of the interest rates implied from the bid and ask prices.

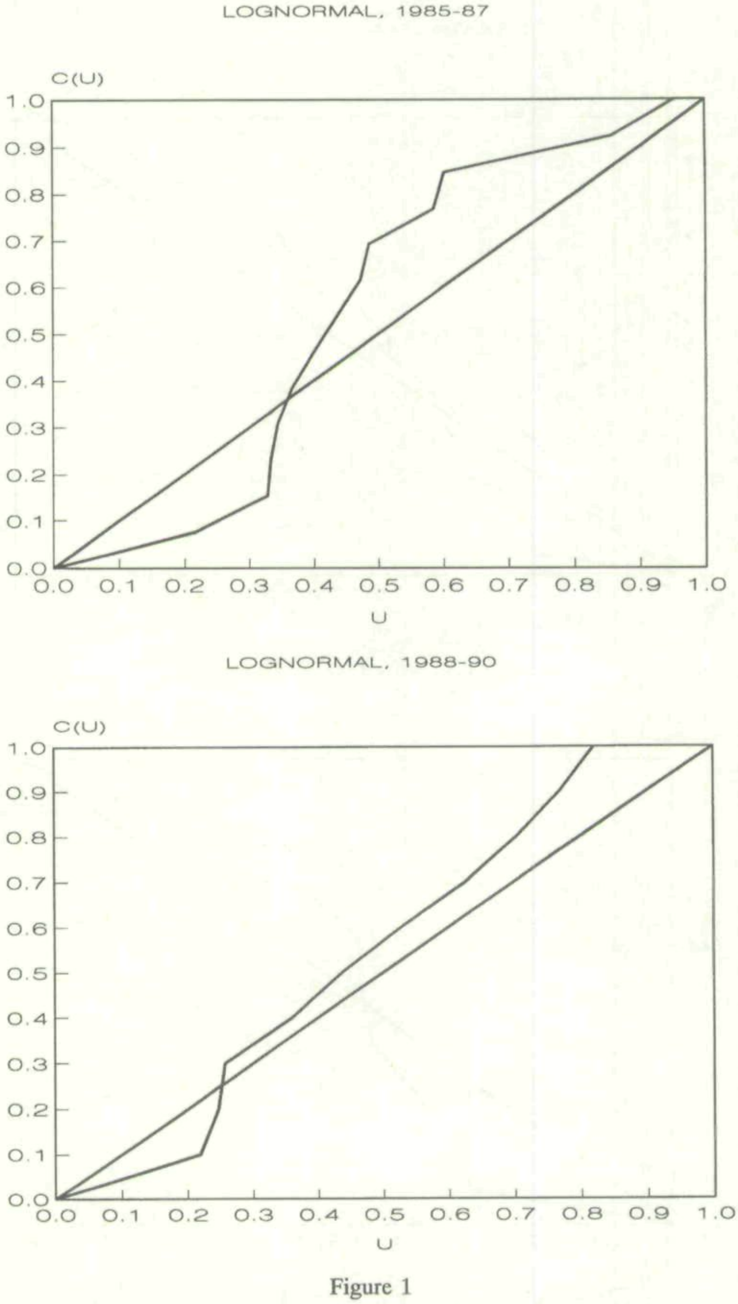


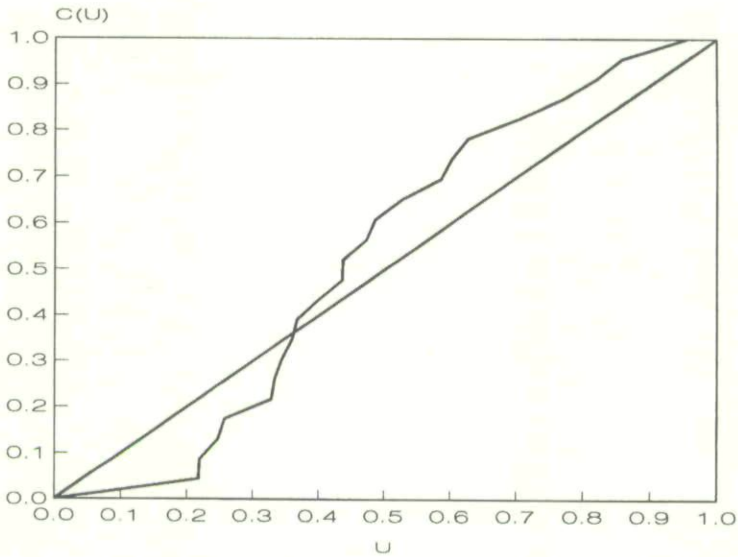
Figure 1
Empirical calibration functions for soybean contract months.

EMPIRICAL RESULTS

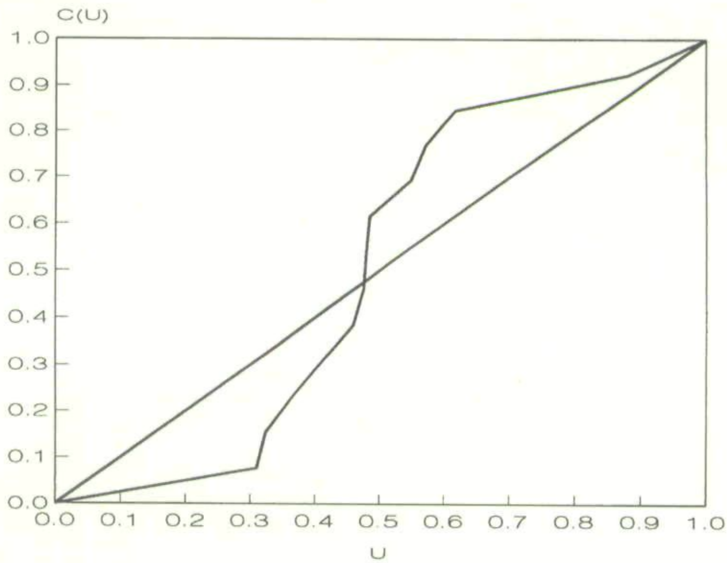
The three calibration curves for soybeans using the lognormal approach and the curve for 1985–1987 using the distribution-free approach are S-shaped, suggesting underconfidence (Fig. 1). However, no specific assessment problem can be identified based on the shapes of the remaining two calibration curves.

The statistical test results also provide evidence of underconfidence for soybean forecasts using both approaches during the entire period 1985–1990 and the subperiod

LOGNORMAL, 1985-90



DISTRIBUTION FREE, 1985-87



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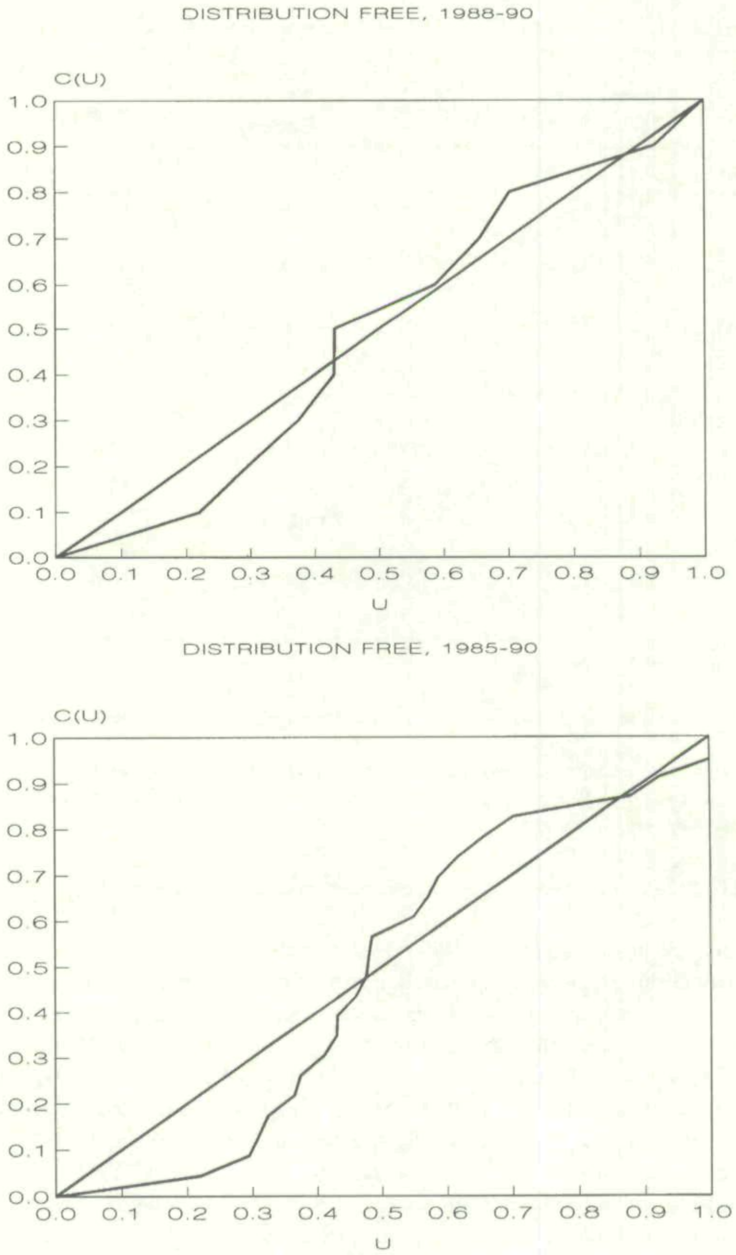


Figure 1 (continued)

1985–1987 (Table III). During both periods, assessment problems in terms of variability are indicated at the 5% level by the Watson test and the sign test in the IQR. For the later subperiod 1988–1990, the deviations from the 45° line are not significant at the 10% level.

For corn, the three calibration curves using the distribution-free approach and the curve for 1988–1990 using the lognormal approach are S-shaped suggesting underconfidence

Table III
STATISTICAL TEST RESULTS FOR SOYBEAN OPTION CONTRACT MONTHS

Sample	Cramer–von Mises W^{2*}	Watson U^{2*}	Anderson–Darling A^2	Sign Tests (%)	
				[0, 0.5]	[0.25, 0.75]
Lognormal					
1985–1987	0.2048	0.2222[a]	1.1678	0.6923 (0.0922)[c]	0.7692[a] (0.0224)
1988–1990	0.0431	0.0692	0.5651	0.5000 (0.7540)	0.6000 (0.3438)
1985–1990	0.2175	0.2272[a]	1.3948	0.6087 (0.2100)	0.6957[a] (0.0346)
Distribution-Free					
1985–1987	0.2647	0.2673[b]	not defined[d]	0.6154 (0.2668)	0.8462[b] (0.0034)
1988–1990	0.0638	0.0483	not defined[d]	0.5000 (0.7540)	0.7000 (0.1094)
1985–1990	0.2972	0.2580[a]	not defined[d]	0.5652 (0.4048)	0.7826[b] (0.0026)

[a]Significance at the 5% level.
[b]Significance at the 1% level.
[c]p-values are shown in parentheses. p-values are calculated considering strict inequalities only, making the test stricter.
[d]In the distribution-free approach, the cumulative probability of the expiration date futures price is one for the July, 1987 and August, 1988 contract months. The Anderson–Darling statistic is not defined if $U_T = 0$ or $U_T = 1$ because the natural logarithm of zero is minus infinity.

(Fig. 2). The remaining two curves do not have shapes indicative of specific assessment problems.

The statistical test results provide no evidence of assessment problems for corn forecasts using the lognormal approach during any time period analyzed (Table IV). However, some evidence of problems in assessing variability is apparent for corn forecasts using the distribution-free approach during 1985–1990 and during the subperiod 1985–1987 (Table IV). During those periods, both the Watson statistic and the sign test statistic in the IQR are significant at the 5% level. In addition, there is limited evidence of problems in assessing location during the period 1985–1990, given the significance of the Cramer–von Mises statistic. However, in the time period 1988–1990, statistical test results indicate no problems in assessing location or variability at the 10% level. Therefore, the deviation of the calibration curves from the 45° line is not statistically significant at the 10% level.

IMPLICATIONS

In 1985–1987, assessment problems (in location and/or variability) are found for soybean option-based probability assessments using both approaches and for corn forecasts if no assumption is made regarding the distribution. However, corn option-based probability assessments are found to be reliable at the 10% significance level if lognormality is assumed. The results found by Fackler and King (1990) for soybean and corn markets during 1985–1988 are corroborated by the results of this study for 1985–1987 for the lognormal approach. In addition, the results of this study indicate that soybean and corn

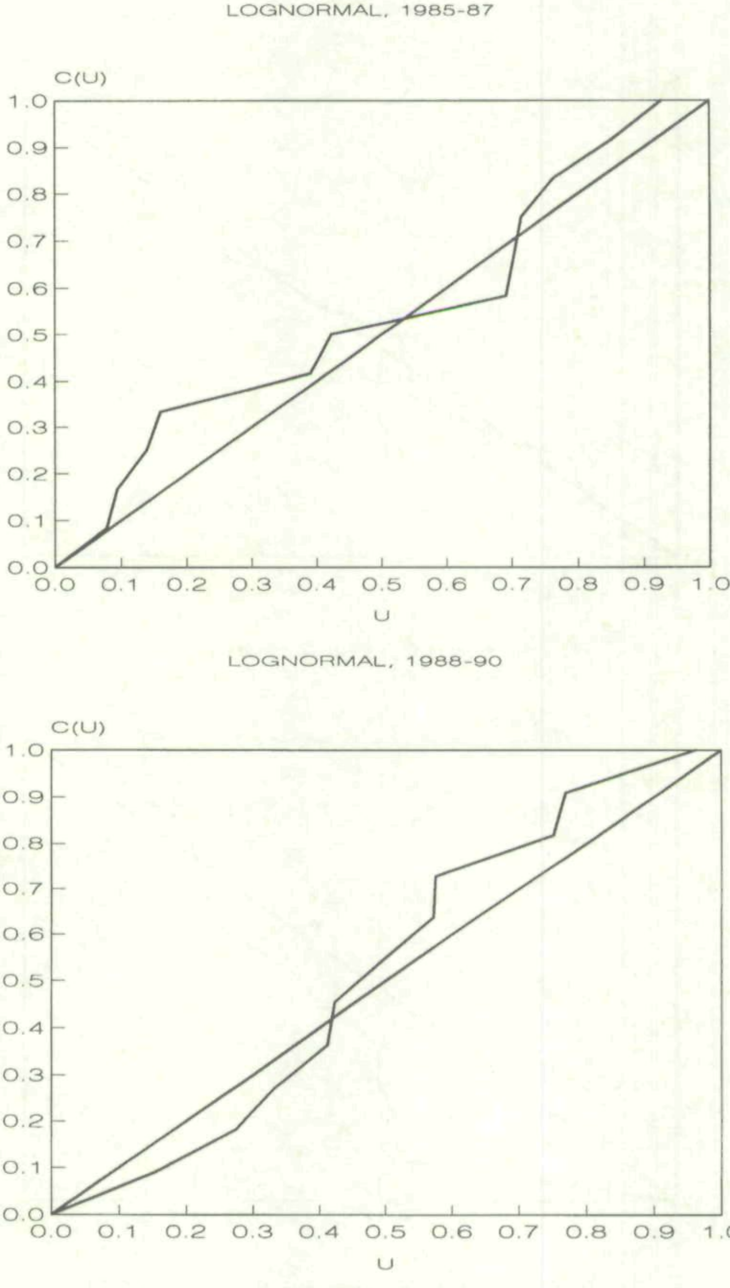


Figure 2
Empirical calibration function for corn contract months.

option-based probability assessments are reliable in the period 1988–1990 regardless of the approach used.[11]

[11]In addition to the analyses reported in this study, similar analyses were conducted for a forecasting date of 120 days using the lognormal assumption and the distribution-free assumption. The general implications from the results are the same as those reported in this article.

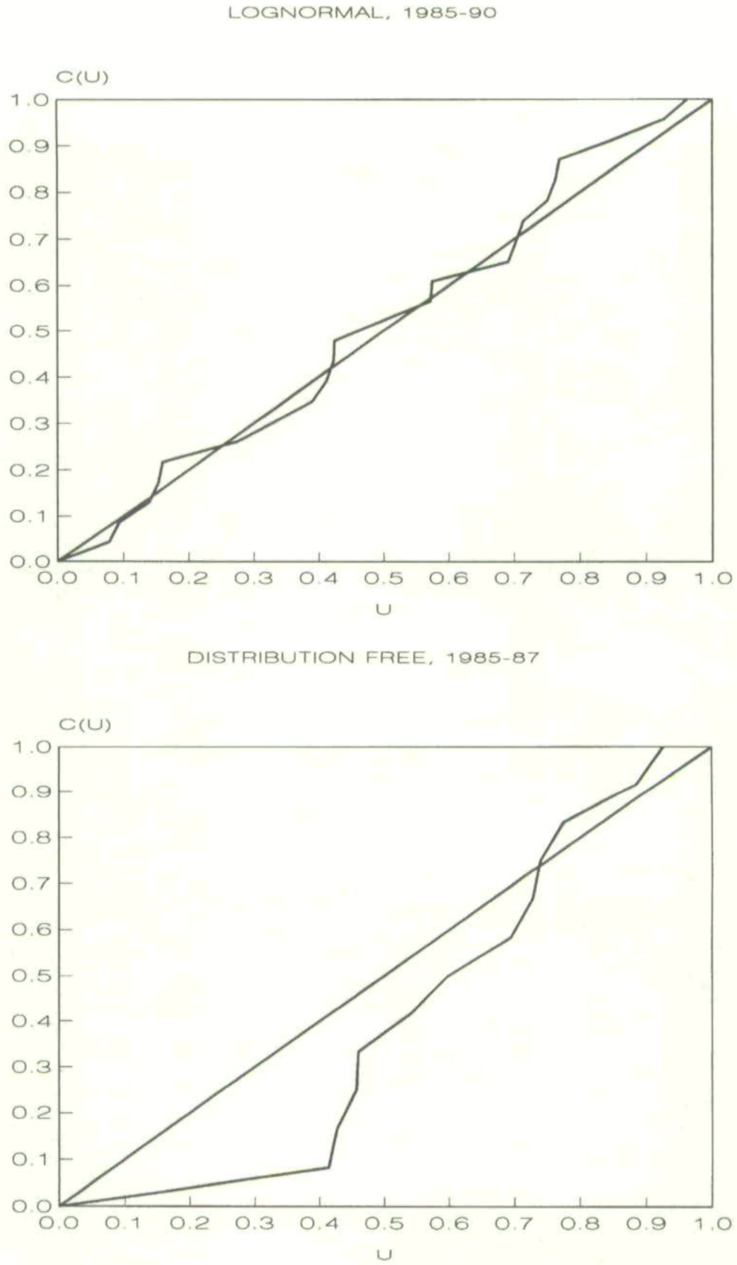
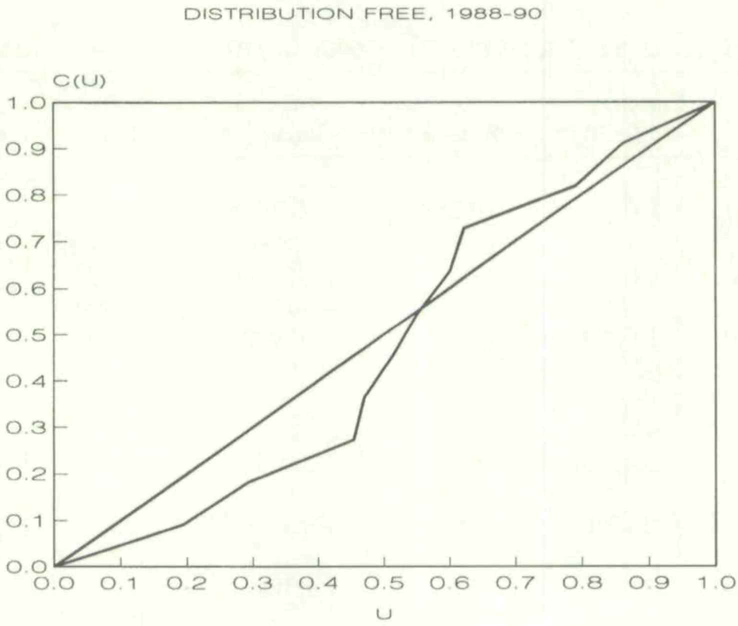


Figure 2 (continued)

The hypothesis that option-based probability assessments become reliable as option markets mature is supported for soybeans regardless of the approach used and for corn using the distribution-free approach. Corn option-based probability assessments using the lognormal assumption are found to be reliable in both periods. Therefore, the calibration results found in the later period indicate that decisionmakers may use soybean and corn price distributions derived directly from futures prices and option premiums, without a need for adjustment. Fackler and King's (1990; 1987) finding that soybean option-based



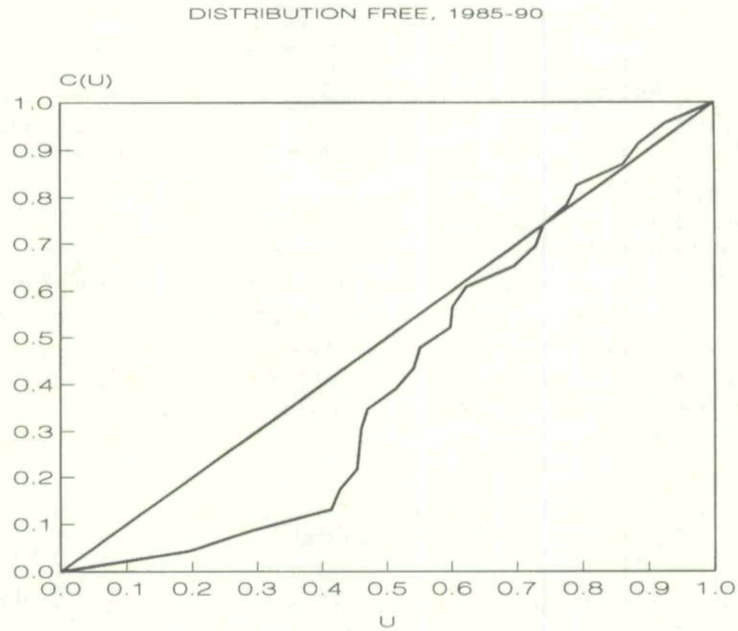


Figure 2 (continued)

probability assessments need adjustment before use is no longer valid in the more recent time period.

The empirical results support the method assuming a lognormal distribution over the method assuming no distributional form to generate reliable corn option-based probability assessments for 1985–1987 and 1985–1990. However, the results found for these periods indicate that, in general, soybean option-based probability assessments must be adjusted

Table IV
STATISTICAL TEST RESULTS FOR CORN OPTION CONTRACT MONTHS

Sample	Cramer–von Mises W^{2*}	Watson U^{2*}	Anderson–Darling A^2	Sign Tests (%) [0, 0.5]	Sign Tests (%) [0.25, 0.75]
Lognormal					
1985–1987	0.0375	0.0595	0.4018	0.5000	0.4167
				(0.7744)[b]	(0.3876)
1988–1990	0.0532	0.0723	0.5046	0.5455	0.6364
				(0.5488)	(0.2266)
1985–1990	0.0209	0.0321	0.2729	0.5217	0.5217
				(0.6776)	(0.6776)
Distribution-Free					
1985–1987	0.4130	0.1888[a]	2.0081	0.3333	0.7500[a]
				(0.1460)	(0.0386)
1988–1990	0.1276	0.0797	not defined[c]	0.3636	0.6364
				(0.2266)	(0.2266)
1985–1990	0.4936[a]	0.2215[a]	not defined[c]	0.3478	0.6957[a]
				(0.0932)	(0.0346)

[a]Significance at the 5% level. No significance is found at the 1% level.

[b]*p*-values are shown in parentheses. *p*-values are calculated considering strict inequalities only, making the test stricter.

[c]In the distribution-free approach, the cumulative probability associated with the expiration date futures price for the May, 1990 contract is equal to one. The Anderson–Darling statistic is not defined for 1988–1990 and 1985–1990 if $U_T = 0$ or $U_T = 1$ because the natural logarithm of zero is minus infinity.

for assessment problems regardless of the method used. In 1988–1990, the results suggest that both the lognormal and the distribution-free assumptions may be used to generate reliable soybean and corn option-based probability assessments. Although the empirical results of this article indicate that the use of either approach leads to reliable probability assessments for the later period, lognormality may be preferred to the distribution-free assumption. The generation of price distributions is simpler and less time consuming when lognormality is assumed than when no assumption is made about the underlying distribution.

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