Cointegration Tests of the Unbiased Expectations Hypothesis in Metals Markets

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INTRODUCTION

The cost of transferring price risk by hedging is measured as the foregone profit from the unhedged spot position as long as (1) the futures contract price reflects an unbiased expectation of the spot price in the contract delivery period and (2) the futures contract price does not contain a systematic risk premium. The cost of hedging increases if a futures contract price reflects a biased expectation or contains a systematic risk premium. This study uses futures contract prices and realized spot prices from four metals markets—silver, copper, platinum, and gold—to test the unbiased expectations and no-risk premium hypotheses. Economic and empirical arguments are presented for the choice of cointegration methodology as an appropriate method of testing the unbiased expectations hypothesis.

PREVIOUS TESTS OF THE UNBIASED EXPECTATIONS HYPOTHESIS

Price dynamics in the metals markets are not characterized by seasonalities in supply and metals can be stored indefinitely with relatively stable storage costs. Despite these convenient features of the data, previous empirical tests of the unbiased expectations hypothesis and its alternative, normal backwardation, give mixed results. A testable implication of normal backwardation is that the difference between the futures and expected spot price increases as the time remaining to contract expiration rises. Regression tests by Kolb, Jordan, and Gay (1983) find that returns from holding gold and silver contracts increase with the time to maturity. Their sample includes prices from the 1972–1980 period for contracts traded at the Chicago Board of Trade. Park (1985), using a sample from July, 1977 through December, 1981, also finds that returns increase as a function of time to maturity in the metals futures markets. Park and Chen (1985) provide a theoretical model which supports normal backwardation in commodity markets. They find that, in general, the covariance between spot commodity prices and default-free discount bond prices is

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less than the variance of the bond prices, implying that at any moment a commodity contract price should be below the expected spot price.

Fama and French (1987) use regression tests to examine whether futures contract prices (their study examines 21 contracts including silver, copper, platinum, and gold) contain evidence of forecast power or systematic risk premiums. Their data set includes futures contract prices through July, 1984. They conclude that copper futures prices contain suggestive evidence of both forecast power and systematic risk premiums. The contract prices for gold and platinum exhibited low basis variability relative to the variability of spot price and futures price changes. For these two metals the regression methodology does not provide reliable tests of the hypotheses. The results for silver are counter-intuitive. A negative and statistically significant relationship between basis (2 months before contract maturity) and spot price change over the remainder of the contract life is found. It is interesting to note the authors do not control for the Hunt brothers episode of 1979–1980.

Kolb (1992) tests three separate implications of the normal backwardation hypothesis for futures contract price changes using 29 futures contracts. He finds that copper is one of nine contracts for which at least two of the three test outcomes are consistent with normal backwardation. He finds evidence that silver, platinum, and gold contract prices provide unbiased expectations of realized spot prices.

COINTEGRATION TESTS OF THE UNBIASED EXPECTATIONS HYPOTHESIS

Two variables which form a cointegrated system are linked by a long-run equilibrium relationship. The short-run deviations from the equilibrium relationship are governed by an error correction process. Estimation of the parameters of the cointegrating vector and testing for cointegration between futures contract prices and realized spot prices have been suggested as means for testing the unbiased expectations hypothesis [see Shen and Wang (1990)].

In a test of unbiased expectations in the forward exchange markets, Hakkio and Rush (1989) reject the joint hypotheses of no predictable risk premium and unbiased expectations for the German Mark and British Pound. Lai and Lai (1991) extend the analysis of Hakkio and Rush by directly testing parameter restrictions on the cointegrating relationship. They use the maximum likelihood estimation methodology and parameter restriction tests suggested by Johansen (1988), Johansen and Juselius (1990), and Johansen (1991) to test the joint hypothesis of unbiased expectations and no-risk premium. The results of their analysis are consistent with Hakkio and Rush (1989); the joint hypothesis of no-risk premium and unbiased expectations is rejected.

Chowdhury (1991) tests the unbiased expectations hypothesis using spot and futures contract prices from the London Metal Exchange (LME) for the four metals: copper, lead, tin, and zinc. The sample consists of 64 non-overlapping observations from the period 1971–1988. The null hypothesis of non-cointegration between futures and realized spot prices is not rejected for lead, tin, and zinc. Non-cointegration is rejected for the copper market, implying the existence of an equilibrium relationship between the LME copper contract price and the realized spot price. Tests of the parameter restrictions implied by the absence of a risk premium and unbiased expectations are rejected for the copper market, implying that the equilibrium relationship between the LME copper contract price and the realized spot price is not consistent with unbiased expectations and implies the absence of a systematic risk premium.

DEVELOPMENT OF TESTABLE HYPOTHESES

Two suppositions form the unbiased expectations hypothesis: the price of a futures contract equals the expected spot market price on the contract delivery date; and, the expectation of the spot price is formed rationally. The first suggests parameter restrictions $\alpha = 0, \beta = 1$ in (1).

$$F_{t-1} = \alpha + \beta(E_{t-1}(S_t)) \tag{1}$$

Because expectations are not directly measurable, realized spot prices are substituted in (1). In this manner the assumption of rational expectations is imposed on the empirical analysis. Relationship (2) is empirically tested in this study.

$$F_{t-1} = \alpha + \beta(S_t) \tag{2}$$

A test of the unbiased expectations hypothesis is of both theoretical and practical significance. The condition of a non-zero risk premium (i.e., normal backwardation, $\alpha < 0$, or contango, $\alpha > 0$) increases the cost of hedging. The possibility of non-zero risk premiums is posited due to the need to compensate speculators for the risk of net long or net short positions when hedgers are net short or net long. If hedgers are paying speculators a risk premium, this would imply that they realize consistent losses on their hedged position. Non-zero risk premiums in futures contract prices will also affect the value of futures contract prices as a means of price discovery.

ARGUMENTS FOR THE COINTEGRATION METHODOLOGY

Many financial market time series are non-stationary. A non-stationary series exhibits infinite variance which violates the central limit theorem and Gauss–Markov assumptions making standard methods for statistical inference inappropriate. The difference operator is commonly applied to achieve stationarity. However, in many applications differenced series are inappropriate for testing parameter restrictions in equilibrium relationships.

When two non-stationary time series (futures contract prices and realized spot prices, for example) are linked by an equilibrium relationship, inference regarding parameters of the equilibrium relationship may be approached using the cointegration methodology. A time series process is said to be integrated of order d, $I(d)$, if dth order differencing produces stationarity. An $I(1)$ series is said to have a unit root. If x_t and y_t are both $I(1)$, then it is generally true that the linear combination $z_t = y_t - \delta x_t$ will also be $I(1)$. However, it is possible that z_t is stationary, i.e., $z_t \sim I(0)$. When this occurs, a special constraint operates on the long-run components of x_t and y_t. Since both x_t and y_t are non-stationary, they will be dominated by trend components; but z_t, being stationary, will not share this property. The stationarity of z_t indicates that y_t and δx_t have long-run co-movements which virtually cancel each other out. In such circumstances, x_t and y_t are said to be cointegrated, with δ being called the cointegrating parameter and $(1, -\delta)$ the cointegrating vector.[1]

JOHANSEN'S PROCEDURE FOR TESTING PARAMETER RESTRICTIONS IN A COINTEGRATED SYSTEM

Johansen (1988, 1991) and Johansen and Juselius (1990) present a methodology for testing parameter restrictions in cointegrated systems. This method enables estimation

[1]Definition: [Engle and Granger (1987)] The components of the vector x_t are said to be co-integrated of order d, b, denoted $x_t \sim CI(d,b)$, if (i) all components of x_t are $I(d)$; (ii) there exists a vector $\delta(0)$ so that $z_t = dx_t \sim I(d - b), b > 0$.

of the cointegration vectors by maximum likelihood and to determine the number of cointegration relationships in the data. The procedure also permits one to test linear parameter restrictions on the cointegration vector(s). A brief summary of the maximum likelihood estimation and testing procedures is given. The reader should consult Johansen (1988) or Johansen and Juselius (1990) for further details.

Let $X_t = (F_{t-1}, S_t)$ and define ∇ to be the difference operator. The general form of a kth order vector autoregression for the realized spot and futures price series is:

$$\nabla X_t = \Gamma_1 \nabla X_{t-1} + \Gamma_2 \nabla X_{t-2} + \ldots + \Gamma_{k-1} \nabla X_{t-k+1} + \Pi X_{t-k} + \mu + \nu_t \qquad (3)$$

where μ is a 2×1 vector of constants and ν_t is a multivariate normal white noise process with mean 0 and finite covariance matrix. Johansen and Juselius (1990) show that the coefficient matrix Π contains the essential information about the relationship between F_{t-1} and S_t. The number of equilibrating relationships is equal to the rank of: if rank(Π) = 2, then S_t and F_{t-1} are jointly stationary; if rank(Π) = 1 then S_t and F_{t-1} are not stationary, but *are* cointegrated; if rank(Π) = 0, then S_t and F_{t-1} are not cointegrated and the relationship between the two time series can be examined using first differenced data.

DATA

The metals markets silver, copper, gold, and platinum are chosen to contrast with previous studies and because there is a lengthy history of futures contract delivery periods from which to construct samples for these metals. Prices of the silver, copper, and gold futures contracts traded at the Commodity Exchange Inc. (COMEX) and the platinum futures contract traded at the New York Mercantile Exchange; the Handy and Harman base price for silver, the producer price of copper, and the London P.M. fixing price of gold are collected from the Dunn & Hargitt Commodity Data Bank, and the Wall Street Journal. A consistent platinum spot price series spanning the sample period was not available; hence, the price of the maturing futures contract on the first day of the delivery month is substituted for the realized spot price. Spot prices for silver, copper, and gold and the expiring platinum futures contract price are recorded on the first trading day of the delivery month.[2] Silver prices on the first day of the delivery month from the period May, 1964 through May, 1992 provide a sample of 85 observations; the sample for copper from January, 1960–May, 1992 provide 98 observations; the sample for gold from June, 1975–June, 1992 provides 52 observations; the sample for platinum from January, 1968–April, 1992 provides 96 observations.

The contract delivery month sequence January, May, September is used for silver and copper; February, June, October for gold; and January, April, July, October for platinum. These sequences provide equally spaced non-overlapping observations of spot and futures prices. Four months calendar time separates the futures contract prices from the realized spot prices for the silver, copper, and gold samples; 3 months of calendar time separates the platinum contract prices from the realized spot prices. For the silver and copper samples, a realized spot price, S_t, futures contract price, F_{t-1}, pair matches the price of the May futures contract observed on the first trading day in January with the spot price of the metal observed on the first trading day of May. Observations corresponding to

[2]Each futures contract specifies a delivery period which extends through a portion of the delivery month. The first delivery notice for the silver futures contract is the last trading day of the month prior to the delivery month; for the copper contract the second-to-last trading day of the month prior to the delivery month; for the gold and platinum contracts the first day of the delivery month.

other delivery months and metals markets are formed analogously. The natural logarithm of prices are used for all series.

RESULTS

Each series is tested for the presence of a unit root. Table I presents the test statistics for the Dickey–Fuller and the Augmented Dickey–Fuller tests. The reported test statistics are generated using *MicroFit 3.0* [Pesaran and Pesaran (1991)]. For each of the eight series tested, the null hypothesis of an $I(1)$ process cannot be rejected; while the null of an $I(2)$ process can. Tests for unit roots in the presence of a linear trend, not reported, are consistent with the non-trended results reported in Table I. Dickey–Fuller and Augmented Dickey–Fuller test results indicate unit root processes for the spot and futures price series of each of the metals markets.

ARE FUTURES REALIZED SPOT PRICES COINTEGRATED?

For each market the cointegrating regression implied by (2) is estimated and the residuals from this regression are tested for stationarity (cointegration) against the null hypothesis of $I(1)$ (non-cointegration). These preliminary tests are designed to detect the existence of a cointegrating relationship. Critical values for Dickey–Fuller statistics computed from sample sizes smaller than 100 are provided in Engle and Yoo (1987). Test results appear in Table II.

The label CRDW is the Cointegrating Regression Durbin Watson test and DF is the Dickey–Fuller test on the cointegrating regression residuals as described in Engle and Granger (1987). The cointegrating regression Durbin–Watson and Dickey–Fuller tests each reject the null hypothesis of non-cointegration for the futures price and realized spot price series in each market.

Table I
DICKEY–FULLER TESTS FOR ORDER OF INTEGRATION

		Ho: $I(1)$ vs. Ha: $I(0)$		Ho: $I(2)$ vs. Ha: $I(1)$	
Market	N	D–F	**ADF(3)**	**DF**	**ADF(3)**
Silver					
$\mathrm{Ln}(S_t)$	85	−1.73	−1.81	−10.07[a]	−4.40[a]
$\mathrm{Ln}(F_{t-1})$	85	−1.71	−1.78	−10.12[a]	−4.32[a]
Copper					
$\mathrm{Ln}(S_t)$	98	−1.38	−1.33	−11.07[a]	−5.24[a]
$\mathrm{Ln}(F_{t-1})$	98	−2	−2.09	−10.48[a]	−6.02[a]
Gold					
$\mathrm{Ln}(S_t)$	52	−1.71	−2.71	−6.35[a]	−3.00[a]
$\mathrm{Ln}(F_{t-1})$	52	−1.55	−2.56	−6.39[a]	−2.96[a]
Platinum					
$\mathrm{Ln}(S_t)$	96	−1.27	−1.66	−9.34[a]	−3.85[a]
$\mathrm{Ln}(F_{t-1})$	96	−1.27	−1.64	−9.20[a]	−3.64[a]

[a]Reject null hypothesis at 5% level.

Table II
ENGLE AND GRANGER TESTS FOR COINTEGRATION

Test	Market			
	Silver	Copper	Gold	Platinum
CRDW	2.16[a]	1.01[a]	1.77[a]	1.78[a]
DF	–9.90[a]	–5.72[a]	–6.41[a]	–8.99[a]

[a]Reject the null hypothesis of non-cointegration at 5% confidence level. Critical values tabled in Engle and Granger (1987).

ANALYSIS OF COINTEGRATING RELATIONSHIPS

Implementation of the Johansen's procedure requires that several decisions be made before estimating the parameters of the vector process. First, the lag length, k, is an unknown parameter and must be estimated. The usual practice is to begin with a maximum possible lag length and to use one of the many model selection rules to reduce the dimension of the parameter space. The maximum possible lag length is set at 4 and the "optimal lag" is selected using Schwarz's (1978) criterion.[3] For each market the optimal lag selected is $k = 2$.

Results of the Johansen test for number of cointegrating relationships in the data are presented in Table III. The first set of tests help establish whether or not F_{t-1} and S_t are cointegrated. Tests are performed to determine the rank referred to as r. At a 5% confidence level there is evidence of one cointegrating vector in the silver, gold, and platinum markets; at the 10% confidence level there is evidence of one cointegrating vector in the copper market.

Maximum likelihood estimates of the parameters of the cointegrating vectors and parameter restriction test statistics are reported in Table IV. For the relationship (2) parameter restrictions $\alpha = 0$ and $\beta = 1$ are tested individually and jointly. The absence of a risk premium, $\alpha = 0$, is not rejected in the silver, gold, and platinum markets. In the copper market, the absence of a risk premium is rejected at the 10% confidence level. The unbiased expectations hypothesis, $\beta = 1$, is rejected at the 5% level for the silver and gold markets; at the 10% level for copper. Interestingly, the joint test, $\beta = 1$ and $\alpha = 0$, is rejected for all markets except copper.

The models for each of the data series are estimated under the assumption of no linear trend in the non-stationary process. This assumption allows one to estimate the parameter in each of the cointegrating relations. The Johansen and Juselius (1990)

Table III
JOHANSEN TESTS FOR NUMBER OF COINTEGRATING VECTORS

Hypothesis		Test Statistic				Critical Values	
Ho:	Ha:	Silver	Copper	Gold	Platinum	95%	90%
$r = 0$	$r = 1$	39.51	15.19	15.17	24.51	15.67	13.75
$r = 1$	$r = 2$	3.47	2.86	6.34	2.08	9.24	7.52
$r < 1$	$r \geq 1$	42.98	18.05	21.51	26.6	19.96	17.85

Critical values from *Microfit 3.0,* Pesaran and Pesaran (1991).

[3]Minimize $2^* \ln(L) - N \ln(T)$ where L is the value of the likelihood function, T is the sample size, and N is the number of parameters in the model.

Table IV
LIKELIHOOD RATIO TESTS OF PARAMETER RESTRICTIONS ON THE COINTEGRATING VECTOR

Market	Lag Length of VAR	Estimates of Eq. (1) α	β	Hypothesis Tests Ho: $\alpha = 0$	Ho: $\beta = 1$	Ho: $\beta = 1$, $\alpha = 0$
Silver ($T = 85$)	2	–0.0198	1.005	1.14	6.05[a]	39.36[a]
Copper ($T = 98$)	2	0.684	0.84	3.44[b]	3.40[b]	3.44
Gold ($T = 52$)	2	–0.0009	1.005	0.0031	8.8[a]	14.79[a]
Platinum ($T = 96$)	2	–0.0098	1.009	0.038	0.01	7.84[a]

[a]Reject at 5% level.
[b]Reject at 10% level.

maximum likelihood estimation procedure permits testing of this maintained assumption. The likelihood ratio test statistics appear in Table V below and each are computed under the assumption of one cointegrating vector. The null hypothesis of no linear trend cannot be rejected at the 5% level in any of the markets.

IMPACT OF THE HUNT EPISODE ON TEST OUTCOMES

Perron (1989) has shown that standard tests of the unit root hypothesis like the ones used above may be unreliable if the non-stationary time series contains a one-time break. The 1979–1980 Hunt brothers' attempt to manipulate prices in the silver market is well known and may have produced such a break. The silver market is used to explore whether the hypothesis tests used in this study are valid and to determine if the results reported in Table IV are affected by this exogenous event.

Perron (1989) and Zivot and Andrews (1992) have suggested tests of the unit root hypothesis in the presence of temporal changes in the stochastic process. Consideration is given first to the case where an exogenous change in the level of the time series, μ, is thought to occur at a known point in time, T_B. Due to the manipulation in the silver market, the break point is thought to have occurred in mid 1979. T_B is chosen

Table V
LIKELIHOOD RATIO TESTS FOR LINEAR TRENDS

Market	Test Statistic
Silver	1.165
Copper	1.134
Gold	0.44
Platinum	0.037

Statistic is distributed as a χ^2 (1).

to be September, 1979.[4] The procedure involves estimating the following augmented regression:

$$y_t = \mu + \theta DU_t + \phi t + dD(T_B) + \psi y_{t-1} + \sum_{j=1}^{k} c_j \nabla y_{t-j} + e_t \quad (4)$$

Where $DU_t = 1$ if $t > T_B$, 0 otherwise; and $D(T_B) = 1$ if $t = T_B$, 0 otherwise. The value of the unknown lag length, $\bar{k}$, is determined using the same procedure described in Perron (1989). First a maximum lag length, k, is chosen. Then, k is selected by working backwards, choosing the first value of k such that the t-statistic is greater than 1.6 in absolute value. Having chosen k, the test statistic for the unit root null hypothesis is represented by the standard t-ratio for testing $\psi = 1$. The null hypothesis of a unit root is rejected if $t_\alpha(\lambda) < k_\alpha(\lambda)$, where $k_\alpha(\lambda)$ denotes the size α critical value from the asymptotic distribution of the t-ratio for a fixed value of $\lambda = T_B/T$. The critical values for this test are tabled in Perron (1989) and Zivot and Andrews (1992). The trend stationary model under the alternative hypothesis is represented by

$$y_t = \mu_1 + \phi t + (\mu_2 - \mu_1)DU_t + e_t \quad (5)$$

where μ_1 and μ_2 represent the level of the time series before and after the structural break. The results of this test are presented in Table VI. For the spot and futures contract prices, the t-ratios associated with the September, 1979 break points are -1.76 and -1.28, respectively. Based on these results, the unit root hypothesis cannot be rejected in the silver spot and futures markets at the 5% confidence level.

The Perron framework can be used also to test the unit root hypothesis under the assumptions that both a structural break occurs at T_B and the rate of growth in the time series is changed. If, for example, the Hunt episode has a permanent impact on the rate of growth in silver prices, then the unit root hypothesis should be tested in a slightly different way. The tests are based on the following regression

$$y_t = \mu + \theta DU_t + \phi t + \gamma DT_t^* + dD(T_B) + \psi y_{t-1} + \sum_{j=1}^{k} c_j \nabla y_{t-j} + e_t \quad (6)$$

where $DT_t^* = t - T_B$ if $t > T_B$, and zero otherwise. The trend stationary alternative hypothesis is

$$y_t = \mu_1 + \phi_1 t + (\mu_2 - \mu_1)DU_t + (\phi_2 - \phi_1)DT^* + e \quad (7)$$

where ϕ_1 and ϕ_2 represent the drift parameters before and after the shift, respectively. Results of unit root tests assuming a structural break and a change in the rate of growth are presented in Table VII. Under this alternative, the unit root hypothesis is rejected in

Table VI
UNIT ROOT TESTS ASSUMING STRUCTURAL CHANGE IN LEVEL OCCURRING SEPTEMBER, 1979

Series	k	α	$t_{0.05}$ (0.5)	$k_{0.05}$ (0.5)[a]
$\mathrm{Ln}(S_t)$	1	0.93	–1.76	–3.76
$\mathrm{Ln}(F_{t-1})$	0	0.929	–1.28	–3.76

[a]Critical values tabled in Perron (1989) and Zivot and Andrews (1992).

[4]Using January, 1980 as a break point does not materially affect the results.

Table VII
UNIT ROOT TESTS ASSUMING CHANGE IN LEVEL AND IN RATE OF CHANGE, BREAKPOINT OCCURRING IN SEPTEMBER, 1979

Series	k	α	$t_{0.05}$ (0.5)	$k_{0.05}$ (0.5)[a]
$\mathrm{Ln}(S_t)$	2	0.684	–3.32	–3.76
$\mathrm{Ln}(F_{t-1})$	4	0.446	–4.81	–3.76

[a]Critical values tabled in Perron (1989) and Zivot and Andrews (1992).

the futures market, but not in the spot market at the 5% confidence level. The outcome of this test complicates the analysis because it casts doubt on whether the cointegration framework is appropriate in the silver market. It is uncertain whether the structural break caused by the Hunt's has had a lasting impact on the rate of change in silver futures contract prices. While it is not difficult to imagine that such a break occurs, it seems unlikely that the silver market would be permanently affected in the way implied under the alternative hypothesis in this test.

On the basis of this reasoning, the next question is addressed: Does the structural change have any effect on the results of the cointegration analysis? A dummy variable is introduced that coincides with the apparent shift in the series caused by the Hunt episode. It is assumed that the events of late 1979 and early 1980 have a fairly long-lasting effect on market participants. Since the duration of the effect is not known, the horizons of 2, 3, 4, and 5 years are used. The dummy variable is centered about its mean and included in eq. (3). The cointegration analysis then continues as before under the assumption that both series are $I(1)$. Evidence is found of a single cointegrating vector (Table VIII). The hypothesis tests $\alpha = 0$; $\beta = 1$; and $\alpha = 0$, $\beta = 1$ are performed on the cointegrating vector (Table IX). Explicitly modeling the structural break presumed to have accompanied the Hunt episode in the silver market affects the test outcomes. The unbiased expectation's hypothesis, $\beta = 1$ can no longer be rejected at standard confidence levels.

CONCLUSION

This study produces convincing evidence of unit root processes in the spot and futures price series from metals markets. This characteristic of the raw series leads naturally to the use of cointegration methodology to examine equilibrium relationships from these markets. Tests for stationarity of the residuals from the cointegrating regression and tests based on the rank of the coefficient matrix from the maximum likelihood estimation of

Table VIII
JOHANSEN TESTS FOR NUMBER OF COINTEGRATING VECTORS

Silver		Structural Break: Sept 1979–May 1983		
Ho:	Ha:	Test Statistic	95% Crit[a]	90% Crit[a]
$r = 0$	$r = 1$	42.2	15.67	13.75
$r = 1$	$r = 2$	0.7305	9.24	7.52
$r < 1$	$r \geq 1$	42.94	19.96	17.85

[a]Critical values from *Microfit 3.0,* Pesaran and Pesaran (1991).

Table IX
LIKELIHOOD RATIO TESTS OF PARAMETER RESTRICTIONS ON THE COINTEGRATING VECTOR

		Estimates of Eq. (1)		Hypothesis Tests		
Market	**Lag Length of VAR**	α	β	**Ho:** $\alpha = 0$	**Ho:** $\beta = 1$	**Ho:** $\beta = 1$, $\alpha = 0$
Silver ($T = 85$)	2	−0.0003	1.005	0.0002	2.34	39.26[a]

[a]Reject at 5% level.

the vector autoregression indicate that futures contract prices and realized spot prices are cointegrated in the silver, copper, gold, and platinum markets.

The Johansen maximum likelihood estimation procedure is used to test the parameter restrictions implied by the unbiased expectations hypothesis. Test outcomes for the hypotheses: absence of a risk premium, unbiased expectations, and a joint test of both are not in complete agreement for any of the metals markets examined. The platinum market provides evidence most consistent with the implied parameter restrictions. The hypothesis of unbiased expectations is rejected for the gold and silver markets in the first models estimated. The estimated value of β in both markets is 1.005. Since this estimate can be statistically distinguished from one, the precision with which the parameters of the cointegrated systems are estimated is quite high.[5] The size of the parameter estimates indicate that a short position initiated 4 months before the contract delivery month will gross one-half percent of the contract price if maintained through the first trading day of the delivery month. At current contract price levels this amount is less than the round trip brokerage commission charged to the public. In addition accounting for an apparent structural break in the silver markets due to the Hunt brother's manipulation changes the tests outcomes. It is not possible to reject the unbiased expectations hypothesis when a centered dummy variable corresponding to the timing of the episode is included in the model. Yet unspecified events in the gold market may have caused a similar structural break which would lead one to reject the unbiased expectations hypothesis in the gold market.

As in previous studies, the cointegration tests indicate that the copper market is one in which economically meaningful departures from the no-risk premium–unbiased expectations paradigm are prevalent. The estimated value of β is significantly less than one indicating the futures price is less than the expected spot price. However, the estimated value of α is significantly greater than zero which by itself is consistent with contango. Taken together these estimated values indicate that the evidence of normal backwardation in the copper market documented in the Kolb study is not due to an additive risk premium of the type generally associated with normal backwardation. A more complex interaction between expectations and possibly non-zero risk premiums is suggested for the process determining copper futures contract prices.

[5]It has been shown that the variance of the ordinary least squares estimates of the parameters of the cointegrating vector converge to zero as a function of the square of the sample size. See sources cited in Engle and Granger (1987).

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