Hedging with Stock Index Futures: Estimation and Forecasting with Error Correction Model

Asim Ghosh

INTRODUCTION

One of the significant innovations of the 1980s is the introduction of stock index futures contracts. Despite some controversy about index arbitrage and program trading that developed after the October stock market crash in 1987, these contracts are very successful and beneficial to stock portfolio managers. Institutional investors use stock index futures as a major trading tool.

Hedging through trading futures is a process used to control or reduce the risk of adverse price changes. Until 1982, market participants could not control market risk of their portfolios. The introduction of stock index futures contracts offers them an opportunity to manage the market risk of their portfolios without changing the portfolio composition. The objective of hedging is to minimize the risk of the portfolio for a given level of return. Index futures is favored as a hedging vehicle because of its liquidity, speed, and lower transaction costs, including bid–ask spread and brokerage commissions. Factors that influence the hedge construction and its effectiveness include basis risk, hedging horizon, and correlation between changes in the futures price and the cash price.

The application of portfolio theory to hedging has attracted a great deal of attention from academics and market participants. Johnson (1960) and Stein (1961) introduced the concept of portfolio theory through hedging the cash position with futures. Edrington (1979) applied this concept in determining a risk-minimizing hedge ratio and derived a measure of hedging effectiveness. These have been followed by numerous studies, including Hill and Schneeweis (1981, 1982); Figlewski (1984, 1985); Witt, Schroeder, and Hayenga (1987); Myers and Thompson (1989); Castelino (1990a, 1990b, 1992); Myers (1991); and Viswanath and Chatterjee (1992).

To control or reduce the risk of the portfolio, the hedger has to determine the optimal hedge ratio. The optimal hedge ratio can differ significantly depending on the estimation technique used. Witt et al. (1987) provide the following specifications to estimate the optimal hedge ratio:

I am grateful to Mark J. Powers and two anonymous referees for their helpful comments and suggestions. I express appreciation to Henry Frank for editorial assistance. I am responsible for any remaining errors.

Asim Ghosh is an Assistant Professor in Finance, Rider College, Lawrenceville, New Jersey.

The Journal of Futures Markets, Vol. 13, No. 7, 743–752 (1993)

CCC 0270-7314/93/070743-10

Price-level Hedge Ratio:

$$C_t = a + bF_t + \varepsilon_i \tag{1}$$

Price Change Hedge Ratio:

$$C_p - C_l = \alpha + \beta(F_p - F_l) + u_i \tag{2}$$

Percentage Change Hedge Ratio:

$$\frac{C_p - C_l}{C_l} = \gamma + \delta\frac{(F_p - F_l)}{F_p} + w_i \tag{3}$$

where

C_t, F_t = cash and futures prices at time t
C_p, F_p = cash and futures prices when initiating the hedge
C_l, F_l = cash and futures prices when lifting the hedge
a, α, γ = intercepts of the regression eqs. (1), (2), and (3)
b, β, δ = slope (or hedge ratios) of eqs. (1), (2), and (3)
ε_i, u_i, w_i = random disturbance terms.

So which one of these methods is going to generate a reliable hedge ratio? The answer is probably none of them. This study attempts to improve the estimation method and proposes a theoretically reasonable method to estimate the optimal hedge ratio.

All empirical studies use one of the above three methods to estimate optimal hedge ratios. Following is an assessment of why each method is misspecified.

Financial time series are nonstationary. To achieve stationarity researchers analyze price change series and/or percentage change series. Classical statistical inference is valid in these situations but at the cost of losing potential valuable long-run information. The theory of cointegration, introduced by Granger (1981) and developed by Engle and Granger (1987), integrates the long-run equilibrium relationship and the short-run dynamics. They maintain that if two series are nonstationary but a linear combination of them is stationary, the two series are said to be cointegrated and there must exist an error correction representation.

Specification (1) is misspecified because it excludes the short-run dynamics and amounts to spurious regression in the sense of Granger and Newbold (1974). Specification (2) is widely used to estimate the optimal hedge ratio [see, for example, Ederington (1979); Figlewski (1984, 1985); Stulz, Wasserfallen, and Stucki (1992); and others]. Regression formulation (2) is misspecified because it excludes an error correction term and as a result it ignores the impact of last period's equilibrium error. Specification (3) is misspecified because it ignores the lagged values, thereby excluding the short-run dynamics. Since these regression formulations are theoretically misspecified, optimal hedge ratios estimated from any of these formulations are likely to be unreliable.

This study estimates the optimal hedge ratio which incorporates nonstationarity, long-run equilibrium relationship, and short-run dynamics using stock index futures. The second section describes the data used in this study and summarizes the theory of cointegration. The empirical evidence in hedging several different stock portfolios with Standard and Poor's 500 (S&P 500) futures contract is then presented. Revised optimal hedge ratios are contrasted with those estimated from regression eq. (2). Out-of-sample forecasts are made which appear to be reasonable in most cases. The final section summarizes the findings of this study.

DATA AND METHODOLOGY

Daily closing prices for the S&P 500 index, Dow Jones Industrial Average (DJIA), New York Stock Exchange (NYSE) composite index, and daily nearby contract settlement prices of S&P 500 index futures contracts are collected from the *Wall Street Journal* covering the time period January 2, 1990 through December 5, 1991. Out of a total of 489 observations, 469 are used for estimation and the remaining 20 are used for out-of-sample forecasting. The logarithms of the prices are analyzed in this study.

Suppose x_t and y_t are two nonstationary time series. If they become stationary after differencing once, they are said to be integrated of order one which is denoted by x_t (or y_t) $\sim I(1)$. If there exists a linear combination of x_t and y_t such that $z_t = y_t - c - dx_t$ is stationary, i.e., $I(0)$, Engle and Granger define them to be cointegrated with cointegrating parameter d. If the system is moving towards equilibrium, economic theory suggests that the long-run relationship between x_t and y_t should be $y_t = c + dx_t$, and z_t measures the equilibrium error. Engle and Granger show that if two series are cointegrated, there must exist an error correction representation; and, conversely, if an error correction representation exists, the two series are coeintegrated.

Before the test of the presence of cointegration is performed, it is necessary to establish that each series is $I(1)$, i.e., each has a unit root in its autoregressive representation. Test for the presence of a unit root is performed by conducting the following augmented Dickey–Fuller (ADF) (1981) regression:

$$\Delta y_t = a_0 + a_1 y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + \varepsilon_t \tag{4}$$

where enough lagged differences are included to ensure ε_t becomes white noise.

In the case of the ADF test, if p is sufficiently large, it reduces the power of the test. Alternative tests that utilize a non-parametric correction for serial correlation for the presence of unit roots as proposed by Phillips (1987) and Phillips and Perron (1988), are conducted by performing the following regression:

$$y_t = \alpha + \beta y_{t-1} + \eta_t \tag{5}$$

where η_t is white noise.

Presence (or absence) of cointegration is investigated by testing for the presence of a unit root in the residuals of the following cointegrating regression:

$$y_t = a + bx_t + u_t \tag{6}$$

The null hypothesis of non-cointegration is conducted by applying the ADF test on the following regression:

$$\Delta u_t = \delta u_{t-1} + \sum_{j=1}^{q} \gamma_i \Delta u_{t-j} + v_t \tag{7}$$

where v_t is white noise. Critical values are obtained from MacKinnon (1991).

Phillips Perron's tests are applied to the cointegrating residuals (u_t) from eq. (6) by conducting the following unit root test:

$$u_t = \mu + \beta u_{t-1} + \xi_t \tag{8}$$

where ξ_t is $I(0)$.

If C_t represents the index spot price series and F_t represents the index of futures price series, then the following error correction model will be estimated:

$$\Delta C_t = \alpha u_{t-1} + \beta \Delta F_t + \sum_{i=1}^{m} \delta_i \Delta F_{t-i} + \sum_{j=1}^{n} \theta_j \Delta C_{t-j} + e_t \tag{9}$$

where enough lagged differences will be added to ensure e_t is white noise. A representative error correction model (ECM) would reflect the change in one variable to the change in the other variable, to past equilibrium errors, and to past changes in both variables.

Parameters of model (9) are estimated by employing the two-step procedure of Engle and Granger. In the first step, cointegrating residuals u_t are collected from eq. (6). In the second step, eq. (9) is estimated using ordinary least squares (OLS) regression and lagged variables are chosen by Akaike information criterion (AIC). Coefficient estimates of eq. (9) are asymptotically efficient, but not the estimate of the cointegrating parameter. In an attempt to add a correction to the two-step estimate of the cointegrating parameter, Engle and Yoo (1991) add a third step.

From the regression eq. (9), the estimated coefficient $\hat{\beta}$ is the desired optimal hedge ratio. Model (9) is compared with model (2) and evaluated by computing the likelihood ratio statistic (λ). $-2 \ln \lambda$ is distributed as χ^2 with degrees of freedom equal to the number of restrictions. If the test statistic is statistically significant, then model (9) is judged superior to model (2). ECM (9) and the traditional model (2) are evaluated by examining the out-of-sample forecasts.

EMPIRICAL EVIDENCE

Table I presents the augmented Dickey–Fuller (ADF) and Phillips and Perron (PP) unit root tests under the assumption that there is no linear trend in the data generation process. It should be noted that the pseudo t values of the level series fail to reject the null hypothesis of the presence of a unit root using ADF and PP tests for all the spot indices and the futures contract. This implies that these time series are nonstationary. When ADF and PP tests are applied to the difference series, the pseudo t values clearly reject the null hypothesis of the presence of a unit root. This shows that the difference series are stationary. This establishes the fact that each series is an $I(1)$ process which is necessary for testing the existence of cointegration.

It is established that the price series of spot indices and futures contract are nonstationary, any linear combination of each spot index with the futures contract is expected to be nonstationary. But if a linear combination is found to be stationary, then those series are cointegrated. The ADF and PP tests for cointegrating regressions are presented in Table II. The PP test rejects the null hypothesis of no cointegration for all the indices.

Table I

AUGMENTED DICKEY–FULLER (ADF) AND PHILLIPS PERRON (PP) TESTS FOR UNIT ROOTS IN THE AUTOREGRESSIVE REPRESENTATIONS OF THE LOGARITHMS OF DAILY CLOSING PRICES OF DJIA, S&P 500, AND NYSE SPOT INDICES AND THE SETTLEMENT PRICES OF NEARBY S&P 500 FUTURES CONTRACTS COVERING THE TIME PERIOD JANUARY 2, 1990 THROUGH DECEMBER 5, 1991

	DJIA Spot		S&P 500 Spot		NYSE Spot		S&P 500 Futures		Critical Value
	ADF	PP	ADF	PP	ADF	PP	ADF	PP	(10%)
Levels	−1.31	−1.30	−1.01	−1.03	−0.99	−1.03	−0.89	−1.10	−2.57
Differences	−4.10	−19.78	−4.05	−19.66	−4.66	−20.53	−4.63	−21.39	−2.57

Out of 489 observations, 469 are used for estimation and the remaining 20 are used for forecasting. Critical values are taken from MacKinnon (1991).

Table II

AUGMENTED DICKEY–FULLER (ADF) AND PHILLIPS PERRON (PP) TESTS FOR COINTEGRATION BETWEEN THE LOGARITHMS OF SPOT INDICES OF DJIA, S&P 500, AND NYSE WITH THE FUTURES PRICES OF S&P 500 COVERING THE TIME PERIOD JANUARY 2, 1990 THROUGH DECEMBER, 1991

Regressand	Regressor		Critical
	S&P 500 Futures		
Spot	ADF	PP	Value (10%)
DJIA	–3.23	–3.22	–3.05
S&P 500	–5.10	–9.50	–3.05
NYSE	–2.19	–9.55	–3.05

Critical values are taken from MacKinnon (1991).

However, the ADF test fails to reject the null hypothesis for all indices except the NYSE spot index. It takes 13 lags to ensure the residuals v_t of eq. (7) are white noise. For sufficiently large lags ADF test trends to loose power. From the evidence it appears that the ADF test fails to detect the presence of cointegration for the NYSE index. The evidence, therefore, suggests that each spot index is cointegrated with the S&P 500 futures contract for the time period investigated.

Engle and Granger show that if a pair of series is cointegrated, then there exists an error correction model. The relevant error correction model, which is of interest here, is represented by eq. (9). In this formulation, change in spot price series is not only a function of change in futures price series, which is commonly used in estimating the optimal hedge ratio represented by eq. (2), but also a function of lagged equilibrium error and lagged values of the changes in spot and futures price series. The error correction term measures the long-run equilibrium relationship. The lagged variables capture the short-run dynamics, and the coefficient of the change in futures price series represents the optimal hedge ratio. The traditional method, used by Ederington, Figlewski and others, of estimating the optimal hedge ratio, i.e., by eq. (2) ignores the error correction term as well as lagged variables if any.

The estimated parameters of the error correction eq. (9) as well as eq. (2) are presented in Table III. Consider now whether model (9) is statistically significantly different from model (2). The likelihood ratio statistics for the spot indices of DJIA, S&P 500, and NYSE are 23.12, 125.08, and 148.58, respectively, and are statistically significant at the 5% level of significance. This evidence shows that model (9) is superior to model (2). This implies that the optimal hedge ratio estimated from eq. (9) is preferable to that from eq. (2).

Error correction coefficients are statistically significant in all cases. This shows that last period's equilibrium error has significant impact in the adjustment process of the subsequent price changes in the spot market. Lagged variables with lags of varying degrees are statistically significant. Evidence indicates that the deviations from equilibrium in the short run in one period are adjusted in the next period. This implies that short run deviations have significant impact on the optimal hedge ratio. The intercepts are statistically insignificant which reinforces the assumption that there is no linear trend in the data generation process. This provides evidence to support that model (9) is better than model (2).

Table III

ESTIMATES OF THE PARAMETERS FROM THE PRICE CHANGE HEDGE RATIO MODEL (2) AND THOSE FROM THE ERROR CORRECTION MODEL (9) FOR DJIA, S&P 500, AND NYSE COMPOSITE SPOT INDICES COVERING THE TIME PERIOD JANUARY 2, 1990 THROUGH DECEMBER 5, 1991

	Regressand: DC_t					
	DJIA Spot		S&P 500 Spot		NYSE Spot	
Regressor	Equation (9)	Equation (2)	Equation (9)	Equation (2)	Equation (9)	Equation (2)
Constant	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.28)[a]	(0.22)	(0.03)	(0.28)	(0.04)	(0.24)
u_{t-1}	−0.0372	—	−0.2251	—	−0.1087	—
	(−2.85)		(−6.55)		(−3.62)	
ΔF_t	0.8825	0.8866	0.8978	0.8825	0.8612	0.8398
	(60.38)	(59.54)	(83.19)	(73.93)	(56.66)	(48.13)
ΔF_{t-1}	0.0543	—	0.1997	—	0.4399	—
	(3.72)		(4.64)		(9.31)	
ΔF_{t-2}	—	—	0.0216	—	0.2733	—
			(1.95)		(5.68)	
ΔF_{t-3}	—	—	—	—	0.1524	—
					(3.57)	
ΔF_{t-7}	—	—	−0.1191	—	—	—
			(−3.15)			

ΔC_{t-1}	—	—	−0.1621 (−3.50)	—	−0.4357 (−8.73)	—
ΔC_{t-2}	—	—	—	—	−0.2709 (−5.34)	—
ΔC_{t-3}	—	—	—	—	−0.1495 (−3.35)	—
ΔC_{t-7}	—	—	0.1149 (2.79)	—	—	—
Log-likelihood	2025.81	2014.25	2179.97	2117.43	2014.22	1939.93
Adjusted R^2	0.8885	0.8833	0.9388	0.9231	0.8757	0.8319
LR[b]	23.12[c]		125.08[d]		148.58[e]	

Out of 489 observations, 469 are used for estimation and the remaining 20 are used for forecasting. Spot index ≡ C and futures contracts ≡ F.

[a]Numbers in parentheses are t-ratios.

[b]The likelihood ratio test statistic: LR = 2 (log-likelihood [model (9)]- log-likelihood [model (2)])

[c]χ^2_2, 0.05 = 5.99.

[d]χ^2_5, 0.05 = 11.07.

[e]χ^2_7, 0.05 = 14.07.

Table IV
SUMMARY STATISTICS FOR TWENTY-ONE-STEP AHEAD FORECASTS FOR DJIA, S&P 500, AND NYSE COMPOSITE SPOT INDICES COVERING THE TIME PERIOD JANUARY 2, 1990 THROUGH DECEMBER 5, 1991

	Δ(DJIA)$_t$		Δ(S&P)500$_t$		Δ(NYSE)$_t$	
Statistic	**Equation (9)**	**Equation (2)**	**Equation (9)**	**Equation (2)**	**Equation (9)**	**Equation (2)**
Mean	–0.0011	–0.0014	–0.0021	–0.0014	–0.0028	–0.0014
SD	0.0099	0.0101	0.0102	0.0201	0.0099	0.0106
RMSE	0.0036	0.0036	0.0020	0.0024	0.0035	0.0041

RMSE Ratios

Ratio	Δ(DJIA)$_t$	Δ(S&P)500$_t$	Δ(NYSE)$_t$
Equation (9)/Equation (2)	1.00	0.83	0.85

Out of 489 observations, 469 are used for estimation and the remaining 20 are used for out-of-sample forecasting.

Estimated optimal hedge ratios from model (9) are higher than those from model (2) in most cases together with adjusted R^2. This provides strong evidence that the optimal hedge ratios obtained by using model (9) do a much better job in reducing the risk of the cash position than those obtained from model (2). This means that, on average, hedgers in stock index futures need more futures contracts to reduce the market risk of their cash portfolios. This implies that their losses are going to be reduced substantially in case of adverse price movements. This means also that it would reduce their overall costs of hedging.

Summary statistics of the forecasts from ECM (9) and the traditional model (2) for all spot indices are presented in Table IV. The standard deviations of the forecasts from model (2) are larger than those from the ECM (9). The forecasting performance is evaluated by the root mean squared error (RMSE). RMSEs of the ECM (9) are smaller than those from model (2) in almost all cases. ECM gives about 17% and 15% reduction in RMSE for the spot indices of S&P 500 and NYSE respectively. From the evidence presented in the out-of-sample forecasts, it is concluded that the ECM (9) is largely successful.

To summarize, from the evidence presented it appears that model (9) provides estimates of the optimal hedge ratios which reduce the risk of the cash portfolio as well as the costs of hedging. Out-of-sample forecasts from model (9) are better than those from model (2).

CONCLUSION

This study finds that hedge ratios obtained from traditional methods are underestimated. The reason for underestimation is that the traditional methods are misspecified.

This study differs from the previous studies. It tests for the presence or absence of cointegrating relation between spot and futures price series. Once it recognizes the presence of cointegration, it estimates the relevant error correction model which incorporates the long-run equilibrium relationship as well as the short-run dynamics. This results in significant improvement in the estimation of the optimal hedge ratio. In contrast, the statistical techniques used in the prior studies ignore the error correction term and/or short-run dynamics.

Evidence shows significant improvement over the results from price change regression. Estimates of the optimal hedge ratio as well as adjusted R^2 from the ECM (9) are higher compared to those from the traditional price change hedge ratio. Error correction terms as well as lagged variables are generally significant. Underestimating the optimal hedge ratios results in a suboptimal hedge of the cash portfolio. Improved optimal hedge ratios appear to reduce considerably the risk of the risk minimizing portfolio. This means loss from a suboptimal hedge is significantly reduced and helps to reduce the impact of the costs of hedging.

Out-of-sample forecasts from the ECM (9) are compared to those from the traditional model (2). Forecasts generated by the estimated ECM are found to be better than the forecasts generated by model (2) based on root mean squared error.

This methodology has the potential for application to foreign currency futures, commodity futures, and interest rate futures.

Bibliography

Akaike, H. (1974): "A New Look at Statistical Model Identification," *IEEE Trans. Auto. Control*, 19:716–723.

Castelino, M. (1990a): "Minimum Variance Hedging with Futures Re-visited," *Journal of Portfolio Management,* 16:74–80.

Castelino, M. (1990b): "Futures Markets and Hedging: The Time Dimension," *Journal of Quantitative Economics,* 6:271–287.

Catelino, M. (1992): "Hedge Effectiveness: Basis Risk and Minimum Variance Hedging," *The Journal of Futures Markets,* 12:187–201.

Dickey, D., and Fuller, W., (1981): "The Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica,* 49:1057–1072.

Ederington, L. (1979): "The Hedging Performance of the New Futures Markets," *Journal of Finance,* 34:157–170.

Engle, R., and Granger, C. (1987): "Cointegration and Error Correction Representation, Estimation, and Testing," *Econometrica,* 55:251–276.

Engle, R., Granger, C., and Yoo, B. (1991): "Cointegrated Economic Time Series: An Overview with New Results," in *Long-run Economic Relationships Readings in Cointegration,* Engle, R., and Granger, C. (eds.), New York: Oxford University Press, pp. 237–266.

Figlewski, S. (1984): "Hedging Performance and Basis Risk In Stock Index Futures," *Journal of Finance,* 39:657–669.

Figlewski, S. (1985): "Hedging with Stock Index Futures: Theory and Application in a New Market," *The Journal of Futures Markets,* 5:183–199.

Granger, C. (1981): "Some Properties of Time Series Data and Their Use in Econometric Model Specification," *Journal of Econometrics,* 16:121–130.

Granger, C., and Newbold, P. (1974): "Spurious Regressions in Econometrics," *Journal of Econometrics,* 2:111–120.

Hill, J., and Schneeweis, T. (1981): "A Note on the Hedging Effectiveness of Foreign Currency Futures," *The Journal of Futures Markets,* 1:659–664.

Hill, J., and Schneeweis, T. (1982): "The Hedging Effectiveness of Foreign Currency Futures," *The Journal of Futures Markets,* 3:95–104.

Johnson, L. (1960): "The Theory of Hedging and Speculation in Commodity Futures," *Review of Economic Studies,* 27:139–151.

MacKinnon, J. (1991): "Critical Values for Cointegration Tests," in *Long-run Economic Relationships Readings in Cointegration,* Engle, R., and Granger, C. (eds.), New York: Oxford University Press, pp. 267–276.

Myers, R. (1991): "Estimating Time-varying Optimal Hedge Ratios on Futures Markets," *The Journal of Futures Markets,* 11:39–53.

Myers, R., and Thompson, S. (1989): "Generalized Optimal Hedge Ratio Estimation," *American Journal of Agricultural Economics,* 71:858–868.

Phillips, P. (1987): "Time Series Regression with a Unit Root," *Econometrica,* 55:277–301.

Phillips, P., and Perron, P. (1988): "Testing for a Unit Root in Time Series Regression," *Biometrika,* 75:335–346.

Stein, J. (1961): "The Simultaneous Determination of Spot and Futures Prices," *American Economic Review,* 51:1012–1025.

Stulz, R., Wasserfallen, W., and Stucki, T. (1992): "Stock Index Futures in Switzerland: Pricing and Hedging Performance, *The Review of Futures Markets,* 9:576–594.

Viswanath, P., and Chatterjee, S. (1992): "Robustness Results for Regression Hedge Ratios: Futures Contracts with Multiple Deliverable Grades," *The Journal of Futures Markets,* 12:253–263.

Witt, H., Schroeder, T., and Hayenga, M. (1987): "Comparison of Analytical Approaches for Estimating Hedge Ratios for Agricultural Commodities," *The Journal of Futures Markets,* 7:135–146.