Utility Maximizing Hedge Ratios in the Extended Mean Gini Framework

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INTRODUCTION

Studies of optimal futures hedging have traditionally aimed either at finding the risk minimizing or the utility maximizing hedge ratio. By far, most articles in the literature seek the risk minimizing hedge ratio, and the preponderance of this literature focuses on mean-variance risk minimization. The popularity of this approach stems mainly from its simplicity. This approach assumes that all investors adopt the spot-futures position that achieves the global minimum variance. This approach stems from the work of Johnson (1960), and it has been extended to financial futures by Ederington (1979) and adopted in a variety of contexts by many other scholars, such as Brown (1985); Bond, Thompson, and Lee, (1987); and Myers and Thompson (1989).

In many cases, assuming that all investors seek to minimize risk is not reasonable, because one intuitively expects that weakly risk averse investors would adopt hedge ratios that differ from those chosen by strongly risk averse investors. This point is recognized in the literature and addressed by Stein (1961); Peck (1975); Rolfo (1980); Kahl (1983); and Cicchetti, Cumby, and Figlewski (1988).

Largely in an effort to overcome the deficiencies of risk minimizing hedging, a number of articles have considered utility functions. This approach allows hedgers to find the hedge ratio that maximizes utility, not merely the hedge ratio that minimizes risk. Utility maximizing hedging strategies require the specification of a utility function, and the most commonly proposed utility functions have been quadratic, logarithmic, and exponential functions. No matter what type of function is chosen, there is always the concern that the proposed function will not accurately describe the preferences of hedgers. For example, a researcher may study quadratic utility functions, but hedgers' tastes may follow a different model. In addition to this general problem, each of the major utility functions that has been proposed has its special difficulties.

Quadratic utility has been criticized by a number of authors. Hanoch and Levy (1969), for example, notes that a quadratic utility function implies disutility to incremental wealth at some wealth levels for any set of parameter values. Pratt (1964) has demonstrated

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that absolute risk aversion increases with increasing wealth under quadratic utility. Logarithmic utility possesses the desirable characteristics of decreasing absolute risk aversion and constant relative risk aversion, but Rolfo (1980) has demonstrated that logarithmic utility function represents only one class of risk averse investor. Maximizing exponential utility preferences is consistent with maximizing quadratic utility function, provided that the joint distribution of returns are normal. However, Helms and Martell (1985) have shown that the joint distribution of futures and spot returns are not normal. Although more general than the risk minimization strategy, the utility maximization approach has its own difficulties in choosing the appropriate utility function among many candidates and in avoiding the specific limitations associated with a particular function.

This article proposes a hedging strategy that maximizes utility but escapes problems associated with specifying a particular utility function. This alternative utility function is a function of the expected level of wealth minus the extended mean Gini coefficient. Utility maximizing hedge ratios correspond to the point of tangency between an indifference curve and the extended mean Gini efficient frontier. This approach is attractive, because extended mean Gini efficient sets are second order stochastic dominant (SSD). Consequently, the utility maximizing hedge ratios that are developed in this article are valid for all risk averse investors.

The next section describes the extended mean Gini model. The section following that provides some empirical evidence on utility maximizing hedge ratios in the extended mean Gini framework.

EXTENDED MEAN GINI MODEL

Yitzhaki (1983) recently proposed that the extended mean Gini coefficient be used as an alternative measure of dispersion in portfolio analysis. He defines the extended mean Gini coefficient as:

$$\Gamma(v) = \mu - a - \int_a^b (1 - F(x))^v \, dx \tag{1}$$

where:

μ = the expected return
a = the lower bound of the distribution
b = the upper bound of the distribution
$F(x)$ = the cumulative probability density function of the random variable x
v = a risk aversion parameter

Yitzhaki has shown that if an uncertain prospect F dominates G by second order stochastic dominance, then the following condition holds for all positive values of v:

$$\int_a^b (1 - F(x))^v \, dx \geq \int_a^b (1 - G(x))^v \, dx \tag{2}$$

Following Yitzhaki (1984), eqs. (1) and (2) are combined and the extended mean Gini efficiency (EMG) condition is defined as $\mu_F \geq \mu_G$ and

$$\mu_F - \Gamma_F(v) \geq \mu_G - \Gamma_G(v) \tag{3}$$

This criterion can be stated verbally as:

If prospect F dominates prospect G by the inequality of eq. (3), then F is contained within the SSD set.

Yitzhaki (1982) has noted the similarity between the EMG efficiency condition of (3) and Baumol's (1963) expected gain confidence efficiency criteria. Baumol suggested that increased dispersion in returns could be offset by a requisite increase in the level of returns. The EMG efficiency condition applies to any probability density function provided that its first absolute moment exists. Clearly this condition is far less restrictive than the corresponding mean variance conditions.

The parameter ν may be interpreted as an index of risk aversion. Yitzhaki (1983) has shown that the certainty equivalent of an investment may be expressed as $\mu - \Gamma(\nu)$. That is, the risk premium required by investors increases for higher degrees of risk aversion ν. Values of ν between 0 and 1 characterize risk seekers. Risk neutral investors have $\nu = 1$, and risk averse investors have values of ν greater than 1.0.

The extended Gini model uses two summary statistics and hence provides a basis for constructing efficient frontiers for any level of risk aversion. As one would expect, the efficient frontier in the return and the extended mean Gini space varies with different levels of risk aversion. Figure 1 depicts EMG efficient frontiers for different levels of risk aversion.

The EMG coefficient can be measured in terms of price levels, price differences, or returns. In the following, the EMG coefficient is measured in terms of price levels. This is done to compare the empirical results from the EMG model with those obtained by Rolfo (1980) in examining utility maximizing hedge ratios for cocoa producing countries.[1] Rolfo's analysis is in terms of price levels. Similar to Rolfo, a portfolio is created that consists of an asset and a short position in a related futures contract. The efficient frontier is found by minimizing the EMG coefficient of the returns of the portfolio subject to the wealth constraint. Figure 1 represents the locus of minimum EMG coefficients subject to the wealth constraint. The EMG frontier is that portion of the curve which lies above the global minimum EMG portfolio. Risk minimizing hedge ratios for different levels of risk aversion are represented by the global minimum extended mean Gini coefficient for varying levels of ν, as depicted by point A in Figure 1. As Figure 1 illustrates, the global minimum EMG coefficient changes for different levels of ν. Consequently, the hedge

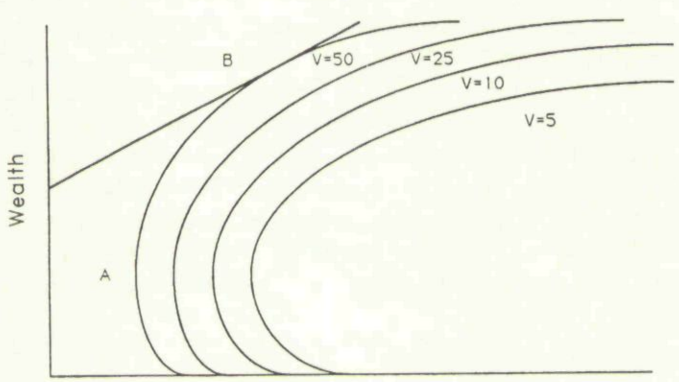

Figure 1
Extended mean Gini efficient frontier.

[1]Rolfo (1980) does not consider that hedgers might adjust their hedges over time. This article employs a similar restriction.

ratio differs for different values of the risk aversion parameter.[2] Figure 1 also shows that utility maximizing hedge ratios will differ from risk minimizing hedge ratios.

The utility function is defined as:

$$U(\tilde{W}) = E(\tilde{W}) - \Gamma_{\nu}(\tilde{W}) \tag{4}$$

This representation is similar in form to the quadratic utility function. As noted previously, the extended Gini coefficient embodies the risk aversion parameter. Indifference curves calculated from eq. (4) will have a slope of one.[3] Therefore, utility maximizing hedge ratios correspond to the point of tangency between the extended mean Gini efficient frontier and indifference curves with a slope of 1.0. Point B in Figure 1 illustrates this relationship for a risk aversion factor of $\nu = 50$.

In general the utility maximizing hedge ratios in the return and extended mean Gini space differ from those that minimize risk. However, for larger values of the risk aversion parameter, the utility maximizing hedge ratio tends toward the risk minimizing hedge ratio. This convergence results because the EMG efficient frontier becomes steeper for increasing ν near the global minimum portfolio. (This point is also illustrated by Fig. 1.)

Quadratic utility generally is defined as:

$$U(\tilde{W}) = E(\tilde{W}) - m\sigma^2$$

where m is the risk aversion parameter. Utility maximizing hedge ratios in the mean-variance space correspond to different points on the mean-variance efficient frontier. By changing the risk aversion parameter, the utility maximizing portfolio moves along the mean-variance efficient frontier reflecting the value of m. For larger values of m, the optimal hedge portfolio moves toward the global minimum variance portfolio. Thus, this article focuses on the extended mean GINI criterion for both risk minimization and utility maximization. This approach contrasts strongly with the familiar global risk minimization strategy that is usually pursued in a mean-variance framework.

SOME EMPIRICAL EVIDENCE

Rolfo (1980) has examined utility maximizing hedging strategies for goods subject to variability in both price and production output for the world's major cocoa producing countries. Ghana, Nigeria, the Ivory Coast, and Brazil produce approximately 80% of the world's cocoa. Specifying quadratic and logarithmic utility functions, Rolfo examined optimal utility maximizing hedging strategies for each of these countries.

Rolfo has described the cocoa production season as commencing late in September (pre-harvest) and extending to the harvest in March. The data he collected represent a time series of futures prices in September, spot prices in March, futures prices in March, and production forecasts in September for March harvests. These prices are quoted in British pounds per ton of cocoa. His data set also included actual production figures for

[2]See Kolb and Okunev (1992).

[3]The slope of the indifference curve is given by:

$$\frac{dE(\tilde{W})}{d\Gamma\nu(\tilde{W})} = -\frac{\dfrac{\partial U}{\partial \Gamma\nu(\tilde{W})}}{\dfrac{\partial U}{\partial E(\tilde{W})}}$$

On the right-hand side, the derivative of the utility function with respect to the EMG coefficient is -1.0, because it is a linear function. In the denominator, the derivative with respect to the expected return is 1.0. Thus, the derivative in the return/EMG plane is 1.0.

March harvests for the years 1952–1976.[4] Using the data from Rolfo's analysis, optimal utility maximizing hedge ratios in the extended mean Gini framework for each of the four major cocoa producing countries are compound.

A farmer may elect to hold n futures contracts to hedge output and price uncertainty. Consequently, the end of period wealth may be expressed as:

$$W = PQ + n(f_s - f_m)$$

where:

P = spot price at harvest (March),
Q = actual production at harvest (March),
f_s = the futures price when the hedge is initiated (September),
f_m = the futures price at harvest (March).

When the farmer initiates the futures position in September, P, Q, and f_m are unknown, but all of these values are fully determined in March.

Extended mean Gini efficient frontiers are generated for each country for various levels of risk aversion. This requires minimizing the extended Gini coefficient subject to a budget constraint which may be stated as follows:[5]

$$\text{minimize } \Gamma_v(\tilde{W}) = -v \operatorname{Cov}\left(\tilde{W}, (1 - F(\tilde{W}))^{v-1}\right)$$

$$\text{subject to: } \tilde{W} = \tilde{P}\tilde{Q} + n(f_s - \tilde{f}_m)$$

This alternative formulation of the EMG coefficient is the covariance between the wealth level and the complement of the CDF raised to the power $v - 1$. To estimate the EMG coefficient empirically, the returns are ranked from lowest to highest. Then the complement of the CDF $(1 - F(w))$ is equivalent to $[N - \text{Rank}(w)]/N$. This expression is then raised to the power $v - 1$. From this it is possible to determine the covariance. Higher ranked returns will have lower complements of the CDF and raising them to a power greater than 1.0 will drive these terms toward zero. So, as v becomes larger, more emphasis is placed upon the lower end of the distribution. In the limit, as v becomes large, investors will adopt a maximum strategy. Increasing v, reflecting greater risk aversion, has the effect of truncating the upper end of the distribution, focusing more attention on the worst outcomes. This is intuitively appealing, because one expects more risk averse investors to give attention to the probability of suffering the worst outcomes. The mean variance approach, by contrast, weights all outcomes equally.

An iterative procedure is used to determine the EMG efficient frontier for various risk aversion parameters.[6] Because there are only two assets, specifying the hedge ratio (n) automatically fixes the level of wealth and also determines the EMG coefficient. Figure 2 represents the EMG efficient frontier of Ghana for risk aversion parameters 5, 10, 25, 50, and 100. As Figure 2 illustrates, the EMG efficient frontier varies for different degrees of risk aversion.

Table I compares risk minimizing hedge ratios and utility maximizing hedge ratios for Ghana. The selection of risk aversion parameters is dictated by the variability of the hedge ratios. Most pronounced variability occurs in the risk aversion parameter range of 1.08 to 5.0.[7] Recall that $v = 1$ represents risk neutrality. For utility maximizing hedge

[4]For a more detailed explanation of the data set, see Rolfo (1980), pp. 113–116.

[5]This alternative formulation of $\Gamma_v(W)$ is equivalent to eq. (1) in Shalit and Yitzhaki (1984).

[6]For a more detailed explanation of the calculation of the extended Gini coefficient see Kolb and Okunev (1992).

[7]Lower values of v produce pronounced reverse hedges.

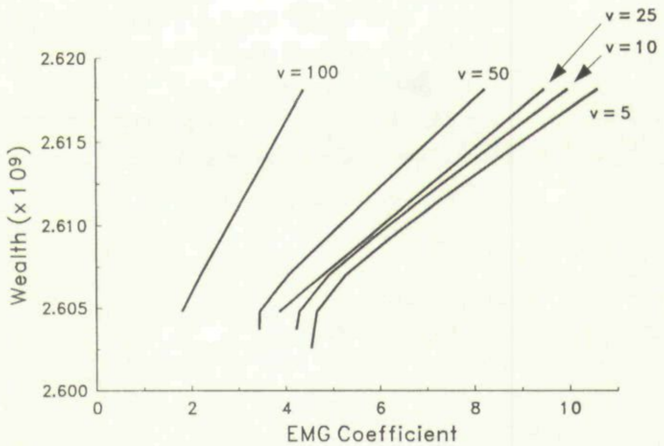

Figure 2

Extended mean Gini efficient frontiers for Ghana. The EMG coefficients should be scaled as follows: for $v = 100$ multiply by 10^6; for all others multiply by 10^7

ratios, Table I shows that reverse hedging occurs when the risk aversion parameter is less than 1.25. Reverse hedging means that the hedger hedges a long position by buying futures, or hedges a short position by selling futures. Above this value of v, the hedge ratio steadily increases and reaches a maximum at $v = 3$. As v increases beyond 3.0, the hedge ratio falls to the final steady state hedge ratio of .621 for values of v greater than 27. The risk minimizing hedge ratio commences at 3.515 for $v = 1.08$ and steadily decreases until a final steady state hedge ratio of .651 is attained for $v = 27$. Note that the utility maximizing and risk minimizing hedge ratios are approximately the same for risk aversion parameters greater than 27, as one would expect. By contrast, for low levels of risk aversion, the two hedge ratios differ significantly. For example, for $v = 1.1$, a risk minimizing approach would sell futures contracts for 3.515 times the level of output, but the utility maximizing approach would buy futures for 7.391 times the level of output. (The negative signs in Table II indicate reverse hedging.) In this particular case, the difference in the level of wealth attained from a risk minimizing and utility maximizing strategy would be 43.9 million British pounds.[8] This represents an increase of 1.7% over the level of wealth from a risk minimizing approach. The utility maximizing approach also involves accepting 3.5 times the level of risk characteristic of the risk minimizing portfolio. Figure 3 graphically presents the information contained in Table I.[9]

Table II displays utility maximizing hedge ratios across the four cocoa producing countries for risk aversion parameters ranging from 1.08 to 27. Large values of v produce similar hedge ratios for each country. For example, risk aversion factors above 15 give the optimal hedge ratios of 0.621 for Ghana, 0.639 for Nigeria, 1.164 for the Ivory Coast, and 1.009 for Brazil. As Table II shows, hedge ratios for risk aversion factors less than $v = 15$ vary considerably. The most pronounced variation occurs when v lies between

[8]This wealth difference is the difference in the value of selling 3.515 times the level of output (for the risk minimizing approach) and buying 7.391 times the level of output (for the utility maximizing approach). As such, it represents the difference in wealth attained by investing in a risk minimizing strategy versus a utility maximizing strategy.

[9]One would expect utility maximizing hedge ratios to be monotonically increasing for increasing v. The empirical results of Table I, the "bulge" in the hedge ratios for risk aversion parameters in the neighborhood of $v = 2$ to $v = 5$ may be due to the small number of sample points used to estimate the CDF.

Table I
COMPARISON OF RISK MINIMIZING AND UTILITY MAXIMIZING HEDGE RATIOS FOR GHANA

Risk Aversion Parameter	Risk Minimizing Hedge Ratio	Utility Maximizing Hedge Ratio
1.08	3.515	−21.877
1.09	3.515	−21.877
1.10	3.515	−7.391
1.11	3.515	−2.661
1.12	3.308	−2.069
1.13	3.196	−2.069
1.14	3.134	−2.069
1.15	2.954	−2.069
1.16	2.924	−2.069
1.17	2.865	−1.774
1.18	2.865	−1.774
1.19	2.863	−1.478
1.20	2.691	−1.183
1.21	2.599	−0.975
1.22	2.392	−0.591
1.23	2.392	−0.354
1.24	2.392	−0.295
1.25	2.392	0.000
1.26	2.392	0.088
1.27	2.245	0.148
1.28	2.156	0.236
1.29	2.156	0.266
1.30	2.128	0.296
1.40	1.833	0.591
1.50	1.744	0.857
1.60	1.655	0.857
1.70	1.567	1.005
1.80	1.537	1.016
1.90	1.537	1.016
2.00	1.537	1.094
3.00	1.360	1.123
4.00	1.331	1.064
5.00	1.331	1.005
6.00	1.301	0.828
7.00	1.301	0.798
8.00	1.271	0.798
9.00	1.212	0.798
10.0	1.064	0.798
11.0	0.828	0.769
12.0	0.798	0.709
13.0	0.798	0.650
14.0	0.798	0.650

(continued)

Table I (continued)

Risk Aversion Parameter	Risk Minimizing Hedge Ratio	Utility Maximizing Hedge Ratio
15.0	0.798	0.650
16.0	0.798	0.650
17.0	0.798	0.650
18.0	0.798	0.650
19.0	0.798	0.650
20.0	0.798	0.650
.	.	.
.	.	.
.	.	.
27.0	0.651	0.621

Optimal hedge ratios are defined as the number of futures contracts divided by expected output. Values of ν larger than 27.0 produced the same hedge ratios.

1 and 2. Reverse hedging for Ghana, Nigeria, the Ivory Coast, and Brazil occur for risk aversion factors less than 1.25. This is a surprising result as Rolfo found that reverse hedging occurred in the mean variance framework at 0.000175, 0.000319, 0.000523, and 0.00003 for Ghana, Nigeria, the Ivory Coast, and Brazil, respectively.

Investors with $1 < \nu < 1.25$ are considered to be weakly risk averse. In Table II, these traders are generally reverse hedgers. Moderately risk averse investors, with $1.25 < \nu < 2.0$, hedge some of their output. Investors with $\nu > 10.0$ are strongly risk averse, and they adopt hedge ratios that are similar to risk minimizing hedge ratios.

Table III presents the implied risk aversion factors for logarithmic utility maximization hedge ratios. Given a particular hedge ratio, Table III shows the risk aversion parameter that is implied by that hedge ratio. The risk aversion parameters implied by logarithmic utility appear to be fairly consistent for the West African countries. These parameters are 1.269, 1.268, and 1.272. For Brazil, $\nu = 1.328$. These results differ from the results of Rolfo, who found the implied mean variance risk aversion factors were 0.000234, 0.000401, 0.000857, and 0.000065 for Ghana, Nigeria, the Ivory Coast, and Brazil, respectively. However, these results support Rolfo's findings that the logarithmic utility represents a subset of risk averse investors, and the empirical evidence implies a risk aversion parameter of approximately 1.3 across the four countries.

Unfortunately, there is no known analytical relationship between the risk aversion parameters of the EMG approach and the mean-variance approach. Nonetheless, it is possible to develop an intuitive understanding of their relationship. Table IV compares mean-variance risk aversion parameter (m) with EMG risk aversion parameters (ν) for identical hedge ratios across the four countries. Table IV provides a means for correlating the risk aversion parameters of the EMG approach with the hedge ratios derived from the mean-variance strategy. Thus, in Table IV, the EMG risk aversion parameters are broadly consistent for given levels of the mean-variance parameters, especially for the West African countries.

By comparing Tables II and IV for strongly risk averse investors, say $\nu > 15$ and $m > 1$, one can see that the EMG and mean-variance hedge ratios are almost identical. For example, in Table IV for Ghana, the mean-variance hedge ratio is consistently equal to 0.609 for high levels of the mean-variance risk parameter m. In Table II the utility maximizing hedge ratio for $\nu = 27$ is 0.621. Nigeria and Brazil show a similar

Table II
UTILITY MAXIMIZING HEDGE RATIOS ACROSS COUNTRIES

	Hedge Ratios			
Risk Aversion Parameter	Ghana	Nigeria	Ivory Coast	Brazil
1.08	−21.877	−21.831	−40.299	−27.641
1.09	−21.877	−21.831	−40.299	−27.641
1.10	−7.391	−7.452	−13.881	−9.592
1.11	−2.661	−2.663	−4.746	−3.281
1.12	−2.069	−2.130	−3.761	−2.651
1.13	−2.069	−2.130	−3.761	−2.524
1.14	−2.069	−2.130	−3.761	−2.524
1.15	−2.069	−2.130	−3.761	−2.524
1.16	−2.069	−2.130	−3.761	−2.524
1.17	−1.774	−2.130	−3.761	−2.524
1.18	−1.774	−1.632	−3.134	−2.146
1.19	−1.478	−1.000	−2.866	−2.019
1.20	−1.183	−1.000	−2.507	−1.767
1.21	−0.975	−0.958	−1.791	−1.262
1.22	−0.591	−0.586	−1.075	−0.757
1.23	−0.354	−0.373	−0.716	−0.505
1.24	−0.295	−0.266	−0.448	−0.377
1.25	0.000	0.000	0.000	0.000
1.26	0.088	0.106	0.179	0.126
1.27	0.148	0.159	0.269	0.252
1.28	0.236	0.266	0.448	0.252
1.29	0.266	0.266	0.537	0.379
1.30	0.296	0.319	0.537	0.379
1.4	0.591	0.586	1.074	0.757
1.5	0.828	0.798	1.552	1.001
1.6	0.857	0.852	1.611	1.135
1.7	1.005	1.118	1.881	1.262
1.8	1.061	1.118	1.881	1.388
1.9	1.061	1.118	1.970	1.388
2.0	1.094	1.118	1.881	1.388
3.0	1.123	1.118	2.059	1.388
4.0	1.064	1.118	1.970	1.388
5.0	1.005	1.012	1.881	1.262
6.0	0.828	1.012	1.522	1.009
7.0	0.798	1.012	1.433	1.009
8.0	0.798	1.012	1.433	1.009
9.0	0.798	1.012	1.433	1.009
10.0	0.798	0.798	1.433	1.009
11.0	0.798	0.798	1.433	1.009
12.0	0.709	0.798	1.343	1.009
13.0	0.650	0.798	1.164	1.009
14.0	0.650	0.798	1.164	1.009
15.0	0.650	0.639	1.164	1.009
27.0[a]	0.621	0.639	1.164	1.009

[a]Optimal hedge ratios remain stable for values of ν greater than 27.0.

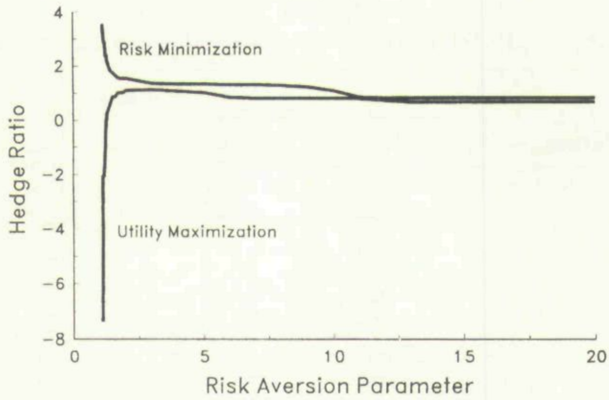

Figure 3
Comparison of risk minimization and utility maximization hedge ratios.

conformity between Tables II and IV. The Ivory Coast presents an exception. For the Ivory Coast with high values of v, the EMG hedge ratio reaches 1.164. By contrast, in Table IV, the highly risk averse mean-variance hedge ratio is 0.778. For other values of v and m, Table IV presents a comparison of equivalent hedge ratios across the four countries. These are broadly consistent across all four countries, and strongly consistent across the three West African countries. For example, with a value of $m = 0.00001$, the four risk aversion parameters range from 1.08 for the Ivory Coast to 1.178 for Brazil, with the average of the West African countries being 1.09. Generally, the African countries have very similar parameters, with the Brazilian parameter being slightly higher.

CONCLUSION

Established futures hedging strategies have focused on risk minimization in a mean-variance framework or on utility maximization under the assumption of a particular utility function, such as a mean-variance, logarithmic, or exponential function. This article presents an alternative utility maximizing approach.

Using the extended mean Gini coefficient as an alternative measure of risk, a utility function is defined in terms of the expected level of wealth and the EMG coefficient. Utility maximizing hedge ratios correspond to the point of tangency between the EMG efficient frontier and an EMG indifference curve. The analysis finds that, for strongly risk averse investors, risk minimizing and utility maximizing hedge ratios are generally similar. For weakly risk averse investors, these hedge ratios differ significantly.

Table III
RISK AVERSION PARAMETERS IMPLIED BY LOGARITHMIC UTILITY

Country	Risk Aversion Parameter	Hedge Ratio
Ghana	1.269	0.151
Nigeria	1.268	0.134
Ivory Coast	1.272	0.303
Brazil	1.328	0.452

The optimal logarithmic hedge ratios are taken from Rolfo (1980), p. 110.

Table IV
Comparison of Mean-Variance and EMG Risk Aversion Parameters

	Ghana		Nigeria		Ivory Coast		Brazil	
Mean-Variance	EMG	Hedge Ratio	EMG	Hedge Ratio	EMG	Hedge Ratio	EMG	Hedge Ratio
1000.0	27.0	0.609	15.0	0.654	13.0	0.778	5.0	0.935
100.0	27.0	0.609	15.0	0.654	13.0	0.778	5.0	0.935
10.0	27.0	0.609	15.0	0.654	13.0	0.778	5.0	0.935
1.0	27.0	0.609	15.0	0.654	13.0	0.778	5.0	0.935
0.1	27.0	0.608	15.0	0.652	13.0	0.773	5.0	0.935
0.01	1.4	0.599	1.42	0.633	1.35	0.737	1.443	0.932
0.001	1.39	0.502	1.37	0.446	1.272	0.370	1.440	0.904
0.0001	1.225	−0.461	1.189	−1.431	1.178	−3.295	1.386	0.622
0.00001	1.098	−10.090	1.091	−20.201	1.08	−39.949	1.178	−2.195

For Ghana, values of v greater than 27 have a hedge ratio of 0.621. For Nigeria, values of v greater than 15 have a hedge ratio of 0.639. For the Ivory Coast, values of v greater than 13 have a hedge ratio of 1.164. For Brazil, values of v greater than 5 have a hedge ratio of 1.009. The mean-variance hedge ratios are adopted from Rolfo (1980).

In comparing utility maximizing hedge ratios across four cocoa producing countries, reverse hedging occurs at $v = 1.25$. For strongly risk averse investors, EMG hedge ratios closely resemble mean-variance hedge ratios in each country, except for the Ivory Coast. The analysis also compares EMG and mean-variance utility maximizing hedge ratios for weakly risk averse investors. The article shows that risk aversion parameters implied by logarithmic utility are consistent across the four countries, and that logarithmic utility represents only a subset of all risk averse investors.

In conclusion, the advantage of the EMG approach over the mean-variance approach is that EMG efficiency implies SSD, whereas the same cannot be said for mean-variance efficiency. Furthermore, EMG efficiency only requires that the first absolute moment exists, and it holds for any utility function. By contrast, mean-variance efficiency requires that the function be fully specified by the first two moments, or that the utility function is quadratic. It appears that the EMG model is a worthwhile alternative utility function and that it warrants further research across different commodities and time periods.

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