Boundary Conditions for Index Options: Evidence from the Finnish Market

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INTRODUCTION

One way to approach the test of options and futures market efficiency is to look for the presence of risk-free arbitrage opportunities in the spirit of Jensen (1978). Most option valuation models, including the famous Black and Scholes (B&S) model, are based on an arbitrage argument. In a continuous time framework, the arbitrage equilibrium is easily determined by a continuous hedging strategy. However, in an incomplete market with only finite opportunities to revise portfolios, no preference-free option price can be constructed with a portfolio of stocks and bonds. Thus, Galai (1983) argues that testing market efficiency with the B&S model is more likely testing the validity of the model itself.

In the discrete time case, Merton (1973) has developed certain boundary conditions for options by using simple dominance argument. Arbitrage strategies based on these conditions do not require continuous portfolio revision but rather the positions are assumed to be held until the expiration. Underlying the establishment of these rational boundaries is the idea of "nondominance" among financial assets. As Galai (1983) has stated, the market cannot be shown to be inefficient (or nonsynchronous) for these weak conditions assumptions, and, at the same time, efficient for compatible but stronger assumptions. Therefore, in a new market it makes sense to start the research with these weaker conditions. This is particularly the case with options written on a stock index.

Most research to date involves call options. Indeed, relatively little has been written about put options. Galai (1983) attributes this to the relatively small volume of put trading and the existence of put–call parity.[1] Perhaps the most important reason, however, is the well-known fact that American put options on even non-dividend-paying stocks may be exercised before maturity. With puts on dividend-paying stocks, the early exercise is even more difficult to assess. Generally, it is more difficult to price American options since early exercise may be optimal before some ex-dividend dates. The pricing becomes even more difficult when stock index options are concerned. In addition, most of the research on options has focused on stock options. Little empirical work has been done with index options. This probably is due to the shorter history of index related products and the difficulties in trading with the index [Evnine and Rudd (1985); Cotner and Horrell (1989)].

[1]For the put–call parity and put–call–futures parity relations, see Moriarty, Phillips, and Tosini (1981).

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The purpose of this article is to examine the boundary conditions on the Finnish Options Market (FOM) which began trading in stock index call and put options and futures contracts May 2, 1988. This study extends the results of prior studies in three ways. Firstly, in addition to boundaries developed by Merton (1973) and Galai (1978), the lower boundaries of FOX calls and puts depend also on the futures price. These conditions are tested in this study. Secondly, published studies to date deal solely with call options when testing the rational boundary conditions of options. This article tests, with modifications, the boundary conditions for put options as well. Thirdly, following the results of Puttonen and Martikainen (1991), this study provides a comprehensive analysis of the relevant transaction costs related to the boundary conditions. Puttonen and Martikainen suggested that a different transaction cost estimate should be employed when examining the upper side of the cost-of-carry model from that used when examining the lower side of it. This was explained by the restrictions on short selling. The same applies for the call option rational boundary, since it may require an arbitrageur to be able to sell the index stocks.

THEORETICAL FRAMEWORK

Merton (1973) derived the rational lower boundary conditions for option prices with respect to their underlying stock prices. The conditions are quite general and they give boundaries for relationships between options which differ by one parameter or more. Galai (1978) extended the boundary conditions for exchange listed American calls. In his study, only conditions for European dividend-unprotected index options are examined[2] because FOX options are such instruments.

The following arbitrage-free condition for a FOX call option can be derived:

$$C(S, K, \tau, D_{t,T}, F) \geq \max[0,\ S - Kr^{-\tau} - D_{t,T},\ (F - K)r^{-\tau}] \tag{1}$$

where $C(S, K, \tau, D_{t,T}, F)$ is the market value of the European call option; S is the index price; K is the striking price; τ is the time to maturity $(T - t)$; F denotes the price of the futures contract with maturity T; r denotes one plus the risk-free rate of interest over a given period; and $D_{t,T}$ is the sum of the present values of all cash dividends to be paid between the current date t and the expiration date T. Let $d_{t'}$ measure the dividend to be paid at t' measured in days after t. The exact value of $D_{t,T}$ can then be expressed as

$$D_{t,T} = \sum_{t'=1}^{T-t} d_{t'} r^{(-t'/365)} \tag{2}$$

The immediate exercise lower boundary tested by Evnine and Rudd (1985) and Chance (1988), i.e., $C > S - K$, does not necessarily hold for European options. Meanwhile, in the FOX markets, the call and put lower boundaries depend not only on the index level but on the futures price. Since this is not addressed in previous studies, this additional condition is shown here for both calls and puts.

Assume that $C < (F - K)r^{-\tau} \rightarrow Cr^{\tau} < F - K$. Buy the call, borrow the money needed at risk-free interest rate r, and sell the futures contract. Hold this portfolio until the expiration date. The cashflows at the expiration are presented in Table I. This strategy is based on an assumption that $Cr^{\tau} < F - K$. Thus, $F - K - Cr^{\tau}$ is positive. Furthermore, if the index value at the expiration (S^*) is less than the striking price (K), then $F - S^* - Cr^{\tau}$ is also positive. The outcome of the strategy is also positive regardless of the terminal value of the index.

[2]See Merton (1973), and Cox and Rubinstein (1985) for proofs of other bounds.

Table I
CALL OPTION LOWER BOUNDARY DETERMINED BY THE FUTURES PRICE

	Current Date	Expiration Date	
		$S^* < K$	$S^* \geq K$
Buy a call	$-C$	—	$S^* - K$
Borrow C	$+C$	$-Cr^{T}$	$-Cr^{T}$
Sell a futures	—	$F - S^*$	$F - S^*$
Total	—	$F - S^* - Cr^{T}$	$F - K - Cr^{T}$

C denotes the current price of a call option with striking price K and time to maturity τ, F is the current price of a futures contract, S^* is the index value at the expiration, and r denotes one plus the risk-free interest rate over the τ.

Correspondingly, the lower boundary for a put option can be written as:

$$P(S, K, \tau, D_{t,T}, F) \geq \max[0, (K - F)r^{-\tau}, D_{t,T} + Kr^{-\tau} - S] \tag{3}$$

For the proof of put lower boundary determined by the futures price, assume $P < (K - F)r^{-\tau}$. Buy the put, borrow the money needed, and buy the futures contract. The expiration cashflows are presented in Table II. The base assumption is $K - F - Pr^{\tau} > 0$. Thus, it is easy to see that there are only positive outcomes from the strategy.

Let e_c be the left-hand side of eq. (1) minus its right-hand side and e_p the difference in eq. (3). A testable hypothesis for well-synchronized stock, futures, and option markets is that e_c and e_p are not less than zero.

Galai (1978) expanded and tested the call boundary conditions of Merton (1973). He examined daily closing prices of the CBOE during the exchange's first 6 months of operation. To overcome some of the problems in dealing with daily closing prices, tests were carried out on transaction-on-transaction prices of few options during October, 1973. The hypothesis that simultaneous closing prices are within theoretical boundaries, was rejected. The frequent deviations could not be explained by either the assumption of perfect knowledge of the future dividends or by the possible inaccuracy of the data.

Using transactions data for options traded on the CBOE for the period August, 1976 to June, 1977, Bhattacharya (1983) found that transaction costs can explain why most of the mispricings are not eliminated by arbitrage. The transaction cost estimate used included the bid–ask spread. Halpern and Turnbull (1985) tested the boundary conditions for transactions data from the Toronto Stock Exchange. It was found that violations did

Table II
PUT OPTION LOWER BOUNDARY DETERMINED BY THE FUTURES PRICE

	Current Date	Expiration Date	
		$S^* < K$	$S^* \geq K$
Buy a put	$-P$	$K - S^*$	—
Borrow P	$+P$	$-Pr^{T}$	$-Pr^{T}$
Buy a futures	—	$S^* - F$	$S^* - F$
Total	—	$K - F - Pr^{T}$	$S^* - F - Pr^{T}$

P denotes the current price of a put option.

occur even after considering tangible transaction costs. Moreover, the results suggest that over the 2-year period, both the frequency of violations and their magnitude tend to increase rather than decrease.

Generally, to demonstrate market inefficiency, an implementable trading rule generating profits should be specified. Galai (1978, 1983) argues that many of these tests of market efficiency do not specify a trading strategy which a trader can duplicate. The point is that observing a price which, ex post, would have allowed a trader to establish a position that generates abnormal profits does not necessarily mean that markets are inefficient. To set up a trading strategy after observing the signal the arbitrageur frequently must place simultaneous orders in at least two different markets. A price change in any of these markets can eliminate the arbitrage opportunity and there is no guarantee of receiving only nonnegative returns. Thus, when testing market efficiency, ex ante tests should be carried out.

Halpern and Turnbull (1985) observed only 16% reduction in the number of profitable arbitrage positions after allowing an execution delay. If a violation signal was observed, a position was taken using the data of the next option trade. They thus concluded that initial violations tend to be followed by a subsequent violation which suggests market inefficiency. Quite the opposite results were reported by Chance (1988) who examined the bid and ask prices of S&P 100 index call options for the first 4 months of 1984. Even absent transaction costs, only few violations were found. Moreover, assuming an execution delay of one day explained most of these violations. Chance therefore concluded that index options appeared to be priced efficiently.

DATA

The data used in this study is from the option and futures contracts based on the Finnish Options Index (FOX). The Finnish markets provide a unique arena for empirical work in this area. (1) The FOX options are European type options, which makes it easier to determine the theoretical price of them. In particular, it makes it possible to carry out the boundary tests with put options. (2) Prior empirical work concentrates on large markets. However, there are differences in economic and institutional environments between large and small markets such as market mechanism and legislation. (3) Finnish companies pay dividends only once a year, usually in March or April. This results in clustered dividend payments on the stock index. (4) There is no institutional framework for short selling in the Helsinki Stock Exchange (HeSE). The continuous underpricing of the FOX futures contract has been explained [Puttonen (1993)] by the difficulties the arbitrageur faces when trying to benefit from arbitrage opportunities. Theoretically, however, the short sales constraints cannot alone explain the substantial mispricings observed. Arbitrageurs with capital in index stocks should sell the stocks needed and purchase the undervalued futures contract. It is also possible for traders to borrow the stocks needed for arbitrage. This would, however, cause additional roundtrip stamp duty and the trader probably would have to pay some premium to the owner of the stocks. In addition to these direct costs, the current stock owner may incur additional taxes upon selling the stocks. (5) The FOX index consists of only 25 stocks, but the index is heavily weighted toward (about half) the five most liquid stocks. Therefore, the effect of tracking error can be reasonably ignored. (6) Preliminary evidence suggests that put–call parity does not hold in the Finnish markets. Puttonen (1992) found several violations from the put–call parity. (7) The first years of index derivatives trading were marked by heavy stock price fluctuations. A great decrease in trading volumes occurred in 1990. Insufficient liquidity widens the arbitrage spread in two ways: it causes index autocorrelation and it increases market impact costs to traders. Generally, illiquidity is more serious for stocks than for futures.

Transaction Price Data

The base figure of the FOX index was set at 500.00 on March 7, 1988. One index point equals 100 Finnish marks.[3] FOX index options and futures contracts expire six times a year in February, April, June, August, October, and December. The last trading day is the fourth Thursday of the month. Since the expiration period of each option contract is 4 months, there is always a supply of instruments with a remaining time to expiration of at least 2 months. The sample period is from May 2, 1988, to December 21, 1990. Opening of the market place defines the beginning date while data availability is responsible for the ending date.

Daily closing prices at 3:30 PM are used. The data set is based on the nearby contracts because they have been more liquid than the next nearest option and futures contracts. A week before maturity the analysis is shifted to the next contracts because as maturity week nears, trading volume shifts gradually into the contracts with the next maturity date. In addition, this makes it possible to avoid any possible expiration week effects.

To examine the effect of moneyness as an important variable when examining violations from rational option pricing [see, e.g., Cotner and Horrell (1989); Halpern and Turnbull (1985)], options are divided into five categories. Following Rubinstein (1985), the ratio of current index price to the striking price determines the call "moneyness" as follows:

1. Deep-out-of-money (0.75–0.85)
2. Out-of-money (0.85–0.95)
3. At-the-money (0.95–1.05)
4. In-the-money (1.05–1.15)
5. Deep-in-the-money (1.15–1.25)

The moneyness for put options is determined the opposite way, i.e., by dividing the striking price with the current index price. Options with a ratio of less than 0.75 or greater than 1.25 are omitted since they can be expected to be highly illiquid contracts. This provides a total of 4840 bid and ask call option prices and 4689 bid and ask put option prices.

Table III presents the summary statistics of daily logarithmic price changes of the FOX index. Means and standard deviations are multiplied by 100. The first years of options and futures trading were marked by drastic stock price fluctuations. On the first trading day, May 2, 1988, the FOX index was at the level of 550.7. The index reached its all-time-high, 697.7 on April 17, 1989. Thereafter, price movements became irregular an at the end of the year, the index stood at 495.1 index points. In 1990, the fall continued and at the end of the year the index was at a 34% lower level than one year earlier.

In addition to index values and returns, the effect of dividend stream on the index is calculated. Finnish companies generally pay dividends only once a year, usually in March or April. This results in clustered dividends on the stock index. Assuming a constant dividend yield would greatly distort the option lower boundary tests. In this study, actual

[3]One Finnish mark used to equal approximately 0.25 USD.

Table III
SUMMARY STATISTICS OF THE FOX INDEX
AND DIVIDEND FLOW ON THE INDEX STOCKS

	Index			Present Value of Dividends			
	Returns		Value at T	Index Points		Percents	
Contract	Mean	Stdv.	S_T	Mean	Maximum	Mean	Maximum
Jun. 88	0.4	0.4	617.01	1.24	2.58	0.22	0.47
Aug. 88	0.0	0.4	616.55	—	—	—	—
Oct. 88	−0.2	0.8	594.15	—	—	—	—
Dec. 88	0.1	0.4	602.19	—	—	—	—
Feb. 89	0.2	0.5	656.11	—	—	—	—
Apr. 89	0.1	0.8	686.44	7.57	12.78	1.14	1.97
Jun. 89	−0.1	0.4	659.71	1.40	1.99	0.21	0.30
Aug. 89	−0.1	0.4	652.33	—	—	—	—
Oct. 89	−0.3	1.2	550.79	—	—	—	—
Dec. 89	−0.3	1.1	495.10	—	—	—	—
Feb. 90	0.2	1.6	541.89	0.18	0.43	0.04	0.09
Apr. 90	−0.3	0.7	465.80	9.16	14.21	1.74	2.66
Jun. 90	−0.1	0.8	449.43	0.92	1.65	0.20	0.36
Aug. 90	−0.1	0.6	419.72	—	—	—	—
Oct. 90	−0.7	1.1	323.16	—	—	—	—
Dec. 90	−0.0	0.8	324.09	—	—	—	—

S_T denotes the index value at the expiration T.

instead of a proxy for expected dividend payments are used. This is because for the next 2 months most of the firms have already declared dividends and the exact payment dates.

Interest Rates

Daily quotations of the Helibor rate are used for the risk-free interest rate. Helibor is commonly used in Finland as the analog of the short-term Treasure bill rate in the U.S. Since there is no interest rate for the maturity exactly matching the option and futures contracts each trading day, the rate nearest to the maturity is used. Although not theoretically correct, the errors induced by this approximation are assumed minimal.

Explicit Transaction Costs

The costs of opening and closing an arbitrage position in the Finnish stock index markets involve commissions of the market place (HeSE and FOM), stamp duties, and market impacts in both the stock and derivatives markets. If the position is held until expiration, no market impact costs are incurred because the stocks, futures, and options can be sold or purchased at the market-close prices. However, there are commissions for liquidating the position. A stamp duty has to be paid for the profit received from the futures contract position.

There are several explicit costs in the Finnish markets that have to be considered.[4] First, if an arbitrageur acts as a broker on the HeSE, the only direct cost incurred in the

[4]Figures are based on conversations with staff members of the FOM and some brokerage houses.

stock market is the stamp duty. The roundtrip stamp duty is 1.0%, consisting of 0.5% when buying (selling) the stock and 0.5% when reversing the position. When building an arbitrage strategy, however, the exact value of index level at the maturity is not known. Thus, a conservative estimate must be used. In this study, the stamp duty is estimated to be 1.0% of the current price of the index stocks. For a non-broker, the costs in the stock market also include a brokerage commission varying between 0.5% and 1.0%. In this study, the former estimate is used.

Traders on FOM can be divided into three categories: market makers, brokers, and final customers (clients). The explicit transaction costs for an arbitrageur acting as a market maker in the options and futures markets consist of fixed commissions of FOM and a stamp duty. Table IV shows how the market place has changed the commissions during the first years of trading. For a non-broker, 60 marks brokerage fee in the futures market is added while in the options market, an estimate of 2.0% brokerage fee is assumed. The total roundtrip transaction costs for a final customer have been as high as 10–13%/premium in options. Although not directly comparable with Phillips and Smith's (1980) results, these figures also indicate the great difference in the costs between a market maker and a final customer.

In the FOX options market, estimating the explicit transaction costs is not as straightforward as in the stock and futures markets. Since 1989 it has been possible for market makers to pay a fixed amount of 15000 mk (45000 mk) per month and lower the commission to 5,5 mk (3.5 mk) per option. This complicates the calculation of arbitrage profits, because the arbitrage should be viewed from the position of the most advantageously placed trader. Phillips and Smith (1980) argue that investment in an exchange seat establishes an opportunity cost. It is argued also that traders incur costs from gathering and monitoring arbitrage opportunities. These costs are extremely difficult to allocate to a particular transaction. In this study, these fixed costs are ignored for simplicity. From the beginning of 1990, the trader had an opportunity to choose among three different commission systems. This study assumes the arbitrageur does not pay any fixed amount each month nor that the arbitrageur is under contract to trade some minimum amount of contracts in a month.

Implicit Transaction Costs

Implicit costs are more difficult to calculate. Implicit costs are usually estimated by the bid–ask spread, the difference between the highest quote to buy, and the lowest offer to sell. Phillips and Smith (1980) and Bhattacharya (1983) were the first who argued that financial assets are generally bought at the ask price and sold at the bid price. As noted by Phillips and Smith, however, estimation of the appropriate magnitude of this cost is no simple matter and is not easily solved. In practice, trades occur also inside the spread, not only at the bid or ask. In this study, for options and futures contracts, the actual bid ask spread is used. Table V presents summary statistics on the spreads at the market close. The difference between bid and ask prices is generally wider for puts than for calls.

The bid and ask quotes of the stock market were not available for this study. Keim (1989) has provided statistics for bid–ask spreads for NYSE, AMEX, and OTC NMC stocks on the U.S. markets. The median spread for the four largest size deciles of the OTC NMC stocks is under 1%. Yadav and Pope (1990) examined the stocks included in the FTSE-100 index quoted on the International Stock Exchange (formerly the London Stock Exchange). They reported an average spread of about 1%. The Helsinki Stock Exchange

Table IV
ONE-WAY TRANSACTION COSTS IN THE FINNISH OPTIONS MARKET (FOM)

FOX Index Options			
Broker		Market Maker	
Commission	Clearing Fee	Commission	Clearing Fee
May 2, 1988 ⇒			
3.0%/premium	1.5%/premium/option min 10 mk, max 26 mk	—	1.5%/premium min 10 mk, max 26 mk
June 1, 1988 ⇒			
1.7%/premium	1.0%/premium min 10 mk, max 26 mk	—	1.0%/premium min 10 mk, max 26 mk
November 1, 1988 ⇒			
1.7%/premium	1.0%/premium min 10 mk, max 26 mk	—	0.5%/premium min 5, max 16 mk
August 14, 1989 ⇒			
1.7%/premium	1.0%/premium min 10 mk, max 26 mk	—	0.7%/premium min 5, max 16 mk
September 18, 1989 ⇒			
1.8%/premium	0.9%/premium min 8 mk, max 26 mk	—	0.7%/premium min 5 mk, max 16 mk
March 1, 1990 ⇒ December 21, 1990			
1.6%/premium	0.9%/premium min 6,5 mk	—	0.7%/premium min 5 mk, max 16 mk

(continued)

Table IV (continued)

FOX Index Future			
Broker		Market Maker	
Commission	Clearing Fee	Commission	Clearing Fee
May 2, 1988 ⇒			
—	60 mk/futures contract	—	24 mk/futures contract
November 1, 1988 ⇒			
—	60 mk/futures contract	—	10 mk/futures contract
August 14, 1989 ⇒			
—	60 mk/futures contract	—	13 mk/futures contract
March 1, 1990 ⇒ December 21, 1990			
—	40 mk/futures contract	—	13 mk/futures contract

Table V
THE AVERAGE BID–ASK SPREAD AT THE MARKET CLOSE (AS A PERCENTAGE OF THE UNDERLYING INDEX)

	Deep-in-the-Money		In-the-Money		At-the-Money		Out-of-the-Money		Deep-out-of-the-Money		Futures Contracts
Contract	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Calls	Puts	Mean (%)
Jun. 88	0.52	—	0.40	0.35	0.21	0.20	0.13	0.15	—	0.10	0.36
Aug. 88	0.59	—	0.46	0.24	0.19	0.15	0.07	0.13	—	0.12	0.28
Oct. 88	—	0.86	0.45	0.60	0.18	0.18	0.09	0.13	0.02	—	0.36
Dec. 88	1.00	—	0.53	0.44	0.12	0.16	0.04	0.09	—	—	0.19
Feb. 89	—	—	0.49	0.44	0.14	0.16	0.05	0.06	—	—	0.23
Apr. 89	1.02	—	0.72	0.39	0.13	0.13	0.06	0.07	—	0.03	0.17
Jun. 89	—	—	0.76	0.38	0.18	0.11	0.03	0.07	—	—	0.19
Aug. 89	—	—	0.48	0.66	0.18	0.11	0.04	0.06	—	—	0.22
Oct. 89	—	2.93	1.16	1.81	0.23	0.26	0.08	0.17	0.04	—	0.44
Dec. 89	1.97	2.29	1.21	1.64	0.29	0.36	0.12	0.19	0.09	0.19	0.56
Feb. 90	2.09	2.73	1.21	2.11	0.29	0.61	0.24	0.25	0.28	0.26	0.66
Apr. 90	1.85	1.70	1.70	1.34	0.42	0.26	0.13	0.31	0.07	0.42	0.50
Jun. 90	—	1.37	1.02	0.99	0.28	0.25	0.11	0.13	0.11	—	0.48
Aug. 90	—	—	0.87	0.85	0.21	0.24	0.07	0.18	—	—	0.53
Oct. 90	2.15	1.90	1.27	1.40	0.37	0.55	0.14	0.30	0.07	0.25	1.30
Dec. 90	1.61	1.74	1.15	1.29	0.24	0.52	0.13	0.16	0.09	0.11	0.82
Average	1.42	1.94	0.81	0.93	0.23	0.27	0.10	0.15	0.10	0.19	0.46

can be regarded as much more thinly traded than its U.S. and English counterparts. On the other hand, FOX index consists of the 25 most liquid stocks. In this study, an estimate of 2% bid–ask spread (i.e. ±1% market impact) is used.

In the case of index derivatives arbitrage, there is also always a choice between trading all the stocks in the index basket or using only some major stocks that closely match the index [see Evnine and Rudd (1985)]. This study ignores the effect of tracking error in the name of simplicity. The difference between borrowing and lending rates is ignored also because it is felt to be a negligible factor for large investors. Option and futures traders are subject also to margin requirements. The margins can be posted in the form of stocks or bonds on which the investor continues to earn dividends and interest. Hence they are not a restriction for those who already own the stocks or bonds.

EMPIRICAL RESULTS

The empirical work in this section is based on Inequalities (1) and (3). The inequalities are here extended to include bid–ask spreads and the relevant transaction costs. For the European call option the lower boundary condition can be stated as

$$e_c \equiv C_a - \max[0, S_b - Kr^{-\tau} - D_{t,T} - TC_{(s,c)}, (F_b - K)r^{-\tau} - TC_{(f,c)}] \geq 0 \quad (4)$$

and for the European put option

$$e_p \equiv P_a - \max[0, (K - F_a)r^{-\tau} - TC_{(f,p)}, D_{t,T} + Kr^{-\tau} - S_a - TC_{(s,c)}] \geq 0 \quad (5)$$

where

$C_{a/b}$ = ask/bid price of a call
$P_{a/b}$ = ask/bid price of a put
$F_{a/b}$ = ask/bid price of a futures contract
$TC_{(c,p,s,f)}$ = roundtrip transaction costs incurred in the options (c) and (p), stock (s) and futures (f) markets.

The testable null hypotheses are that the price of an option is not less than the rational lower boundary, i.e., e_c and e_p are not less than zero. Daily closing bid and ask prices are checked to test the hypothesis.

Ex Post Tests

Only one violation of the put lower boundary is found. Furthermore, no violations are found when the futures call lower boundary is tested. Meanwhile, several violations from the call lower boundary determined by the underlying index ($S_b - Kr^{-\tau} - D_{t,T} - TC_{(s,c)}$) are observed. The findings have two main implications. First, the possible inaccuracy of the data is not the only factor affecting the results. Since the cash market data is the same for both puts and calls, violations in call prices are expected to appear also in put prices. Second, the call lower boundary determined by the underlying index, is the only condition that requires selling the index to benefit from the violations. The evidence suggests that short selling restrictions in the Finnish stock market affect the arbitrage process between the stock and options markets. The lack of arbitrage allows option prices to deviate from their theoretical values.

The results for calls are shown in Table VI. The profits represent the results of the positions of a market maker and are calculated on a per contract basis. All the violations occur in Deep-in-the-money to At-the-money categories.[5] It also seems that the more in-the-money the option is, the greater is the magnitude of violation. Galai (1978) suggests that this can be explained by the fact that deep-in-the-money option prices tend to follow the price of the underlying security very closely. Under these circumstances, it is difficult to maintain the stock markets in a continuous equilibrium or even within boundary conditions.

The most viable discrepancies occur in the February, 1990 contract. This can be explained partly by the strike of bank employees at that time. The stock exchange then was highly illiquid making trading in the stocks difficult. At that time, the actual market impact was probably more than 1%. Other large and frequent violations are found for the last two contracts in each year. It is notable that only a few violations occur during the April and June contracts, which suggests that dividends have no effect if option pricing. Similar to the findings of Halpern and Turnbull (1985) the results suggest that over a 3-year period, both the frequency and their magnitude tend to increase rather than vice versa. Empirical evidence thus contradicts the claim that markets grow more efficient over time while traders become more familiar with new instruments. The results thus suggest that market liquidity is a more important factor in arbitrage pricing.

Ex Ante Tests

Following Galai (1978) and Halpern and Turnbull (1985), ex ante tests are carried out to test the efficiency of the markets. The null hypothesis of efficient market here is that a trading rule based on boundary conditions does not yield above the risk-free interest rate. Upon observing a violation at time t, the arbitrageur enters the markets at $t + 1$. Only the positions yielding a profit are reported here. The rest of the trading based on observed arbitrage possibilities at time t produce a loss. The results of the tests are summarized in Table VII.

The statistical significance of ex ante arbitrage profits is tested by the nonparametric Sign test. The test is employed when at least 10 violations are reported in a category. Unlike the more common parametric tests, nonparametric tests are distribution-free since they assume nothing about the population from which the sample is taken. Even though nonparametric tests have less power, these tests have an advantage of symmetric weighting. Also, outliers do not affect the results. The test results suggest that in almost every case the arbitrage strategies are more often profitable than unprofitable when one day's execution lag is assumed.[6] Initial violations tend to be followed by a subsequent violation. Thus, even allowing an execution delay of one day does not reduce the arbitrage profits significantly.

[5]As suggested by Halpern and Turnbull (1985), the low frequency of violations for out-of-the-money options is not surprising. If call violates its lower boundary, then $C < S - Kr^{-\tau} - D_{t,T}$. This can be written as

$$C + (K - S) < -Kr^{-\tau} - D_{t,T} + K$$

As the call is out-of-money, then $K - S$ is greater than zero. Then, the call price can only satisfy the above inequality when no dividends are paid and the time to maturity is long.

[6]The Sign test statistics is calculated as

$$\text{Sign} = \frac{U + 1/2 - n/2}{\sqrt{n/4}}$$

where U = number of positive returns and N = number of observations. The Sign is approximately a standard normal variable.

Table VI
MISPRICING SIGNALS FOR THE EUROPEAN CALL LOWER BOUNDARY FROM THE POSITION OF A MARKET MAKER: EX POST TESTS (IN FINNISH MARKS)

	Deep-in-the-Money				In-the-Money				At-the-Money			
Contract	#	%	Mean	Median	#	%	Mean	Median	#	%	Mean	Median
Jun. 88	—	—	—	—	2	3.0	35.54	35.54	—	—	—	—
Aug. 88	—	—	—	—	—	—	—	—	—	—	—	—
Oct. 88	—	—	—	—	6	21.4	67.29	52.79	5	3.9	70.16	51.89
Dec. 88	—	—	—	—	34	38.6	394.78	389.98	2	1.7	88.82	88.82
Feb. 89	—	—	—	—	2	4.7	174.95	174.95	—	—	—	—
Apr. 89	1	4.8	65.83	65.83	4	3.8	109.08	83.09	1	0.7	110.32	110.32
Jun. 89	—	—	—	—	—	—	—	—	—	—	—	—
Aug. 89	—	—	—	—	—	—	—	—	—	—	—	—
Oct. 89	—	—	—	—	10	34.5	638.21	734.31	7	5.2	276.65	299.13
Dec. 89	5	22.7	364.03	451.78	15	27.8	470.94	457.30	8	8.1	215.44	215.46
Feb. 90	54	68.4	1148.46	1071.86	57	63.3	786.59	739.06	7	6.4	295.65	197.28
Apr. 90	13	41.9	201.85	230.09	3	3.5	106.8	98.83	—	—	—	—
Jun. 90	—	—	—	—	1	1.7	16.77	16.77	—	—	—	—
Aug. 90	—	—	—	—	20	35.1	244.05	167.22	9	8.7	195.31	138.01
Oct. 90	14	70.0	815.11	758.79	37	69.8	455.26	426.52	6	6.7	182.92	148.82
Dec. 90	16	42.1	637.00	789.19	24	39.3	447.07	448.06	7	10.0	100.80	87.57

% denotes the percentage of the total observations in each category.

Table VII
THE PROFITABILITY AND STATISTICAL SIGNIFICANCE OF EX ANTE STRATEGY (IN FINNISH MARKS)

	Deep-in-the-Money				In-the-Money				At-the-Money			
Contract	Mean	Min	Max	Sign	Mean	Min	Max	Sign	Mean	Min	Max	Sign
Jun. 88	—	—	—	—	−145.80	−282.86	−8.73		—	—	—	—
Aug. 88	—	—	—	—	—	—	—	—	—	—	—	—
Oct. 88	—	—	—	—	−107.03	−482.53	151.05	—	−269.79	−1218.09	78.55	—
Dec. 88	—	—	—	—	329.88	−273.35	1068.55	4.63[c]	−243.97	−314.87	−173.07	—
Feb. 89	—	—	—	—	−16.41	−279.27	246.46	—	—	—	—	—
Apr. 89	−526.55	−526.55	−526.55	—	−257.76	−499.60	194.7	—	−486.12	−486.12	−486.12	—
Jun. 89	—	—	—	—	—	—	—	—	—	—	—	—
Aug. 89	—	—	—	—	—	—	—	—	—	—	—	—
Oct. 89	—	—	—	—	519.47	−479.83	1120.47	2.85[c]	−4.32	−391.93	491.88	—
Dec. 89	316.57	−523.85	632.87	—	134.41	−789.75	996.55	0	−87.63	−831.00	398.65	—
Feb. 90	1031.96	−542.89	2849.49	5.49[c]	660.17	−717.25	2046.14	5.30[c]	76.38	−1062.27	978.50	—
Apr. 90	46.27	−289.01	353.69	1.11	101.14	−328.96	98.83	—	—	—	—	—
Jun. 90	—	—	—	—	−36.18	−36.18	−36.18	—	—	—	—	—
Aug. 90	—	—	—	—	199.12	−176.39	542.49	5.38[c]	114.65	−174.19	507.69	—
Oct. 90	720.86	−151.75	1967.31	3.47[c]	377.09	−104.20	1389.77	5.26[c]	−145.18	−823.83	738.86	—
Dec. 90	501.73	−288.48	1127.52	3.25[c]	424.31	−115.35	975.04	3.88[c]	109.50	−163.32	363.08	—

Sign = the nonparametric test statistics under the null hypothesis that the number of positive violations is equal to the number of negative violations. H1: number of positive violations is greater than number of negative violations. The letters [a, b,] and [c] suggest that the null hypothesis is rejected at 0.10, 0.05, and 0.01 levels of significance.

Table VIII
MISPRICING SIGNALS FOR THE EUROPEAN CALL LOWER BOUNDARY FROM THE POSITION OF A FINAL CUSTOMER (IN FINNISH MARKS)

	Deep-in-the-Money				In-the-Money				At-the-Money			
Contract	#	%	Mean	[Mean]	#	%	Mean	[Mean]	#	%	Mean	[Mean]
Jun. 88	—	—	—	—	—	—	—	—	—	—	—	—
Aug. 88	—	—	—	—	—	—	—	—	—	—	—	—
Oct. 88	—	—	—	—	—	—	—	—	—	—	—	—
Dec. 88	—	—	—	—	10	11.4	215.49	33.49	—	—	—	—
Feb. 89	—	—	—	—	—	—	—	—	—	—	—	—
Apr. 89	—	—	—	—	—	—	—	—	—	—	—	—
Jun. 89	—	—	—	—	—	—	—	—	—	—	—	—
Aug. 89	—	—	—	—	—	—	—	—	—	—	—	—
Oct. 89	—	—	—	—	7	24.1	356.20	69.15	2	1.5	66.03	−361.02
Dec. 89	—	—	—	—	8	14.8	281.91	−80.75	—	—	—	—
Feb. 90	35	44.3	829.11	552.16[b]	36	40.0	530.17	388.81[c]	3	2.75	167.32	207.26
Apr. 90	—	—	—	—	—	—	—	—	—	—	—	—
Jun. 90	—	—	—	—	—	—	—	—	—	—	—	—
Aug. 90	—	—	—	—	3	5.3	154.06	−185.01	1	1.0	144.81	−210.71
Oct. 90	11	55.0	563.41	447.66[c]	23	43.3	349.94	222.54[c]	2	2.2	138.07	−639.84
Dec. 90	10	26.3	534.72	387.17[b]	15	24.6	336.85	276.21[b]	2	2.9	32.97	−127.03

Mean = mean arbitrage profit in ex post tests, [mean] = mean arbitrage profit is ex ante tests. The letters [a], [b], and [c] after the [mean] denote rejection of the null hypothesis at 0.10, 0.05, and 0.01 levels of significance. The null hypothesis is that the number of positive violations is equal to the number of negative violations.

Table VIII provides results from positions taken by a final customer. The number of violations decrease from 370 to 168 cases when larger transaction costs are assumed. However, there still seems to be profitable ex ante profits available for February 1989 and October–December 1990 contracts. This further suggests that the owners of large stock portfolios could benefit from the violations. They should have made extra profits by selling the index stocks and buying the undervalued calls.

To examine the relationship between days to maturity and the frequency of the violations, the violations are put in order according to the time left to maturity. Figure 1 shows that there exists no relationship between these variables. In categories of less than 20 days or more than 60 days to maturity, slightly fewer violations exist, but no other conclusions can be made.

Time to Maturity

Concerning the average size of violations, the correlation coefficient between the size and the corresponding time to maturity is computed. The statistically insignificant coefficient of 0.016 suggests that no relationship between the maturity of an option and the average size of violation exists. A closer look indicates, however, that there may be some kind of nonlinear relationship as Figure 2 shows.

It seems that the violations in categories 18 to 30 and 51 to 56 days until expiration are of greater magnitude than other violations. The limited number of observations prevent the assertion of further conclusions about the relationship between the number of days until the maturity and the size of violations. A larger data set is necessary for further conclusions.

Some shortcomings of the data and methodology of this study should be discussed. The use of daily closing price data may be problematic for a study of mispricings because closing prices for different instruments may reflect trades made at different points in time. In the Finnish markets, the major difficulty concerns the relatively illiquid stock market. In sharply moving markets the bias in favor of inefficient markets is greater. Thus, carefulness is needed when interpreting the results in times of heavy stock price fluctuations. Even though the frequent violations suggest that the option market is inefficient, the "inefficiency" probably results from illiquid underlying cash market and the difficulty of arbitrage trading of index stocks.

Deep-in-the-money and deep-out-of-money options, in particular, may not be traded for long time periods. However, the bid and ask quotations are binding for the market makers who are committed to make at least one trade (10 contracts) at these prices. Whether

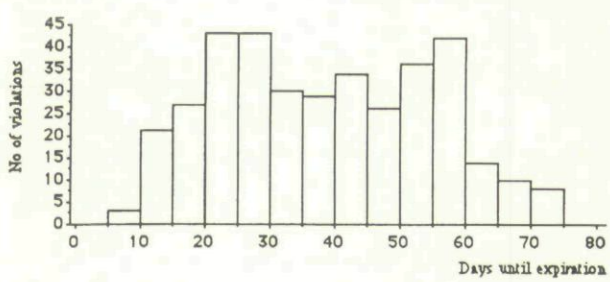

Figure 1
Frequency of violations and the number of days to expiration.

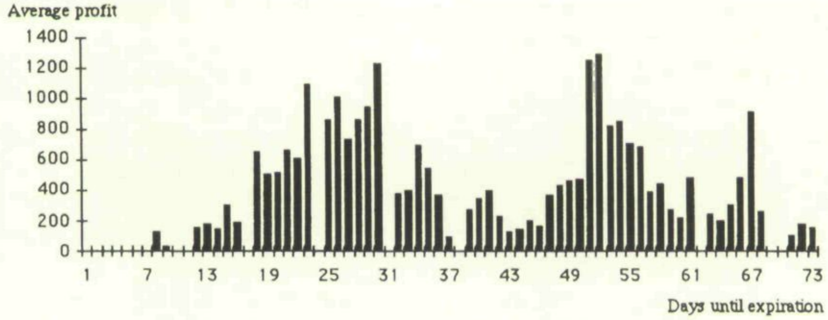

Figure 2
Average arbitrage profit and the number of days to expiration.

more than one set of contracts could be traded at these prices cannot be ascertained.[7] The ex ante tests may overstate the delay in the execution when daily data is used. Contrary to the findings of Chance (1988), however, it is found here that even ex ante profits are available in the Finnish markets.

In this study, the actual bid and ask prices are used. By working only with the widest band of available bid–ask spread the tests may be biased in favor of the null hypothesis of efficient or synchronized markets [Phillips and Smith (1980); Bhattacharya (1983)]. The data available do not allow the use of any other estimate. One the other hand, it is possible that only one trade could be completed at any given bid or ask price.

CONCLUSIONS

Two rational boundary condition tests are performed for both call and put options. Violations from call lower boundary are reported for in-the-money options. These are consistent with the findings in the U.S. stock options markets except that violations on the FOX market are of greater magnitude. All the violations are from the lower boundary determined by the underlying index. Put options are found to have traded clearly above their rational lower boundary.

Many of the reported arbitrage opportunities appear too large to be the result of transaction costs only. Therefore, the effect of transaction costs on a final customer (a portfolio manager) is examined. The results suggest that an arbitrageur with FOX stocks gain additional risk-free profits. It must be remembered, however, that there are certain problems related to stock owners' arbitrage. The stock owners may find it inappropriate to sell the stocks due to, e.g., voting power connected with the stocks or fiscal reasons. The large stock owners may also find it impossible to sell the stocks due to the illiquidity of the market.

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[7]It was possible to implement trades in FOM either electronically via computer or via telephone by using a so-called block-order function during the time period covered by this study. The block-order function matches larger blocks (minimum 10) of contracts. A major share of trading was transacted using that function during the research period. Since then, an electronic system has been used.

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