How Price Discovery by Futures Impacts the Cash Market

James T. Witherspoon

INTRODUCTION

Futures markets serve as an important medium for price discovery. This has been well-documented by many studies (especially for stock index futures) using a variety of methods, for example, Herbst et al. (1987), Kawaller et al. (1987), Ng (1987), Swinnerton et al. (1988), Harris (1989), Amihud and Mendelson (1989), and Stoll and Whaley (1990).

The Garbade and Silber (1983) model (GS model) for imperfect arbitrage between a futures market and its corresponding cash market is particularly useful for measuring, simultaneously, the source of price discovery and the elasticity of cash/futures arbitrage. Garbade and Silber used their model to study the commodity markets—wheat, corn, oats, orange juice, copper, gold, and silver. They found futures to be the primary source of price discovery for the corn, wheat, gold, and silver markets, and they found cash/futures arbitrage elasticities to be distinctly higher for the gold and silver markets than for the other commodities studied. Several other studies of price discovery have employed the GS model.[1] Merrick (1987) used the GS model to study stock index futures, finding futures to be the primary source of price discovery by 1985, which coincided with the futures market assuming leadership over the cash market in dollar volume terms. Laatsch and Schwarz (1988) studied the Major Market Index using one-minute differencing intervals; they found futures to be the primary source of price discovery by 1986. Oellermann et al. (1989), using daily differencing intervals from 1979 to 1986, found futures to be the primary source of price discovery for the feeder cattle market. Schwarz and Laatsch (1991), using weekly, daily, and 5-minute differencing intervals, found that futures became the primary source of price discovery for the Major Market Index after just 10 months of trading the futures contract. High levels of price discovery by futures were coincident with high relative volume in the futures market, especially in the months preceding the 1987 crash. However, they found that the futures market does not maintain its leading role as the source of price discovery at all times intraday. Khoury and Yourougou (1991) found futures to be the primary source of price discovery for the Canadian barley, oats, and canola markets. Schroeder and Goodwin (1991) used

The author is grateful to the editor and an anonymous referee for encouragement and helpful suggestions. Any remaining errors are the responsibility of the author.

[1]The GS model also has been used to study other cash/futures market relationships. For example, Lien (1987) used GS to study the inventory effect in commodity futures markets.

James T. Witherspoon is a Research Associate at Witherspoon Investment Counsel, St. Louis.

The Journal of Futures Markets, Vol. 13, No. 5, 469–496 (1993)

CCC 0270-7314/93/050469–28

the GS model to study the live hog market on a daily basis from 1975 to 1989; futures were the primary source of price discovery except for 5 of the 15 years. Quan (1992) used GS and other methods to study the crude oil futures market on a monthly basis from 1984 to 1989 and found the cash market to be the primary source of price discovery.

Price discovery by futures has important implications which have not been recognized for cash market volatility and autocorrelation. The purpose of this article is:

i. to show that the GS model is a very special case of a much larger class of models that can be studied in a canonical manner;
ii. to explore the implications of the GS model for cash market volatility and autocorrelation and the sensitivity of the model to changes in its parameters, and to identify the most interesting critical and limiting cases;
iii. to identify useful regulatory policies based on the implications of the GS model.

The results show that if price discovery by futures exceeds certain critical threshold levels of dominance, the implications for the cash market are progressively deleterious—increased autocorrelation, volatility, illiquidity, and instability. On the other hand, if price discovery by futures is not excessive, the implications for the cash market are salutary. These results challenge the findings of previous studies—namely, that futures trading:

i. increases market efficiency [Burns (1979, pp. 74–77); Zeckhauser and Niederhoffer (1983); Silber (1985); MacKinlay and Ramaswamy (1988); Stoll and Whaley (1988); Brorsen (1991)],
ii. has no impact on market volatility [Santoni (1987); Aggarwal (1988); Edwards (1988a); Edwards (1988b); Grossman (1988)],
iii. will neither cause nor exacerbate a crash [Miller et al. (1989)],
iv. increases market liquidity [Silber (1985); Kwast (1986); Kling (1986)].

Ironically, the role futures play in price discovery has been cited frequently as a benefit of futures markets [e.g., Kahl et al. (1985); Herbst et al. (1987); Amihud and Mendelson (1989); Stoll and Whaley (1990); Brorsen (1991)]. The results of this analysis show that under certain conditions the "benefit" may turn out to be a detriment to efficiently functioning markets.

The results differ distinctly from previous studies of the relationship between cash and futures markets. For example, for storable commodities, Turnovsky (1983) concluded that the futures market *stabilizes* the cash market. Kawai (1983) showed that the futures market may destabilize the cash market when the inventory demand disturbance is preponderant. But neither Turnovsky's nor Kawai's model embraces the concepts of cash/futures arbitrage and price discovery—both of which are key to GS.

This article is organized as follows. The next section presents a generalization of the GS model for imperfect arbitrage between a futures market and its corresponding cash market. The third section derives the implications of the standard GS model for the volatility, autocorrelation, and stability of the cash market. The fourth section discusses the results in light of previous research and the implications of the results for regulatory policy. All proofs are in the appendix.

THE GARBADE–SILBER MODEL

Garbade and Silber (1983) derived the following model for the relationship between a futures market and the corresponding cash market. Let

ΔT be a given sampling time interval (the length of each period) and s a given positive integer multiple of ΔT,

C_k be the log of the cash market price in period k,

F_k be the log of the cash equivalent futures market price in period k,

N_C be the number of agents[2] trading exclusively in the cash market,

N_F be the number of agents trading exclusively in the futures market,

A_C be the mean elasticity of demand of agents trading exclusively in the cash market ($0 < A_C < \infty$),

A_F be the mean elasticity of demand of agents trading exclusively in the futures market[3] ($0 < A_F < \infty$),

H be the elasticity of demand for cash/futures arbitrage services ($0 < H < \infty$),

R_k^C be the cash market mean reservation price in period k,

R_k^F be the futures market mean reservation price in period k.

There are an *unspecified* number of agents trading in *both* cash and futures markets (e.g., arbitrageurs, hedgers, etc.).

Garbade and Silber [1983, eqs. (4) and (5)] showed that the supply/demand equilibrium equations ensuring the clearing of markets are

$$\begin{aligned} A_C N_C(C_k - R_k^C) &= H(F_k - C_k) \\ A_F N_F(F_k - R_k^F) &= -H(F_k - C_k) \end{aligned} \tag{1}$$

for the cash and futures markets, respectively. Assume that the cash and futures mean reservation prices in period k are[4]

$$\begin{aligned} R_k^C &= C_{k-1} + \alpha_{CC}(C_{k-1} - C_{k-2}) + \alpha_{CF}(F_{k-1} - F_{k-2}) + v_k^C \\ R_k^F &= F_{k-1} + \alpha_{FC}(C_{k-1} - C_{k-2}) + \alpha_{FF}(F_{k-1} - F_{k-2}) + v_k^F \end{aligned} \tag{2}$$

where $\{v_k^C\}$ are independent, identically distributed random variables (cash market innovations) with means $\Delta T \mu_C$ and variances $\Delta T \sigma_C^2$, and $\{v_k^F\}$ are independent, identically distributed random variables (futures market innovations) with means $\Delta T \mu_F$ and variances $\Delta T \sigma_F^2$.[5] Garbade and Silber treated just the special case with $\alpha_{CC} = \alpha_{CF} = \alpha_{FC} = \alpha_{FF} = 0$, which is referred to henceforth as the "standard" model, as opposed to the "extended" model presented here. The extended model allows mean reservation prices, for cash and for futures, to depend linearly on the most recent *changes* in cash prices *and* futures prices [eq. (2)]. Assume that

$$\begin{aligned} \mathrm{Cov}(v_k^C, v_k^F) &= r\sigma_C\sigma_F \\ \mathrm{Cov}(v_k^C, v_j^F) &= 0 \text{ for } k \neq j \end{aligned} \tag{3}$$

where r is the correlation between contemporaneous cash and futures innovations. These innovations represent information or noise shocks (or both) entering the cash and futures

[2]"Number of agents" could be interpreted either, literally, as the number of *different* agents trading, or alternatively, as the *amount of capital* traded. These interpretations are not necessarily equivalent.

[3]Garbade and Silber assumed $A_F = A_C$.

[4]These equations are simply generalizations of Garbade and Silber's eqs. (9a) and (9b).

[5]Garbade and Silber assumed these random variables are *normal*—an overly stringent assumption.

markets. Garbade and Silber considered just the special case where

$$\begin{aligned} v_k^C &= v_k + w_k^C \\ v_k^F &= v_k + w_k^F \end{aligned} \tag{4}$$

and $\mathrm{Cov}(w_k^C, w_k^F) = 0$, $\mathrm{Cov}(v_k, w_k^C) = 0$, and $\mathrm{Cov}(v_k, w_k^F) = 0$. The random variables w_k^C and w_k^F represent *noise* shocks to the cash and futures markets, respectively; whereas, v_k represents an *information* shock common to both cash and futures markets. Note that there is more noise in futures market innovations than cash market innovations if and only if $\sigma_F/\sigma_C > 1$.

Using the supply/demand equilibrium eq. (1) and the equations for mean reservation prices (2), the following generating equations for cash and futures prices may be derived:[6]

$$\begin{aligned} C_k &= (1-a)[C_{k-1} + \alpha_{CC}(C_{k-1} - C_{k-2}) + \alpha_{CF}(F_{k-1} - F_{k-2})] \\ &\quad + a[F_{k-1} + \alpha_{FC}(C_{k-1} - C_{k-2}) + \alpha_{FF}(F_{k-1} - F_{k-2})] + u_k^C \\ F_k &= (1-b)[F_{k-1} + \alpha_{FC}(C_{k-1} - C_{k-2}) + \alpha_{FF}(F_{k-1} - F_{k-2})] \\ &\quad + b[C_{k-1} + \alpha_{CC}(C_{k-1} - C_{k-2}) + \alpha_{CF}(F_{k-1} - F_{k-2})] + u_k^F \end{aligned} \tag{5}$$

where

$$\begin{aligned} a &\overset{\text{def}}{=} \frac{H/(N_C A_C)}{1 + H/(N_C A_C) + H/(N_F A_F)} \\ b &\overset{\text{def}}{=} \frac{H/(N_F A_F)}{1 + H/(N_C A_C) + H/(N_F A_F)} \end{aligned}$$

and

$$\begin{aligned} u_k^C &\overset{\text{def}}{=} (1-a)v_k^C + a v_k^F \\ u_k^F &\overset{\text{def}}{=} (1-b)v_k^F + b v_k^C \end{aligned}$$

Note that $0 < a < 1$, $0 < b < 1$, and $a + b < 1$. The following transformations of a and b have particularly significant interpretations:

$$\begin{aligned} \theta &\overset{\text{def}}{=} a/(a+b) = \frac{1}{1 + (N_C A_C)/(N_F A_F)} \\ \delta &\overset{\text{def}}{=} 1 - a - b = \frac{1}{1 + H/(N_C A_C) + H/(N_F A_F)} \end{aligned} \tag{6}$$

Garbade and Silber proposed θ as a measure of the source of price discovery. At one extreme, $\theta \approx 0$, N_C is large relative to N_F; and, so, the cash market is the primary source. At the other extreme, $\theta \approx 1$, N_F is large relative to N_C; and, so, the futures market is the primary source. From eqs. (5),

[6] These equations correspond to Garbade and Silber's eq. (10).

$$F_k - C_k = \delta\big[(F_{k-1} - C_{k-1}) + (v_k^F - v_k^C) + (\alpha_{FC} - \alpha_{CC})(C_{k-1} - C_{k-2}) + (\alpha_{FF} - \alpha_{CF})(F_{k-1} - F_{k-2})\big] \tag{7}$$

The premium between the cash equivalent futures and the cash price at period k is $F_k - C_k$. An inverse measure of the elasticity of cash/futures arbitrage is δ. At one extreme, $\delta \approx 0$, eq. (7) implies the premium is reduced to zero in one period, ΔT (i.e., cash/futures arbitrage is perfectly elastic). At the other extreme, $\delta \approx 1$ and if $\alpha_{FC} - \alpha_{CC} = 0$, $\alpha_{FF} - \alpha_{CF} = 0$, and $v_k^F - v_k^C = 0$, the premium is propagated intact to the next period (i.e., cash/futures arbitrage is completely inelastic). Garbade and Silber did not carry the theoretical development of the model further than this. Yet, further theoretical development is quite tractable and revealing.

DEFINITION 1. *A* realization *of the GS model is specified uniquely by the values for the nine parameters:* $\sigma_C, \sigma_F, r, \theta, \delta, \alpha_{CC}, \alpha_{CF}, \alpha_{FC}$, and α_{FF}.

DEFINITION 2. *For a given realization of the GS model there is a corresponding* cash-dominant model, *specified uniquely by the criterion that all parameters have the same value as for the given realization except that* $\theta = 0$.[7]

Let V_s be the steady-state volatility of cash market price differences measured over sampling interval $s\,\Delta T$:

$$V_s \stackrel{\text{def}}{=} \lim_{k\to\infty} \operatorname{Var}(C_{k+s} - C_k) \tag{8}$$

Similarly, let ρ_s be the steady-state first-order autocorrelation of cash market price differences over sampling interval $s\,\Delta T$:

$$\rho_s \stackrel{\text{def}}{=} \lim_{k\to\infty} \frac{\operatorname{Cov}(C_{k+2s} - C_{k+s}, C_{k+s} - C_k)}{\operatorname{Var}(C_{k+s} - C_k)} \tag{9}$$

For a given realization of the GS model, let $\mathcal{U}_s$ be the relative measure of cash market steady-state volatility directly attributable to price discovery by futures:

$$\mathcal{U}_s \stackrel{\text{def}}{=} \frac{V_s}{V_s|_{\theta=0}} - 1 \tag{10}$$

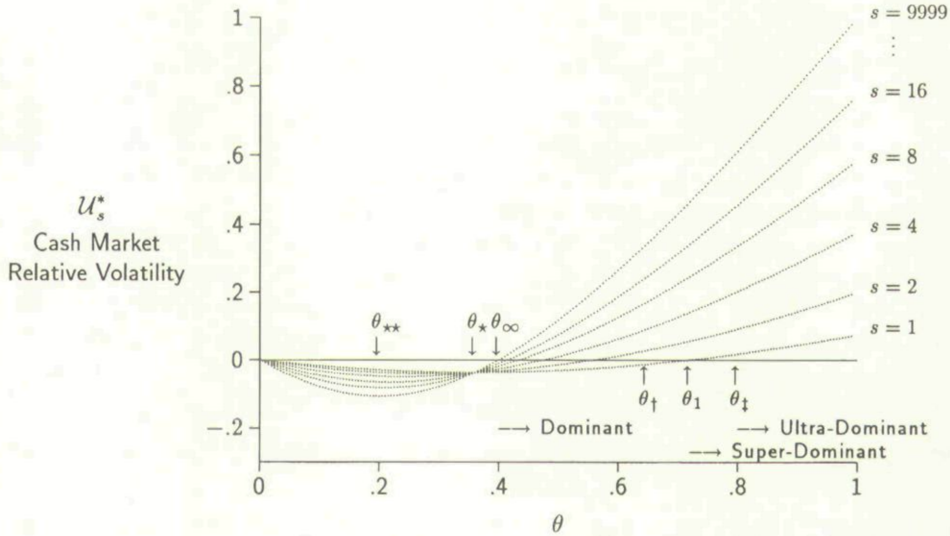
This measure can be expressed in terms of standard deviations rather than variances by the transformation

$$\mathcal{U}_s^* \stackrel{\text{def}}{=} (\mathcal{U}_s + 1)^{\frac{1}{2}} - 1$$

Similarly, for a given realization of the GS model and sampling interval $s\Delta T$, let $\Delta\rho_s$ be the component of cash market steady-state autocorrelation directly attributable to price discovery by futures:

$$\Delta\rho_s \stackrel{\text{def}}{=} \rho_s - \rho_s|_{\theta=0}$$

[7]If $\alpha_{CC} = \alpha_{CF} = 0$, then for the cash market, the cash-dominant model is equivalent to a random walk.



$\delta = .8,\ \sigma_F/\sigma_C = 2,\ r = 0,\ \alpha_{CC} = \alpha_{CF} = \alpha_{FC} = \alpha_{FF} = 0$

Figure 1

Impact of price discovery by futures, θ, on cash market relative volatility $\mathcal{U}_s^*$.

The following terminology is useful (see Figs. 1 and 2). Let

$\mathcal{U}_\infty \stackrel{\text{def}}{=} \lim_{s\to\infty} \mathcal{U}_s$,

θ_s be the largest value of θ for which $\mathcal{U}_s \leq 0$,

θ_* be the value of θ that minimizes relative cash market short-term volatility, $\mathcal{U}_1$,

θ_{**} be the value of θ that minimizes relative cash market long-term volatility, $\mathcal{U}_\infty$,

$\theta_\dagger$ be the largest value of θ for which $\partial\mathcal{U}_1/\partial\delta \geq 0$,

$\theta_\ddagger$ be the maximum of $\limsup_{\delta\to 1} \theta_1$ and $\limsup_{\delta\to 1} \theta_\dagger$,

$\theta_\sim$ be the largest value of θ for which $\rho_1 = \rho_1\,|_{\,\theta=0}$, i.e., $\theta_\sim$ is the largest value of θ for which the autocorrelation of the cash market is the same as it is for the cash-dominant model.

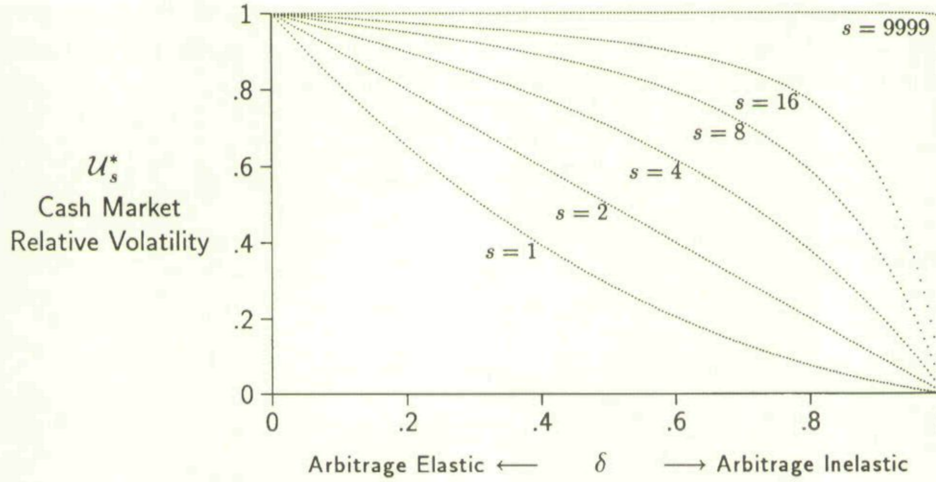
DEFINITION 3. *Price discovery by futures*

i. dominates *the cash market if* $\theta > \theta_\infty$,[8]
ii. super-dominates *the cash market if* $\theta > \theta_1$,
iii. ultra-dominates *the cash market if* $\theta > \theta_\ddagger$.

Following the rationale of Hasbrouck and Schwartz (1988), *short-term* volatility serves as a proxy for illiquidity.[9] If futures dominate ($\theta > \theta_\infty$), then cash market long-term

[8]Previous GS research has equated dominance by futures with the condition $\theta > 1/2$ and dominance by the cash market with the condition $\theta < 1/2$. Unfortunately, the traditional definition of dominance by futures is misleading because it does not *always* imply a deleterious impact on the cash market (e.g., if $1/2 < \theta < \theta_\infty$). The definition of dominance employed here is *coincident* with excess volatility and autocorrelation in the cash market.

[9]Intuitively, liquidity means the ability to buy or sell promptly, with minimal impact on market price [Silber (1985, p. 90); Kwast (1986, p. 8)]. Faithfully translating this intuitive understanding into precise mathematical terms is not clear, as Lippman and McCall (1986) and also Grossman and Miller (1988) have observed. On the other hand, proximate measures such as this one are convenient and useful.



$\theta = 1,\ \sigma_F/\sigma_C = 2,\ r = 0,\ \alpha_{CC} = \alpha_{CF} = \alpha_{FC} = \alpha_{FF} = 0$

Figure 2

Impact of arbitrage inelasticity, δ, on cash market relative volatility $\mathcal{U}_s^*$.

volatility is greater than it would be without the impact of price discovery by futures. If futures super-dominate ($\theta > \theta_1$), then cash market short-term volatility (illiquidity) is greater than it would be without the impact of price discovery by futures. Whenever $\theta_\infty < \theta < \theta_1$, the impact of futures is ambivalent—decreasing cash market *short-term* volatility (i.e., increasing liquidity) but increasing cash market *long-term* volatility. At $\theta_\dagger$ the sensitivity of $\mathcal{U}_1$ (i.e., illiquidity) to cash/futures arbitrage elasticity (as measured by δ) changes sign: if $\theta > \theta_\dagger$, liquidity is *improved* by *inhibiting* cash/futures arbitrage elasticity (i.e., increasing δ).

DEFINITION 4. *A* crash (*or a* bubble) is *characterized by* $\rho_s > 0$ *for all* $s \geq 1$ *and* $\limsup_{s\to\infty} \rho_s > 0$.

In other words, a crash (or bubble) is characterized by downside (upside) price momentum (i.e., positive autocorrelation in successive price changes) that is *persistent* in the sense that it does not damp-out as the sampling interval multiple s increases.[10] In the third section it is shown that ultra-dominance by futures is likely to induce either a crash or a bubble.

The presence of nonzero autocorrelation is a common test for weak-form market inefficiency [LeRoy (1989, p. 1602); Harris (1989, p. 94); Fama (1991, pp. 1578–1581)]. Therefore, the impact of price discovery by futures on cash market efficiency may be inferred from its impact on cash market autocorrelation. Assuming the cash market is already efficient without the impact of price discovery by futures, clearly the optimal value of θ for cash market weak-form efficiency is $\theta_\sim$.

The task now is to derive specific formulas for $\mathcal{U}_s$ and ρ_s in terms of the parameters of the GS model to determine the impact of θ, the level of price discovery by futures, on cash market volatility and autocorrelation.

[10]Of course, a crash (or bubble) will not last *forever*, so this definition characterizes an *idealized* crash (or bubble).

GARBADE–SILBER MODEL REDUX

The GS model may be represented in the form of the linear stochastic evolutionary system (Appendix A) by making the following identifications:

$$\boldsymbol{X}_k = \begin{pmatrix} x_{1k} \\ x_{2k} \\ x_{3k} \end{pmatrix} = \begin{pmatrix} C_k - C_{k-1} \\ F_k - F_{k-1} \\ F_k - C_k \end{pmatrix} \text{ and } \mathcal{E}_k = \begin{pmatrix} \varepsilon_{1k} \\ \varepsilon_{2k} \\ \varepsilon_{3k} \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} v_k^C \\ v_k^F \end{pmatrix} \tag{11}$$

where

$$\boldsymbol{P} = \begin{pmatrix} (1-a) & a \\ b & (1-b) \\ -\delta & \delta \end{pmatrix} \tag{12}$$

and

$$\boldsymbol{A} = \begin{pmatrix} 1-a & a & a \\ b & 1-b & -b \\ -\delta & \delta & \delta \end{pmatrix}\begin{pmatrix} \alpha_{CC} & \alpha_{CF} & 0 \\ \alpha_{FC} & \alpha_{FF} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Clearly,

$$C_{k+s} - C_k = \left(\sum_{i=0}^{s-1} \boldsymbol{X}_{k+i+1}\right)_1 \tag{13}$$

As noted in Appendix A, the stability of the system is measured by $\Omega(\boldsymbol{A})$, the spectral radius of $\boldsymbol{A}$. For the standard model ($\alpha_{CC} = \alpha_{CF} = \alpha_{FC} = \alpha_{FF} = 0$), clearly $\Omega(\boldsymbol{A}) = \delta$. The following proposition clarifies the behavior of $\Omega(\boldsymbol{A})$ in general.

PROPOSITION 1. *If* $|\alpha_{CC}|$, $|\alpha_{CF}|$, $|\alpha_{FC}|$, *and* $|\alpha_{FF}|$ *are sufficiently small, then* $\Omega(\boldsymbol{A}) < 1$, *and* $\lim_{\delta \to 1} \Omega(\boldsymbol{A}) = 1$.

Thus, even for the extended model, the stability of the system is governed largely by the behavior of δ: as cash/futures arbitrage becomes less elastic (i.e., $\delta \to 1$), the system becomes increasingly unstable.

The following derivations apply only for the standard model;[11] results may be obtained numerically for the extended model. For convenience, let

$$\phi_s = \phi(\delta, s) \overset{\text{def}}{=} \frac{1-\delta^s}{1-\delta}$$

It is easy to show:

$$(I - \boldsymbol{A})^{-1} = \begin{pmatrix} 1 & 0 & a/(1-\delta) \\ 0 & 1 & -b/(1-\delta) \\ 0 & 0 & 1/(1-\delta) \end{pmatrix}, \qquad \boldsymbol{A}^s = \begin{pmatrix} 0 & 0 & \delta^{s-1}a \\ 0 & 0 & -\delta^{s-1}b \\ 0 & 0 & \delta^s \end{pmatrix} \tag{14}$$

$$(I - \boldsymbol{A})^{-1}(I - \boldsymbol{A}^s) = \begin{pmatrix} 1 & 0 & a\phi_{s-1} \\ 0 & 1 & -b\phi_{s-1} \\ 0 & 0 & \phi_s \end{pmatrix}$$

[11]As pointed out by a referee, *direct* derivations are feasible for the standard GS model. Nevertheless, it is instructive to consider the derivations within the generalized framework.

$$A(I - A)^{-1}(I - A^s) = \begin{pmatrix} 0 & 0 & a\phi_s \\ 0 & 0 & -b\phi_s \\ 0 & 0 & \delta\phi_s \end{pmatrix}$$

$$A\left[(I - A)^{-1}(I - A^s)\right]^2 = \begin{pmatrix} 0 & 0 & a\phi_s^2 \\ 0 & 0 & -b\phi_s^2 \\ 0 & 0 & \delta\phi_s^2 \end{pmatrix}$$

$$(I - A)^{-1}\left[sA - A(I - A)^{-1}(I - A^s)\right] = \begin{pmatrix} 0 & 0 & \dfrac{a(s - \phi_s)}{1 - \delta} \\ 0 & 0 & \dfrac{-b(s - \phi_s)}{1 - \delta} \\ 0 & 0 & \dfrac{\delta(s - \phi_s)}{1 - \delta} \end{pmatrix} \tag{15}$$

Also, from eq. (A2)

$$G = (g_{ij}) = \mathrm{Cov}(\mathcal{E}_k, \mathcal{E}_k) = \Delta T \; P\begin{pmatrix} \sigma_C^2 & r\sigma_C\sigma_F \\ r\sigma_C\sigma_F & \sigma_F^2 \end{pmatrix}P' \tag{16}$$

Note that $\mathrm{Var}(v_k^F - v_k^C) = \Delta T(\sigma_F^2 - 2r\sigma_C\sigma_F + \sigma_C^2)$. Formulas for the elements of G may be derived from eqs. (12) and (16):

$$\begin{aligned} g_{11} &= \Delta T((1 - a)^2\sigma_C^2 + 2a(1 - a)r\sigma_C\sigma_F + a^2\sigma_F^2) \\ g_{31} &= \Delta T((1 - a)\delta(-\sigma_C^2 + r\sigma_C\sigma_F) + a\delta(\sigma_F^2 - r\sigma_C\sigma_F)) \\ &= \Delta T\delta(-\sigma_C^2 + r\sigma_C\sigma_F + a(\sigma_F^2 - 2r\sigma_C\sigma_F + \sigma_C^2)) \\ g_{33} &= \Delta T\delta^2(\sigma_C^2 - 2r\sigma_C\sigma_F + \sigma_F^2) \end{aligned} \tag{17}$$

From eqs. (A4) it is clear that

$$\begin{aligned} q_{11} - a^2 q_{33} &= g_{11} \\ q_{31} - a\delta q_{33} &= g_{31} \\ q_{33} - \delta^2 q_{33} &= g_{33} \end{aligned} \tag{18}$$

Solving these equations for the q's and substituting the g's:

$$\begin{aligned} q_{33} &= \frac{g_{33}}{1 - \delta^2} \\ &= \frac{\delta^2}{1 - \delta^2}\Delta T(\sigma_F^2 - 2r\sigma_C\sigma_F + \sigma_C^2) \\ q_{11} &= g_{11} + a^2 q_{33} \\ &= \Delta T\left[\sigma_C^2 + \theta(1 - \delta)\left[-2(\sigma_C^2 - r\sigma_C\sigma_F) + \frac{\theta}{1 + \delta}(\sigma_F^2 - 2r\sigma_C\sigma_F + \sigma_C^2)\right]\right] \\ q_{31} &= g_{31} + a\delta q_{33} \\ &= \Delta T\delta\left[-(\sigma_C^2 - r\sigma_C\sigma_F) + \frac{\theta}{1 + \delta}(\sigma_F^2 - 2r\sigma_C\sigma_F + \sigma_C^2)\right] \end{aligned} \tag{19}$$

Now it is possible to derive formulas for cash market volatility and autocorrelation.

Volatility

Using eqs. (8), (A6), (11), (13), and (14), a formula for the cash market steady-state volatility, V_s may be derived:

$$V_s = \lim_{k\to\infty} \mathrm{Var}(C_{k+s} - C_k) = \lim_{k\to\infty}\left[\mathrm{Cov}\left(\sum_{i=0}^{s-1} \boldsymbol{X}_{k+i+1}, \sum_{i=0}^{s-1} \boldsymbol{X}_{k+i+1}\right)\right]_{1,1}$$

$$= sq_{11} + 2\frac{a(s-\phi_s)}{1-\delta}q_{31}$$

$$= s\Delta T\sigma_C^2 + \theta\Delta T\Big[-2(s-\delta\phi_s)(\sigma_C^2 - r\sigma_C\sigma_F) + \theta\left(s - \frac{2\delta\phi_s}{1+\delta}\right)(\sigma_F^2 - 2r\sigma_C\sigma_F + \sigma_C^2)\Big] \tag{20}$$

In particular, for $\theta = 0$,

$$V_s|_{\theta=0} = s\Delta T\sigma_C^2 \tag{21}$$

From eqs. (10), (21), and (20), the relative steady-state volatility of the cash market is

$$\mathcal{U}_s\left(\theta, \delta, r, \frac{\sigma_F}{\sigma_C}\right) = \theta\left[-2\left(1-\delta\frac{\phi_s}{s}\right)\left(1 - r\frac{\sigma_F}{\sigma_C}\right) + \theta\left(1 - \left(\frac{2\delta}{1+\delta}\right)\frac{\phi_s}{s}\right)\left(\left(\frac{\sigma_F}{\sigma_C}\right)^2 - 2r\frac{\sigma_F}{\sigma_C} + 1\right)\right] \tag{22}$$

Clearly, $\mathcal{U}_s$ is a quadratic in θ if and only if $\mathrm{Var}(v_k^F - v_k^C) = \sigma_F^2 - 2r\sigma_C\sigma_F + \sigma_C^2 > 0$. Using the definitions from the second section, it is straightforward to show

$$\mathcal{U}_\infty\left(\theta, r, \frac{\sigma_F}{\sigma_C}\right) = \theta\left[-2\left(1 - r\frac{\sigma_F}{\sigma_C}\right) + \theta\left(\left(\frac{\sigma_F}{\sigma_C}\right)^2 - 2r\frac{\sigma_F}{\sigma_C} + 1\right)\right]$$

$$\theta_\infty = \max\left[\frac{2\left(1 - r\dfrac{\sigma_F}{\sigma_C}\right)}{\left(\left(\dfrac{\sigma_F}{\sigma_C}\right)^2 - 2r\dfrac{\sigma_F}{\sigma_C} + 1\right)}, 0\right]$$

$$\theta_* = \frac{1+\delta}{2}\theta_\infty$$

$$\theta_{**} = \theta_\infty/2$$

$$\theta_s = \frac{1 - \delta\dfrac{\phi_s}{s}}{1 - \dfrac{2\delta}{1+\delta}\dfrac{\phi_s}{s}}\theta_\infty$$

$$\theta_1 = (1+\delta)\theta_\infty$$

$$\theta_\dagger = (1+\delta)\theta_1/2 = (1+\delta)^2\theta_\infty/2$$

$$\theta_\ddagger = 2\theta_1/(1+\delta) = 2\theta_\infty$$

The following theorem catalogs the key properties of the cash market relative steady-state volatility $\mathcal{U}_s$.

THEOREM 2. *Given the standard GS model relating cash and futures markets, if* $\mathrm{Var}(v_k^F - v_k^C) > 0$, *and* $\theta_\infty > 0$ *then*

i. $\mathcal{U}_s$ *is a quadratic in* θ *with roots at* 0 *and* θ_s *and is minimized for* $\theta = \frac{1}{2}\theta_s$, *and*

$$\mathcal{U}_s\left(\frac{1}{2}\theta_s, \delta, r, \frac{\sigma_F}{\sigma_C}\right) = -\frac{1}{2}\theta_s\left(1 - \delta\frac{\phi_s}{s}\right)\left(1 - r\frac{\sigma_F}{\sigma_C}\right) < 0;$$

ii. *if* $\theta > \theta_\dagger$, *then* $\partial\mathcal{U}_1/\partial\delta < 0$; *also,* $\theta_\dagger < \theta_1 < \theta_\ddagger$, *and if* $\delta > \sqrt{2} - 1$, *then* $\theta_\infty < \theta_\dagger$;
iii. $\mathcal{U}_s(\theta_*, \delta, r, \sigma_F/\sigma_C) = -\theta_*(1 - \delta)(1 - r\sigma_F/\sigma_C) < 0$;
iv. $\mathcal{U}_\infty(1, r, \sigma_F/\sigma_C) = (\sigma_F/\sigma_C)^2 - 1$;
v. $\theta_\infty < 1$ *if and only if* $\sigma_F/\sigma_C > 1$;
vi. $\theta_{**} < \theta_* < \theta_\infty < \theta_{s+1} < \theta_s < \theta_1$ *for every* $s > 1$;
vii. *for* $\theta > \theta_*, \mathcal{U}_s(\theta, \delta, r, \sigma_F/\sigma_C)$ *increases with s to* $\mathcal{U}_\infty(\theta, r, \sigma_F/\sigma_C)$, *which is independent of* δ;
viii. *if* $\sigma_F/\sigma_C > 1$, *then* θ_*, θ_∞ *and* θ_s *decrease as r increases;*
ix. $\mathcal{U}_s$ *increases as r increases;*
x. *if either* $r \leq 0$ *or* $\sigma_F/\sigma_C > 1$, *then* θ_*, θ_∞ *and* θ_s *decrease as* σ_F/σ_C *increases;*
xi. *if either* $r \geq 0$ *or* $\theta > \frac{1}{2}\theta_s$, *then* $\mathcal{U}_s$ *increases as* σ_F/σ_C *increases.*

Figures 1 and 2 illustrate some of the key results of Theorem 2. Clearly, the assumption $\mathrm{Var}(v_k^F - v_k^C) > 0$ is quite reasonable, for otherwise, the cash market and the futures market would receive the exact same shocks concurrently and respond identically. This is unlikely because of the very difference in the mechanisms of the two markets. Moreover, since the set of agents trading in the two markets is different, the responses of each market will be different.

Clearly, $\theta_\infty > 0$ if and only if $1 - r\sigma_F/\sigma_C > 0$. In particular, if the correlation between futures and cash market innovations is non-positive (i.e., $r \leq 0$), then $\theta_\infty > 0$. If $r > 0$, one still may have $\theta_\infty > 0$, depending on the level of the relative noise in futures prices (σ_F/σ_C) as well as r. If $\theta_\infty = 0$, then futures are ultra-dominant a fortiori.

The following point-by-point discussion of each item of Theorem 2 clarifies the implications and significance of GS for cash market volatility:

i. Cash market relative volatility ($\mathcal{U}_s$) is a quadratic in the price discovery parameter (θ) and has a minimum that is *negative*. This result shows that cash market s-period volatility is less than it would be without the impact of futures whenever the level (θ) of price discovery by futures is between the two roots (0 and θ_s) of this quadratic. But price discovery by futures has a progressively deleterious impact on s-period volatility the more θ exceeds θ_s (see Fig. 1). Thus, policies that promote *unconstrained* price discovery by futures may be misguided.
Nevertheless, this result also provides significant justification for the existence of futures markets. For simplicity, consider just the special case when cash and fu-

tures market innovations are uncorrelated ($r = 0$) and $\sigma_F/\sigma_C = 2$. Observe, for example, that when $\theta = \theta_{**}$ there would be a maximum potential 20% decrease ($\mathcal{U}_\infty = -0.20$) in cash market long-term volatility.

ii. If the level (θ) of price discovery by futures is greater than $\theta_\dagger$, then decreasing the elasticity of cash/futures arbitrage (i.e., increasing δ) will *decrease* cash market short-term relative volatility ($\mathcal{U}_1$). The result is a paradox—a decrease in stability implies a *decrease* in short-term volatility. The paradox is resolved simply by observing that the notion of "instability" should be kept *logically* distinct from the notion of "volatility." Since instability can precipitate a crash, which is usually associated with an increase in volatility, indeed it is understandable why the two notions sometimes get mixed up. The critical level, $\theta_\dagger$, is sandwiched between θ_∞ and θ_1. In particular, if futures are not dominant (i.e., $\theta < \theta_\infty$), then this paradoxical result is strictly hypothetical. On the other hand, this property is crucial for proving Theorem 6.

iii. The level of price discovery, θ_*, that minimizes cash market *short-term* relative volatility ($\mathcal{U}_1$) is salutary also for cash market s–period relative volatility for *every* s (i.e., $\mathcal{U}_s(\theta_*, \delta, r, \sigma_F/\sigma_C) < 0$). In particular, a policy that minimizes short-term relative volatility can not err by worsening long-term volatility (see Fig. 1).

iv. At the maximum level of price discovery by futures ($\theta = 1$), cash market long-term relative volatility depends only on the ratio σ_F/σ_C—and neither on r nor δ. In particular, for this special case, cash market volatility is made worse if and only if the noise in futures prices is greater than the noise in cash prices (i.e., $\sigma_F/\sigma_C > 1$) (see Fig. 2).

v. It is *possible* for price discovery by futures to be deleterious for cash market volatility if and only if the noise in futures prices is greater than the noise in cash prices ($\sigma_F/\sigma_C > 1$). Otherwise, $\theta_\infty \geq 1$, and thus $\theta \leq \theta_\infty$ a fortiori.

vi. There is a natural ordering of the respective critical levels: θ_s decreases monotonically from θ_1 to θ_∞. In particular, price discovery by futures may be ambivalent—salutary for cash market short-term volatility and deleterious for long-term volatility (e.g., $\theta_\infty < \theta < \theta_1$) (see Fig. 1). The optimal level of θ for cash market long-term volatility is θ_{**}; the optimal level for short-term volatility is θ_*. But since $\theta_{**} < \theta_*$, θ cannot assume both of these levels simultaneously. Therefore, the interval $[\theta_{**}, \theta_*]$ constitutes an optimal range for θ—a compromise between optimal long-term volatility and optimal short-term volatility.

vii. For $\theta > \theta_*$, cash market relative volatility ($\mathcal{U}_s$) increases with s. So the effect of a given deleterious level of price discovery will be more pronounced for long-term volatility than for short-term volatility (see Fig. 1). Moreover, long-term relative volatility ($\mathcal{U}_\infty$) is completely independent of the elasticity of cash/futures arbitrage (δ). Therefore, policies that regulate solely the elasticity of cash/futures arbitrage (δ) should have no effect on *long-term* volatility.

viii. If there is more noise in futures prices than cash prices (i.e., $\sigma_F/\sigma_C > 1$), then an increase in the correlation between cash and futures market innovations (r) implies a decrease in the critical level θ_∞, thus making it more likely for futures to be dominant (i.e., $\theta > \theta_\infty$).

ix. An increase in the correlation between cash and futures market innovations (r) implies an increase in cash market relative volatility ($\mathcal{U}_s$). Therefore, policies designed to increase the correlation between information flowing to the cash and futures markets may be counterproductive. In particular, an increase in r would simply exacerbate the deleterious impact of dominance by futures.

x. If either the correlation between cash and futures market innovations is nonpositive ($r \leq 0$) or there is more noise in futures prices than cash prices ($\sigma_F/\sigma_C > 1$), then an increase in the relative noise in futures prices (σ_F/σ_C) implies a decrease in the dominance critical level (θ_∞), thus making it more likely for futures to be dominant (i.e., $\theta > \theta_\infty$).

xi. Cash market s-period relative volatility ($\mathcal{U}_s$) increases with the relative noise in futures prices (σ_F/σ_C), except possibly if price discovery by futures is exceptionally low (namely, $\theta < \frac{1}{2}\theta_s$) and the correlation between cash and futures market innovations is negative ($r < 0$). The exception represents extraordinary ideal conditions that seldom may be satisfied. This result suggests that a policy whose goal is to decrease σ_F/σ_C would decrease the deleterious impact of futures on cash market volatility. Such a policy, however, could ensure *elimination* of the deleterious impact only if the ratio σ_F/σ_C could be decreased to 1 (in which case it is impossible for futures to be dominant [see (v)]).

Autocorrelation

Using formulas (9), (A6), (A7), (11), and (13), the steady-state first-order autocorrelation of the cash market may be derived:

$$\begin{aligned}
\rho_s\left(\theta, \delta, r, \frac{\sigma_F}{\sigma_C}\right) &= \lim_{k\to\infty} \frac{\operatorname{Cov}(C_{k+2s} - C_{k+s}, C_{k+s} - C_k)}{\operatorname{Var}(C_{k+s} - C_k)} \\
&= \lim_{k\to\infty} \frac{\left[\operatorname{Cov}\left(\sum_{i=0}^{s-1} X_{k+s+i+1}, \sum_{j=0}^{s-1} X_{k+j+1}\right)\right]_{1,1}}{V_s} \\
&= \frac{a\phi_s^2 q_{31}}{sq_{11} + 2\theta(s - \phi_s)q_{31}} \\
&= \frac{\theta(1 - \delta)(\phi_s^2/s)q_{31}/q_{11}}{1 + 2\theta(1 - \phi_s/s)q_{31}/q_{11}}
\end{aligned} \tag{23}$$

Clearly, for $\theta = 0$, $\rho_s|_{\theta=0} \equiv 0$. The following theorem catalogs key properties of the cash market steady-state autocorrelation ρ_s.

THEOREM 3. *Given the standard GS model relating cash and futures markets, if* $\operatorname{Var}(v_k^F - v_k^C) > 0$, *and* $\theta_\infty > 0$, *then*

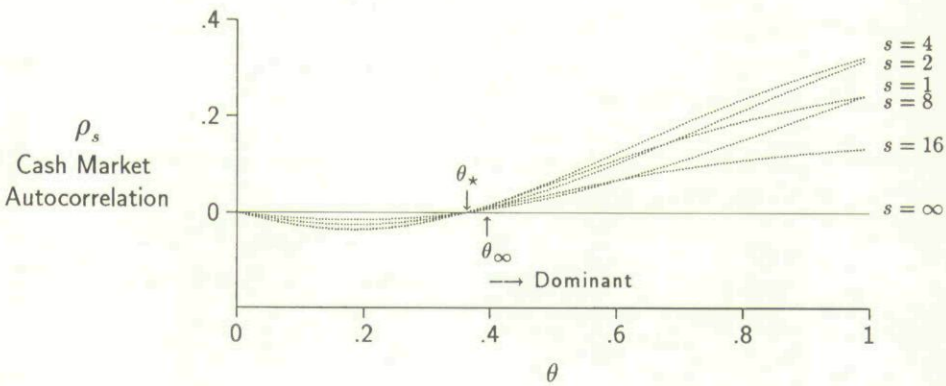
i. $\rho_s < 0$ *for* $\theta < \theta_*$, $\rho_s = 0$ *for* $\theta = \theta_*$, *and* $\rho_s > 0$ *for* $\theta > \theta_*$;
ii. *if* $\theta_* < 2 - (1 - \delta)(1 - r\sigma_F/\sigma_C)$, *then* ρ_s *is a unimodal function of* θ *for* $0 < \theta < 1$, *and* ρ_s *increases with* θ *for* $\theta > \theta_*$;
iii. *for* fixed $\delta < 1$, $\lim_{s\to\infty} \rho_s = 0$;
iv. $\lim_{\sigma_F/\sigma_C\to\infty} \rho_1 = \delta$;
v. $\limsup_{s\to\infty} \rho_s > 0.2035\mathcal{U}_\infty/(1 + 0.4308\mathcal{U}_\infty)$;
vi. $\lim_{\sigma_F/\sigma_C\to\infty} (\limsup_{s\to\infty} \rho_s) = \limsup_{s\to\infty} (\lim_{\sigma_F/\sigma_C\to\infty} \rho_s) = 1$;
vii. *if* $\theta > \theta_*$, ρ_s *is a unimodal function of* s, *having a maximum at some value* $s = \bar{s} > 0$. *Moreover,* $\bar{s}$ *increases as* δ *increases, and* $\bar{s}$ *decreases as* θ *increases.*

COROLLARY 4. *A crash or a bubble due to dominance by futures is not possible if δ is bound away from* 1.

COROLLARY 5. *Either a crash or a bubble will occur if futures remain dominant and* $\delta \to 1$.

The following point-by-point discussion of each item of Theorem 3 clarifies the implications and significance of GS for cash market autocorrelation:

i. For each s, cash market s-period autocorrelation (ρ_s) is zero precisely when the level of price discovery by futures coincides with θ_* (the level of price discovery that minimizes short-term volatility). Thus, there is an *equivalence* between the optimum level of price discovery for s-period efficiency and the optimum level of price discovery for short-term volatility (liquidity). In particular, since $\theta_* < \theta_\infty$, dominance by futures (i.e., $\theta > \theta_\infty$), implies cash market inefficiency (i.e., $\rho_s > 0$). See Figure 3.
ii. Subject to the mild condition on θ_*, cash market s-period autocorrelation (ρ_s) increases monotonically with the level of price discovery by futures (θ), for $\theta > \theta_*$, implying a progressive increase in cash market inefficiency (see Fig. 3).
iii. For a *fixed* level of cash/futures arbitrage elasticity (i.e., δ fixed and $\delta < 1$), cash market s-period autocorrelation (ρ_s) converges to zero as s increases without bound (see Fig. 4). This, of course, is exactly what one would expect.
iv. For $\theta > 0$, cash market short-term autocorrelation (ρ_1) converges to δ as the relative noise in futures prices measured by the ratio σ_F/σ_C increases without bound. Thus, an unbounded increase in σ_F/σ_C would be benign for cash market short-term autocorrelation only if $\delta = 0$ (i.e., only if cash/futures arbitrage were perfectly elastic). It is interesting that this result holds for *any* positive level of price discovery by futures.
v. If futures are dominant, then cash market s-period autocorrelation (ρ_s) does not converge to 0 *uniformly* as s increases without bound (in contrast to the simplistic result, iii). As δ increases from 0, ρ_s increases from 0, reaches a maximum and then decreases to 0 as δ approaches 1. As s increases, the value



$\delta = .8,\ \sigma_F/\sigma_C = 2,\ r = 0,\ \alpha_{CC} = \alpha_{CF} = \alpha_{FC} = \alpha_{FF} = 0$

Figure 3
Impact of price discovery by futures, θ, on cash market autocorrelation ρ_s.

of δ for which ρ_s reaches its maximum monotonically increases to 1 and the corresponding maximum (a function of $\mathcal{U}_\infty$) *does not* decrease to 0 (Fig. 4). Thus, cash market long-term autocorrelation due to price discovery by futures can be substantial. Namely, if futures are dominant and there is not a consistent policy for maintaining elasticity of cash/futures arbitrage, (i.e., keeping δ constrained away from 1) then *large positive* ρ_s could be induced for *ever greater* multiples s. This situation conforms with Definition 4 for a market crash or a bubble.

vi. This result shows that if the elasticity of cash/futures arbitrage is not maintained (i.e., if δ is not constrained away from 1) and if, simultaneously, the relative noise in futures prices measured by the ratio σ_F/σ_C increases without bound, then cash market long-term autocorrelation could approach its theoretical maximum (namely, $\lim_{\sigma_F/\sigma_C \to \infty} \limsup_{s \to \infty} \rho_s = 1$). This situation conforms, with Definition 4, for a market crash or a bubble (in an even stronger sense than the previous result).

vii. The shape of cash market autocorrelation as a function of the period multiple, s, is unimodal with maximum at $s = \bar{s}$ (see Fig. 5); moreover, $\bar{s}$ increases as cash/futures arbitrage elasticity decreases (i.e., as δ increases). This demonstrates once more the danger of policies inhibiting the elasticity of cash/futures arbitrage—doing so would stretch out the period ($\bar{s}$) for maximum positive cash market autocorrelation, thus precipitating a crash or bubble (Definition 4). Although $\bar{s}$ decreases as price discovery by futures (θ) increases, it would be erroneous to conclude that increasing price discovery by futures would be beneficial. In fact, as already shown, if $\theta > \theta_*$, an increase in θ implies an increase in cash market autocorrelation (ρ_s).

A nominal level of dominance by futures, although not optimal, nevertheless may be tolerable. But excessive dominance by futures is ominous as the following theorem proves.

THEOREM 6. *If cash and futures markets conform to the standard GS model, with which all cash/futures arbitrageurs are cognizant, then ultra-dominance by futures will induce either a crash or a bubble.*

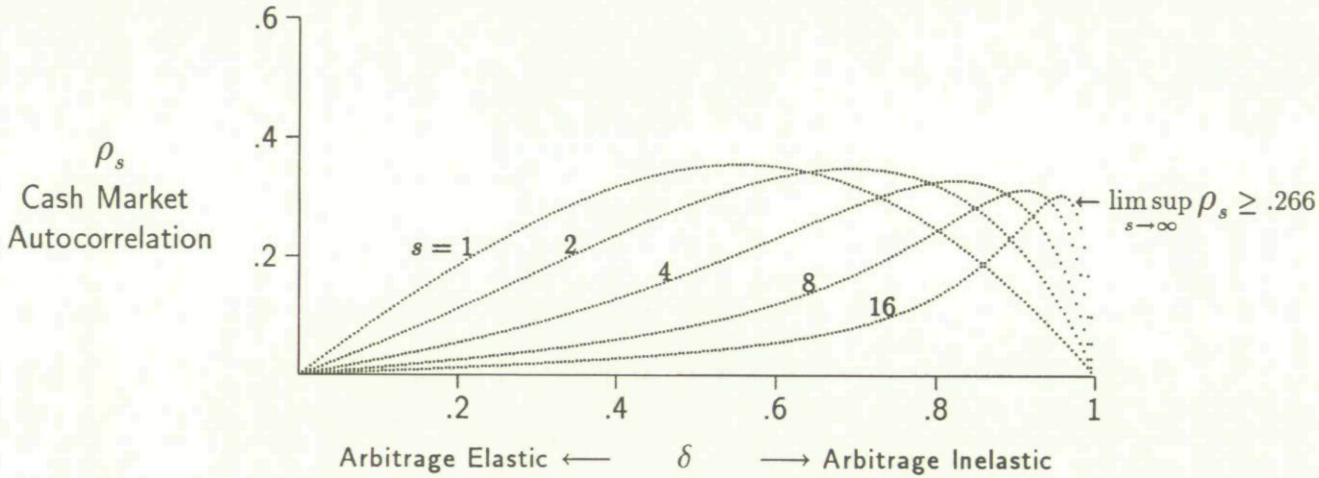
In essence, Theorem 6 reveals that the markets' commonly presumed "self-correcting" nature is nullified by ultra-dominance by futures.[12] Ultimately, of course, a crash or bubble will spend itself; this will occur when futures fail to remain super-dominant.

The conditions of Theorem 6 are *sufficient* but not *necessary* for inducing a crash or bubble. Other conditions or events, either fundamental or structural, also might be capable of playing this role. However, once a crash or bubble has been triggered, dominance by futures will magnify cash market autocorrelation and long-term volatility.

IMPLICATIONS

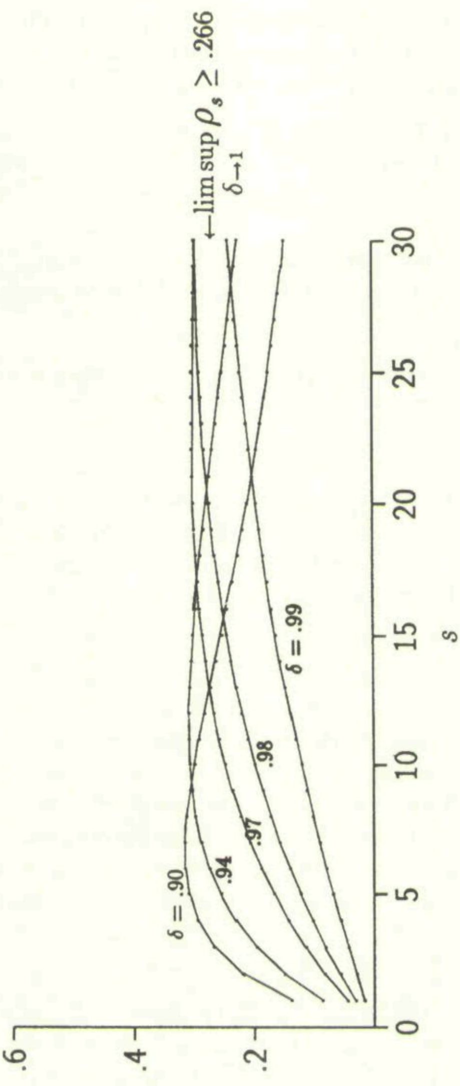
The theory has important implications for regulatory policy. If price discovery by futures is not dominant, the impact on the cash market is *salutary*—increased liquidity, decreased volatility, better efficiency. More particularly, the most favorable impact is when the price discovery parameter, θ, is within its optimal range $[\theta_{**}, \theta_*]$. If θ exceeds its optimal range, clearly there are just two ways to restore the salutary impact—decrease θ or increase θ_*.

[12]Ultra-dominance is akin to a spring in a mechanical system being stretched beyond its limit—the spring loses sufficient elasticity to restore the system to equilibrium.



$\theta = 1$, $\sigma_F / \sigma_C = 2$, $r = 0$, $\alpha_{CC} = \alpha_{CF} = \alpha_{FC} = \alpha_{FF} = 0$

Figure 4
Impact of arbitrage inelasticity, δ, on cash market autocorrelation ρ_s.



$\theta = 1$, $\sigma_F/\sigma_C = 2$, $r = 0$, $\alpha_{CC} = \alpha_{CF} = \alpha_{FC} = \alpha_{FF} = 0$

Figure 5
Cash market autocorrelation ρ_s.

Margins

There is no dispute that increasing the initial margin requirement for futures would decrease the price discovery parameter, θ:

> Federalizing control over futures margins will simply raise the cost of trading index futures and lower the quantity of futures contracts traded.
> That such must be the case is almost axiomatic [Miller (1991, p. 17)].

Also, Kahl et al. (1985, p. 111) conclude: "Low performance margins induce a relatively high volume of speculation and facilitate price discovery." Similarly, Kalavathi and Shanker (1991, p. 230) conclude from their simulations:

> The results indicate that the demand for S&P 500 spot index futures contracts drop[s] substantially when margin requirements are increased to be compatible with the stock market.

Schwarz and Laatsch (1991, p. 682) found that an increase in futures market trading volume relative to cash market trading volume is consistent with higher levels of price discovery by futures:

> ... the next five months (those immediately preceding the Stock Crash of 1987) exhibit a large divergence in the trading volume of the two markets. The futures market continues to grow 41.6% versus a 4% decline for the cash market. During this period, a large increase in price leadership for the futures market is observed, especially when measured on a daily basis.

Pliska and Shalen (1991, p. 141) conclude "The combined effect ... [the effects of higher margins] is unambiguous, as they all concur in decreasing the size of individual speculative demands." Significantly, Hartzmark (1986) found that increasing the initial margin requirement affects the *composition* of futures traders—driving out the less sophisticated trader.[13] Each of these conclusions is entirely consistent with the thesis that increasing the initial margin requirement for futures would decrease price discovery by futures. In terms of this analysis, increasing the margin requirement for futures would decrease N_F; which, in turn, would decrease the price discovery parameter θ [eq. (6)].

Noise

What might explain the sharp contrast between previous work and the results presented here concerning the *merit* of price discovery by futures? "Price discovery" comprises both "noise discovery" and "information discovery." Unfortunately, previous studies of price discovery have ignored the possibility that noise could be an important component. If price discovery by futures has deleterious implications, excessive noise in futures prices could be a logical explanation.

Fama (1989, p. 72) argues "... economic efficiency is served by the immediate adjustment of prices to fundamental values." But he also observes (p. 74):

> Noise is socially wasteful. It obscures the information in prices about resource allocation. It can increase the expected returns investors require to hold stock, which means higher costs-of-capital and less capital investment.

[13]However, Hartzmark's data set is minimal—consisting of just 13 margin changes spread over four different futures contracts (wheat, feeder cattle, pork bellies, and T-bonds).

Powers (1970, p. 461) also recognized the virtue in reducing the noise component in prices:

> ...the real concern with the pricing system is to improve its efficiency as a communications and allocative device by reducing the noise factor, the random elements in prices, while maintaining the responsiveness of the systematic component of fundamental economic conditions.

He correctly identified the potential benefit that futures might decrease the noise factor but did not consider the possibility that futures also might exacerbate the noise factor if price discovery is primarily noise discovery.

Black (1986) also emphasized the distinction between the effects of trading on noise and trading on information. This distinction provides a basis for explaining the deleterious consequences of price discovery dominance by stock index futures: noise traders are short-term oriented, trade frequently, and base their trading decisions on technical trading rules; information traders are long-term oriented, trade infrequently, and base their decisions on fundamental economic information and company specific information. Since trading costs are far less in the futures market than in the cash market, noise traders naturally would concentrate their trading in futures rather than the cash market [Kling (1986, pp. 53–54)]. Conversely, to the extent that selection of value is accomplished best by the *choices* available in the cash market, information traders would concentrate their trading in the cash market.[14] Thus, if the margin differential between cash and futures markets is sufficiently large, excessive noise trading speculation in futures relative to value trading in the cash market likely will be de-stabilizing. More specifically, whenever $\sigma_F > \sigma_C$, there is more noise in futures prices than in cash prices [eq. (4)]. That, by itself, is not sufficient to cause trouble, but it does lower the critical levels for dominance [Theorem 2(x)]. However, as N_F increases relative to N_C, the price discovery parameter, θ, increases. Therefore, sufficient excess noise in futures prices (i.e., a sufficiently low critical level θ_∞) *together with* a sufficiently large proportion of futures traders (sufficiently high θ) will induce the deleterious market behavior identified in this study.

It is worth emphasizing that noise in futures prices is instrumental in determining *only* the critical level(s) for dominance; it does *not* determine whether price discovery by futures is sufficiently high to exceed the critical level(s). If there is no excess noise in futures prices (i.e., $\sigma_F \leq \sigma_C$), then the critical level, θ_∞, is greater than 1, which is outside the feasible range for θ; hence, in this case, *any* feasible level of price discovery by futures would not imply dominance. In other words, excessive noise in futures prices is *necessary* but not *sufficient* for price discovery by futures to be excessive.

Would increasing the initial margin requirement for futures decrease the level of noise in futures prices, indicated by σ_F, and thereby increase the critical levels [Theorem 2(x)]? The prospect for dampening stock market volatility by increasing the initial margin requirement for stocks [Hardouvelis (1988); Hardouvelis (1990)] suggests that similar results might obtain for futures markets. However, Hardouvelis' results have been challenged aggressively [Hsieh and Miller (1990); Kupiec (1989); Salinger (1989); Schwert (1989)]. The effect of margin policy on excess noise in futures prices is still an open question. Even if increasing margin requirements for futures might not decrease

[14] As Black (1986) points out, there will always be ambiguity about *who* is a noise trader and *who* is an information trader.

excess noise in futures prices, using margin policy to maintain price discovery by futures (θ) below the critical level (θ_∞) will render any excess noise in futures prices benign.

De Long et al. (1991) argue that the activities of noise traders are not inconsequential:

> ...noise traders can come to dominate the market in that the probability that they eventually have a high share of total wealth is close to one.... We conclude that the case against their long-run viability is not as clear-cut as is commonly supposed [p. 1].
>
> If idiosyncratic risk is large, each individual noise trader with high probability fails to survive in the market, but noise traders as a whole can nevertheless survive. Evolution may leave an ever-shrinking army of ever-richer fools who collectively dominate the market [p. 4].

Thus, the presence of excessive noise trading in the futures markets is a worry—price discovery by futures may be little more than noise discovery.

Arbitrage

If price discovery by futures is dominant, then maintaining the elasticity of cash/futures arbitrage (i.e., keeping $\delta \ll 1$) is *essential.* Otherwise, either a crash or a bubble is likely (Corollary 5 and Figs. 4 and 5). This result lends support to the conclusion of Harris (1989) and Fama (1989) that the 1987 stock market crash might not have been as severe if more orderly trade mechanisms had been maintained. "Circuit-breakers" and "shock absorbers" have been proposed for preventing a crash by the New York Stock Exchange Blue-Ribbon Panel on Volatility and Investor Confidence [NYSE (1990)]. But neither circuit-breakers nor shock absorbers qualify as viable means for decreasing price discovery dominance by futures. If implemented, such measures likely would disrupt cash/futures arbitrage (increase δ), which could induce a crash (Corollary 5).

The Brady commission [Brady (1988)] blamed the severity of the 1987 stock market crash on the breakdown of the "linkage" between cash and futures markets. The GS model clarifies the meaning of "linkage" in terms of the parameters θ and δ: in particular, cash and futures markets may become *too closely linked* in the sense that price discovery by futures is *dominant* over the cash market ($\theta > \theta_\infty$), but may become *too loosely linked* in the sense that cash/futures arbitrage is inelastic (δ too close to 1). If futures become ultra-dominant, then cash/futures linkage may be inferior in *both* respects—which would induce either a crash or a bubble.

CONCLUSIONS

A futures market provides important salutary benefits to the corresponding cash market—lower long-term volatility, more liquidity, better efficiency—if price discovery by futures does not *dominate* (in the specific sense of this article) the cash market.

If price discovery by futures exceeds the *dominant* critical level, then cash market autocorrelation and long-term volatility are increased. If the *super-dominant* critical level is exceeded, cash market liquidity is decreased. If the *ultra-dominant* critical level is exceeded, either a crash or a bubble is likely. Excessive noise in futures prices is *necessary* but not *sufficient* for price discovery by futures to dominate the cash market. Increasing the initial margin requirement for futures would decrease excessive price discovery by futures. Price discovery by futures serves a valuable function when it is primarily "information discovery;" it is counterproductive when it is primarily "noise discovery."

Further research is desirable. In particular, the following areas should be explored:

i. theoretical results for the extended GS model comparable to the results presented here for the standard model;
ii. empirical tests of both the standard and the extended GS model for various markets;
iii. empirical confirmation of the impact of futures margin requirements on the level of price discovery by futures.

APPENDIX

The Linear Stochastic Evolutionary System

The GS model may be represented mathematically as a special case of the linear stochastic evolutionary system

$$X_{k+1} = AX_k + \mathcal{E}_{k+1} \tag{A1}$$

where X_k is an m-vector, A is a constant $m \times m$ matrix and $\mathcal{E}_k$ is an m-vector of random variables where

$$\begin{aligned} G &\overset{\text{def}}{=} \text{Cov}(\mathcal{E}_k, \mathcal{E}_k) \quad \text{for all } k, \\ \text{Cov}(\mathcal{E}_k, \mathcal{E}_j) &= 0 \quad \text{for } j \neq k \\ \text{Cov}(\mathcal{E}_k, X_j) &= 0 \quad \text{for } j < k \end{aligned} \tag{A2}$$

This system is *stable* if and only if $\Omega(A)$, the spectral radius of A, is less than 1, in which case, as $k \rightarrow \infty$ the system converges to a unique stochastic steady-state from any given initial condition, X_0. This study is concerned only with analysis of the stochastic steady-state and not with the transient behavior of the system in arriving at such a state. From eqs. (A1) and (A2),

$$\text{Cov}(X_{k+1}, X_{k+1}) = A\text{Cov}(X_k, X_k)A' + G \tag{A3}$$

Letting $Q = (q_{ij}) \overset{\text{def}}{=} \lim_{k\to\infty} \text{Cov}(X_k, X_k)$, and using (A3):

$$Q - AQA' = G \tag{A4}$$

Equation (A4) is a linear system of equations in the elements of Q; it is nonsingular provided $\Omega(A) < 1$, in which case a solution Q exists and is unique.

From eqs. (A1) and (A2) it follows that

$$\text{Cov}(X_{k+j}, X_k) = A^j \text{ Cov}(X_k, X_k) \tag{A5}$$

The following formulas for the applicable steady-state covariances follow directly from eqs. (A4) and (A5) and the fact that *any* square matrix A commutes with $(I - A)^{-1}$ provided $(I - A)^{-1}$ exists:

$$\lim_{k\to\infty} \mathrm{Cov}\left(\sum_{i=0}^{s-1} X_{k+i}, \sum_{j=0}^{s-1} X_{k+j}\right) = sQ + \sum_{n=1}^{s-1}(s-n)[A^n Q + QA'^n]$$
$$= sQ + (I-A)^{-1}[sA - A(I-A)^{-1}(I-A^s)]Q$$
$$+ [(I-A)^{-1}[sA - A(I-A)^{-1}(I-A^s)]Q]' \tag{A6}$$

$$\lim_{k\to\infty} \mathrm{Cov}\left(\sum_{i=0}^{s-1} X_{k+s+i}, \sum_{j=0}^{s-1} X_{k+j}\right) = \sum_{i=0}^{s-1}\sum_{j=0}^{s-1} A^{s+i-j}Q$$
$$= A[(I-A)^{-1}(I-A^s)]^2 Q \tag{A7}$$

Proofs

Proof of Proposition 1:

The proposition is an immediate consequence of the continuity of $\Omega(A)$ in each of the parameters, $\delta, \alpha_{CC}, \alpha_{CF}, \alpha_{FC}$, and α_{FF}.

Proof of Theorem 2:

i. This follows directly from eq. (22).

ii. From eq. (22), $\partial \mathcal{U}_s/\partial\delta$ is a quadratic in θ; in particular:

$$\frac{\partial \mathcal{U}_1}{\partial \delta} = 2\theta\left[\left(1 - r\frac{\sigma_F}{\sigma_C}\right) - \frac{\theta}{(1+\delta)^2}\left(\left(\frac{\sigma_F}{\sigma_C}\right)^2 - 2r\frac{\sigma_F}{\sigma_C} + 1\right)\right]$$

the results follow immediately.

iii. Substitute θ_* for θ in $\mathcal{U}_s(\theta, \delta, r, \sigma_F/\sigma_C)$ and simplify.

iv. Substitute 1 for θ in $\mathcal{U}_\infty(\theta, r, \sigma_F/\sigma_C)$ and simplify.

v. Set $\theta_\infty < 1$ and solve this inequality for σ_F/σ_C.

vi. It suffices to show $\partial\theta_s/\partial s < 0$. Let $y = \phi_s/s$; then $\partial\theta_s/\partial s = (\partial\theta_s/\partial y)(\partial y/\partial s)$. It is straightforward to show $\partial\theta_s/\partial y > 0$. Thus, it suffices to show $\partial y/\partial s < 0$:

$$\frac{\partial y}{\partial s} = \frac{s\partial\phi_s/\partial s - \phi_s}{s^2} = \frac{-s\dfrac{\delta^s \ln\delta}{1-\delta} - \phi_s}{s^2} = \frac{\delta^s(-s\ \ln\delta + 1) - 1}{s^2(1-\delta)}$$

For fixed s, let $w(\delta) = \delta^s(-s\ \ln\delta + 1) - 1$. Thus, to show $\partial y/\partial s < 0$, it suffices to show $w(\delta) < 0$ for $0 < \delta < 1$. Since $w(1) = 0$, it suffices to show $w'(\delta) > 0$ for $0 < \delta < 1$:

$$w'(\delta) = s\delta^{s-1}(-s\ \ln\delta + 1) - \delta^s s/\delta = s\delta^{s-1}(-s\ \ln\delta) > 0$$

vii. Let $y = \phi_s/s$; as noted previously $\partial y/\partial s < 0$. Set $\partial\mathcal{U}_s/\partial s > 0$ and solve this inequality for θ:

$$\frac{\partial \mathcal{U}_s}{\partial s} = 2\delta\theta\frac{\partial y}{\partial s}\left[\left(1 - r\frac{\sigma_F}{\sigma_C}\right) - \frac{\theta}{1+\delta}\left(\left(\frac{\sigma_F}{\sigma_C}\right)^2 - 2r\frac{\sigma_F}{\sigma_C} + 1\right)\right] > 0$$

Thus,

$$\theta > \frac{(1 + \delta)\left(1 - r\dfrac{\sigma_F}{\sigma_C}\right)}{\left(\dfrac{\sigma_F}{\sigma_C}\right)^2 - 2r\dfrac{\sigma_F}{\sigma_C} + 1} = \frac{1 + \delta}{2}\,\theta_\infty = \theta_*$$

viii. It suffices to show $\partial\theta_\infty/\partial r < 0$; this is straightforward.

ix. It suffices to show $\partial\mathcal{U}_s/\partial r > 0$. Let $y = \phi_s/s$; then

$$\begin{aligned}
\frac{\partial\mathcal{U}_s}{\partial r} &= 2\theta\frac{\sigma_F}{\sigma_C}\left[(1 - \delta y) - \theta\left(1 - \frac{2\delta y}{1 + \delta}\right)\right] \\
&> 2\theta\frac{\sigma_F}{\sigma_C}\left[(1 - \delta y) - \left(1 - \frac{2\delta y}{1 + \delta}\right)\right] \\
&= 2\theta\delta y\frac{\sigma_F}{\sigma_C}\left[\frac{2}{1 + \delta} - 1\right] > 0
\end{aligned}$$

x. Let $x = \sigma_F/\sigma_C$. Set $\partial\theta_\infty/\partial x < 0$ and solve this inequality for x:

$$\frac{\partial\theta_\infty}{\partial x} = 2\frac{(x^2 - 2rx + 1)(-r) - (1 - rx)(2x - 2r)}{(x^2 - 2rx + 1)^2} < 0$$

which is equivalent to $rx^2 - 2x + r < 0$. This inequality is always satisfied if $r \leq 0$. Otherwise, it is satisfied provided

$$\frac{1 - \sqrt{1 - r^2}}{r} < x < \frac{1 + \sqrt{1 - r^2}}{r}$$

The right-hand inequality is satisfied because $1 - rx > 0$, since $\theta_\infty > 0$. The left hand-inequality is satisfied because $x \geq 1$.

xi. Let $x = \sigma_F/\sigma_C$ and $y = \phi_s/s$. Set $\partial\mathcal{U}_s/\partial x > 0$ and solve this inequality for x:

$$\frac{\partial\mathcal{U}_s}{\partial x} = \theta\left[-2(1 - \delta y)(-r) + \theta\left(1 - \frac{2\delta y}{1 + \delta}\right)(2x - 2r)\right] > 0$$

which is equivalent to

$$x - r > \frac{-(1 - \delta y)r}{\theta\left(1 - \dfrac{2\delta y}{1 + \delta}\right)}$$

or

$$x > r\left(1 - \theta_\infty^{-1}\frac{\theta_s}{\theta}\right) \tag{A8}$$

Clearly, this inequality is satisfied if $r \geq 0$. On the other hand, if $r < 0$ and $\theta > \frac{1}{2}\theta_s$, then inequality (A8) is satisfied whenever

$$x > r\left(1 - 2\theta_\infty^{-1}\right) \tag{A9}$$

since $\theta_s/\theta < 2$. Substituting for θ_∞, inequality (A9) is equivalent to

$$x > r\left(1 - 2\frac{x^2 - 2rx + 1}{2(1 - rx)}\right)$$

which is equivalent to $1 > r^2$, which is always satisfied.

Proof of Theorem 3:

i. This follows from eqs. (23) and (19) showing that q_{31} is linearly increasing in θ with a root at θ_*.

ii. From eq. (23) it is clear that $\partial\rho_s/\partial\theta$ is a rational function of θ for which the denominator is positive and the numerator is a quadratic in θ. It is easy to show that this numerator is negative at $\theta = \frac{1}{2}\theta_*$, positive at $\theta = \theta_*$, and provided the stipulated condition is satisfied, positive at $\theta = 1$. Thus, $\partial\rho_s/\partial\theta > 0$ for $\theta > \theta_*$.

iii. Obvious from eq. (23).

iv. It is straightforward to show

$$\lim_{\sigma_F/\sigma_C\to\infty} \frac{q_{31}}{q_{11}} = \frac{\delta}{\theta(1 - \delta)}$$

Therefore,

$$\lim_{\sigma_F/\sigma_C\to\infty} \rho_s = \frac{\delta\phi_s^2/s}{1 + 2\dfrac{\delta}{1 - \delta}(1 - \phi_s/s)} \tag{A10}$$

The result follows immediately on taking $s = 1$.

v. For an arbitrary $h > 0$, take $\delta = 1 - h/s$. Then

$$\limsup_{s\to\infty}\ \rho_s \geq \limsup_{s\to\infty} \frac{\theta\dfrac{(1 - (1 - h/s)^s)^2}{h}\dfrac{q_{31}}{q_{11}}}{1 + 2\theta\left(1 - \dfrac{(1 - (1 - h/s)^s)}{h}\right)\dfrac{q_{31}}{q_{11}}}$$

$$= \frac{\dfrac{(1 - e^{-h})^2}{h}\dfrac{1}{2}\mathcal{U}_\infty}{1 + \left(1 - \dfrac{(1 - e^{-h})}{h}\right)\mathcal{U}_\infty} \tag{A11}$$

since $\lim_{\delta\to1} q_{31}/q_{11} = \mathcal{U}_\infty/(2\theta)$. Let h^* be the value of h that maximizes $(1 - e^{-h})^2/h$. It is easily shown that h^* satisfies $e^{-h^*} = 1/(2h^* + 1)$, which has the solution $h^* = 1.257$. The result follows directly.

vi. From inequality (A11), and for arbitrary $h > 0$, and since $\lim_{\sigma_F/\sigma_C\to\infty} \mathcal{U}_\infty = \infty$,

$$\lim_{\sigma_F/\sigma_C\to\infty}\left(\limsup_{s\to\infty} \rho_s\right) \geq \frac{\dfrac{1}{2}\dfrac{(1 - e^{-h})^2}{h}}{1 - \dfrac{(1 - e^{-h})}{h}}$$

from which the result follows immediately upon taking the limit as $h \rightarrow 0$. Similarly, using eq. (A10) with $\delta = 1 - h/s$, simplifying and taking the limit as $s \rightarrow \infty$:

$$\limsup_{s\rightarrow\infty}\left(\lim_{\sigma_F/\sigma_C\rightarrow\infty}\rho_s\right) \geq \frac{\frac{1}{2}\frac{(1-e^{-h})^2}{h}}{1-\frac{(1-e^{-h})}{h}}$$

from which the result follows immediately upon taking the limit as $h \rightarrow 0$.

vii. First, note that $q_{31} > 0$ since $\theta > \theta_*$, and hence $\rho_s > 0$. Differentiating ρ_s with respect to s, equating to 0 and simplifying yields

$$\psi(s) \stackrel{\text{def}}{=} \frac{\partial\rho_s}{\partial s} = s + \frac{\delta^{-s}-1}{2\ln\delta} - \frac{\theta q_{31}/q_{11}}{1+2\theta q_{31}/q_{11}}\left(\frac{1-\delta^s}{1-\delta}\right) = 0$$

It is easy to show $\psi(0) = 0$, $\psi'(0) > 0$, and $\lim_{s\rightarrow\infty}\psi(s) = -\infty$. Therefore, there *does* exist a value $\bar{s}$, $0 < \bar{s} < \infty$, such that $\psi(\bar{s}) = 0$. Furthermore, it may be shown that $\psi''(s) < 0$ for all $s \geq 0$ by showing $\psi''(0) < 0$ and $\psi'''(s) < 0$ for all $s \geq 0$. Therefore, ψ has one *and only one* positive root, $\bar{s}$; thus, ρ_s is unimodal with a maximum at $\bar{s}$. To show $\bar{s}$ increases as δ increases, it suffices to show $\partial\psi/\partial\delta > 0$, which is straightforward. Finally, to show $\bar{s}$ decreases as θ increases it suffices to show $\partial\psi/\partial\theta < 0$, which also is straightforward.

Proof of Corollary 4:

The result follows directly from Theorem 3(iii) and Definition 4.

Proof of Corollary 5:

The result follows directly from Theorem 3(v) and Definitions 3 and 4.

Proof of Theorem 6:

If futures become ultra-dominant, then futures are both super-dominant, and dominant, a fortiori. If futures are super-dominant, cash/futures arbitrageurs will perceive a decrease in cash market liquidity ($\mathcal{U}_1 > 0$), which will make cash/futures arbitrage less viable, thereby causing δ to increase. A discrete increase in δ will move the system out of its *steady-state* and into a *transient state* that, as time passes, will converge to a new steady-state, again with $\mathcal{U}_1 > 0$ and $\partial\mathcal{U}_1/\partial\delta < 0$. Although $\partial\mathcal{U}_1/\partial\delta < 0$, $\mathcal{U}_1$ can not be decreased to 0 by *any* feasible increase in δ because futures are ultra-dominant (Definition 3). Therefore, by Corollary 5, either a crash or a bubble will be induced, as δ increases arbitrarily close to 1 (unsuccessfully attempting to restore the nominal level of liquidity, i.e., decrease $\mathcal{U}_1$ to 0).

Bibliography

Aggarwal, R. (1988): "Stock Index Futures and Cash Market Volatility," *Review of Futures Markets,* 7:290–299.

Amihud, Y., and Mendelson, H. (1989): "Index and Index-Futures Returns," Technical Report, Salomon Brothers Center of the Study for Financial Institutions.

Black, F. (1986): "Noise," *Journal of Finance,* 41:529–543.

Brady, N. F. (1988): *Report of The Presidential Task Force on Market Mechanisms,* Washington, D.C.

Brorsen, B. W. (1991): "Futures Trading, Transaction Costs, and Stock Market Volatility," *The Journal of Futures Markets,* 11:153–163.

Burns, J. M. (1979): *A Treatise on Markets,* American Enterprise Institute for Public Policy Research, Washington, D.C.

De Long, J. B., Shleifer, A., Summers, L. H., and Waldmann, R. J. (1991): "The Survival of Noise Traders in Financial Markets," *Journal of Business,* 64:1–19.

Edwards, F. R. (1988a): "Does Futures Trading Increase Stock Market Volatility?" *Financial Analysts Journal,* January/February:63–69.

Edwards, F. R. (1988b): "Futures Trading and Cash Market Volatility: Stock Index and Interest Rate Futures, *The Journal of Futures Markets,* 8:421–439.

Fama, E. F. (1989): "Perspectives on October, 1987, or, What Did We Learn from the Crash?" in *Black Monday and the Future of Financial Markets,* Kamphuis, Jr., R. W., Kormendi, R. C., and Watson, J. H. (eds.), Homewood, IL: Dow Jones-Irwin, pp. 71–82.

Fama, E. F. (1991): "Efficient Capital Markets: II," *Journal of Finance,* 46:1575–1617.

Garbade, K. D., and Silber, W. L. (1983): "Price Movements and Price Discovery in Futures and Cash Markets," *Review of Economics and Statistics,* 65:289–297.

Grossman, S. J. (1988): "Program Trading and Market Volatility: A Report on Interday Relationships," *Financial Analysts Journal,* July/August:18–28.

Grossman, S. J., and Miller, M. H. (1988): "Liquidity and Market Structure," *Journal of Finance,* 43:617–637.

Hardouvelis, G. A. (1988): "Margin Requirements and Stock Market Volatility," *Federal Reserve Bank of New York Quarterly Review,* 12:80–89.

Hardouvelis, G. A. (1990): "Margin Requirements, Volatility, and the Transitory Component of Stock Prices," *American Economic Review,* 80:736–762.

Harris, L. (1989): "The October 1987 S&P 500 Stock-Futures Basis," *Journal of Finance,* 44:77–99.

Hartzmark, M. L. (1986): "The Effects of Changing Margin Levels on Futures Market Activity, the Composition of Traders in the Market, and Price Performance," *Journal of Business,* 59:S147–S180, Supplementary.

Hasbrouck, J., and Schwartz, R. A. (1988): "Liquidity and Execution Costs in Equity Markets," *Journal of Portfolio Management,* Spring:10–16.

Herbst, A., McCormack, J. P., and West, E. N. (1987): "Investigation of a Lead–Lag Relationship between Spot Stock Indices and Their Futures Contracts," *The Journal of Futures Markets,* 7:373–381.

Hsieh, D. A., and Miller, M. (1990): "Margin Regulation and Stock Market Volatility," *Journal of Finance,* 45:3–29.

Kahl, K. H., Rutz, R. D., and Sinquefield, J. C. (1985): "The Economics of Performance Margins in Futures Markets," *The Journal of Futures Markets,* 5:103–112.

Kalavathi, L., and Shanker, L. (1991): "Margin Requirements and the Demand for Futures Contracts," *The Journal of Futures Markets,* 11:213–237.

Kawai, M. (1983): "Price Volatility of Storable Commodities Under Rational Expectations in Spot and Futures Markets," *International Economic Review,* 24:435–458.

Kawaller, I. G., Koch, P. D., and Koch, T. W. (1987): "The Temporal Price Relationship between S&P 500 Futures and the S&P 500 Index," *Journal of Finance,* 42:1309–1329.

Khoury, N. T., and Yourougou, P. (1991): "The Informational Content of the Basis: Evidence from Canadian Barley, Oats, and Canola Futures Markets," *The Journal of Futures Markets,* 11:69–80.

Kling, A. (1986): "Futures Markets and Transaction Costs," in *Financial Futures and Options in the U.S. Economy, A Study by the Staff of the Federal Reserve System,* Kwast, M. L. (ed.), Federal Reserve Board of Governors, Washington, D.C., pp. 41–54.

Kupiec, P. H. (1989): "Initial Margin Requirements and Stock Returns Volatility: Another Look," *Journal of Financial Services Research,* 3:287–301.

Kwast, M. L. (1986): "Futures Markets and Transaction Costs," in *Financial Futures and Options in the U.S. Economy, A Study by the Staff of the Federal Reserve System,* Kwast, M. L. (ed.), Federal Reserve Board of Governors, Washington, D.C., pp. 1–38.

Laatsch, F., and Schwarz, T. V. (1988): "Price Discovery and Risk Transfer in Stock Index Cash and Futures Markets," *Review of Futures Markets,* 7:272–289.

LeRoy, S. F. (1989): "Efficient Capital Markets and Martingales," *Journal of Economic Literature,* 27:1583–1621.

Lien, D. D. (1987): "The Inventory Effect in Commodity Futures Markets: An Empirical Study," *The Journal of Futures Markets,* 7:637–652.

Lippman, S. A., and McCall, J. J. (1986): "An Operational Measure of Liquidity," *American Economic Review,* 76:43–58.

MacKinlay, A. C., and Ramaswamy, K. (1988): "Index-Futures Arbitrage and the Behavior of Stock Index Futures Prices," *Review of Financial Studies,* 1:137–158.

Merrick, Jr., J. J. (1987): "Price Discovery in the Stock Market," Working Paper Series 416, Salomon Brothers Center for the Study of Financial Institutions.

Miller, M. H. (1991): "International Competitiveness of U.S. Futures Exchanges," *Journal of Applied Corporate Finance,* 3:6–20.

Miller, M. H., Malkiel, B., Scholes, M., and Hawke, Jr., J. D. (1989): "Stock Index Futures and the Crash of '87," *Journal of Applied Corporate Finance,* 1:6–17.

Ng, N. (1987): "Detecting Spot Price Forecasts in Futures Prices Using Causality Tests," *Review of Research in Futures Markets,* 6:251–276.

NYSE (1990): "New York Stock Exchange Blue-Ribbon Panel Report on Market Volatility and Investor Confidence," Technical Report, New York Stock Exchange.

Oellermann, C. M., Brorsen, B. W., and Farris, P. L. (1989): "Price Discovery for Feeder Cattle," *The Journal of Futures Markets,* 9:113–121.

Pliska, S. R., and Shalen, C. T. (1991): "The Effects of Regulations on Trading Activity and Return Volatility in Futures Markets," *The Journal of Futures Markets,* 1:135–151.

Powers, M. J. (1970): "Does Futures Trading Reduce Price Fluctuations in the Cash Market?" *American Economic Review,* 60:460–464.

Quan, J. (1992): "Two-Step Testing Procedure for Price Discovery Role of Futures Prices," *The Journal of Futures Markets,* 12:139–149.

Salinger, M. A. (1989): "Stock Market Margin Requirements and Volatility: Implications for Regulation of Stock Index Futures," *Journal of Financial Services Research,* 3:121–138.

Santoni, G. J. (1987): "Has Programmed Trading Made Stock Prices More Volatile?" *Federal Reserve Bank of St. Louis Review,* 69:18–29.

Schroeder, T. C., and Goodwin, B. K. (1991): "Price Discovery and Cointegration for Live Hogs," *The Journal of Futures Markets,* 11: 685–696.

Schwarz, T. V., and Laatsch, F. (1991): "Price Discovery and Risk Transfer in Stock Index Cash and Futures Markets," *The Journal of Futures Markets,* 11:669–683.

Schwert, G. W. (1989): "Margin Requirements and Stock Volatility," *Journal of Financial Services Research,* 3:153–164.

Silber, W. L. (1985): "The Economic Role of Financial Futures," in Peck, A. (ed.), *Futures Markets: Their Economic Role,* 83–114, American Enterprise Institute for Public Policy Research, Washington, D.C.

Stoll, H. R., and Whaley, R. E. (1988): "Volatility and Futures: Message Versus Messenger," *Journal of Portfolio Management,* Winter:20–22.

Stoll, H. R., and Whaley, R. E. (1990): "The Dynamics of Stock Index and Stock Index Futures Returns," *Journal of Financial and Quantitative Analysis,* 25:441–468.

Swinnerton, E. A., Curcio, R. J., and Bennett, R. E. (1988): "Index Arbitrage Program Trading and the Prediction of Intraday Stock Price Changes," *Review of Futures Markets,* 7:300–323.

Turnovsky, S. J. (1983): "The Determination of Spot and Futures Prices with Storable Commodities," *Econometrica,* 51:1363–1387.

Zeckhauser, R., and Niederhoffer, V. (1983): "The Performance of Market Index Futures Contracts," *Financial Analysts Journal,* January/February:59–65.