Optimal Hedging When Preferences Are State Dependent

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INTRODUCTION

For a farmer growing corn, the news that farm prices are up certainly would seem to be good news. The Farmer's income for the year would probably be higher. Farmers are, however, consumers as well so that the above news would likely result in higher grocery bills. In fact, it might be that corn prices are up only slightly while other agricultural prices are dramatically higher. The point is that it is not known whether or not the farmer is better off due to the higher farm prices. Indeed, it may be that nonagricultural prices are up as well. This article focuses not on the farmer's welfare per se, but rather on how the farmer might alter hedging strategies due to such price movements.

The problem in the above scenario is that ceteris is not paribus. The farmer's wealth changes with a change in the price of corn. However, the consumption-opportunity set (i.e., the set of affordable bundles of consumption goods) also changes when prices change. Thus, the farmer's utility for a given level of wealth is dependent upon the prices of various goods and services in the marketplace. In a world of uncertainty, preferences over various probability distributions of final wealth alone would only seem to make sense if the prices for goods and services remain fixed. If prices vary, both wealth and prices must be considered in comparing probability distributions. In particular, suppose that each state of nature yields both a wealth level and a set of prices for goods and services. Then, if one views utility as a function of wealth alone, the utility function itself will be different in each state of the world, reflecting the different price levels.[1]

The purpose of this article is to incorporate state-dependent preferences into a simple hedging model. The fact that preferences are state dependent is shown to systematically affect the optimal hedging strategy. As a simple example, consider the corn farmer in

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[1]Readers well-versed in economics will recognize this as nothing more than viewing the indirect utility function with prices suppressed. In other words, utility of wealth represents the best achievable utility of consumption at a given wealth level *and* a given set of prices.

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a world where all prices are perfectly positively correlated and where futures prices are unbiased estimates of future spot prices. If the farmer's sole source of income is the corn crop, the optimal hedging strategy is to stay out of the futures market. The reason for this strategy is simple. Although the farmer faces an income risk, there is no consumption risk, since the farmer can afford the same bundles of consumption goods in any state of the world. In other words, the farmer's income rises and falls with other prices and, thus, the farmer's real purchasing power is constant.

The model here is simplified to focus on the particular effects of state-dependent preferences. The case of a monoperiodic, expected-utility maximizer, who holds a nontradable (at least in the very short-run) position in an asset, which is to be sold at the end of the current period is used for study. There are only two states of nature. Each state of nature yields a different set of prices within the economy. The individual's wealth is determined by the realized price of the (nontraded) asset and by any profit or loss made in a futures market on that asset. However, the consumption-opportunity set also varies across states, due to the price differences. This yields a type of state dependency in the utility of wealth.[2]

In this setting, the optimal hedge ratio is shown to consist of three components: the so-called pure hedge ratio, which protects against the spot-price income risk; a price-compensation component, which protects against changes in the consumption-opportunity set; and a speculative component, which is due to any bias that might exist in futures-market prices. The logic here is similar to that found in Merton's (1973) three-fund separation theorem.

This study finds that a key result in the optimal-hedging literature by Benninga, Eldor, and Zilcha (1984)—that the expected-utility-maximizing hedging strategy also minimizes the variance of wealth across states of nature if futures-market prices are unbiased—does not hold in the above stated setting. Instead, the optimal hedging strategy is to minimize the variation of marginal utility across states of nature, a point made by Ingersoll (1987) in a portfolio context.

Of course, state dependency of preferences might occur for many reasons and not just because of a changing consumption-opportunity set. For example, Cook and Graham (1977) and Shavell (1978) consider the effects of state-dependent preferences in an insurance context. In particular, Cook and Graham consider the case where some components of wealth are "irreplaceable" in that they have a large nonpecuniary sentimental value. Shavell presents a model where utility depends not only on one's level of wealth, but also on one's physical health. Indeed, so long as the consumer's preferences depend upon the realized state of nature in addition to the level of wealth in that state, this analysis is applicable.[3]

[2]The mathematical relationship between a small change in price and the corresponding marginal utility of wealth follows from Roy's identity: $\partial u(y)/\partial y = -(1/x_i)[\partial u(y)/\partial p_i]$, where y denotes wealth and x_i denotes demand for the ith commodity $i = 1, \cdots, m$. [See, for example, Varian (1992).] Obviously, a discrete change in prices involves a complicated integration across such price changes, which is beyond our present concern in this article.

[3]More formally, consider a fixed probability space (ϕ, β, π), where ϕ is a finite set, β its power set, and π a probability measure on β. A final wealth prospect is a random variable Y: $\phi \rightarrow \mathbb{R}_+$. State independence of preferences implies that an individual is indifferent to any two random variables with identical cumulative distribution functions over $\mathbb{R}_+$. If preferences also depend upon the state of nature realized, then they are called *state dependent*. This article considers only the preference functional induced by an expected-utility representation of preferences. The model could be extended to nonexpected utility representations so long as a general preference functional induces a preference ordering that satisfies stochastic dominance preference and that is quasiconcave when viewed as a function of state-contingent claims. See Karni (1985) for further justifications of the state-dependent-preferences approach.

The second section describes the basic model. The third section shows that state-independent marginal utility is sufficient to obtain the same hedge ratio as with state-independent utility itself and the fourth section examines the optimal hedge ratio when marginal utility can be ordered across states of nature. An example is then presented to illustrate the importance of the results. The final section summarizes the results and presents some further avenues for research.

THE BASIC MODEL

Consider a risk-averse individual who owns Q units of an asset subject to price risk. The individual expects to sell the Q units at time $t = 1$ at the then prevailing spot-market price, p_s. Q is considered as endogenous and nonrandom [see Adler and Detemple (1988) for a discussion]. The simplest possible case is considered where the probability space contains only two states of nature yielding two vectors of prices in the economy. State L occurs with probability π and is characterized by the price vector $P_L = (p_{L1}, p_{L2}, \ldots, p_{Lm})$, where P_{Li} denotes the price of the ith commodity in state L, $i = 1, \ldots, m$. State H occurs with probability $1 - \pi$ and is characterized by the vector of commodity prices P_H, defined in a similar manner. Without loss of generality, it is assumed that the asset owned by the individual is asset 1 and that $p_{L1} < p_{H1}$. It is assumed also that $p_{Li} \neq p_{Hi}$ for some $i > 1$. Since reference is made only to individual commodity prices for commodity 1, the "1" subscript is dropped for ease of notation. p_L and p_H denote the price of commodity 1 in the respective states of nature. The reader is cautioned that, although $p_L < p_H$ by assumption, no other assumption is made about how the prices in price vector P_L compare to those in P_H. In particular, it is not assumed that *all* prices are necessarily lower in state L than in state H. The designation of the states as L and H is made for convenience, with an obvious reference to the relative price of commodity 1. Note also that the value of Q units of the spot asset is either $p_L Q$ or $p_H Q$, depending upon which state of nature is realized.

The realization of a particular state of nature affects not only the individual's wealth, but also his or her consumption-opportunity set for spending that wealth. The individual's preferences are given by the utility function $V(y)$ if the price vector P_L prevails, where y denotes total wealth. If P_H prevails, preferences are represented by the utility function $U(y)$. Both U and V are assumed to be increasing and concave, as well as twice differentiable with continuous first derivatives. The reader may wish to think of U and V as the individual's indirect utility function viewed over the price vectors P_H and P_L, respectively.[4] The concavity of U and V ensures risk-averse behavior based on expected-utility preference over lotteries. If U and V are identical, the usual model of state-preference theory obtains (with preferences between states dependent only upon the wealth in that state). The only difference between the assumptions of this study and those from the traditional hedging literature is that U and V can differ.

If $V(y) > U(y)\ \forall y$, it is possible that the individual is better off in state L, with less wealth, perhaps due to lower overall prices. However, neither U nor V need be greater than the other for all values of y. Moreover, the concern of this study is with the optimal hedge, which will depend on the relative magnitudes of the marginal utilities, U' and V'. This will become apparent later.

The individual can hedge the price risk by taking a short position, i.e., selling contracts, in the futures market. It is assumed that the current futures market price, F, relates to

[4]For a more detailed mathematical treatment of state-dependent preferences, the reader is referred to Karni (1985). Also, see the next section.

the delivery of one unit of the spot asset at time $t = 1$, $p_L < F < p_H$. The assumption of a contract for delivery at $t = 1$ is obviously not innocuous as it eliminates basis risk in this model. This helps to simplify the analysis and focus on the particular effects of state-dependent preferences. The effects of adding basis risk can be substantial. Interested readers are referred to Benninga, Eldor, and Zilcha (1984) and to Briys, Crouhy, and Schlesinger (1993).[5] In this model, the futures price converges to the spot price at $t = 1$. By selling N contracts (where one contract is for one unit of the spot asset) in the futures market, the individual's expected utility is given by

$$EU \equiv (1 - \pi)U[W + p_H Q + N(F - p_H)] + \pi V[W + p_L Q + N(F - p_L)] \quad (1)$$

where W represents a nonrandom background wealth.

The first-order condition for maximizing (1) is

$$dEU/dN = (1 - \pi)(F - p_H)U'(y_H) + \pi(F - p_L)V'(y_L) = 0 \quad (2)$$

where y_H and y_L denote final wealth when the realized spot price is p_H and p_L, respectively. N^* denotes the optimal number of futures contracts, i.e., using N^* in y_L and y_H will satisfy (2). The first term of the middle expression in (2) represents the expected marginal utility cost from increasing N, while the second term represents the expected marginal utility gain from an increase in N. The second-order condition follows easily from the concavity of U and V. The case where U and V are identical is just a two-state version of the standard model considered by Benninga, Eldor, and Zilcha (1984). The next sections consider how the outcome changes when U and V differ.

THE OPTIMAL HEDGE RATIO: STATE-INDEPENDENT MARGINAL UTILITY

For now, suppose the futures market is unbiased; that is, the current futures price is an unbiased predictor of the spot price at time $t = 1$. Thus, $F = E(p_s) = (1 - \pi)p_H + \pi p_L$. Most of this study focuses on this case. Its simplicity allows isolation of the effects of state-dependent preferences. However, this assumption is relaxed later. Under this assumption, the first-order condition (2) implies that marginal utility should be equated across states, i.e.,

$$U'(y_H) = V'(y_L) \quad (3)$$

For the case where preferences are state independent (U and V are identical) eq. (3) implies that $y_H = y_L$. From the definitions of y_H and y_L, this only occurs if $N^* = Q$. This result is simply a special case of Benninga, Eldor, and Zilcha (1984) and is analogous to a well-known result of Mossin (1968) for insurance markets, that a risk averter will fully insure at an actuarially fair price. Here, the unbiasedness guarantees an actuarially fair price, and the absence of basis risk implies that hedging is fully effective, similar to Mossin's insurance.[6]

From (3), it is evident that even in the case of state-dependent preferences, a condition that $U'(y) = V'(y)$ for all y is sufficient to yield the same optimal hedge, namely $N^* = Q$. Thus, if *marginal* utility is state independent, the individual will fully hedge

[5]These articles use state-independent preferences. Obviously incorporating state-dependent preferences into more realistic, complicated settings would be valuable, but the goal here is to keep the base model as simple as possible.

[6]"Fully effective" means that it is possible, by setting $N = Q$, to eliminate all wealth risk. Note that when $N = Q$, $y_L = y_H$ obtains. If insurance contracts are not fully effective, e.g., due to insolvency risk, this result need not hold. See Doherty and Schlesinger (1990). See also Adler and Detemple (1988) who make this same point for a futures market.

if futures prices are unbiased and there is no basis risk. Note that if utility between states differs by a constant $k \neq 0$, $U(y) = V(y) + k$, then utility is state dependent, but marginal utility is independent of the state of nature.

Suppose marginal utility is state independent and the futures market is not unbiased, but exhibits normal backwardation. In the simple model of this study, this implies $F < E(p_s)$. Then, from (2), it is easy to see that $dEU/dN < 0$ when evaluated at $N = Q$. Consequently, $N^* < Q$. Here, the so called "pure hedge" is still $N = Q$, since there is no basis risk; but, due to the bias in the futures market, there is now a speculative component to the hedge, $(N^* - Q) < 0$. Although one cannot have $F < p_L$ (otherwise infinite arbitrage profits are possible), if F is low enough, the optimal futures-market position will entail a long position, $N^* < 0$. This yields a so-called "Texas Hedge" and basically implies that the expected profit from a long futures-market position more than offsets any potential hedging benefits to the individual.

In a similar manner, it follows that $N^* > Q$ when the futures market exhibits contango, $F > E(p_s)$. In this case, the individual not only fully hedges, but takes an additional speculative short position in the futures market as well.

The results for an unbiased futures market are depicted graphically in Figure 1. This figure views the individual's final wealth position as a contingent claim. The individual's endowed position is the claim $E = (\bar{y}_H, \bar{y}_L)$ where $\bar{y}_i$ denotes $W + p_i Q$, $i = H, L$. A futures position of shorting N contracts entails a state-contingent loss of $N(p_H - F)$ should state H occur, and a state-contingent gain of $N(F - p_L)$ should state L occur. Since U and V both are concave, it follows that indifference curves (iso-expected utility contours) are convex, with the marginal rate of substitution of state H contingent claims for state L contingent claims given by MRS $= [(1 - \pi)U'(y_H)]/[\pi V'(y_L)]$. If $U'(y) = V'(y)\ \forall y$, a ray-homotheticity results whereby MRS $= (1 - \pi)/\pi$ for all claims along the 45° certainty line, exactly as with state-independent preferences.[7]

The price line represents all contingent claims obtainable by the individual. If the market is unbiased, as depicted in Figure 1, with $F = E(p_s)$, then the slope of the price line equals $-(1 - \pi)/\pi$. In this case, the price line is referred to as the "fair price line." This terminology, borrowed from game theory, relates to the fact that buying or selling futures contracts represents a "fair gamble" (i.e., zero expected gain or loss) in an unbiased futures market. As a consequence, expected utility is maximized at claim A in Figure 1, where $N^* = Q$. At a claim with $N < Q$, such as at claim B, MRS $< (1 - \pi)/\pi$, and hence more futures contracts should be sold. Note in Figure 1 that, at claim A, $y^*_L = y^*_H = W + FQ$.

The main result of this section can be summarized as follows: If utility is state dependent but marginal utility is state independent, i.e., $U(y) = V(y) + k$, hedging behavior is unaffected by the state dependence of preferences.

THE OPTIMAL HEDGE RATIO: STATE-DEPENDENT MARGINAL UTILITY

If marginal utility is not state independent, then the above analysis need not hold. Consider the case where $U'(y) > V'(y)\ \forall y$. In other words, marginal utility is higher for any

[7]The MRS is calculated by setting EU as in (1) equal to a constant and then solving for the slope, dy_L/dy_H, for this equation. It represents the number of state-H dollars that an individual would be willing to swap for one state-L dollar, at the margin. For example, MRS = 1.2 indicates that exchanging \$1.20 in state H for \$1.00 in state-L would leave the individual at the same expected utility level. The ray-homothetic property implies that for all certain state-claims (i.e., claims where income is equal in both states), the MRS is a constant equal to the ratio of probabilities $(1 - \pi)/\pi$. This result is well-known in the state-preference literature for the usual case of state-independent preferences.

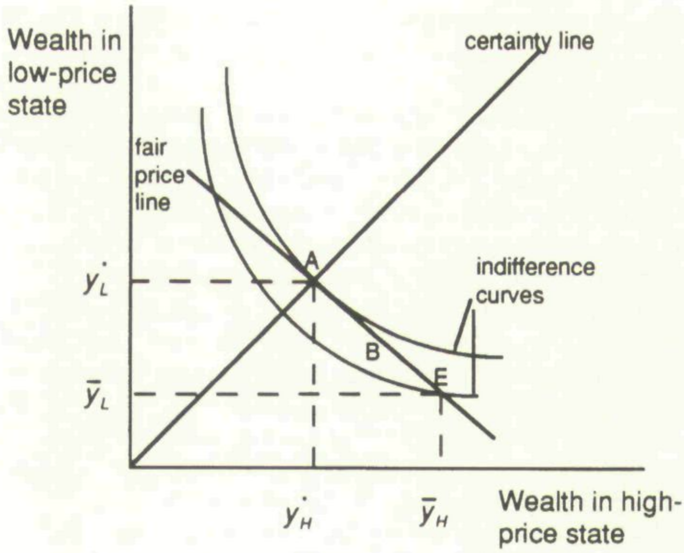

Figure 1

given wealth level in state H than in state L. Define the reference set, as in Karni (1985), by

$$\Omega \equiv \{(y_1, y_2) \in \mathbb{R}^2_+ \mid U'(y_2) = V'(y_1)\} \tag{4}$$

Thus, Ω is the set of contingent wealth claims such that marginal utility is the same in both states of the world. Since both U and V are concave and since it is assumed that $U'(y) > V'(y)$, Ω will lie everywhere below the certainty line, such as depicted in Figure 2. Given the initial claim E and the reference set Ω depicted in Figure 2, and given an unbiased futures market, the individual would choose to move to claim D. Once again, choices along the fair-price line represent contingent wealth claims for the various levels of N. Note that fully hedging with $N = Q$, i.e., moving from E to A in Figure 2, is no longer optimal, as is the case when marginal utility is state independent. Indeed, at a riskless claim such as A, $U' > V'$ by the assumption of this section. Using the first-order condition (2), this implies that $dEU/dN < 0$. Hence, the optimal hedging strategy entails selling fewer than Q contracts, i.e., $N^* < Q$. This is evident in Figure 2 from the properties of Ω and the fact that preferences are strictly convex.

The reasoning behind this result is as follows. Although futures-market prices are actuarily fair, the individual has a higher marginal utility of wealth in the state of the world in which p_H prevails (state H). As a result, an individual who has fully hedged (to claim A in Fig. 2) will find it worthwhile transferring some of his or her wealth from state L to state H. The optimal hedge can be decomposed into two components: The first is the pure hedge, which in this model with no basis risk is to go short Q futures contracts. This pure hedge eliminates all of the variation in income. In Figure 2, the pure hedge is reflected as a movement from E to A. The second component is a price-compensation term equal to $N^* - Q$, which is negative when $U'(y) > V'(y)\ \forall y$. This component captures the effect of changes in the individual's opportunity set in different states of the world. In Figure 2, this effect is represented by the movement from A to D. In particular, it compensates for the change in the prices of other assets between the two states of the world. Note that the optimal hedge, at claim D, does not eliminate the variation in

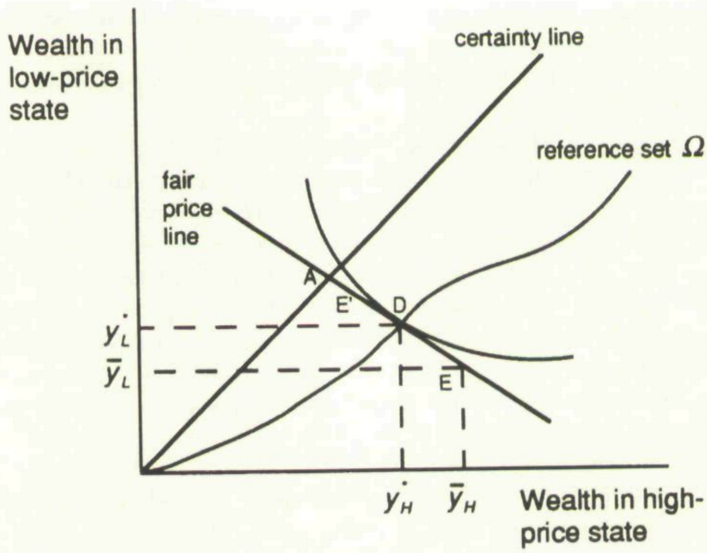

Figure 2

final wealth. Rather, it eliminates the variation in marginal utility between states.[8] If the individual had eliminated the variation in wealth between states by fully hedging via a short position in futures contracts, as would be optimal with state-independent preferences, then $U' > V'$. Thus, marginal utility of a dollar in state H exceeds that in state L; so that taking contingent dollars away from state-L wealth and moving them to state-H wealth by going long in futures contracts would increase the individual's expected utility. This long position in the futures market, starting from a fully-hedged position, is precisely what is defined here as the price-compensation component of the optimal hedge. The optimal hedge is thus the sum of the pure hedge (i.e., the fully-hedged short position) and the price-compensation component (i.e., the long position to compensate for state dependence of preferences).

If in Figure 2, the individual's initial claim were at E' rather than at E with reference set Ω, a case would result where a long futures market position is optimal, $N^* < 0$, even though the futures market is unbiased and the individual can perfectly hedge via a short position. In this case, the price-compensation component (A to D) of the optimal hedge outweighs the pure-hedge component (E' to A), so that the optimal net position is long in futures contracts.

For markets exhibiting normal backwardation it follows readily from the first-order condition (2) that the optimal final contingent claim will lie below the reference set Ω. Depending on the location of the initial contingent claim, one obtains the two hedging affects mentioned above, plus a third hedge component: a speculative component that moves the final claim below the reference set Ω, due to the bias in the futures market price. For the case of normal backwardation, $F < E(p_s)$, the speculative component is a long position (since going long yields an expected profit). The optimal hedge is thus "longer" (i.e., either net short by fewer contracts or net long by more contracts) than is

[8]Obviously, the simplification of no basis risk is important here. If basis risk is added, then hedging is no longer fully effective. In this case, variations in final wealth or marginal utility are minimized rather than eliminated.

the case in an unbiased futures market. If the futures market exhibits contango, a similar reasoning shows that the speculative component is a short position (since going short yields an expected profit) and the optimal hedge is thus "shorter" than in an unbiased futures market.

The above analysis is all based on the assumption that $U'(y) > V'(y)\ \forall y$. This would make sense, for example, if all prices are generally higher in state H. In that case, an individual would be "poorer" in real terms in state H; and given diminishing marginal utility of wealth, marginal utility in state H would be higher. Of course other scenarios are possible. If $U'(y) < V'(y)\ \forall y$, then the individual would prefer to have more wealth in state L and less wealth in state H to protect against changes in the consumption-opportunity set. This might occur, for instance, if other prices are negatively correlated to the price of commodity 1. In this case, the reference set would lie completely above the certainty line in Figure 2 and the price-compensation component would be a short position in the futures market.

A summary of the price-compensation effects and how they relate to the pure hedge and the speculative component are presented in Table I. The optimal hedge in each case is simply the sum of these three effects. Of course, the three cases considered are not exhaustive, and it is possible that $U'(y) > V'(y)$. for some values of y while $U'(y) < V'(y)$ for other value of y. In this case, the reference set would cross the certainty line as represented in Figure 2 (perhaps several times) and an analysis of the optimal hedge would depend upon where the reference lies with regards to where it intersects the relevant price line.

AN EXAMPLE

This section considers a simple example where prices are perfectly correlated. For concreteness, say that all prices are 20% higher in state H than in state L, which can be interpreted as general inflation. Assume, also, that the futures market is unbiased with both states equally likely, $\pi = 1 - \pi = \frac{1}{2}$. Thus:

$$F = (1.1)p_L \tag{5}$$

$$\overline{y}_L = W + p_L Q \tag{6}$$

$$\overline{y}_H = W + p_H Q = W + (1.2)p_L Q \tag{7}$$

Furthermore, $U(1.2y) = V(y)$, since 120% of state-L wealth will leave the individual with exactly the same purchasing power in state H. It thus follows that the reference set Ω, as defined in (4), is given by $y_H = 1.2y_L$. Note that $U'(y) > V'(y)$ for each y.

It follows from the first-order condition (2), that $y^*_H = (1.2)y^*_L$; or equivalently

$$W + (1.2)p_L Q + N^*(-0.1p_L) = (1.2)[W + p_L Q + N^*(0.1p_L)] \tag{8}$$

Solving for the optimal hedge,

$$N^* = -(^{10}/_{11})^{W}/_{p_L} \tag{9}$$

Of importance in (9), note that $N^* \leq 0$, with $N^* = 0$ only in the case where $W = 0$.

Consider first the case where $W = 0$. In this case note that $\overline{y}_H = 1.2\overline{y}_L$. In other words, the individual's initial contingent wealth claim lies on the reference set. In state H, the individual has 20% more wealth, but prices are 20% higher. Thus, real purchasing power is the same in both states. In essence, the price fluctuation in commodity 1 acts as a perfect hedge against price inflation. Ignoring prices other than commodity 1 and

Table I
SUMMARY OF HEDGE COMPONENTS

	$U' = V'$			$U' > V'$			$U' < V'$		
	Pure Hedge	**Price Comp.**	**Speculative**	**Pure Hedge**	**Price Comp.**	**Speculative**	**Pure Hedge**	**Price Comp.**	**Speculative**
Unbiased market	S	0	0	S	L	0	S	S	0
Normal backwardation	S	0	L	S	L	L	S	S	L
Contango	S	0	S	S	L	S	S	S	S

This table shows whether each of the three hedging components is short, long, or nil under various settings. The total effect is the sum of the three components.

using a state-independent-preference analysis would lead to an optimal hedge of $N^* = Q$, which fully eliminates the variation in income between states. Thus, a common state-independent framework would totally ignore the fact that wealth is naturally hedging against inflation.

If $W > 0$, then $\overline{y}_H < 1.2\overline{y}_L$, since only part of the individual's wealth adjusts for inflation. In this case, the individual can compensate by going *long* $({}^{10}/_{11})({}^{W}/_{p_L})$ contracts. This will cause total wealth y_H to equal 1.2 times y_L and provide a perfect hedge against inflation. Once again, the standard state-independent preference model would ignore the effects of inflation and only consider the pure hedge of $N = Q$. Not only is this hedge suboptimal, but it entails a short position in the futures market to protect against price risk for commodity 1 alone; whereas the true optimal hedge would protect against the inflation risk and entail a net long position in the futures market.

The example is illustrated in Figure 3, which is a special case of the model illustrated in Figure 2. When $W = 0$, the initial contingent claim lies on the reference set, such as at point E. Here the pure hedge would entail moving from E to A, but the price-compensation component would entail moving from A back to E, so the net effect is not to hedge at all. In the case where $W > 0$, E' denotes the initial claim. In this case the pure hedge (moving from E' to A) is dominated by the price-compensation component (moving from A to E), and the optimal hedge is a long position (moving from E' to E).

CONCLUDING REMARKS

This study presents a fairly simple model to show how optimal hedging strategies are affected by state-dependent preferences. In particular, futures-market contracts might have an ability to hedge not only against price fluctuations for an owned asset, but also against changes in the consumption-opportunity set that arise from changing prices.

In this simple model, where all correlations are perfect, the individual fully insures in the futures market whenever futures prices are unbiased and *marginal* utility is state independent. This result is a slight generalization of that under state-independent preferences, since marginal utility being state independent also allows for utility to differ

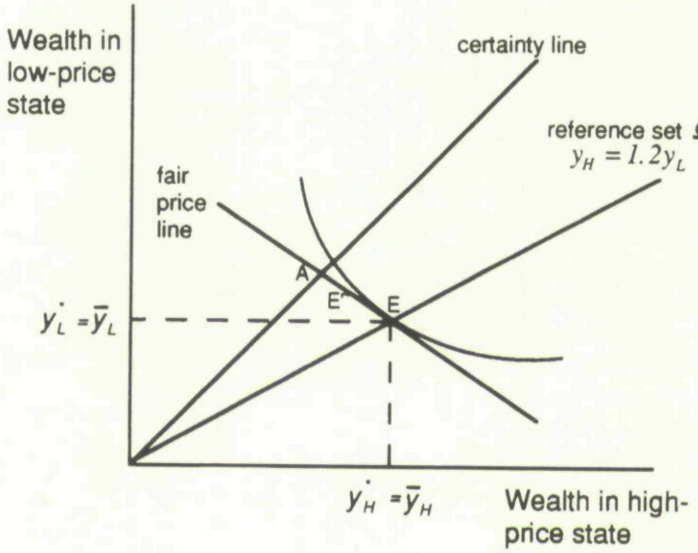

Figure 3

by a constant in both states of nature. If futures prices are unbiased but marginal utility is state dependent, the individual eliminates the variation in marginal utility across states. The optimal hedging strategy leaves the individual with a contingent claim contained in Karni's (1985) reference set, which does not generally minimize the variance of wealth across states.

The optimal hedge is shown to have three components. In this simple model, the first component fully eliminates price risk for the owned asset (the pure-hedge component), the second deviates from the pure hedge to eliminate risk associated with changes in the consumption-opportunity set (the price-compensation component), and the third deviates from the reference set whenever futures prices are biased (the speculative component).

Although the model is simple, it does capture the essence of how state-dependent preferences change several well-known results. In a more realistic setting, the price correlations would not be so perfect. How such correlations affect a model with state-dependent preferences is certainly an area deserving of study. Also, basis risk needs to be considered. Thus, the pure hedge will not be "full insurance," but will depend on the covariance between the futures and spot prices; and the covariance between the owned-asset spot prices and other commodity prices will also not allow one to achieve a zero variation of marginal utilities across states. The model is best viewed as illustrating how the rudiments of more realistic (and, hence, more complicated) optimal-hedging models are altered a by changing consumption-opportunity set, which yields a state dependence in preferences.

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