# Do the Options Markets Really Overreact?

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## INTRODUCTION

In a recent article, Stein (1989) derives and tests a model that describes the relationship between implied volatilities on options of different maturities. Under the assumption that volatility follows a mean-reverting process with a constant long-run mean and a constant coefficient of mean reversion, it is shown that for a given change in the implied volatility of a short maturity option, there should be a smaller change in the implied volatility of a longer maturity option. Stein also conducts tests of his model using a sample of S&P 100 (OEX) options and concludes that long-term volatilities overreact to changes in short-term volatilities in a manner that is inconsistent with the predictions of his model.

This article identifies some problems associated with the above mentioned empirical tests, and conducts an alternate set of tests. The results provide no indication of overreaction in the market for index options.

The following section provides a summary of Stein's mean reverting volatility model and tests. The third section contains an alternate set of tests of the overreactions hypothesis and a description of the data used in this study, and the fourth section presents the results of the tests. The final section contains conclusions and some suggestions for further research.

## THE MEAN REVERTING VOLATILITY MODEL

Under the assumption that implied volatility is equal to the average instantaneous volatility over the life of the option and the null hypothesis that instantaneous volatility is reverting at a constant rate to a constant long-run mean volatility ($\overline{\sigma}$), the following elasticity relationship can be derived:[1]

$$\left[i_t^d - \overline{\sigma}\right] = \frac{m_t^n\left[\rho_D^{m_t^d} - 1\right]}{m_t^d\left[\rho_D^{m_t^n} - 1\right]}\left[i_t^n - \overline{\sigma}\right] \qquad (1)$$

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[1]The reader is referred to Stein (1989) for details of the derivation. Equation (1) is simply Stein's eq. (4) with time measured in days.

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where $i_t^n$ is the implied volatility of a short maturity option at time $t$ with $m_t^n$ days left to maturity, $i_t^d$ is the implied volatility of a longer maturity option at time $t$ with $m_t^d$ days left to maturity, and $\rho_D$ is the first order (daily) autocorrelation coefficient of implied volatility. Assuming a constant difference ($\Delta m$) between the maturities of the two options, eq. (1) may be expressed as:

$$[i_t^d - \overline{\sigma}] = \beta(m_t^n, \rho_D)[i_t^n - \overline{\sigma}] \tag{2}$$

where:

$$\beta(m_t^n, \rho_D) = \frac{m_t^n\left[\rho_D^{m_t^{n+\Delta m}} - 1\right]}{m_t^{n+\Delta m}\left[\rho_D^{m_t^n} - 1\right]} \tag{3}$$

Using weekly autocorrelations, Stein calculates "theoretical" values of $\beta$ under the null hypothesis for a range of values of the autocorrelation coefficient, states that the elasticity relation can then be tested by comparing the theoretical $\beta$ to the slope coefficients obtained from regressing $[i_t^d - \overline{\sigma}]$ on $[i_t^n - \overline{\sigma}]$, and presents a table of regressions of $i_t^d$ on $i_t^n$. After comparing the estimated betas from the regressions with the estimates under the null hypothesis he concludes that "... long-maturity options tend to 'overreact' to changes in the implied volatility of short-maturity options."

### Methodological Caveats

A closer examination of this methodology reveals some caveats. Regressing $[i_t^d - \overline{\sigma}]$ on $[i_t^n - \overline{\sigma}]$ is, in principle, not possible unless the unobservable long run mean volatility $\overline{\sigma}$ is known. However, under the null hypothesis, eq. (2) implies the following model:

$$i_t^d = [1 - \beta(m_t^n, \rho_D)]\overline{\sigma} + \beta(m_t^n, \rho_D)i_t^n \tag{4}$$

Assuming that $m_t^n = m$, after reparameterizing and simplifying the notation, eq. (4) can be rewritten as:[2]

$$i_t^d = \alpha + \beta i_t^n \tag{5}$$

Equation (5) forms the basis for Stein's empirical model which can be written in empirical form as:[3]

$$i_t^d = \alpha + \beta i_t^n + e_t \tag{6}$$

where:

$$\mathbf{E}[e_t] = 0, \qquad \mathbf{E}[e_t e_s] = 0 \quad \forall\, t \neq s$$
$$\mathbf{E}[e_t^2] = \sigma_e^2 \qquad e_t \sim iid\mathbf{N}(0, \sigma_e^2)$$

Stein estimates (6) using ordinary least squares (OLS) and corrects the standard errors of the parameter estimates for autocorrelation in the estimated residuals.[4]

This study questions whether (6) is the correct model specification. Observed serial correlation in the estimated residuals can generally be construed as a result of model misspecification. In other words, the presence of serial correlation will generally imply a model which cannot be taken to represent the data generating process. The remedy to this is a revised specification, but not re-estimation [see Hendry and Mizon (1978)]. As discussed by Granger and Newbold (1974), the form of the misspecification can be considered to be either (i) the omission of relevant variables; (ii) the inclusion

[2]To obtain a constant $\beta$, Stein assumes that $m_t^n$ and $m_t^d$ are constants.

[3]This is the model implicit in Stein's estimator selection.

[4]Although the properties of the error term $e_t$ are not stated or assumed explicitly by Stein, his use of the results in hypothesis tests confirms this error structure.

of irrelevant variables; or (iii) autocorrelated residuals. The next section outlines an alternative empirical specification of (6).

## ALTERNATIVE OVERREACTION TESTS

### Model Specification

As specified in (4), the theoretical relationship between longer and shorter implied volatility is a function of the time to maturity, $m_t^n$. As the time to maturity of both options increases, the slope in (4) increases, while the intercept decreases, implying that long and short implied volatility are more "tightly" related. Conversely, the relationship becomes "weaker" as the time to maturity of both options decrease. The empirical consequence of this theoretical regularity is that the error term in (6) will be serially correlated.

To demonstrate this point a mean reverting process for short implied volatility is simulated and a long maturity implied volatility series based on the theoretical model in (2) is generated.[5] Model (6) is estimated using the simulated data. The OLS estimates confirm that even when the theoretical model is correctly specified, the estimation of (6) produces serially correlated residuals. The second panel of Table I contains the results of estimating (6) using first differences instead of levels. The use of first differences in volatility effectively eliminates the observed serial correlation, without significantly altering the estimate of $\beta$. This is consistent with the notion that a correctly specified theoretical model can be estimated using either levels or first differences without significantly altering the estimated slope coefficients. The results of these simulations motivate an alternative empirical specification of (6):

$$i_t^d = \alpha + \beta i_t^n + e_t \tag{7}$$

where:

$$e_t = \phi e_{t-1} + u_t, \qquad \mathbf{E}[u_t] = 0$$
$$\mathbf{E}[u_t u_s] = 0 \quad \forall\, t \neq s, \qquad \mathbf{E}[u_t^2] = \sigma_u^2$$
$$u_t \sim iid\mathrm{N}(0, \sigma_u^2)$$

**Table I**
**OLS ESTIMATES OF $\beta$ USING SIMULATED IMPLIED VOLATILITY SERIES IN LEVELS AND FIRST DIFFERENCES[a]**

| | $\hat{\alpha}$ | $\hat{\beta}$ | D.W. | $\hat{\phi}$ |
|---|---|---|---|---|
| Levels | 0.076[b] | 0.550[b] | 0.55[b] | 0.74[b] |
| | (0.0004) | (0.0025) | | (0.05) |
| Differences | 0.00 | 0.564[b] | 2.17 | |
| | (0.00006) | (0.0063) | | |

[a]The estimated equation is $i_t^d = \hat{\alpha} + \hat{\beta} i_t^n + e_t$, $\hat{\phi}$ is the maximum likelihood autocorrelation coefficient for the error term. The short implied volatility series, $i_t^n$, is simulated as the following autoregressive process: $i_t^n = 0.17 + e_t(1 - 0.95B)$, where $B$ is the backshift operator, $e_t \sim N(0, \sigma_e^2)$ and $\sigma_e^2 = 0.0001$. The volatility of the middle maturity option, $i_t^d$ is generated following the theoretical model $[i_t^d - \overline{\sigma}] = \beta(m_t^n, \rho_D)[i_t^n - \overline{\sigma}]$ Standard errors are in parentheses beneath the coefficients. $N = 200$

[b]Significant at the 1% level.

[5]The parameters of the autoregressive process are chosen to approximate the process followed by the data used in this study.

The model defined in (7) is an explicit representation of the null hypothesis for the correct empirical specification. Under the hypothesis that this model is correct it can be rewritten as:

$$i_t^d = \alpha(1 - \phi) + \phi i_{t-1}^d + \beta i_t^n - \phi\beta i_{t-1}^n + u_t \tag{8}$$

The unrestricted version of (8) may be written as:

$$i_t^d = \gamma_0 + \gamma_1 i_{t-1}^d + \gamma_2 i_t^n + \gamma_3 i_{t-1}^n + u_t \tag{9}$$

and the hypothesis of first order serial correlation is only acceptable if the restriction:

$$\gamma_1\gamma_2 + \gamma_3 = 0 \tag{10}$$

is satisfied. This test is designed to assess the adequacy of Stein's empirical specification of the model. The results of Wald tests of restriction (10) that suggest deviations from the null hypothesis are presented in the next section.

An estimator that specifically takes into account the structure of (7) is the estimated generalized least squares estimator (EGLS). Although both OLS and EGLS estimators are unbiased, EGLS is more efficient.[6] Point estimates however, can be very different depending on the degree of autocorrelation $\phi$ of the error term and may lead to contradictory statistical conclusions about overreaction.[7] Estimates of the empirical elasticity coefficient $\hat{\beta}$ using the two different estimators are presented in the next section. The results clearly indicate that our conclusions about "overreaction" are contradictory. Estimating the model with OLS supports the notion that the options markets overreact. On the other hand, the EGLS results support the conclusion that the options markets slightly underreact.

Testing the null hypothesis that volatility follows a mean reverting process requires tests of hypotheses on the two parameters of the assumed underlying process: the mean reversion parameter $\rho$, and the long-run mean volatility $\overline{\sigma}$. Since both parameters are unobservable, these hypotheses cannot be tested directly. The relevant hypothesis on $\rho$ can, however, be reduced to testing for equality between a "theoretical" elasticity coefficient ($\beta$) and its empirical counterpart ($\hat{\beta}$).[8] The relevant hypothesis on the long-run mean volatility is that it is time invariant. This hypothesis has not been tested previously.

Under the assumptions that $m_t^n = m$ and that $\Delta m$ is constant over time, eq. (2) can be rewritten as:

$$i_t^d - \overline{\sigma} = \beta[i_t^n - \overline{\sigma}] \quad \forall\ t \tag{11}$$

Since the overreactions hypothesis is expressed in terms of an elasticity relationship, perhaps a more natural approach to testing the model would be to examine changes in volatility. As noted by Granger and Newbold (1974), if a "good" theory holds for levels, but is unspecific about the time-series properties of the residuals, then an equivalent theory holds for changes so that nothing is lost by model building with both levels and changes.[9] Since (11) is assumed to hold for all $t$, taking first differences yields the following testable relationship:

$$i_t^d - i_{t-1}^d = \beta[i_t^n - i_{t-1}^n] \quad \forall\ t \tag{12}$$

[6]See for example Harvey and McAvinchey (1978). Aware of this fact Stein corrects only the standard errors of his OLS estimates.

[7]It is a well known result that if one's variables are near random walks, OLS regressions in levels will result in "spurious relationships" between them. See Box and Newbold (1971), Granger and Newbold (1974), and the whole body of literature on cointegration, which is summarized in Granger (1986).

[8]In this article *theoretical* means the elasticity under the null hypothesis. This is given in eq. (3) where $\rho_D$ is replaced with $\hat{\rho}_D$.

[9]In fact, EGLS and first differences results are expected to be very close due to the high serial correlation in the residuals observed when the correctly specified model is estimated.

Based on (12), and the simulation results, a second alternative empirical model is estimated to be:[10]

$$i_t^d - i_{t-1}^d = \alpha + \beta[i_t^n - i_{t-1}^n] + e_t \tag{13}$$

where

$$e_t \sim iid\mathbf{N}(0, \sigma_e^2), \qquad \mathbf{E}[e_t] = 0$$
$$\mathbf{E}[e_t e_s] = 0 \quad \forall\, t \neq s, \qquad \mathbf{E}[e_t^2] = \sigma_e^2$$

This model has several empirical advantages. First, since the implied volatility series are highly serially correlated, by specifying the model in first differences the problem of serially correlated errors is eliminated. Under this alternative specification elasticity estimates will be invariant to the estimator chosen.[11] Second, implicit in the direct testing of eq. (7) is the assumption that the unobservable long-run mean volatility is identical to the sample mean volatility for the period. Taking first differences eliminates the unobservable long-run mean from the empirical model. Third, under the null hypothesis, $\alpha$ should be zero. A non-zero $\alpha$ would indicate that the long-run mean volatility $\overline{\sigma}$ may not be constant over time.[12] In other words, the long-run mean volatility may be subject to structural change. Note, however, that the test for structural change in long-run volatility is very conservative because it is based on a "local" trend (daily differences). These local trends are likely to be rather small and statistically insignificant unless there is a large and consistent global trend. Finally, the estimated parameters will be best linear unbiased.

The results of the estimation of eq. (13) are presented in the next section. Chow (1960) tests are also conducted to determine whether there are structural changes in the relationship between "long" and "short" maturity implied volatilities. These tests are performed to check for structural change between the overlapping and nonoverlapping subperiods, and within each of the subperiods.

## The Data

The implied volatilities used in this study are calculated from transactions data and averaged to obtain estimates of daily implied volatilities. Average bid–ask quotes are used as the market option price to avoid the negative serial correlation that would be induced by using last sale prices.[13]

Since the component stocks in the S&P 100 index do not trade simultaneously in response to new information, recorded index levels may lag behind the true index level being used by traders in the options market. If the recorded level is too low, call option implied volatilities calculated from the recorded index level will be upwardly biased, while put option implied volatilities will be biased downward. The opposite will occur

[10]Under the null, $\alpha$ is a "nuisance" parameter. Incorporating it in the regression model does not affect the properties of $\beta$.

[11]OLS and EGLS estimates of (8) are virtually identical for the data used in this study. Furthermore, given the strong serial correlation of the errors, the EGLS results and the results obtained using OLS on first differences should be consistent.

[12]To see this point consider the alternative hypothesis

$$[i_t^d - \overline{\sigma}_t] = \beta[i_t^n - \overline{\sigma}_t]$$

Taking first differences yields

$$i_t^d - i_{t-1}^d = [1 - \beta][\overline{\sigma}_t - \overline{\sigma}_{t-1}] + \beta[i_t^n - i_{t-1}^n]$$

and, $\text{sign}(\alpha) = \text{sign}(\overline{\sigma}_t - \overline{\sigma}_{t-1})$.

[13]The tests conducted in this study are repeated using trade prices for the calculation of the implied volatilities. This modification did not significantly change any of the results.

if the recorded index level is greater than the true index level.[14] Following Stein (1989), to avoid the potential biases induced by stale index levels, we use averages of put and call implied volatilities.

Option quotes, index levels, and trading volume for the period from 12/1/85–11/29/88 are from the CBOE's Market data retrieval (MDR) tapes. The original data set is composed of approximately 35 million records containing time stamped OEX last sale transactions and market maker's bid/ask quotes. Each record also contains the most recently recorded S&P 100 index level.

Bid and ask quotes on the short maturity and middle maturity put and call options for the striking price that is closest to the money at the start of the day are collected at 15-minute intervals during the period that both the index stocks and the OEX options are trading.[15] The first observation is collected 15 minutes after the New York Stock Exchange opens, and the last observation is recorded 15 minutes before the CBOE close. To avoid problems associated with asynchronous option prices, quotes recorded closest to the end of the 15-minute interval are used, and if the last available quote is not recorded within 5 minutes of the end of the interval, the observation is not used. Since significant increases in implied volatility occur during the week prior to maturity [see Day and Lewis (1988); Stoll and Whaley (1987), for example], both the short and medium maturity series are switched to the next expiration date 7 days prior to the expiration of the shortest maturity options.[16]

Averages of bid and ask discounts on maturity matched daily U.S. Treasury bill rates are used to estimate annualized risk-free rates. Daily dividends on the S&P 100 index are from a series constructed by Bob Whaley and Campbell Harvey.[17]

Since optimal early exercise cannot be categorically ruled out for both the calls and the puts in the sample, the Cox, Ross, and Rubinstein (1979) binomial American approximation is used to calculate the implied volatilities in this study. The binomial index lattice is adjusted for cash dividends on a daily basis using Roll's (1977) escrowed dividend technique. The optimality of early exercise is checked at the end of each period. Since roughly 300,000 binomial implied volatilities are calculated in this study, enormous amounts of computer time are required. To increase the efficiency of the estimation process, dividend adjusted Black–Scholes implied volatilities are used to provide a starting point for the American binomial implied volatility search. This initial implied volatility is used to calculate a 12-period binomial implied volatility accurate to three decimal places. The number of periods is increased to 25, 50, and finally 100 periods, while the error tolerance is correspondingly reduced to 0.0001, 0.00001, and finally 0.000001. This sequential procedure is a more efficient means of achieving the desired degree of accuracy than a straight forward numerical search using 100 periods.

[14]The reader is referred to Harvey and Whaley (1991c) for a more detailed discussion.

[15]The CBOE usually closes 15 minutes after the NYSE. The CBOE closed early for a period of time following the October, 1987 crash. OEX refers to the ticker symbol for the S&P100 index. Following the 1987 crash, options with ticker symbols OEX and OEZ both traded on the CBOE. The S&P100 index option is used that is closest to the money, regardless of whether it is part of the OEX or OEZ series.

[16]Other highly volatile periods are not eliminated because both the long maturity and short maturity series are affected in these periods. Expiration week effects on volatility are present in the expiring series but not on the non-expiring longer maturity series.

[17]The authors are grateful to Bob Whaley and Campbell Harvey for providing the S&P 100 dividend series used in this study and for suggesting the use of escrowed dividends to avoid the recombination problems created by the use of cash dividends in the binomial model. Details on the construction of the series are provided in Harvey and Whaley (1991a).

## RESULTS

Table II contains autocorrelations and partial autocorrelations for the short-term implied volatility series, $i_t^n$, at various lags for the full sample (12/1/85–11/29/88), the portion of the sample that overlaps Stein's sample (12/1/85–9/30/87), and the portion of the sample that does not overlap Stein's (10/1/87–11/29/88). The correlograms are generally consistent with Stein's proposed AR(1) process for implied volatility. Dickey–Fuller (1979, 1981) tests are performed to test the null hypotheses that the implied volatility series are non-stationary. The unit root hypothesis is rejected for several different random walk specifications as it was in Stein (1989).

Table III contains the results of the Wald specification tests of the alternative empirical model. The results of these tests suggest that, for the two sub-periods considered, the error structure of model (6) is misspecified, and they support the error structure of the alternative model specified in (7). While none of the $\chi^2$ test statistics are significant at the 1% level, the test statistic is significant at the 5% level for the full sample period. The apparent rejection of the proposed model for the full sample period, and the lack of rejection for each of the sub-periods, is consistent with the presence of structural change in the relationship between implied volatilities. The significant Durbin h statistic for the full sample is also consistent with structural change. Direct evidence of the presence of structural change is provided when the model is tested using a first difference specification.

The second panel of Table III contains the Wald test results for the time series simulated under the assumption that the short maturity implied volatility follows an AR(1) process, and that the long maturity implied volatility is generated by relation (2). The results of the specification tests for this constructed series indicate that if Stein's theoretical model is correctly specified, empirical model (6) is misspecified, while the alternative model (7) is correctly specified. Also, the signs and magnitudes of the coefficients are generally consistent with the estimates for the actual sample.

Table IV contains OLS and EGLS estimates of the slope coefficient ($\hat{\beta}$) of eq. (7), as well as estimates of the theoretical elasticity coefficient ($\beta$), estimated as the sample mean from eq. (3). The data sources and pricing models used here differ from those used by Stein. To facilitate comparisons with his results, estimates for 1986 are presented

**Table II**
**AUTOCORRELOGRAMS AND PARTIAL CORRELOGRAMS FOR $i_t^n$ SERIES[a]**

| | Sample Period | | | | | |
|---|---|---|---|---|---|---|
| | Full Sample | | Overlapping | | Non-Overlapping | |
| Lag in Days | $\hat{\rho}_D$ | Partial | $\hat{\rho}_D$ | Partial | $\hat{\rho}_D$ | Partial |
| 1 | 0.9402 | 0.9402 | 0.9331 | 0.9331 | 0.9207 | 0.9207 |
| 5 | 0.7227 | 0.0704 | 0.7222 | −0.0589 | 0.6288 | 0.0714 |
| 10 | 0.6176 | 0.0701 | 0.5090 | 0.0270 | 0.4991 | 0.0704 |
| 15 | 0.5418 | 0.0682 | 0.3740 | −0.0438 | 0.4489 | 0.0354 |
| 20 | 0.4759 | −0.0068 | 0.1637 | −0.0200 | 0.4067 | 0.0047 |
| 25 | 0.4471 | −0.0147 | −0.0068 | −0.0538 | 0.4103 | −0.0222 |
| 30 | 0.4084 | −0.0109 | −0.0597 | −0.0529 | 0.3600 | −0.0065 |
| 35 | 0.3732 | −0.0080 | −0.1079 | −0.0011 | 0.3325 | −0.0077 |

[a] $i_t^n$ is the implied volatility of the short maturity OEX series for day $t$. $\hat{\rho}_D$ is the autocorrelation of the series at lag D. The full period is 12/1/85–11/29/88. The sample period overlapping Stein's (1989) sample period is 12/1/85–9/29/87. The non-overlapping period is 10/1/87–11/29/88.

**Table III**
**WALD TESTS FOR $H_o : \hat{\gamma}_1\hat{\gamma}_2 + \hat{\gamma}_3 = 0$[a]**

| Sample Period | $\hat{\gamma}_1$ | $\hat{\gamma}_2$ | $\hat{\gamma}_3$ | $h$ | $\hat{\gamma}_1\hat{\gamma}_2 + \hat{\gamma}_3$ | $\chi^2$ |
|---|---|---|---|---|---|---|
| Full | 0.8473 | 0.5679 | −0.4183 | −3.36[b] | 0.0628 | 4.39[c] |
| | (0.0149) | (0.0115) | (0.0177) | | | |
| Overlapping | 0.8923 | 0.4490 | −0.3501 | −0.14 | 0.0505 | 3.76 |
| | (0.0144) | (0.0116) | (0.0153) | | | |
| Non-overlapping | 0.7379 | 0.5946 | −0.3463 | −0.98 | 0.0924 | 2.51 |
| | (0.0345) | (0.0121) | (0.0327) | | | |
| Simulated series | 0.7295 | 0.5634 | −0.4149 | 0.76 | −0.0031 | 0.005 |
| | (0.0489) | (0.0059) | (0.0277) | | | |

[a]The estimated model is $i_t^d = \gamma_0 + \gamma_1 i_{t-1}^d + \gamma_2 i_t^n + \gamma_3 i_{t-1}^n + e_t$. Where $i_t^d$ is the implied volatility of the middle maturity option on day $t$, and $i_t^n$ is the implied volatility on the short maturity option on day $t$. $\chi^2$ is the test statistic for the null hypothesis. The simulated series are generated as follows: the short implied volatility series, $i_t^n$, is simulated as the following autoregressive process: $i_t^n = 0.17 + e_t/(1 - 0.95B)$, where $B$ is the backshift operator, $e_t \sim N(0, \sigma_e^2)$ and $\sigma_e^2 = 0.0001$. The volatility of the middle maturity option, $i_t^d$ is generated following the theoretical model $(i_t^d - \overline{\sigma}) = \beta(m_t^n, \rho_D)(i_t^n - \overline{\sigma})$. $h$ is the Durbin's $h$ statistic for serial correlation when the model contains a lagged dependent variable. $h$ has a standard normal distribution under the null hypothesis. Standard errors are in parenthesis.

[b] and [c] indicate significance at the 1% and 5% levels, respectively.

All $\gamma$ coefficients are significant at the 1% level.

**Table IV**
**OLS AND EGLS ESTIMATES OF $\beta$[a]**

| | $\hat{\beta}$ | | | | |
|---|---|---|---|---|---|
| Year | OLS | EGLS | $\beta$ | $\hat{\phi}$ | $\hat{\rho}_D$ |
| 1986 | 0.7732 | 0.3971 | 0.4860 | 0.967 | 0.918 |
| | (0.0187) | (0.0194) | | (0.016) | |
| 1987 | 0.8220 | 0.4595 | 0.5456 | 0.955 | 0.941 |
| | (0.0145) | (0.0228) | | (0.011) | |

[a]The estimated equation is $i_t^d = \hat{\alpha} + \hat{\beta} i_t^n + e_t$, $\hat{\phi}$ is the maximum likelihood autocorrelation coefficient for the error term. $\beta$ is the sample mean of $m_t^n(\hat{\rho}_D^{m_t^{n+\Delta m}} - 1)/m_t^{n+\Delta m}(\hat{\rho}_D^{m_t^n} - 1)$, where $\hat{\rho}_D$ is the estimated first-order autocorrelation coefficient of the short-term implied volatility series, $m_t^n$ is the number of days to maturity for the short maturity option, and $m_t^{n+\Delta m}$ is the number of days to maturity for the middle maturity option. The 1987 sample is from 1/1/87 to 9/29/87 to allow direct comparison with Stein's results. Standard errors are in parenthesis.

All coefficients are significant at the 1% level.

as well as for the 9 months he uses for 1987. A comparison of the OLS regression coefficients ($\hat{\beta}$) to the "theoretical" elasticities ($\beta$) provides evidence of overreaction in both years which is consistent with Stein's findings. The OLS betas are not significantly different from the ones Stein estimates. The EGLS estimates, however, support a different hypothesis. In both years the EGLS estimates are lower than the "theoretical" elasticities. Assuming that the theoretical model is correctly specified, the EGLS results suggest the presence of underreaction.

Since the OLS and EGLS results differ so dramatically, it is necessary to consider the reason for these differences. While both estimators are unbiased in the case of a correctly specified model, specification errors can result in point estimates that are significantly different. Using transactions data, Harvey and Whaley (1991c) document the presence of negative serial correlation in S&P 100 short maturity implied volatility changes. The presence of negative serial correlation in first differences is indicative of a moving average (MA) component in the short maturity implied volatility series. Furthermore, Diz and Finucane (1991) show that outliers in the volatility series can induce a moving average component. They show also that while short maturity S&P 100 implied volatility series appear to contain a strong MA component, longer maturity series do not. Because of the high vega ($\delta V/\delta\sigma$) of the short maturity at-the-money options, the short maturity implied volatility series is more sensitive to measurement errors in the option and underlying asset prices than the long maturity series. Outliers caused by measurement errors and volatility shocks have a much stronger effect upon the short maturity series than the long maturity series.

If the short maturity series contains a moving average component induced by the presence of these outliers, OLS and EGLS estimates based upon the assumption of AR(1) processes for both series can result in significantly different parameter estimates. To illustrate this point an AR(1) process is simulated for the long maturity implied volatility series using relation (2) and an AR(1) process for the short maturity volatility $[i_t^n = 0.17 + 1/(1 - 0.95B)]$. A new short maturity series is constructed from the same simulated errors that are used to generate the original short maturity series, but a moving average component is introduced into the new series $i_t^{n*}$, $(i_t^{n*} = 0.17 + \theta e_t/(1 - 0.95B))$. Then both the OLS and EGLS models are used in an attempt to recover the theoretical $\beta$ of .55 that is assumed in the construction of the series. The results are presented in Table V. Using EGLS the coefficient estimates are not significantly different from the theoretical value of .55, while the OLS estimates are inversely related to the

**Table V**

**OLS AND EGLS ESTIMATES OF $\beta$ USING SIMULATED IMPLIED VOLATILITY SERIES UNDER ALTERNATIVE ARMA PROCESSES FOR SHORT VOLATILITY[a]**

| | $\hat{\beta}$ | | |
|---|---|---|---|
| $\theta$ | OLS | EGLS | $\hat{\phi}$ |
| 0 | 0.550 | 0.557 | 0.74 |
| | (0.0025) | (0.0047) | (0.0485) |
| −0.15 | 0.629 | 0.542 | 0.88 |
| | (0.0044) | (0.0091) | (0.0336) |
| −0.20 | 0.663 | 0.532 | 0.913 |
| | (0.0053) | (0.0084) | (0.0294) |
| −0.25 | 0.714 | 0.536 | 0.958 |
| | (0.0058) | (0.0113) | (0.0223) |
| $\beta$ under null: 0.55 | | | |

[a]The estimated equation is $i_t^d = \hat{\alpha} + \hat{\beta} i_t^{n*}$. The short implied volatility series, $i_t^{n*}$, is simulated as the following general ARMA(1, 1) process: $i_t^{n*} = 0.17 + (\theta e_t^*)/(1 - 0.95B)$, where $\theta$ is a moving average parameter, $B$ is the backshift operator, $e_t^* \sim N(0, \sigma_e^{*2})$ and $\sigma_e^{*2} = 0.0001$. The volatility of the middle maturity option, $i_t^d$ is generated following the theoretical model $[i_t^d - \overline{\sigma}] = \beta(m_t^n, \rho_D)[i_t^n - \overline{\sigma}]$ where $i_t^n$ follows the following AR(1) process: $i_t^n = 0.17 + (e_t)/(1 - 0.95B)$, and $e_t^* = e_t$. $\hat{\phi}$ is the maximum likelihood autocorrelation coefficient for the error term. Standard errors are in parentheses beneath the coefficients. $N = 200$.

All coefficients are significant at the 1% level.

moving average parameter $\theta$. For non-zero but reasonable values of theta, the OLS estimates are substantially larger than the EGLS estimates.[18]

Table VI contains the results of the maximum likelihood estimation of the alternative empirical model in first differences. The estimated $\beta$ coefficients are not significantly different from the EGLS estimates of $\beta$ ($\hat{\gamma}_2$) presented in Table III. Furthermore, for both the overlapping and non-overlapping sub-periods, $\hat{\alpha}$ is not significantly different from zero. This result is consistent with a constant long-run mean volatility for both sub-periods.

Evidence of structural change in the relationship between shorter and longer maturity implied volatilities is contained in the results of the Chow tests for structural change. Significant structural differences in the estimated models are found for the full sample. These results indicate that the relationship between short and medium implied volatility may change over time. Empirical evidence concerning overreaction is considered next.

The thrust of Stein's empirical argument depends upon the comparison of the "theoretical" elasticity coefficient, calculated from the serial correlation coefficients of the implied volatility series, with the indirect estimates of this coefficient provided by the regression $\hat{\beta}$. Stein finds that his regression estimates of $\hat{\beta}$ are considerably higher than the elasticity coefficients, and concludes that the market overreacts. Estimating the model in first differences [eq. (13)] produces very different results.

Table VII presents point and confidence interval estimates for both the elasticity coefficients ($\beta$), estimated as the sample mean value of $\beta$ from eq. (3), and the regression coefficients ($\hat{\beta}$) from the maximum likelihood estimation of eq. (12). The results in Table VII do not support overreaction. None of the regression coefficients are significantly larger than the elasticity coefficients. For the non-overlapping subperiod, the elasticity and regression coefficients are not significantly different. For the overlapping sample, the regression coefficient is actually significantly lower than the elasticity coefficient, which appears to indicate a slight underreaction.

**Table VI**
**MAXIMUM LIKELIHOOD REGRESSION OF DAILY CHANGES IN MIDDLE MATURITY IMPLIED VOLATILITIES ON DAILY CHANGES IN SHORT MATURITY IMPLIED VOLATILITIES AND AN INTERCEPT TERM**[a]

| Sample Period | $\hat{\alpha}$ | $\hat{\beta}$ | $\overline{R}^2$ | D.W. | Chow Test $F$ Statistic |
|---|---|---|---|---|---|
| Full | −0.00009 | 0.5574 | 0.745 | 2.34 | 28.97[b] |
| | (0.00014) | (0.0122)[b] | | | |
| Overlapping | 0.00008 | 0.4329 | 0.629 | 2.16 | 0.99 |
| | (0.00014) | (0.0156)[b] | | | |
| Non-overlapping | −0.00032 | 0.5710 | 0.704 | 2.33 | 1.29 |
| | (0.00038) | (0.0222)[b] | | | |

[a]The estimated equation is $i_t^d - i_{t-1}^d = \hat{\alpha} + \hat{\beta}(i_t^n - i_{t-1}^n)$ where $i_t^d$ is the implied volatility of the middle maturity option on day $t$, and $i_t^n$ is the implied volatility on the short maturity option on day $t$. $\overline{R}^2$ is the adjusted coefficient of determination, D.W. is the Durbin–Watson statistic, and standard errors are in parentheses. The full sample period is 12/1/85–11/29/88. The sample period overlapping Stein's (1989) sample period is 10/1/85–9/30/87. The non-overlapping period is 10/1/87–11/29/88.

[b]Significant at the 1% level.

[18]Estimating the model in first differences provides results that are virtually identical to the estimates obtained using EGLS for the simulated series.

**Table VII**
**POINT AND CONFIDENCE INTERVAL ESTIMATES OF THE "THEORETICAL" ELASTICITY COEFFICIENTS ($\beta$) AND MAXIMUM LIKELIHOOD REGRESSION COEFFICIENTS ($\hat{\beta}$)[a]**

| Sample Period | $\beta$ | $\hat{\beta}$ |
|---|---|---|
| Full<br>$\hat{\rho}_1 = 0.94$ | 0.54<br>(0.51, 0.57) | 0.56<br>(0.53, 0.58) |
| Overlapping<br>$\hat{\rho}_1 = 0.93$ | 0.51<br>(0.48, 0.55) | 0.43<br>(0.40, 0.47) |
| Non-overlapping<br>$\hat{\rho}_1 = 0.92$ | 0.49<br>(0.45, 0.54) | 0.57<br>(0.52, 0.62) |

[a]Reported $\beta$ is the sample mean of $m_t^n(\rho_1^{m_t^{n+\Delta m}} - 1)/m_t^{n+\Delta m}(\hat{\rho}_1^{m_t^n} - 1)$, where $\hat{\rho}_1$ is the estimated daily first-order autocorrelation coefficient of the short-term implied volatility series, $m_t^n$ is the number of days to maturity for the short maturity option, and $m_t^{n+\Delta m}$ is the number of days to maturity for the middle maturity option. $\hat{\beta}$ is the slope coefficient from the maximum likelihood regression estimates provided in Table VI. The full sample period is 12/1/85–11/29/88. The overlapping sample period is 12/1/85–9/30/87, and the non-overlapping period is 10/1/87–11/29/88. Confidence intervals for $\beta$ are based on Fisher's Z transform ($\alpha : 0.01$), and confidence intervals for $\hat{\beta}$ are based on $t_{\infty,\alpha=0.01}$. Confidence intervals are in parenthesis.

Since the evidence concerning overreaction differs so dramatically from Stein's, the reasons for such different conclusions deserve additional consideration. One explanation might be that measurement error, introduced through reporting errors, asynchronous data, or misspecification of the pricing model, might create a larger downward bias when the regressions are done in first differences than when they are done in levels. This explanation, however, seems doubtful.

First, there is no reason to conclude that measurement error induces a downward bias in the slope coefficients. Unless the measurement errors of the dependent and independent variables are uncorrelated, $\beta$ may be underestimated or overestimated.[19] Stein argues that his tests are biased toward underreaction. But, if the measurement errors are correlated, the tests may in fact be biased toward overreaction.

If one accepts the assumption of uncorrelated measurement errors, the degree of the downward bias in the slope coefficient will be an increasing function of the ratio of the variance of the measurement error of the independent variable to the variance of the independent variable. Since the variance of the independent variable is larger when levels are used, the bias may be stronger in the regressions on first differences that are used here. If this is true, taking differences over longer periods should increase the variance of the independent variable, decrease the size of the bias, and provide results that are closer to those obtained using levels.[20] To examine this possibility, eq. (12) is re-estimated using weekly, rather than daily, differences. To avoid the problems induced by overlapping observations, a weekly series of volatilities is constructed, arbitrarily selecting each Thursday from the daily series. The results obtained using the first differences from this weekly series are presented in Table VIII.[21] Contrary to the hypothesis that measurement

[19]Maddala (1977, p. 302) demonstrates that in the presence of correlated measurement errors, either under or overestimation of $\beta$ may occur. Correlated measurement errors would be expected for a number reasons, including asynchronous index levels, and the possibility that market option prices reflect information other than past and present volatility.

[20]The variance of the measurement error of the independent variable would not be expected to change.

[21]Very similar results for the weekly differences are obtained when other days of the week are used to form the weekly series.

**Table VIII**
**MAXIMUM LIKELIHOOD REGRESSION OF WEEKLY CHANGES IN MIDDLE MATURITY IMPLIED VOLATILITIES ON WEEKLY CHANGES IN SHORT MATURITY IMPLIED VOLATILITIES AND AN INTERCEPT TERM[a]**

| Sample Period | $\hat{\alpha}$ | $\hat{\beta}$ | $\bar{R}^2$ | D.W. |
|---|---|---|---|---|
| Full | −0.00031 | 0.6821 | 0.852 | 2.50 |
| | (0.0005) | (0.0236)[b] | | |
| Overlapping | 0.00033 | 0.6005 | 0.821 | 2.21 |
| | (0.0006) | (0.0290)[b] | | |
| Non-overlapping | −0.00099 | 0.7409 | 0.881 | 2.58[b] |
| | (0.0011) | (0.0384)[b] | | |

[a]The estimated equation is $i_t^d - i_{t-1}^d = \hat{\alpha} + \hat{\beta}(i_t^n - i_{t-1}^n)$ where $i_t^d$ is the implied volatility of the middle maturity option for **week** $t$, and $i_t^n$ is the implied volatility of the short maturity option for **week** $t$. $\bar{R}^2$ is the adjusted coefficient of determination. Standard errors are in parentheses. The full sample period is 12/1/85–11/29/88. The sample period overlapping Stein's (1989) sample period is 10/1/85–9/30/87. The non-overlapping period is 10/1/87–11/29/88.

[b]Significant at the 1% level.

error is responsible for the difference between the results using levels and first differences, the results using weekly differences are consistent with the results presented in Table VI for daily differences. None of the regression coefficients are significantly different from the elasticity coefficients as shown in Table IX.

## SUMMARY AND CONCLUSIONS

This study reexamines Stein's finding that the implied volatilities of long-term S&P 100 index options overreact to changes in short-term volatilities. Using an alternative specification for the empirical model, no evidence of overreaction is found. For a

**Table IX**
**POINT AND CONFIDENCE INTERVAL ESTIMATES OF THE "THEORETICAL" ELASTICITY COEFFICIENTS ($\beta$) AND MAXIMUM LIKELIHOOD REGRESSION COEFFICIENTS ($\hat{\beta}$) USING WEEKLY DATA[a]**

| Sample Period | $\beta$ | $\hat{\beta}$ |
|---|---|---|
| Full | 0.74 | 0.68 |
| $\hat{\rho}_W = 0.85$ | (0.67, 0.80) | (0.53, 0.74) |
| Overlapping | 0.62 | 0.60 |
| $\hat{\rho}_W = 0.74$ | (0.52, 0.72) | (0.40, 0.67) |
| Non-overlapping | 0.70 | 0.74 |
| $\hat{\rho}_W = 0.82$ | (0.57, 0.81) | (0.65, 0.83) |

[a]Reported $\beta$ is the sample mean of $m_t^n(\rho_W^{m_t^{n+\Delta m}} - 1)/m_t^{n+\Delta m}(\hat{\rho}_W^{m_t^n} - 1)$, where $\hat{\rho}_W$ is the estimated weekly (every Thursday) first-order autocorrelation coefficient of the short-term implied volatility series, $m_t^n$ is the number of weeks to maturity for the short maturity option, and $m_t^{n+\Delta m}$ is the number of weeks to maturity for the middle maturity option. $\hat{\beta}$ is the slope coefficient from the maximum likelihood regression estimates provided in Table VI. The full sample period is 12/1/85–11/29/88. The overlapping sample period is 12/1/85–9/30/87, and the non-overlapping period is 10/1/87–11/29/88. Confidence intervals for $\beta$ are based on Fisher's Z transform ($\alpha : 0.01$), and confidence intervals for $\hat{\beta}$ are based on $t_{\infty,\alpha=0.01}$. Confidence intervals are in parenthesis.

subperiod that overlaps the sample period examined by Stein, evidence of underreaction is found. No evidence of under or overreaction is found for the non-overlapping sample period. The results clearly reject Stein's conclusion that the implied volatilities of longer maturity options move almost in lockstep with the implied volatilities of shorter maturity options.

The results also suggest that structural change in the relationship between long and short maturity implied volatilities is present for the sample examined. Most importantly, the tests indicate that a simple mean reverting model of implied volatility may not adequately characterize the time series properties of implied volatility.

Stein argues that there is one clear source of information concerning future volatility: the implied volatility of the short maturity option. Given the short-term implied volatility, he then proposes a simple functional relationship for the implied volatilities of longer maturity options. In essence, he is proposing a single factor model of the "term structure" of volatilities, where longer maturity implied volatilities contain no information concerning expected future volatilities that is not already impounded in short-term implied volatilities. The results of this study suggest that short-term implied volatilities are not the only relevant source of information.

Future research needs to identify the factors that are relevant to the formation of expected volatilities, and, by extension, to implied volatilities. While very little work has been done in this area, Harvey and Whaley (1991b) identify several factors which, in addition to lagged implied volatilities, appear to be significant determinants of implied volatilities for short maturity index options. A potentially promising area for future research would involve an examination of the determinants of longer maturity implied volatility and of the term structure of implied volatility.

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