The Distribution of Standardized Futures Price Changes

Meenakshi Venkateswaran
B. Wade Brorsen
Joyce A. Hall

INTRODUCTION

Knowledge of the underlying distribution of futures price changes is required in several applications in the fields of finance and marketing, such as hypothesis testing, option pricing models, and risk management studies. For instance, portfolio analysis based on the assumption that asset returns have a constant finite variance may give misleading results if this assumption is false. Also, option pricing models are often derived assuming returns are distributed normally with constant variance. Yet, research on the distribution of futures and stock price changes indicate that price changes are not distributed normally, but are leptokurtic; i.e., they are characterized by a higher peak and thicker tails than the normal distribution [Cornew et al. (1984); Fama (1963); Hudson et al. (1987); Mandelbrot (1963); Officer (1972)].

Fama (1963) and Mandelbrot (1963) first suggested the family of stable paretian distributions to describe the distribution of stock and futures price changes.[1] In this family, if the variance is finite, the sum converges in a normal distribution. All non-normal distributions have infinite variance. Other research has proposed alternatives to the stable distribution that explains the observed leptokurtosis yet maintains finite variance. Much of this research has described returns as mixed distributions using subordinated stochastic processes, with returns generated independent of past returns. For example, Clark (1973) describes the distribution of returns to be subordinate to a normal distribution conditioned on its variance. Similar tests of subordinated stochastic processes of prices and trading volume are presented by Epps and Epps (1976), Harris (1987), Morgan (1976), and Westerfield (1977). Other subordinated stochastic processes include Press (1967) compound events model and the students' t-distribution proposed by Blattberg and Gonedes (1974) and Praetz (1972).

[1]Gribbin et al. (1992) argue that futures prices are not stable paretian distributed. But, Liu and Brorsen (1992) argue that Gribbin et al. have shown that futures prices are not identically independently stably distributed, but that a stable distribution with time varying scale parameter cannot be rejected using Gribbin et al.'s approach.

Meenakshi Venkateswaran is a Post-Doctoral Research Fellow, Department of Agricultural Economics and Rural Sociology, Auburn University.

B. Wade Brorsen is an Associate Professor, Department of Agricultural Economics, Oklahoma State University.

Joyce A. Hall is an Assistant Professor, Department of Agricultural and Resource Economics, Colorado State University.

The Journal of Futures Markets, Vol. 13, No. 3, 279–298 (1993)

CCC 0270-7314/93/030279-20

Several researchers have attributed the observed non-normality explicitly to non-constant variance. For example, tests using the stability under addition property of stable distributions performed for stock returns [Fielitz and Rozelle (1983)] and futures returns [Hall et al. (1989)] suggest that price changes are not independent and the most likely reason is serially correlated variances. Similar results are reported by Perry (1983), using tests for successfully larger sample variances of securities. Likewise, Bollerslev (1987) and Taylor (1986) suggest that adjacent price changes are not independent. Much of the above research fails to take into account the conditional dependence in the second moments [Bollerslev (1987)]. Several models have been advanced which recognize this dependence and allow for changing conditional variance as a function of past returns. These include the autoregressive conditional heteroskedastic (ARCH) [Engle (1983)], generalized autoregressive conditional heteroskedastic (GARCH) [Bollerslev (1986)] and the exponentially weighted moving average (EWMA) [Taylor (1986)] models.

If the departure of futures price distribution from the normal distribution is entirely due to changing conditional variance, then correction for the heteroskedasticity will result in a normal distribution and thus permit the use of familiar statistical tools. Taylor (1986) suggested that standardized (or rescaled) returns would possess a reasonably homogenous variance. Previous research with futures prices have considered rescaling with ARCH or GARCH models [Brorsen and Yang (1989); Fujihara and Park (1990); Randolph and Najand (1991); Baldauf and Santoni (1991); Yang and Brorsen (1992)]. This past research has generally found that GARCH models can be rejected as a model of futures prices.

This article standardizes futures returns using the EWMA model suggested by Taylor (1986) and tests the standardized returns and the raw returns for convergence to a normal distribution. Taylor's EWMA model has been ignored in recent research. The data for the study consist of daily futures prices for 31 commodities including agricultural commodities, livestock, metals, and financial instruments. For both the raw and rescaled returns, the test is performed on the entire data set and also on a subset (1981–1988) of the data. If the departure of daily futures returns from normality is due only to changing variances, and the EWMA model is able to correctly forecast the conditional variance, then correction for heteroskedasticity using the forecasted conditional variance will result in a normal distribution. If the rescaled returns are still non-normal, then alternative models for forecasting conditional variance and additional causes of the observed leptokurtosis must be considered.

DATA AND PROCEDURE

The data used in this study are the first differences of natural logarithms of the daily closing futures prices which may be interpreted as the returns to futures contracts held for a day under continuous compounding. The data set has 31 commodities including agricultural commodities, livestock, metals, and financial instruments. The number of years of data available vary with the commodity, but data for each commodity are available through 1988 (Table I). To maintain continuity in the data and minimize differences in maturity, the log changes in closing prices are used from the contract with the nearest delivery month, until the third Tuesday of the month prior to delivery, then the next delivery month is used. This procedure is repeated as each nearby contract approaches delivery. The differences are taken before linking the contracts together.

Taylor (1986) suggests that rescaled daily returns are more likely to be independently and identically distributed than the actual daily returns itself, and thus have a reasonably

Table I
COMMODITIES INCLUDED, TIME PERIOD, AND NUMBER OF OBSERVATIONS

Commodities	Period	Number of Observations: All Data	Number of Observations: 1981–1988
I. Agricultural commodities			
Cocoa	1960–88	7229	2009
Coffee	1979–88	2510	2007
Corn	1960–88	7302	2022
Lumber	1974–88	3794	2020
Oats	1960–88	7299	2020
Orange juice	1968–88	5269	2010
Soybean meal	1960–88	7300	2022
Soybean oil	1960–88	7300	2022
Soybeans	1960–88	7315	2021
Sugar	1960–88	6872	2007
Wheat (Chicago)	1960–88	7302	2022
II. Livestock			
Feeder cattle	1974–88	3783	2022
Live cattle	1966–88	6044	2023
Live hogs	1970–88	4786	2021
Pork bellies	1965–88	5790	2021
III. Metals			
Copper	1960–88	7246	2015
Gold	1975–88	3437	2017
Palladium	1977–88	2968	1997
Platinum	1969–88	5151	2005
Silver	1965–88	6240	2010
IV. Financial instruments			
British pound	1977–88	3026	2022
Canadian dollar	1977–88	3028	2022
Deutsche mark	1977–88	3028	2022
European dollar	1982–88	1768	1768
Japanese yen	1977–88	3027	2021
Swiss franc	1977–88	3029	2023
Treasury bills	1976–88	3251	2021
Treasury bonds	1978–88	2814	2022
Treasury notes	1983–88	1679	1679
V. Miscellaneous			
Crude oil	1984–88	1422	1422
Heating oil	1980–88	2259	2005

homogenous variance.[2] The rescaled series $\{U_t\}$ is given by

$$u_t = \frac{(x_t - \mu)}{v_t} \tag{1}$$

[2]For a more detailed description of the procedure, readers are referred to Taylor (1986), particularly chapters 4 and 5. Readers may find it worthwhile to note that the notation used here differs slightly from that of Taylor; i.e., θ is used for δ, and ρ is used for θ.

where u_t is the true rescaled return, x_t is the actual return on day t, μ is the population mean over the entire period, and v_t is the actual conditional standard deviation of the return on day t. In (1), u_t is a strict white noise distributed normally with zero mean and unit variance, and is independent of v_t and all x_{t-j}, for j strictly greater than zero. As v_t and μ are not observable, u_t is approximated by

$$y_t = \frac{(x_t - \bar{x})}{\hat{v}_t} \tag{2}$$

where $\bar{x}$ is the sample mean and $\hat{v}_t$ is an estimate of the actual conditional standard deviation on day t. The approximated rescaled return will be similar to the true rescaled return if $\hat{v}_t$ is a good forecast.

Noting that one cannot observe the forecast v_{t+1} at time t or calculate the forecast errors $(v_{t+1} - \hat{v}_{t+1})$, Taylor (1986) defines an alternate forecast m_{t+1} of an observable series as

$$m_{t+1} = |x_{t+1} - \mu| = v_{t+1}|u_{t+1}| \tag{3}$$

and lets $\hat{m}_{t+1}$ provide information about $\hat{v}_{t+1}$. Drawing upon the classical result in statistical forecasting which states that optimal forecasts are expected values conditional upon the available information I_t, (3) can be written as

$$E(m_{t+1} \mid I_t) = E(v_{t+1} \mid I_t) \bullet E(|u_{t+1}| \mid I_t) \tag{4}$$

The expected value of $|u_{t+1}|$ in (4) is its unconditional mean, θ, whatever the available information I_t. Equation (4) can thus be rewritten as

$$E(m_{t+1} \mid I_t) = \theta \bullet E(v_{t+1} \mid I_t) \tag{5}$$

Since u_t is assumed to be distributed normally, $\theta = 0.798$. The best forecast of v_{t+1} can thus be indirectly obtained using forecasts of m_{t+1} and using the relation

$$\hat{v}_{t+1} = \frac{\hat{m}_{t+1}}{\theta} \tag{6}$$

Taylor compared the forecasts of m_{t+1} for each of 40 spot and futures series derived using six different forecasting methods. Five of these forecasts, namely, the mean, two versions of product process forecasts having log-normal, AR(1) conditional standard deviation and two ARMACH models were obtained using the stationarity assumption. The sixth method, the exponentially weighted moving average (EWMA) model, allowed non-stationarity of the returns generating process.

Taylor (1986) found the forecasts from the EWMA model to be marginally better than those obtained using the other five methods. In addition to this, the EWMA model has two other advantages. First, it requires fewer parameters to be estimated compared to the other methods. Second, the model does not require the stationarity assumption. Therefore, if non-stationarity is introduced due to the changing unconditional variance, the forecasts obtained using this model can accurately reflect this change. Hence, Taylor (1986) recommended that v_{t+1} be determined using the forecast of m_{t+1} which was obtained by using an exponentially weighted moving average of the present and past absolute returns. In other words, he used an EWMA model. Following Taylor (1986), (6) can therefore be rewritten as

$$\hat{v}_{t+1} = \rho \sum_{i=0}^{\infty} \frac{1}{\theta}(1 - \rho)^i |x_{t-1} - \bar{x}| \tag{7}$$

$$= (1 - \rho)\hat{v}_t + \left(\frac{1}{\theta}\right)\rho|x_t - \bar{x}| \tag{8}$$

where ρ is a smoothing parameter. Taylor (1986) derived an initial value for $\hat{v}_t$ as

$$\hat{v}_t = \left(\frac{1}{\theta}\right)\sum_{t=1}^{20} \frac{|x_t - \bar{x}|}{20} \tag{9}$$

The $\hat{v}_t$ in (9) is the conditional standard deviation of the 21st day. The forecast of conditional standard deviation from the 22nd day onwards is given by (8). θ is 0.798 as defined earlier.

Taylor (1986) derived the best back-optimized estimate for ρ for each complete series by minimizing the criterion,

$$\sum_{t=21}^{n-1} (m_{t+1} - \hat{m}_{t+1})^2 \tag{10}$$

where $\hat{m}_{t+1}$ is a function of ρ. For the futures series in this study, the estimated ρ ranges from 0.04 in the case of treasury bonds to 0.19 in the case of coffee. Based on the estimated values and the realization that the best past ρ is not necessarily the best ρ for future forecasts, Taylor (1986) recommended $\rho = 0.1$ for all futures series. The average Root Mean Square Error (RMSE) in Taylor's study, showed that setting $\rho = 0.1$ for all futures series was as accurate as seeking an optimal ρ for each series. In the present study, the characteristic exponents, the α's, are obtained first by setting $\rho = 0.1$ as recommended by Taylor. Following this, ρ is estimated individually for each commodity and the α's recomputed using the estimated values of ρ. Results obtained in the two cases are then compared.

The raw data series and the rescaled data using (i) $\rho = 0.1$ and (ii) using the value of ρ estimated individually for each commodity, are tested for convergence to normality using the stability under addition property of stable distributions.[3] The stable distributions are characterized by four parameters, namely, α, β, γ and δ. The parameter α is the characteristic exponent of the stable distribution and determines the rate at which the tails of the distribution taper off [McCulloch (1986)]; β is the skewness or symmetry parameter of the distribution. γ or c, where $\gamma = c^\alpha$ is the scale parameter of the distribution and δ is a location parameter. The ranges for these parameters are $0 < \alpha \leq 2$, $-1 \leq \beta \leq 1$, $\gamma > 0$, and $-\infty < \delta < \infty$ [Fama (1963)]. When $\alpha = 2.0$, the resulting distribution is normal, with mean δ and variance $2c^2$. When $1 < \alpha < 2$, the variance is infinite, but the mean of the distribution is still equal to δ. However, when $\alpha \leq 1$, the tail(s) are so heavy that even the mean does not exist [McCulloch (1986)].

Stable distributions are invariant under addition. In other words, the distribution of sums of independent observations from an identical stable distribution will also be stable with the same value of α and β. This is referred to as the stability under addition property of stable distributions. The convergence to normality can be tested by estimating α for the entire sample and for non-overlapping sums from the sample. From the central limit theorem, if $\tilde{x}_{it}$ is the return for commodity i on day t, and $\tilde{x}_{i1}, \tilde{x}_{i2}, \ldots, \tilde{x}_{iT}$ is a sequence of independent random variables, then the distribution of the sum

$$S_i = \sum_{t=1}^{T} \tilde{x}_{it} \tag{11}$$

will converge to a normal distribution as T tends to ∞ [Feller (1958)].

The test for convergence is performed on the entire data set and also on a subset of the data (1981–1988). Under this test, the individual adjacent returns in each series is

[3]Since $\hat{v}_t$ is available only from the 21st day onwards, the rescaled series have 20 observations less than the original series.

summed into non-overlapping groups of 1, 5, 10, and 20 observations. Sum size one is the initial rescaled data series and represents one-day continuously compounded returns, and so on.

If the underlying distribution is stable and normal, $\hat{\alpha}$ will be equal to two. However, if the distribution is stable with infinite variance, the estimated value of α will not converge to normality; $\hat{\alpha}$ will be less than two and remain relatively stable over the sums. On the other hand, if the underlying distribution is a mixture of distributions with finite variances, the $\hat{\alpha}$'s will converge to normality (i.e., $\hat{\alpha} \rightarrow 2.0$) as the sum size increases [Fama and Roll (1971)]. The stability under addition test is valid only for independent random variables. Since the raw data series are not expected to be independent due to serially correlated variances, the raw series may not converge to normality even if the underlying distribution possesses a finite variance. However, if the rescaling results in an independent data series, the resulting series will converge to normality.

Most prior empirical studies have obtained the parameters of the stable distribution by using Fama and Roll's (1968, 1971) methodology. In this method, simple functions of predetermined order statistics are used to obtain consistent estimates of δ and almost consistent (i.e., with at most a small asymptotic bias) estimates of α and γ. However, this method is restricted to the symmetrical case, i.e., where $\beta = 0$, and is further restricted to α values in the range $[1,2]$.

McCulloch (1986) provides a generalization of the Fama and Roll (1971) approach to provide consistent estimators of all the four parameters, with β in its full permissible range $[-1,1]$, and α in the range $[0.5,2]$. This method eliminates the small asymptotic bias in the Fama and Roll (1971) estimators of α and β and at the same time relaxes their restrictions on α and β. The maximum likelihood approach of Brorsen and Yang (1990) is not used because it is restricted to symmetric distributions and is considerably more difficult to compute.

Let x_p be the pth population quantile, so that $S(x_p;\alpha,\beta,\gamma,\delta) = p$. Let $\hat{x}_p$ be the corresponding sample quantile, suitably corrected for continuity.[4] McCulloch (1986) maintains that $\hat{x}_p$ is a consistent estimator of x_p. McCulloch (1986) then defines an index v_α to be

$$v_\alpha = \frac{x_{0.95} - x_{0.05}}{x_{0.75} - x_{0.25}} \tag{12}$$

Let $\hat{v}_\alpha$ be the corresponding sample value, i.e.,

$$\hat{v}_\alpha = \frac{\hat{x}_{0.95} - \hat{x}_{0.05}}{\hat{x}_{0.75} - \hat{x}_{0.25}} \tag{13}$$

where $\hat{v}_\alpha$ is a consistent estimator of the index v_α. McCulloch (1986) defines another index v_β to be

$$v_\beta = \frac{x_{0.95} + x_{0.05} - 2x_{0.5}}{x_{0.95} - x_{0.05}} \tag{14}$$

and lets $\hat{v}_\beta$ be the corresponding sample value. $\hat{v}_\beta$ is a consistent estimator of index v_β. v_α and v_β are both functions of α and β, but do not depend on either γ or δ. In other words, $v_\alpha = \psi_1(\alpha,\beta)$ and $v_\beta = \psi_2(\alpha,\beta)$. The values of the indexes v_α and v_β expressed as functions of α and β are tabulated in McCulloch (1985). This relationship can be inverted to obtain $\alpha = \phi(v_\alpha, v_\beta)$. The α values for given values of v_α and v_β are presented in Table III in McCulloch (1985). Hence, the value of α corresponding

[4]For details of the correction procedure, readers are referred to McCulloch (1986).

to the computed values of $\hat{v}_\alpha$ and $\hat{v}_\beta$ is obtained from Table III [McCulloch (1985)] by linear interpolation of presented values.[5]

RESULTS

Tables II and III present the estimated characteristic exponents, $\hat{\alpha}$'s, for the entire data set using raw and rescaled data, respectively. Results for subsets of the data for raw and rescaled returns are presented in Tables IV and V, respectively. Descriptive statistics such as tests of normality of underlying distributions, skewness, and kurtosis, are also presented in each table. McLeod–Li's chi-square is used to test the null hypothesis of no correlation in squared returns.

In the case of all raw data (Table II), 19 of the 31 commodities have a skewness statistic significantly different from zero at the 5% significance level. In the case of all rescaled data, (Table III), 23 commodities have a skewness statistic significantly different from zero at the 5% significance level. In the case of subsets of raw and rescaled data, the skewness statistic is significantly different from zero in 16 and 18 cases, respectively. In other words, the underlying distributions are not symmetric either before or after rescaling the data. In general, positive skewness appears to be more prevalent than negative skewness.

The independence of adjacent returns up to lag 12 are tested in this study, using the McLeod–Li χ^2. All χ^2 values in the case of all raw returns (Table II) and subset of the data (Table IV) are significantly different from zero at the 1% significance level. This indicates that returns up to lag 12 are not independently distributed in the case of raw data. In the case of rescaled data however, the results are slightly different. In only 16 commodities, when considering all data, (Table III) and 7 commodities, when considering a subset of the rescaled data (Table V), are the χ^2 values significantly different from zero at the 5% level. This indicates that rescaling the returns reduces dependence in adjacent returns. Thus, the returns in 15 commodities in the all rescaled data and 24 commodities in the case of the subset of the rescaled data, are independently distributed. However, Taylor's rescaling procedure is not entirely successful in eliminating dependence of adjacent returns. Yang (1989) found a GARCH process removed conditional heteroskedasticity in a similar percentage of futures price series.

The $\hat{\alpha}$'s, are first obtained using 0.1 as the value of the smoothing parameter, ρ, as suggested by Taylor (1986). Following this, the value of ρ is estimated separately for each commodity, using the procedure described in the previous section. The estimated values of ρ for the entire data set and for the subset of the data, for each of the commodities are presented in Table VI. The $\hat{\alpha}$'s are then recomputed using the estimated values of ρ. The $\hat{\alpha}$'s estimated using the two methods are identical in 107 of the 124 cases (86%), supporting Taylor's claim that using $\rho = 0.1$ for futures series is as good as using the values of ρ estimated individually for each commodity. Hence, only the results for rescaled data using $\rho = 0.1$ is used and presented and discussed and used in comparisons with the results obtained for raw data.

The $\hat{\alpha}$'s for the raw and rescaled data for the entire data set, are presented in Tables II and III, respectively. In the case of all raw data, $\hat{\alpha} = 2.0$ in only 4 out of the 124 cases (3.2%) and $\hat{\alpha}$ is ≥ 1.90 in 7 cases (5.6%). The $\hat{\alpha}$'s range from 1.128 to 2.00 and show a slight tendency to converge to two as the sum size increases from one to 20. In the case

[5]Following McCulloch (1985), if the computed value of v_α for a given value of v_β is less than the lowest value of v_α given in the table, the value of α is treated as two. Similarly, for a given value of v_α, if the value of v_β exceeds its maximum value of one, it is treated as being equal to one.

Table II

TESTS OF INDEPENDENTLY AND IDENTICALLY DISTRIBUTED NORMAL DISTRIBUTION FOR ALL RAW FUTURES DATA

Commodities	K–S Test of[a] Normality	Skewness	Relative[a] Kurtosis	McLeod–Li[a] Chi-Square	Characteristic Exponents			
					$n = 1$	$n = 5$	$n = 10$	$n = 20$
I. Agricultural commodities								
Cocoa	0.0289	0.030	0.373	2246.41	1.638	1.729	1.862	1.563
Coffee	0.0635	−0.279[b]	3.771	845.72	1.460	1.437	1.503	1.460
Corn	0.0841	0.135[b]	2.923	9947.96	1.363	1.335	1.363	1.391
Lumber	0.0210	0.035	−0.482	726.01	2.000	1.916	2.000	1.563
Oats	0.0619	0.056[b]	1.957	7649.95	1.437	1.484	1.503	1.484
Orange juice	0.0738	0.039	2.898	1976.85	1.437	1.414	1.563	1.588
Soybean meal	0.0896	0.052	3.882	9999.99	1.335	1.437	1.363	1.414
Soybean oil	0.0570	0.021	1.997	4061.85	1.460	1.563	1.588	1.638
Soybeans	0.0910	−0.039	2.478	9999.99	1.260	1.307	1.279	1.241
Sugar	0.0359	−0.019	1.456	1475.68	1.638	1.664	1.664	1.768
Wheat (Chicago)	0.0832	0.327[b]	7.205	2887.60	1.414	1.460	1.363	1.437
II. Livestock								
Feeder cattle	0.0293	−0.094[b]	0.317	2082.46	1.696	1.664	1.523	1.437
Live cattle	0.0487	−0.111[b]	0.519	4026.65	1.484	1.588	1.543	1.563
Live hogs	0.0327	−0.049	0.211	2633.92	1.613	1.808	1.862	1.916
Pork bellies	0.0236	−0.022	−0.429	2578.89	2.000	1.808	1.768	1.613

III. Metals								
Copper	0.0621	−0.191[b]	2.595	3886.69	1.460	1.460	1.563	1.664
Gold	0.0840	−0.074	3.572	2299.74	1.363	1.437	1.664	1.563
Palladium	0.0713	−0.074	1.889	816.57	1.391	1.437	1.307	1.203
Platinum	0.0555	−0.148[b]	1.403	2056.30	1.484	1.563	1.503	1.664
Silver	0.0845	−0.200[b]	2.242	3381.97	1.335	1.335	1.363	1.523
IV. Financial instruments								
British pound	0.0615	0.174[b]	3.057	464.01	1.523	1.664	1.808	1.808
Canadian dollar	0.0615	−0.086[b]	4.851	292.05	1.543	1.808	1.729	1.768
Deutsche mark	0.0624	0.204[b]	2.436	366.64	1.423	1.638	1.808	1.808
European dollar	0.0979	0.564[b]	9.986	271.28	1.460	1.503	1.696	1.696
Japanese yen	0.0589	0.285[b]	2.881	174.91	1.543	1.613	1.862	1.729
Swiss franc	0.0459	0.154[b]	1.721	330.45	1.638	1.729	1.916	1.664
Treasury bills	0.1219	0.141[b]	4.605	3171.81	1.203	1.460	1.363	1.128
Treasury bonds	0.0604	0.095[b]	1.487	818.89	1.484	1.523	1.729	1.588
Treasury notes	0.0498	0.304[b]	2.240	235.72	1.588	1.808	2.000	1.664
V. Miscellaneous								
Crude oil	0.1168	0.102	6.350	1510.82	1.260	1.241	1.200	1.279
Heating oil	0.0709	−0.029	3.720	2687.88	1.437	1.460	1.523	1.696

[a]The null hypothesis of an i.i.d. normal distribution is rejected for every commodity in the column.
[b]The null hypothesis of no skewness is rejected at 5% level.

Table III
TESTS OF INDEPENDENTLY AND IDENTICALLY DISTRIBUTED NORMAL DISTRIBUTION FOR ALL RESCALED FUTURES DATA

Commodities	K–S Test of[a] Normality	Skewness[b]	Relative[c] Kurtosis	McLeod–Li[d] Chi-Square	Characteristic Exponents			
					$n = 1$	$n = 5$	$n = 10$	$n = 20$
I. Agricultural commodities								
Cocoa	0.0184	−0.060	0.838	30.82*	2.000	2.000	2.000	1.862
Coffee	0.0378	−0.212	1.868	66.27*	1.664	1.916	2.000	2.000
Corn	0.0369	0.284	2.670	26.96*	1.862	1.862	2.000	1.916
Lumber	0.0139	0.051*	−0.141	47.33*	2.000	2.000	2.000	1.768
Oats	0.0416	−0.061	1.810	6.48	1.808	1.808	1.916	1.916
Orange juice	0.0541	−0.094	5.783	22.81*	1.638	1.638	1.729	2.000
Soybean meal	0.0361	0.441	3.667	27.04*	1.768	1.916	1.916	2.000
Soybean oil	0.0197	0.090	1.973	11.92	1.916	1.916	1.916	2.000
Soybeans	0.0260	0.059	1.774	14.47	1.916	2.000	1.862	2.000
Sugar	0.0241	0.088	1.756	44.55*	1.916	2.000	2.000	2.000
Wheat (Chicago)	0.0257	0.028*	1.835	21.51*	1.808	1.808	1.862	2.000
II. Livestock								
Feeder cattle	0.0190	−0.124	0.324	13.74	2.000	2.000	1.808	1.916
Live cattle	0.0164	−0.064	0.972	25.77*	1.916	2.000	2.000	1.916
Live hogs	0.0179	−0.112	0.527	25.94*	1.916	2.000	2.000	1.916
Pork bellies	0.0214	0.010*	0.288	45.03*	2.000	2.000	2.000	2.000

III. Metals								
Copper	0.0261	−0.121	1.965	17.39	1.808	1.916	1.916	2.000
Gold	0.0696	−0.196	5.872	14.14	1.503	1.563	1.808	1.563
Palladium	0.0493	0.052*	2.827	73.49*	1.563	1.543	1.460	1.437
Platinum	0.0420	−0.163	3.068	17.82	1.729	1.808	1.696	1.808
Silver	0.0487	1.383	25.116	40.75*	1.768	1.808	1.638	1.729
IV. Financial instruments								
British pound	0.0567	0.936	14.830	3.15	1.613	1.729	2.000	2.000
Canadian dollar	0.0518	−0.502	3.502	16.01	1.638	1.808	1.768	2.000
Deutsche mark	0.0483	0.092	1.366	10.81	1.588	1.808	1.862	1.862
European dollar	0.0548	0.348	4.026	11.54	1.523	2.000	2.000	2.000
Japanese yen	0.0567	0.900	8.571	32.23*	1.613	1.808	1.768	1.729
Swiss franc	0.0358	0.151	1.118	14.02	1.696	1.916	2.000	1.768
Treasury bills	0.0392	0.151	2.752	23.83*	1.638	2.000	2.000	2.000
Treasury bonds	0.0290	0.035*	0.869	32.63*	1.729	2.000	2.000	1.729
Treasury notes	0.0485	−0.063*	1.855	10.85	1.588	2.000	2.000	1.768
V. Miscellaneous								
Crude oil	0.0425	−0.103*	2.117	15.16	1.696	1.808	1.916	2.000
Heating oil	0.0274	−0.090*	1.210	19.19	1.768	1.768	1.862	2.000

[a]The null hypothesis of an i.i.d. normal distribution is rejected for every commodity in the column.
[b]The null hypothesis of no skewness is rejected at 5% level for all commodities not marked with an asterisk.
[c]The null hypothesis of an i.i.d. normal distribution is rejected at 1% level for all commodities except lumber.
[d]The null hypothesis of an i.i.d. normal distribution is rejected at 5% level for those commodities marked with an asterisk.

Table IV
TESTS OF INDEPENDENTLY AND IDENTICALLY DISTRIBUTED NORMAL DISTRIBUTION FOR RAW DATA (1981–1988)

Commodities	K–S Test of[a] Normality	Skewness[b]	Relative[c] Kurtosis	McLeod–Li[a] Chi-Square	Characteristic Exponents			
					$n = 1$	$n = 5$	$n = 10$	$n = 20$
I. Agricultural commodities								
Cocoa	0.0362	−0.003	0.794	221.87	1.588	1.916	1.916	1.768
Coffee	0.0708	−0.343*	4.505	675.87	1.437	1.335	1.363	1.203
Corn	0.0557	0.082	2.224	2061.19	1.613	1.523	1.414	1.363
Lumber	0.0257	0.024	−0.493	294.81	1.916	1.768	2.000	1.664
Oats	0.0366	0.157*	1.397	1404.70	1.638	1.729	1.484	1.279
Orange juice	0.0759	0.252*	3.309	1357.43	1.391	1.460	1.437	1.588
Soybean meal	0.0548	0.043	1.849	1236.59	1.543	1.588	1.664	1.916
Soybean oil	0.0308	0.115*	0.530	563.61	1.638	1.862	2.000	1.808
Soybeans	0.0555	−0.138*	1.572	1380.26	1.563	1.664	1.862	1.808
Sugar	0.0317	0.023	1.651	169.66	1.768	2.000	1.588	1.808
Wheat (Chicago)	0.0579	0.978*	19.214	325.61	1.768	2.000	1.613	2.000
II. Livestock								
Feeder cattle	0.0336	−0.082	0.086	553.70	1.588	1.729	2.000	1.916
Live cattle	0.0313	−0.088	0.060	326.48	1.664	1.862	2.000	2.000
Live hogs	0.0265	−0.038	−0.069	189.81	1.588	2.000	2.000	1.808
Pork bellies	0.0374	−0.018	−0.737	684.93	2.000	2.000	1.638	1.664

III. Metals								
Copper	0.0679	−0.284*	4.580	522.41	1.588	1.563	1.862	1.808
Gold	0.0848	0.046	3.083	528.28	1.335	1.484	1.664	1.503
Palladium	0.0748	−0.043	2.234	392.91	1.363	1.363	1.363	1.437
Platinum	0.0532	−0.075	1.337	566.95	1.543	1.523	1.613	1.563
Silver	0.0630	−0.235*	2.126	321.11	1.484	1.484	1.523	1.437
IV. Financial instruments								
British pound	0.0503	0.255*	2.590	261.15	1.588	1.808	1.768	1.916
Canadian dollar	0.0792	−0.153*	6.135	201.53	1.460	1.503	1.808	2.000
Deutsche mark	0.0558	0.265*	1.931	146.74	1.563	1.808	1.916	2.000
European dollar	0.0979	0.564*	9.986	271.28	1.460	1.503	1.696	1.729
Japanese yen	0.0621	0.371*	4.037	81.34	1.523	1.638	1.523	1.588
Swiss franc	0.0464	0.213*	1.614	129.77	1.664	1.696	1.696	2.000
Treasury bills	0.1138	0.145*	5.120	1422.66	1.279	1.523	1.363	1.437
Treasury bonds	0.0420	0.006	1.028	242.97	1.613	2.000	1.916	1.768
Treasury notes	0.0498	0.304*	2.240	235.72	1.588	1.808	2.000	1.664
V. Miscellaneous								
Crude oil	0.1168	0.102	6.350	1510.82	1.260	1.241	1.203	1.307
Heating oil	0.0699	−0.023	3.516	2325.68	1.460	1.484	1.414	1.664

[a]The null hypothesis of an i.i.d. normal distribution is rejected for every commodity in the column.

[b]The null hypothesis of no skewness is rejected at 5% level for those commodities marked with an asterisk.

[c]The null hypothesis of an i.i.d. normal distribution is rejected at 1% level for all commodities except feeder cattle, live cattle, and live hogs.

Table V
TESTS OF INDEPENDENTLY AND IDENTICALLY DISTRIBUTED NORMAL DISTRIBUTION FOR RESCALED DATA (1981–1988)

Commodities	K–S Test of[a] Normality	Skewness[b]	Relative[a] Kurtosis	McLeod–Li[c] Chi-Square	Characteristic Exponents			
					$n = 1$	$n = 5$	$n = 10$	$n = 20$
I. Agricultural commodities								
Cocoa	0.0305	−0.216*	1.220	20.56	1.808	2.000	2.000	2.000
Coffee	0.0434	−0.296*	2.127	64.67*	1.664	1.729	1.916	1.638
Corn	0.0290	0.051	1.655	5.06	1.916	1.916	1.862	1.696
Lumber	0.0141	0.098	−0.003	42.41*	2.000	2.000	2.000	1.588
Oats	0.0376	−0.275*	1.842	15.49	1.768	2.000	1.696	1.391
Orange juice	0.0556	0.445*	6.036	8.81	1.588	1.808	1.543	1.862
Soybean meal	0.0394	0.202*	1.579	14.31	1.664	1.916	1.664	2.000
Soybean oil	0.0261	0.204*	0.759	12.47	1.862	2.000	2.000	2.000
Soybeans	0.0238	0.025	0.978	15.81	1.916	1.862	2.000	2.000
Sugar	0.0337	0.031	1.486	14.92	1.768	2.000	1.664	1.696
Wheat (Chicago)	0.0290	0.084	1.396	18.62	1.862	2.000	1.862	1.862
II. Livestock								
Feeder cattle	0.0358	−0.193*	0.475	15.98	1.768	2.000	2.000	2.000
Live cattle	0.0297	−0.198*	0.503	30.94*	1.729	2.000	2.000	2.000
Live hogs	0.0278	−0.149*	0.337	35.32*	1.664	2.000	2.000	2.000
Pork bellies	0.0338	−0.014	−0.528	24.01*	2.000	2.000	1.768	1.588

III. Metals								
Copper	0.0437	−0.361*	2.977	13.05	1.729	1.862	2.000	2.000
Gold	0.0775	0.182*	5.570	12.32	1.460	1.563	1.638	1.729
Palladium	0.0578	0.057	2.368	29.76*	1.543	1.543	1.543	1.484
Platinum	0.0529	−0.156*	2.005	15.60	1.664	1.613	1.916	2.000
Silver	0.0561	−0.116*	3.022	11.66	1.588	1.664	1.638	1.808
IV. Financial instruments								
British pound	0.0523	−0.209*	2.163	18.94	1.638	1.729	2.000	1.664
Canadian dollar	0.0639	−0.710*	4.454	15.92	1.613	1.916	1.664	2.000
Deutsche mark	0.0449	0.029	1.146	8.22	1.588	2.000	1.916	1.916
European dollar	0.0548	0.348*	4.026	11.54	1.523	2.000	2.000	2.000
Japanese yen	0.0630	1.065*	10.822	13.64	1.613	1.729	1.523	1.588
Swiss franc	0.0337	0.094	1.106	12.60	1.696	2.000	1.862	2.000
Treasury bills	0.0484	0.267*	3.724	19.14	1.588	2.000	1.916	2.000
Treasury bonds	0.0337	0.019	1.161	23.77*	1.696	2.000	2.000	2.000
Treasury notes	0.0485	−0.063	1.855	10.85	1.588	2.000	2.000	1.768
V. Miscellaneous								
Crude oil	0.0425	−0.103	2.117	15.16	1.696	1.808	1.916	2.000
Heating oil	0.0288	−0.096	1.098	16.48	1.729	1.729	2.000	2.000

[a]The null hypothesis of an i.i.d. normal distribution is rejected for every commodity in the column.
[b]The null hypothesis of no skewness is rejected at 5% level for those commodities marked with an asterisk.
[c]The null hypothesis of an i.i.d. normal distribution is rejected at 1% level for those commodities marked with an asterisk.

of rescaled data however, there is a marked increase in the values of the $\hat{\alpha}$'s (Table III) compared to the corresponding values for the raw data (Table II). The rescaled data have a greater tendency to converge towards normality as evidenced by their larger values of $\hat{\alpha}$'s. In 123 out of 124 cases in the rescaled data, $\hat{\alpha}$ is greater than or equal to the corresponding values for the raw data (Table II).

Table VI
ESTIMATED VALUES OF ONE MINUS THE SMOOTHING PARAMETER[a]

Commodity	All Data	1981–1988 Data
I. Agricultural commodities		
Cocoa	0.949	0.960
Coffee	0.885	0.875
Corn	0.901	0.897
Lumber	0.978	0.974
Oats	0.912	0.900
Orange juice	0.867	0.882
Soybean meal	0.897	0.915
Soybean oil	0.929	0.944
Soybeans	0.904	0.924
Sugar	0.900	0.922
Wheat (Chicago)	0.900	0.900
II. Livestock		
Feeder cattle	0.916	0.938
Live cattle	0.950	0.956
Live hogs	0.947	0.965
Pork bellies	0.948	0.934
III. Metals		
Copper	0.900	0.953
Gold	0.900	0.900
Palladium	0.827	0.900
Platinum	0.900	0.936
Silver	0.900	0.900
IV. Financial instruments		
British pound	0.900	0.925
Canadian dollar	0.900	0.900
Deutsche mark	0.934	0.959
European dollar	0.952	0.952
Japanese yen	0.900	0.905
Swiss franc	0.924	0.962
Treasury bills	0.969	0.981
Treasury bonds	0.959	0.962
Treasury notes	0.962	0.962
V. Miscellaneous		
Crude oil	0.886	0.886
Heating oil	0.900	0.900

[a]The null hypothesis of an i.i.d. normal distribution is rejected for every commodity—all data and 1981–1988 data.

For the rescaled data (Table III), only 10 out of the 124 estimates (8.1%) have an $\hat{\alpha}$ value less than 1.6. However, the corresponding number for the raw data (Table II) is 77, or 62.1% of the estimates. The estimated value of α for the rescaled data (Table III) reaches the value of two for 24 out of the 31 commodities included in the study, compared to just 3 commodities for the raw data (Table II). The average $\hat{\alpha}$ for one-day rescaled returns in Table III is 1.76, compared to 1.89 for the 20-day rescaled returns. Thus, for the rescaled data, the 20-day returns are closer to a normal distribution than the 1-day returns. This implies that there could be some short-run changes in variance (e.g., day of the week effects) which the rescaling procedure fails to capture. However, the rescaling procedure may be well suited for fortnightly or monthly data.

The rescaled returns show a greater tendency to converge to normality than the raw returns, as evidenced by the larger values of the $\hat{\alpha}$'s for the rescaled returns. The rescaled returns, though not normal, are less leptokurtic than the raw returns. The failure of the rescaled returns to be perfectly normally distributed can be attributed to several reasons. For instance, there may be factors in addition to the changing variance (e.g., presence of fewer observations in the tail than in the normal distribution and non-stationarity of scale) that contribute to the observed leptokurticity in the data. Also, rescaling the data using the conditional standard deviation obtained using Taylor's EWMA model may not be an appropriate rescaling factor for all commodities tested. Similar results have been obtained by other researchers. For example, McCulloch (1985) estimated values of the characteristic exponents ranging between 1.614 and 1.714 for six selected financial assets using monthly data and an adaptive conditionally heteroskedastic (ACH) model, implying that the ACH model did not correct the observed leptokurticity completely. Similarly, Bollerslev (1987) concluded that the ARCH or GARCH models with conditionally normal errors do not fully capture the leptokurtosis of the distribution of speculative price changes. Yang and Brorsen (1992) found that the GARCH with a conditional t-distribution can be rejected as a model of futures prices.

A comparison of the rescaled (Table IV) and raw (Table V) subset data reveal a similar relationship as that observed in Tables II and III, i.e., the rescaled returns for the subset data, though not perfectly normal, converge to the normal distribution more often than the raw returns (Table V). In 90 (72.6%) of the 124 cases, the $\hat{\alpha}$'s in Table IV are greater than the corresponding values in Table II (and are closer to two) and hence more closely approximate normal distributions. Similar patterns are observed in the case of rescaled data. Thus, the estimated $\hat{\alpha}$'s in the subset data, have a greater tendency to converge to two compared to the entire data.

Changing conditional variances appear to be an important factor in the observed departure of futures returns from the normal distribution. However, results obtained in this study indicate that there are additional factors contributing to the observed leptokurticity of futures returns and/or models other than the EWMA model may be better choices for rescaling for some commodities. The study results also indicate that the rescaled data used in this study are an improvement over using the raw data.

CONCLUSION

This article corrects for heteroskedasticity in daily futures returns data following the procedure suggested by Taylor (1986). Daily futures returns data for 31 commodities are used in the study. The raw data are corrected for heteroskedasticity by dividing each observation by its forecast conditional standard deviation generated by an exponentially weighted moving average (EWMA) model. The raw and rescaled data series are tested for convergence to normality using the stability under addition test of the stable distributions.

The EWMA model seems to work about as well as the GARCH models considered in past research.

The results indicate that the underlying distribution of the rescaled returns, corrected for changing conditional variances, are less leptokurtic than the raw data but are still not normal. In only 16 commodities when considering all data (Table III) and 7 commodities when considering the subset of the rescaled data, can the null hypothesis of no conditional heteroskedasticity be rejected. This indicates that rescaling the returns reduces dependence in adjacent returns to a great extent. Thus, the returns in 15 commodities in the all rescaled data and 24 commodities in the case of the subset of rescaled data, are independently distributed. The remaining conditional heteroskedasticity suggests that the model is misspecified for some commodities. Even though the procedure used in this study is not able to fully correct the observed leptokurtosis, it provides a more appropriate data series than the raw data and thus yields more accurate results.

There may be factors in addition to the changing conditional variances than contribute to the observed leptokurticity in the data, for example, the day of the week effect and non-stationarity of scale [So (1987)]. An investigation into these factors is a possible avenue for further research. A stable distribution with time varying scale parameter [Liu and Brorsen (1992)] is a possible model for some commodities. But, the stability under addition tests on the rescaled data suggests that a finite variance process is appropriate for most commodities.

Bibliography

Baldauf, B., and Santoni, G. J. (1991): "Stock Price Volatility: Some Evidence from an ARCH Model," *The Journal of Futures Markets*, 11:191–200.

Blattberg, R. C., and Gonedes, N. J. (1974): "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices," *The Journal of Business*, 47:244–280.

Bollerslev, T. (1986): "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, 31:307–327.

Bollerslev, T. (1987): "A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return," *The Review of Economics and Statistics*, 69:542–547.

Brorsen, B. W., and Yang, S. R. (1989): "Generalized Autoregressive Conditional Heteroskedasticity as a Model of the Distribution of Futures Returns," in *Applied Commodity Price Analysis, Forecasting, and Market Risk Management*, Hayenga, M. (ed.), Ames, IA: Iowa State University.

Brorsen, B. W., and Yang, S. R. (1990): "Maximum Likelihood Estimates of Symmetric Stable Distribution Parameters," *Communications in Statistics—Simulation and Computation*, 19:1459–1464.

Clark, P. K. (1973): "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices," *Econometrica*, 41:135–155.

Cornew, R. W., Town, D. F., and Crowson, L. D. (1984): "Stable Distributions, Futures Prices, and the Measurement of Trading Performance," *The Journal of Futures Markets*, 4:531–557.

Engle, R. F. (1983): "Estimates of the Variance of U.S. Inflation Based upon the ARCH Model," *Journal of Money, Credit and Banking*, 15:286–301.

Epps, T. W., and Epps, M. L. (1976): "The Stochastic Dependence of Security Price Changes and Transaction Volumes: Implications for the Mixture-of-Distributions Hypothesis," *Econometrica*, 44:305–321.

Fama, E. F. (1963): "Mandelbrot and the Stable Paretian Hypothesis," *The Journal of Business*, 36:420–429.

Fama, E. F., and Roll, R. (1968): "Some Properties of Symmetric Stable Distributions," *Journal of the American Statistical Association*, 63:817–836.

Fama, E. F., and Roll, R. (1971): "Parameter Estimates for Symmetric Stable Distributions," *Journal of the American Statistical Association*, 66:331–338.

Feller, W. (1958): *An Introduction to Probability Theory and Its Applications*, (Volume 1, 2nd ed.). New York: Wiley.

Fielitz, B. D., and Rozelle, J. P. (1983): "Stable Distributions and the Mixtures of Distributions Hypotheses for Common Stock Returns," *Journal of the American Statistical Association*, 78:28–36.

Fujihara, R., and Park, K. (1990): "The Probability Distribution of Futures Prices in the Foreign Exchange Market: A Comparison of Candidate Processes," *The Journal of Futures Markets*, 10:623–641.

Gribbin, D. W., Harris, R. W., and Lau, H.-S. (1992): "Futures Prices are Not Stable-Paretian Distributed," *The Journal of Futures Markets*, forthcoming.

Hall, J. A., Brorsen, B. W., and Irwin, S. H. (1989): "The Distribution of Futures Prices: A Test of the Stable Paretian and Mixture of Normals Hypotheses," *Journal of Financial and Quantitative Analysis*, 24:105–116.

Harris, L. (1987): "Transaction Data Tests of the Mixture of Distributions Hypothesis," *Journal of Financial and Quantitative Analysis*, 22:127–141.

Hudson, M. A., Leuthold, R. M., and Sarassoro, G. F. (1987): "Commodity Futures Price Changes: Recent Evidence for Wheat, Soybeans and Live Cattle," *The Journal of Futures Markets*, 7:287–301.

Liu, S.-M., and Brorsen, B. W. (1992): "Maximum Likelihood Estimation of the Stable Distribution with a Time-Varying Scale Parameter," Unpublished Working Paper, Purdue University.

Mandelbrot, B. (1963): "The Variation of Certain Speculative Prices," *The Journal of Business*, 36:394–419.

McCulloch, J. H. (1985): "Interest-Risk Sensitive Deposit Insurance Premia: Stable ACH Estimates," *Journal of Banking and Finance*, 9:137–156.

McCulloch, J. H. (1986): "Simple Consistent Estimators of Stable Distribution Parameters," *Communications in Statistics: Simulation and Computation*, 15:1109–1136.

McLeod, A. J., and Li, W. K. (1983): "Diagnostic checking ARMA Time Series Models Using Squared-Residual Autocorrelations," *Journal of Time Series Analysis*, 4:269–273.

Morgan, I. G. (1976): "Stock Prices and Heteroskedasticity," *The Journal of Business*, 49:496–508.

Officer, R. R. (1972): "The Distribution of Stock Returns," *Journal of the American Statistical Association*, 67:807–812.

Perry, P. R. (1983): "More Evidence on the Nature of the Distribution of Security Returns," *Journal of Financial Quantitative Analysis*, 18:211–221.

Praetz, P. D. (1972): "The Distribution of Share Price Changes," *The Journal of Business*, 45:49–55.

Press, J. S. (1967): "A Compound Events Model for Security Prices," *The Journal of Business*, 40:317–335.

Randolph, W. L., and Najand, M. (1991): "A Test of Two Models in Forecasting Stock Index Futures Price Volatility," *The Journal of Futures Markets*, 11:179–190.

So, J. C. (1987): "The Sub-Gaussian Distribution of Currency Futures: Stable Paretian or Nonstationary?" *The Review of Economics and Statistics*, 69:100–107.

Taylor, S. J. (1986): *Modelling Financial Time Series,* New York: Wiley.

Westerfield, R. (1977): "The Distribution of Common Stock Price Changes: An Application of Transactions Time and Subordinated Stochastic Models," *Journal of Financial and Quantitative Analysis,* 12:743–765.

Yang, S. R. (1989): "The Distribution of Speculative Price Changes," Unpublished Ph.D. Dissertation, Purdue University.

Yang, S. R., and Brorsen, B. W. (1992): "Nonlinear Dynamics of Daily Futures Prices: Conditional Heteroskedasticity or Chaos?" Unpublished Working Paper, North Dakota State University.