A Test of the Intertemporal Hedging Model of the Commodities Futures Markets

Stacie E. Beck

INTRODUCTION

A number of studies have tested the efficiency of commodities futures markets by estimating a variant of the equation

$$E_t(S_{t+N}) = K_0 + K_1 F_t^N \tag{1}$$

where S_{t+N} = commodity spot price in $t + N$; F_t^N = the price in t of a futures contract expiring in $t + N$. Market efficiency is equated with unbiasedness, hence the null hypothesis is $K_0 = 0$ and $K_1 = 1$. Most studies reject the hypothesis, particularly for long forecast horizons [e.g., Bigman, Goldfarb, Schechtman (1983); Martin and Garcia (1981); Leuthold (1974)]. The conclusion that futures markets are inefficient is conditional on the assumption that there is no risk premium. However, rejection of the null hypothesis is consistent with the intertemporal hedging theory which implies that risk premia cause $K_0 \neq 0$ and $K_1 \neq 1$ even if markets are efficient. The intertemporal model is an extension of the theory of normal backwardation, advanced by Keynes (1930), which hypothesizes that hedging pressure creates a differential between the futures price and the expected spot price at contract expiration. This risk premium compensates speculators for bearing spot price risk.

Another criticism of studies that have estimated (1) is that the problem of overlapping observations has been ignored [Elam and Dixon, (1988); Hein, Ma and MacDonald (1990); Kahl and Tomek (1986)]. The problem arises when the data has an observation interval shorter than the forecast horizon which introduces a moving average process in the residuals of (1). This implies that only short forecast horizons can be tested, or that observations must be deleted to test long forecast horizons. In this study, the problem is solved with the application of a technique developed by Hansen and Hodrick

Data were provided by the Center for the Study of Commodities Markets, Columbia University. Financial assistance was provided by the University of Delaware General Research Foundation. Comments by seminar participants at the University of Pennsylvania and Rutgers University and by referees on earlier versions are gratefully acknowledged. All errors remain the author's responsibility.

Stacie E. Beck is an Assistant Professor of Economics at the University of Delaware.

The Journal of Futures Markets, Vol. 13, No. 3, 223–236 (1993)

CCC 0270-7314/93/030223-14

(1980) which allows pooled data to be used in tests of the risk premium at long forecast horizons by correcting for inefficiency caused by residual correlation. This technique is more powerful than ordinary least squares estimated with nonoverlapping observations. In addition, an important implication of the intertemporal theory, that the risk premium is a function of the futures price at long forecast horizons, can be tested by estimating (1) at various forecast horizons. As with other tests for the existence of risk premia, this test is limited because the hypothesis is conditional on the maintained assumption of market efficiency. Nevertheless, one can determine whether estimates of K_0 and K_1 are consistent with the intertemporal hedging theory.

PREVIOUS RESEARCH AND THEORETICAL CONSIDERATIONS

Two theories indicate risk premia may exist in commodities futures markets. Asset equilibrium models imply risk premia are determined by commodity price correlations with returns on other assets or factors affecting asset returns. Most evidence fails to support this view [e.g., Dusak (1973); Baxter, Conine, and Tamarkin (1985); Ehrhardt, Jordan, and Walkling (1987)]. The theory of normal backwardation hypothesizes that net hedging pressure biases the futures prices above or below the expected spot price depending on whether net hedging is long or short.[1] Evidence on normal backwardation is inconclusive [e.g., Park (1985); Kolb (1992); Fama and French (1987)]. Attempts to explicitly link the risk premium to measures of hedging demand have also had mixed results. For example, Maddala and Yoo (1991) find such a link using spot price variance; whereas, Raynauld and Tessier (1984) find risk premia do not depend on inventory levels.[2]

The theory of normal backwardation is based on a simple hedging model where agents are assumed to make production and consumption plans independently of current futures prices and hedge these positions in the futures markets. This implies that the intercept, K_0, is biased away form zero and the futures price coefficient, K_1, equals one. The intertemporal hedging theory [Danthine (1978)] asserts that production, inventory, and hedging decisions are made jointly and depend on the current futures price, F_t^N. Hence, the risk premium has a portion dependent on the futures price causing K_1 to be biased away from one and K_0 away from zero.

The difference between the simple and intertemporal hedging models can be illustrated with an example. Suppose there are three types of agents in the market: commodity producers (farmers), consumers, i.e., intermediate good producers (buyers), and inventory holders. Assume all agents completely hedge their commodity positions and also take speculative positions depending on $[E_t(S_{t+1}) - F_t]$. Then

$$n^f[-E_t(S_{t+1}) + F_t] + Y_{t+1} = X_t^f \tag{2a}$$

$$n^b[-E_t(S_{t+1}) + F_t] - Q_{t+1} = -X_t^b \tag{2b}$$

$$n^i[-E_t(S_{t+1}) + F_t] + I_t = X_t^i \tag{2c}$$

[1]The normal backwardation hypothesis originally assumed that producers and inventory holders are hedgers, hence the futures price is biased below expected spot prices.

[2]Breeden's (1980) consumption CAPM could also be viewed as a consumption hedging model since risk premia are functions of aggregated consumption deviations. He finds significant risk premia for some commodities.

where

Y_{t+1} = commodity output planned in t and produced in $t + 1$;

Q_{t+1} = commodity purchases planned in t and made in $t + 1$;

I_t = inventory holdings at the beginning of period $t + 1$;

X_t^k = aggregate futures contract supply by farmers, commodity buyers, and and inventory holders ($k = f, b, i$);

n^k = a measure of risk aversion, assumed constant, for $k = f, b, i$.

The futures market condition is

$$X_t^f + X_t^i - X_t^b = 0 \tag{3}$$

hence

$$E_t(S_{t+1}) = F_t + \frac{1}{n}(Y_{t+1} + I_t - Q_{t+1})$$

where $n = n^f + n^b + n^i$.

Here $K_1 = 1$ and K_0 is the second term on the right-hand side. Because Q_{t+1}, Y_{t+1}, and I_t are predetermined, they appear only in the intercept of (1). Instead, suppose Q_{t+1}, Y_{t+1}, and I_t are determined jointly with X_t^k. Therefore, they depend on expected spot price and/or current futures price.[3] In the extreme case where future spot price uncertainty is the agents' only source of uncertainty, current futures price completely replaces the expected spot price in agents' decisions; hence:

$$Y_{t+1} = bF_t + e_{t+1}^f \tag{4a}$$

$$Q_{t+1} = A - aF_t + e_{t+1}^b \tag{4b}$$

$$I_t = d(F_t - S_t) \tag{4c}$$

where e^k = random errors representing production uncertainty for farmers and buyers ($k = f, b$) and are independently and identically distributed with zero means and constant variances. Expressions (4a–c) replace Q_{t+1}, Y_{t+1}, and I_t in (2a–c) which are then substituted into (3) to get

$$nE_t(S_{t+1}) = (n + d + b + a)F_t - dS_t - A \tag{5}$$

The rational expectations solution to the spot market equilibrium condition is

$$E_t(S_{t+1}) = \lambda S_t + \frac{\lambda A}{(1 - \lambda)}\left(\frac{1}{d} + \frac{1}{n}\right) \tag{6}$$

where $0 \leq \lambda \leq 1$. Equation (6) is substituted for S_t in (5) to get

$$E_t(S_{t+1}) = \frac{\lambda A}{n(1 - \lambda)} + \frac{[\lambda(n + a + b + d)]}{\lambda n + d}F_t \tag{7}$$

The first term on the right-hand side is K_0 which is nonzero, and the coefficient of F_t is K_1 which is not equal to one. It can be shown that $K_1 < 1$ (see Appendix). These results hold even when the model is generalized to include multiple sources of uncertainty so that (4a–c) depend on both $E_t(S_{t+1})$ and F_t and commodity positions are not completely

[3]Functions (2a–c) and (4a–c) can be derived from utility maximization conditions. These derivations are in a separate Appendix available from the author upon request. This version of the intertemporal hedging model is described in Turnovsky, 1983 and Kawai, 1983.

hedged.[4] At short horizons commodity producers and consumers (intermediate producers) are assumed to have already planned production and consumption because of production lags, so the price dependent portion of the risk premium should disappear and $K_1 = 1$ (see discussion in the Appendix).

To summarize, the intertemporal hedging theory implies that $K_0 \neq 0$ and $K_1 < 1$ at long horizons; $K_0 \neq 0$ but $K_1 = 1$ at short horizons. This hypothesis is conditional on the maintained assumption that markets are efficient. Alternatively, the same results could simply reflect inefficiency in the absence of a risk premium.

DATA CHARACTERISTICS

The five commodities are the live hog and live cattle contracts from the Chicago Mercantile Exchange, the silver and soybean contracts from the Chicago Board of Trade, and the frozen orange juice concentrate contract from the New York Cotton Exchange.[5] These commodities are chosen because they have six futures contracts equally spaced throughout the year. The February, April, June, August, October, and December contracts are pooled to obtain the hog, cattle, and silver price series covering 1966–1987, 1967–1987, and 1970–1978, respectively. The January, March, May, July, September, and November contracts are pooled to get the orange juice and soybean series which cover 1968–1987 and 1973–1987. Since six contracts per year are pooled, the observation interval is 2 months. The spot price series is actually the futures price on the day of contract expiration because arbitrage will drive the two prices together at that time.[6] The futures prices were selected at four forecast horizons; 8, 12, 24, and 40 weeks prior to expiration. If the day chosen fell on a holiday, the following business day was used. The observation was not used if there was an absence of trades.[7] Prices are the day's settlement prices.

All price series follow an upward trend because of inflation over the period, so the producer price index is used to convert prices to their 1982–1983 values. This conversion expresses prices in constant dollars to eliminate this trend.

The Dickey–Fuller τ-statistics are computed to determine whether the constant price series follow processes with unit roots [Dickey, Bell, and Miller (1986)]. When trends implied by the existence of unit roots are ignored, testing will overstate the strength of the relationship between the spot price and the futures price and understate evidence of a risk premium [Granger and Newbold (1977); Plosser and Schwert (1978)]. The null hypothesis that a unit root exists is rejected at the 2.5% level by four orange juice series and at the 5% level by all five. Three hog series rejected the hypothesis at the 2.5% level, four rejected it at the 5% level; and one marginally failed to reject it at the 5% level. The hypothesis is rejected at the 2.5% level by one cattle and one soybean series; it is rejected at the 5% level for two soybean and two cattle series. The hypothesis is not rejected for the silver data.

[4] Proofs for the generalized model are in a separate Appendix available from the author upon request.

[5] Data are from the Center for the Study of Futures Markets, Columbia University.

[6] By using the futures price on expiration day rather than the price quoted in the spot market, the problem of basis risk is avoided if these two commodities differ in some way, e.g., grade or location. However, the delivery basis risk, which arises when futures contracts allow delivery of several varieties in several locations is not avoided. This may create a risk premium also. For example, Kamara (1990) discovered such a risk premium in the soybean market. The risk premia found in this study may be caused by delivery risk. This is an area for future research.

[7] There are 2 and 22 such observations for hogs at the 24- and 40-week horizon respectively; 1 for orange juice at the 40-week horizon; 1 for soybeans at the 12-, 24-, and 40-week horizon; and 2 for silver at the 40-week horizon.

Since these results indicate the cattle, soybean, and silver series may have trends, the Φ-statistic is computed to determine whether the trends are deterministic or stochastic, i.e., random walks [Dickey and Fuller (1981)]. The random walk hypothesis is rejected for all soybean and three cattle series. It is not rejected by the silver series nor two cattle series.[8] A linear time trend is used to detrend data for all three commodities. Although the silver and two cattle series do not reject the random walk hypothesis, economic theory indicates that constant prices are determined by marginal costs in the long run; hence, it is implausible they are random walks.[9] However, since there will be strong serial correlation in the residuals if a deterministic trend is used when the actual trend is stochastic, this assumption's validity can be checked [(Plosser and Schwert (1978); Durlauf and Phillips (1988)]. In Table I the Durbin–Watson statistic indicates that the deterministic trend assumption is valid in the 8 week silver regression. The moving average process in the residuals, introduced by overlapping observation intervals, renders this statistic useless in the regressions where the forecast horizon exceeds 8 weeks. For these cases, the Durbin–Watson statistic is computed for regressions estimated on subsets of the data. For example, in 12 week regressions, data pooled over three contracts with expiration dates 3 months apart are used. The Durbin–Watson statistic supports the use of the linear trend variable in all but two regressions.[10]

ESTIMATION TECHNIQUE

When (1) is estimated at a forecast horizon greater than the observation interval, a moving average process will be introduced in the residuals. For example, at the 8-week horizon there will be no residual correlation, but at the 12-week horizon, the residuals follow a moving average process of order one because the forecast horizon overlaps two observation intervals [see Granger and Newbold (1977, p. 115)]. Since the form of the serial correlation is known, an obvious solution is to use generalized least squares. However, this implies estimation of

$$\sum_{i=0}^{\infty}(-\phi)^i S_{t+N-i} = K_0 \sum_{i=0}^{\infty}(-\phi)^i + K_1 \sum_{i=0}^{\infty}(-\phi)^i F_{t-i}^N + e_{t+N} \tag{8}$$

Lagged values of the spot price and the futures price appear on the left-hand and right-hand sides of (8), respectively. Because common economic processes determine both variables, least squares estimates will be inconsistent. The Hansen–Hodrick (1980) technique entails estimating (1) with ordinary least squares and then replacing the usual diagonal residual variance–covariance matrix with one that includes estimates of the off-diagonal elements implied by the known structure. For example, if the residuals follow

[8]The spot, 8- and 12-week futures price series reject the hypothesis; the 24- and 40-week futures series fail to reject the hypothesis in the cattle market.

[9]The linear trend is estimated with the spot price series for each commodity and used to detrend all five series since the maintained hypothesis is that futures prices are unbiased estimates of expected spot prices.

[10]The exceptions are one silver and one cattle regression estimated with a subset of data at the 24-week horizon. However, the regressions estimated on other subsets at the same horizon did support the hypothesis. Therefore, it is assumed that when all the contracts are pooled, the deterministic trend is a reasonable assumption.

Table I
ESTIMATION WITH SERIAL CORRELATION CORRECTION
$(S_{t+N} - F_t^N) = K_0 + (K_1 - 1)F_t^N + e_{t+N}$

Forecast Horizon (N = obs)	Intercept (K_0)	Futures Price Coefficient ($K_1 - 1$)	R^2	Residual Standard Deviation	Dependent Variable Mean	Durbin–Watson
Hogs						
8 weeks ($N = 127$)	3.081 (3.436)	−0.013 (0.059)	0.0007	0.684	2.263	1.775
12 weeks ($N = 127$)	9.512[a] (4.284)	−0.116 (0.076)	0.0314	8.972	2.509	1.397
24 weeks ($N = 125$)	11.157 (8.046)	−0.119 (0.147)	0.0157	11.200	4.165	0.715
40 weeks ($N = 105$)	13.216 (11.063)	−0.137 (0.202)	0.0126	13.247	5.442	0.543
Orange Juice						
8 weeks ($N = 118$)	8.713 (9.321)	−0.054 (0.081)	0.0068	18.445	1.823	1.915
12 weeks ($N = 118$)	21.047[a] (10.266)	−0.146[b] (0.088)	0.0363	21.350	2.727	1.310
24 weeks ($N = 118$)	41.360[a] (12.798)	−0.299[a] (0.103)	0.0738	26.854	4.512	0.692
40 weeks ($N = 117$)	78.338[a] (22.610)	−0.585[a] (0.176)	0.1638	30.744	8.745	0.417
Soybeans						
8 weeks ($N = 83$)	329.314[a] (110.587)	−0.407[a] (0.138)	0.0224	101.903	−7.122	1.754

Table I (continued)

Forecast Horizon (N = obs)	Intercept (K_0)	Futures Price Coefficient ($K_1 - 1$)	R^2	Residual Standard Deviation	Dependent Variable Mean	Durbin–Watson
12 weeks	540.298[a]	−0.662[a]	0.3506	120.779	−1.376	1.308
(N = 82)	(107.106)	(0.131)				
24 weeks	806.558[a]	−0.988[a]	0.5107	128.963	−18.503	0.811
(N = 82)	(131.548)	(0.152)				
40 weeks	920.550[a]	−1.125[a]	0.6045	127.768	−15.485	0.808
(N = 82)	(79.951)	(0.086)				
Live Cattle						
8 weeks	15.386[b]	−0.191[b]	0.0610	6.369	1.169	1.675
(N = 124)	(5.079)	(0.068)				
12 weeks	26.977[a]	−0.343[b]	0.1441	7.363	1.641	1.363
(N = 124)	(8.903)	(0.124)				
24 weeks	37.985[a]	−0.484[a]	0.1856	8.343	2.828	0.795
(N = 124)	(9.588)	(0.135)				
40 weeks	52.774[a]	−0.686[a]	0.2191	9.093	3.317	0.595
(N = 124)	(9.523)	(1.135)				

TABLE I (continued)

Forecast Horizon (N = obs)	Intercept (K_0)	Futures Price Coefficient ($K_1 - 1$)	R^2	Residual Standard Deviation	Dependent Variable Mean	Durbin–Watson
Silver						
8 weeks	1286.250[b]	−0.160[a]	0.0426	998.291	241.681	1.933
(N = 57)	(466.428)	(0.077)				
12 weeks	1091.550	−0.118	0.0140	1111.570	331.078	1.244
(N = 57)	(721.889)	(0.124)				
24 weeks	4079.700[a]	−0.576[a]	0.2080	1393.390	425.970	0.711
(N = 57)	(843.616)	(0.102)				
40 weeks	5273.200[a]	−0.757[a]	0.2988	1481.670	663.180	0.514
(N = 55)	(996.970)	(0.129)				

[a]Indicates significance at the 95% confidence level; [b]indicates significance at the 90% level. H_0: $K_0 = 0$ and $K_1 = 1$.
Note: The figures in parentheses, ( ), are standard errors. The t-statistics and coefficient standard errors are adjusted using the procedure described in the article. The R^2 and D.W. statistics are not adjusted. The soybean, cattle, and silver price series are detrended as described in the article.

a moving average process of degree one, the covariance matrix is

$$\Omega = \begin{bmatrix} \sigma_e^2 & \tilde{w} & 0 & \cdots & 0 \\ w & \sigma_e^2 & \tilde{w} & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & w & \sigma_e^2 & \tilde{w} \\ 0 & \cdots & 0 & w & \sigma_e^2 \end{bmatrix}$$

where $w = \text{cov}(e_{t-1}e_t)$ and $\tilde{w} = \text{cov}(e_t e_{t+1})$. This method incorporates knowledge about the true residual covariance matrix into estimates of the coefficient covariance matrix to improve the efficiency of the estimates. Because estimates of Ω are not necessarily positive semi-definite, a modification which imposes a system of declining weights on off-diagonal covariances is used to ensure positive semi-definiteness [see Newey and West (1987)].

Heteroskedasticity in the residuals of (1) is detected using White's (1980) test so a variation of the Hansen–Hodrick technique which incorporates a conditional heteroskedasticity correction is used (Hansen, 1982). Since a study by Mishkin (1990) suggests that corrections for heteroskedasticity when its form is unknown can cause misleading results, estimates are obtained without a heteroskedastic correction as well. However, significance results are similar.[11]

RESULTS

Table 1 reports the regression estimates using pooled data for each commodity at each forecast horizon. The dependent variable is $[E_t(S_{t+1}) - F_t]$ to highlight the role of the price-dependent component of the risk premium $(K_1 - 1)F_t$ which acts as an offset to the constant component K_0 on the right-hand side. Residual serial correlation caused by overlapping observations is reflected in the Durbin–Watson statistics for all horizons greater than 8 weeks. The coefficient variances and t-statistics are obtained with the method described above.

There are significant constant risk premia (i.e., nonzero intercepts) in: the cattle and soybean regressions; the 12-, 24-, and 40-week orange juice regressions; the 8-, 24-, and 40-week silver regressions; and the 12-week hog regression. Positive intercepts imply that hedging demand is net short, so production and inventory hedging dominate consumption hedging. The size of the constant risk premium decreases at shorter forecast horizons except in the silver market.

The model predicts that K_1 is below one at long forecast horizons, i.e., before consumption and production plans are final. At short horizons, $K_1 = 1$ after plans are made and hedged in the futures markets. Soybean, silver, cattle, and frozen orange juice estimates show this pattern; K_1 is below one and becomes closer to one at shorter forecast horizons. Hog regressions do not show this pattern, possibly because hog production/consumption plans are made in advance of the 40-week horizon and yield is predetermined. The cattle estimates are inconsistent with hog estimates if this is so, since cattle consumption lags are probably similar and the production cycle is longer than that for hogs. The cattle results, however, are supported by Kahl and Tomek's

[11]When uncorrected standard errors are used, K_0 becomes significantly positive in the 24-week hog regression and K_1 becomes significantly below one in the 12-week hog regression at the 10% level. In the 8 week silver regression, K_1 becomes insignificantly different from one at the 5% level. All other significance results are unchanged.

(1986) research on market efficiency in the cattle and soybean markets. They found similar results for these two markets at the 2- and 8-month horizon using the seemingly unrelated regression (SUR) approach. They viewed their estimates as evidence rejecting market efficiency. However, because the hypothesis is joint, their evidence also can be interpreted as support for the existence of risk premia.

Kolb (1992) and Park (1985) also studied the correlation of normal backwardation, i.e., risk premia, with forecast horizon using a different technique.[12] However, they test the significance of the constant risk premium K_0 (expressed as a rate of return) but not the price-dependent risk premium $(K_1 - 1)F_t$. Results may change if this term is omitted since it offsets the constant term K_0. When the price-dependent term is omitted here, K_0 becomes insignificant in the soybean and orange juice markets, marginally significant at the 8- and 12-week horizon in the silver market, and remains significant in the cattle market. Interestingly, K_0 becomes significant in the hog market when $(K_1 - 1)F_t$ is omitted. Although the results are consistent with Kolb for these commodities, except orange juice; futures price coefficient estimates in Table I suggest that $K_1 = 1$ is a valid assumption only in the hog market.

CONCLUSIONS

This study differs from previous research on the normal backwardation hypothesis at long forecast horizons by applying the more powerful Hansen–Hodrick technique rather than ordinary least squares. This technique yields consistent estimates from pooled data while correcting for inefficiency caused by the attendant residual correlation. Price data are also converted to constant prices and, where necessary, detrended to achieve stationarity and avoid misleading significance results. Based on the intertemporal model, risk premia are hypothesized to have two components, a constant component and a price-dependent component which diminishes as forecast horizon decreases. Significant constant risk premia are found in five commodity markets. Four markets also exhibit significant price-dependent risk premia with behavior consistent with the predictions of the model. However, conclusions regarding the existence of risk premia are conditional on the assumption of market efficiency.

Appendix

The Futures Price Coefficient at Long Forecast Horizons

In this section (5) through (7) are derived and the results are used to prove that $K_1 < 1$ at long forecast horizons. The market clearing conditions for the futures and spot markets are (3) and

$$Q_{t+1} + I_{t+1} = Y_{t+1} + I_t \tag{A1}$$

After substituting (2a–c) and (4a–c) into (3) and (A1) these equilibria are (5) and

$$A - aF_t + d(F_{t+1} - S_{t+1}) = bF_t + d(F_t - S_t) + e_{t+1} \tag{A2}$$

where $e_{t+1} = e^f_{t+1} - e^b_{t+1}$ and $n = n^f + n^b + n^i$.

[12] Fama and French (1987) also studied the behavior of the premium at different horizons for a number of commodities. However, they treated the premium as a function of the basis; whereas here, the risk premium is hypothesized to be composed of a constant and price-dependent term.

Equation (5) is substituted into (A2) for F_{t+1} and F_t, which is rearranged to get

$$E_t(S_{t+2}) - \frac{(n + a + b)}{n} S_{t+1} - \frac{(a + b + d)}{d} E_t(S_{t+1}) + S_t$$

$$= -A\left(\frac{1}{n} + \frac{1}{d}\right) - \frac{(n + a + b + d)}{nd} e_{t+1} \qquad \text{(A3)}$$

Using a factoring technique described by Sargent (1979, p. 178), $E_t(S_{t+1})$ is expressed in terms of S_t and the lag operator is applied to each side of (A3) to get (6) where

$$\lambda = \frac{n}{d} \frac{[d(1 - \lambda) - \lambda(a + b)]}{n(1 - \lambda) + a + b}$$

and $0 < \lambda < 1$. Since the parameters on the right-hand side are positive, the roots are real. One root lies above unity, the other, below unity. The bounded solution requires the coefficient of the larger root to vanish, therefore λ is set equal to the smaller root $\lambda < 1$.

Substitution of (6) into (5) yields the joint equilibrium condition (7). The risk premium has a constant portion, reflected in $K_0 = \lambda A[n(1 - \lambda)]^{-1}$, and a price-dependent portion, which biases K_1 away from one. To illustrate, suppose there is no hedging demand, hence, no risk premium. Then, only the speculative components of (2a–c) remain so that (7) is $E_t(S_{t+1}) = F_t$ and $K_0 = 0$ and $K_1 = 1$.

If the risk premium biases K_1 below one, the inequality

$$K_1 = \frac{\lambda(n + a + b + d)}{\lambda n + d} < 1$$

must hold. Dividing both sides by $(\lambda n + d)$ and rearranging gives

$$0 < d(1 - \lambda) - \lambda(a + b)$$

Using the expression for λ obtained for (6) (see above), solving for $d(1 - \lambda) - \lambda(a + b)$ and substituting into the inequality above gives

$$0 < \frac{d}{n}[n(1 - \lambda) + a + b]$$

which is positive so the inequality holds.

The Futures Price Coefficient at Short Forecast Horizons

It can be shown that when producers are not influenced by the futures price at all, the price-dependent portion of the risk premium disappears and K_1 equals one. Assume that producers must commit themselves to a production level before actual production occurs because of lags. At very short forecast horizons aggregate production and consumption are planned, hence, independent of price, so $Y_{t+1} = \overline{Y}_{t+1}$ and $Q_{t+1} = \overline{Q}_{t+1}$. The futures and spot market equilibria are now

$$nE_t(S_{t+1}) = (n + d)F_t - dS_t + \overline{Y}_{t+1} - \overline{Q}_{t+1} \qquad \text{(A6a)}$$

$$\overline{Q}_{t+1} + d(F_{t+1} - S_{t+1}) = \overline{Y}_{t+1} + d(F_t - S_t) + e_{t+1} \qquad \text{(A6b)}$$

where, without loss of generality it is again assumed there is only spot price uncertainty. The same procedure used to obtain (7) can be used again to express $E_t(S_{t+1})$ as a function of S_t. Equation (A6a) is solved for F_t and F_{t+1} and substituted in (A6b). This substitution, with rearranging, yields

$$nE_{t+1}(S_{t+2}) - nS_{t+1} - nE_t(S_{t+1}) + nS_t = \overline{Y}_{t+2} - \overline{Q}_{t+2}$$

$$\left(\frac{n}{d}\right)(\overline{Y}_{t+1} - \overline{Q}_{t+1}) + \left(\frac{n+d}{d}\right)e_{t+1} \tag{A7}$$

After the expectations operator and the factoring technique is applied (A7) becomes

$$E_t(S_{t+1}) = \lambda S_t + \left(\frac{1}{n} + \frac{1}{d}\right)\sum_{j=0}^{\infty}\left(\frac{1}{\lambda}\right)^j\left(\overline{Y}_{t+1-j} - \overline{Q}_{t+1-j}\right)$$

$$- \frac{1}{d}(\overline{Y}_{t+1} - \overline{Q}_{t+1})$$

which is

$$E_t(S_{t+1}) = S_t + \left(\frac{1}{n}\right)(\overline{Y}_{t+1} - \overline{Q}_{t+1}) + \left(\frac{1}{d} + \frac{1}{n}\right)I_t$$

After substituting the above expression for S_t in (A6a) the following is obtained

$$E(S_{t+1}) = F_t + \left(\frac{1}{n}\right)(I_t + \overline{Y}_{t+1} - \overline{Q}_{t+1})$$

The price-dependent risk premium is zero, i.e., $K_1 = 1$. The constant risk premium, K_0, is the net of the predetermined levels of consumption, production, and inventory hedging.

Intuitively, if the forecast horizon is short enough so that some producers have made decisions but others have not, the price-dependent risk premium should be smaller, but should not vanish. The aggregate supply and demand functions in this case are

$$Y_{t+1} = \sum_{j=1}^{m^f - \overline{m}^f} \overline{Y}_{j,t+1} + \sum_{j=1}^{\overline{m}^f} b_j F_t$$

and

$$Q_{t+1} = \sum_{j=1}^{m^b - \overline{m}^b} \overline{Q}_{j,t+1} + \sum_{j=1}^{\overline{m}^b} (A_j - a_j F_t)$$

where $\overline{m}^k$ = producers uncommitted to a production level for $k = f, b$; $\overline{Y}_{j,t+1}$ = planned output by farmer j; and $\overline{Q}_{j,t+1}$ = planned purchased by commodity buyer j. Consumption and production are less dependent on futures prices (parameters a and b decrease) which should be reflected in a smaller price-dependent risk premium at short horizons.

No conclusion can be drawn concerning the behavior of K_0 over contract lifetime. Equation (7) indicates that K_0 is positive when there is only spot price risk, but its sign is ambiguous when external risk exists. At shorter horizons K_0 depends on $\overline{Q}_{t+1}$ and $\overline{Y}_{t+1}$, i.e., the relative amount of long and short hedging of predetermined production levels. Thus, a nonzero K_0 indicates the existence of a risk premium due to hedging demand, but a K_0 equal to zero may either indicate no hedging demand or hedging demand balanced between short and long hedging.

Bibliography

Baxter, J., Conine, T., and Tamarkin, M. (1985): "On Commodity Market Risk Premiums: Additional Evidence," *The Journal of Futures Markets,* 5:121–125.

Bigman, D., Goldfarb, D., and Schechtman, E. (1983): "Futures Market Efficiency and the Time Content of the Information Sets," *The Journal of Futures Markets,* 3:321–334.

Breeden, D. (1980, May): "Consumption Risk in Futures Markets," *Journal of Finance,* 35:503–520.

Danthine, J. (1978): "Information, Future Prices, and Stabilizing Speculation," *Journal of Economic Theory,* 17:79–98.

Dickey, D. A., Bell, W. R., and Miller, R. B. (1986, February): "Unit Roots in Time Series Models: Tests and Implications," *The American Statistician,* 40:12–26.

Dickey, D. A., and Fuller, W. A. (1981, July): "Likelihood Ratio Statistics for Autoregressive Time Series With a Unit Root," *Econometrica,* 49:1057–1072.

Durlauf, S., and Phillips, P. (1988, November): "Trends versus Random Walks in Time Series Analysis," *Econometrica,* 56:1333–1354.

Dusak, K. (1973): "Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums," *Journal of Political Economy,* 81:1387–1406.

Ehrhardt, M., Jordan, J., and Walkling, R. (1987): An Application of Arbitrage Pricing Theory to Futures Markets: Tests of Normal Backwardation," *The Journal of Futures Markets,* 7:21–34.

Elam, E., and Dixon, B. (1988): "Examining the Validity of a Test of Futures Market Efficiency," *The Journal of Futures Markets,* 8:365–372.

Fama, E. F., and French, K. R. (1987, January): "Commodities Futures Prices: Some Evidence on Forecast Power, Premiums and the Theory of Storage," *Journal of Business,* 60:55–73.

Granger, C. W., and Newbold, P. (1977): *Forecasting Economic Time Series,* New York: Academic Press.

Hansen, L. P. (1982, July): "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica,* 50:1029–1054.

Hansen, L. P., and Hodrick, R. J. (1980): "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," *Journal of Political Economy,* 88:829–853.

Hein, S., Ma, C., and MacDonald, S. (1990): "Testing Unbiasedness in Futures Markets: A Clarification," *The Journal of Futures Markets,* 10:555–562.

Kahl, K., and Tomek, W. (1986): "Forward-Pricing Models for Futures Markets: Some Statistical and Interpretive Issues," *Food Research Institute Studies,* 20:71–85.

Kamara, A. (1990, March): "Delivery Uncertainty and the Efficiency of Futures Markets" *Journal of Financial and Quantitative Analysis,* 25:45–64.

Kawai, M. (1983, June): "Price Volatility of Storable Commodities Under Rational Expectations in Spot and Futures Markets," *International Economic Review,* 24:435–459.

Keynes, J. M. (1930): *A Treatise on Money, Vol. 2,* London: MacMillan.

Kolb, R. (1992): "Is Normal Backwardation Normal?" *The Journal of Futures Markets,* 12:75–91.

Leuthold, R. (1974): "The Price Performance in the Futures Markets of a Nonstorable Commodity: Live Beef Cattle,": in *Selected Writings on Futures Markets, Vol. 2,* Peck, A. (ed.), Chicago Board of Trade.

Maddala, G., and Yoo, J. (1991, April): "Risk Premia and Price Volatility in FuturesMarkets," *The Journal of Futures Markets,* 11:165–178.

Martin, L., and Garcia, P. (1981): "Price Forecasting Performance of Futures Markets for Live Cattle and Hogs," *Journal of Agricultural Economics,* 63:209–215.

Mishkin, F. (1990): "Does Correcting for Heteroskedasticity Help?" *Economic Letters,* 34:351–356.

Newey, W., and West, K. (1987): "A Simple, Positive Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix" *Econometrica,* 53:703–708.

Park, H. (1985): "Reexamination of Normal Backwardation Hypothesis in Futures Markets," *The Journal of Futures Markets,* 5:505–515.

Plosser, C., and Schwert, G. (1978): "Money, Income and Sunspots: Measuring Economic Relationships and the Effects of Differencing," *Journal of Monetary Economics,* 4:637–660.

Reynauld, J., and Tessier, J. (1984): "Risk Premiums in Futures Markets: An Empirical Investigation," *The Journal of Futures Markets,* 4:188–209.

Sargent, T. (1979): *Macroeconomic Theory,* New York: Academic Press.

Turnovsky, S. (1983, September): "The Determination of Spot and Futures Prices with Storable Commodities," *Econometrica,* 51:1363–1387.

White, H. (1980, May): "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica,* 48:817–838.