Nonlinear Dynamics of Daily Futures Prices: Conditional Heteroskedasticity or Chaos?

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INTRODUCTION

Research on futures prices [Hudson et al. (1987); Cornew et al. (1984); Gordon (1985); Hall et al. (1988)] has found that the distribution of futures prices is not normal but leptokurtic. Specifically, the empirical distributions of daily price changes have more observations around the means and in the extreme tails than does a normal distribution. Leptokurtosis also appears in stock returns [Fama (1965); Brock et al. (1991)] and exchange rate changes [Hsieh (1988); Friedman and Vandersteel (1982)]. Further, nonlinear dependence has been found in futures price changes [Taylor (1985); Blank (1990); Fujihara and Park (1990)]. Yet, empirical research on market anomalies has either ignored the non-normality and dependence or resorted to nonparametric tests which generally are less powerful than parametric tests.

Past research suggests distributions such as the stable paretian [Mandelbrot (1963); Fama (1965)] and more recently a diffusion-jump process [Akgiray and Booth (1988)] as models of speculative prices. These distributions partly explain leptokurtosis, but since they assume that successive observations are independent, these distributions are inconsistent with empirical work that has found linear and nonlinear dependence. The Generalized Autoregressive Conditional Heteroskedastic (GARCH) model, however, can explain both nonlinear dependence and leptokurtosis.

Various time varying variance models have been considered by several authors [Friedman and Vandersteel (1982); McCulloch (1985); Taylor (1986); Akgiray (1989); Brorsen and Yang (1989); Fujihara and Park (1990); Randolph and Najand (1991); Baldauf and Santoni (1991)]. Results suggest such an approach looks promising. While a GARCH or similar time varying model looks promising, the adequacy of such models has not been tested rigorously.

Savit (1988, 1989) suggests that asset returns may not follow a stochastic process. Rather, they might be generated by deterministic chaos in which the forecasting error

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grows exponentially so that the process appears stochastic. Savit (1989) shows the effect of chaos on pricing options. Frank and Stengos (1986) find evidence of nonlinear structure for gold and silver markets. Sheinkman and LeBaron (1989) find some support for the hypothesis that stock returns follow a nonlinear dynamic system. Blank (1990) finds results consistent with deterministic chaos in futures prices.

The objective of this research is to test both the GARCH and deterministic chaos processes for a large sample of daily futures price changes. No past model successfully explains non-normality and dependence in speculative price changes. This study considers market anomalies including, seasonality, day-of-the-week, and maturity effects. Previoùs studies using simple ARCH or GARCH models [Bollerslev et al. (1990)] do not fully describe the underlying return generating process.

This study extends previous work in several ways. It provides a more appropriate test of market anomalies because the estimates of the GARCH models are heteroskedasticity and autocorrelation-adjusted. Numerous studies have tested a single market anomaly. This work considers seasonality, day-of-the-week, and maturity effects at the same time. This study uses goodness-of-fit tests to determine the adequacy of the GARCH model. It does not simply select a model which is more descriptive among alternatives, but tests whether the model can generate the sample data.[1] In addition, the hypothesis that futures returns are generated by a nonlinear deterministic process is tested using standardized residuals. Since conditional heteroskedasticity is a type of nonlinear dynamics, adjusting for conditional heteroskedasticity could lead to very different conclusions about deterministic chaos than when no adjustment is made.

STATISTICAL MODELS AND ESTIMATION PROCEDURES

The GARCH Process

The GARCH model [Bollerslev (1986)] generates data with fatter tails than the normal distribution. The GARCH process implies serial dependence in the second moment. The GARCH process can model factors that violate the i.i.d assumption other than conditional heteroskedasticity. The GARCH model of this study includes the well-documented market anomalies of day-of-the-week effects [Chiang and Tapley (1983); Junkus (1986)], seasonality in variance [Anderson (1985); Kenyon et al. (1987)] and maturity effects [Milonas (1986)]. Ten lagged dependent variables are included in the model to allow for autocorrelation in the mean [Taylor (1986)]. Finally, to test for the risk-return relationship, this GARCH model includes the conditional standard deviation in the mean equation.

Bollerslev (1986) suggested that the simplest but often very useful GARCH model is the GARCH(1, 1) process. This study follows Bollerslev's proposition. By specifying the model, a priori, and not using the data to select the model, in-sample hypothesis tests remain valid. The specific GARCH models to be estimated are

$$y_t = a_0 + \sum_{s=1}^{10} a_s y_{t-s} + a_{11} D_M + a_{12} D_T + a_{13} D_W + a_{14} D_H + a_{15} h_t + e_t ,$$

[1]These are tests of whether the model is "well-calibrated." Bunn defines a model as well-calibrated if, over the long run, for all propositions assigned a given probability, the proportion that is true equals the probability assigned.

$$
\begin{aligned}
h_t^2 = {} & b_0 + \alpha e_{t-1} + \beta h_{t-1}^2 + b_1 D_M + b_2 D_T + b_3 D_W + b_4 D_H + b_5 D_{\text{hol}} \\
& + b_6 M_t + b_7 \cos(2\pi I/126) + b_8 \cos(2\pi I/252) + b_9 \cos(2\pi I/126) \\
& + b_{10} \text{SIN}(2\pi I/252), \\
e_t|\psi_{t-1} \sim {} & N(0, h_t^2) \qquad (1)
\end{aligned}
$$

When y_{t-s} denotes the lagged dependent variable, the dummy variables for each day-of-the-week are $D_M = 1$ if Monday, $D_T = 1$ if Tuesday, D_W if Wednesday, D_H if Thursday. They are 0 otherwise. M_t indicates remaining days to contract maturity. I in the sine and cosine functions is the number of trading days after Jan. 1 of the particular year. This avoids any bias from different numbers of trading days in different years. Denominators in the sine and cosine functions are the specified cycle length in days, so 126 indicates a 6-month cycle and 252 a 1-year cycle.

The GARCH models are estimated by maximum likelihood using the algorithm developed by Berndt, Hall, Hall, and Hausman (1974) with a numerical gradient method. Hypothesis tests are conducted with Wald tests using the cross product of the gradients as the estimate of the information matrix.

If the GARCH models are a well-calibrated model of the data, the residuals from the models should be i.i.d. normal. To test for normality, the hypothesis that the standardized residuals, $\hat{e}_t/\hat{h}_t$ are normal is tested with the Kolmogorov–Smirnov D-statistic. The estimated kurtosis and skewness are calculated also and compared to those of the original data.

Deterministic Chaos

It is widely assumed that certain characteristics of the economy such as taste, productivity, and technology are random. However, there is an alternative to the stochastic assumption to explain economic fluctuations. Many early economists tried to identify the internal mechanisms that could explain the observed variations in price movements [Hayek (1933)]. Deterministic chaos allows for a nonlinear dynamic model which is deterministic with respect to initial conditions, but errors made in estimating parameters and initial conditions can accumulate into forecasting errors [Brock (1986)]. This makes the process look random. Deterministic processes that look stochastic are referred to as *deterministic chaos.* The deterministic process cannot be distinguished from the stochastic process by statistical tests based on spectral densities or autocovariances [Brock and Dechert (1986)].

Consider a time series, $\{x_t\}$ and a sequence of m-histories, $X_t^m = (x_t, x_{t+1}, \ldots, x_{t+m-1})$ that is, the m-dimensional vectors obtained by putting m consecutive observations together. The embedding theorem [Takens (1980)] says that the behavior of an m-history will mimic that of the underlying unknown dynamic process if m is large enough. To illustrate, a good random number generator for i.i.d. uniform $[0, 1]$ should fill the closed interval $[0, 1]$. If the numbers clump together on a few points, then the series is said to be low dimensional. If the 2-dimensional vectors $[(X_t^m, X_{t+1}^m)]$ fill out the unit square of $[0, 1]^2$, the series $\{x_t\}$ has correlation dimension ≥ 2. Likewise, a good random number generator provides a random series which fills m-cubes for any embedding dimension, m. If a series is generated by a deterministic process, m-dimensional vectors may not fill m-cubes but clump onto a low-dimensional subset or a few points. Consequently, the objective is to determine whether m-histories of $\{x_t\}$ fail to fill m-cubes for large embedding dimensions, or, equivalently, whether the estimated dimension is well below the embedding dimension.

The correlation dimension is estimated using the correlation integral $[C_m(\varepsilon)]$:

$$C_m(\varepsilon) = \lim_{T\to\infty}\{\#\text{ of }(i,j)\text{ for which }\ \| X_i^m - X_j^m \| < \varepsilon, i \neq j\}/T^* \tag{2}$$

where m is an embedding dimension, and ε is a sufficiently small number. $T^* = (T^2 - T)/2$, where T is the number of m-histories that can be made out of a series of length N. From the sample size N, $T = N - (m + 1)$ m-histories can be made. The norm ($\| \cdot \|$) used in the computer program is the sup norm. The correlation integral measures the asymptotic probability that the distance between any two m-histories is less than ε. If the data are generated by a deterministic process, the correlation integral in (2) will be independent of m but will increase with ε. The correlation dimension (SC_m) is defined as:

$$SC_m = [\ln C_m(\varepsilon_i) - \ln C_m(\varepsilon_{i-1})]/[\ln \varepsilon_i - \ln \varepsilon_{i-1}] \tag{3}$$

Eleven values of ε are used: $0.9^1, 0.9^2, \ldots, 0.9^{11}$. Four embedding dimensions are used: 2, 4, 6, and 8. The median of the ten estimates of SC_m for each m is employed as the estimate of the correlation dimension since the distribution of estimates of SC_m for each embedding dimension has great dispersion and skewness.

One test of deterministic chaos is the residual test suggested by Brock (1986). He shows that if time series data are chaotic, the estimated dimension of residuals from the best fitting time series model is the same as that of the original data. If the data are stochastic, the dimension of the residuals will increase since they have less structure than the original data. Brock's residual test is easy to apply and especially useful for this study since the GARCH processes provide residuals which are adjusted for possible linear and quadratic dependence. The drawback of Brock's residual test is that no formal hypothesis testing is possible. Therefore, the stronger test of Brock, Dechert, and Sheinkman (1986), which is called the BDS test, is used as well.

The BDS statistic is used to test the null hypothesis of an independent identical distribution [Brock et al. (1991)]. The BDS test has power against chaotic alternatives, unlike conventional tests. The statistic is based on the correlation integral. Under the null hypothesis that $\{x_t\}$ is i.i.d., the BDS statistic is,

$$W_m(\varepsilon, T) = T^{1/2}[C_m(\varepsilon, T)^m - C_1(\varepsilon, T)^m]/\sigma_m(\varepsilon) \tag{4}$$

This statistic converges in distribution to a standard normal random variable under the null hypothesis. Based on Monte Carlo results by Hsieh and LeBaron (1988), the epsilon chosen is $\varepsilon = \sigma$ with embedding dimensions, $m = 3$, 6, and 9, where σ is the standard deviation of the data series. A rejection of the null hypothesis may be due to linear or nonlinear dependence or to chaotic structure.

The BDS statistic converges in distribution to a standard normal random variable under the null hypothesis. But, Brock et al. (1991) show that the distribution of the statistic changes when applied with GARCH residuals. They use Monte Carlo methods to obtain the distribution of the BDS statistic applied with GARCH(1, 1) standardized residuals and 2500 observations. The hypothesis tests for raw data are based on the asymptotic distribution; but for the GARCH residuals, the table in Brock et al. (1991) is used.

SAMPLE DATA

The data are the first differences of the natural logarithms of daily closing futures prices. Changes in the logarithms of the prices can be interpreted as percentage changes in

Table I
SAMPLE PERIOD AND CONTRACT MONTHS FOR FUTURES COMMODITIES

Commodity	Sample Period	Contract Months	Price Limit[a]
Corn	1/79–12/88	Mar May Jul Dec	yes
Coffee	1/79–12/88	Mar May Jul Sep Dec	yes
Oats	1/79–12/88	Mar May Jul Sep Dec	yes
Soybean	1/79–12/88	Jan Mar May Jul Nov	yes
Soybean meal	1/79–12/88	Mar May Jul Dec	yes
Wheat (Chicago)	1/79–12/88	Mar Jul Dec	yes
Wheat (Kansas City)	2/79–12/88	Mar Jul Dec	yes
Copper	1/79–12/88	Mar May Jul Sep Dec	yes
Gold (NY)	1/79–12/88	Feb Apr Jun Aug Sep	yes
Palladium	1/79–12/88	Mar Jun Sep Dec	yes
Platinum	1/79–12/88	Jan Apr Jul Oct	yes
Silver	1/79–12/88	Mar May Jul Sep Dec	yes
NYSE	1/84–12/88	Mar Jun Sep Dec	no
S&P 500	1/83–12/88	Mar Jun Sep Dec	no
Value Line	1/84–12/88	Mar Jun Sep Dec	no

[a]Source: Commodity Trading Manual.

continuous time. Daily returns are multiplied by 100 to avoid possible scaling problems in estimation and to express them in percentage terms. The data set is composed of 15 contracts actively traded in U.S. futures markets. The list of data and sample periods of each commodity are shown in Table I. The data are for the 10 years from January 1979 to December 1988, except for contracts which began trading after January 1979. Only the three stock indexes began trading after January 1979. For these contracts, data is included for the first full year of trading through December 1988 (when the data are available). The more than 2500 observations for each commodity (except for the stock indexes) provide enough degrees of freedom that it seems reasonable to use tests that are only asymptotically valid.

A continuous series of data is constructed. The data consist of changes in the log of daily closing prices of the futures contract closest to delivery until the third Tuesday of the month prior to delivery, after which the log changes in the next nearest delivery month are used. A few low volume contracts such as September corn are not included in the dataset. Since all data are close to maturity, the maturity effect should be less important in this data. The data are from the Dunn & Hargitt Commodity Data Bank.

Table II provides summary statistics of the data. All means are not statistically different from zero. Sample sizes vary slightly by commodity because the various exchanges observe different holidays and because markets are closed occasionally when exchanges fear traders may panic in response to some new announcement. Estimated variances are very different from commodity to commodity even though the data are unitless due to using log changes. Twelve of 15 contracts are skewed. The skewness is mostly negative, but sometimes positive. All are leptokurtic at conventional levels of significance. Three stock index futures, NYSE, S&P 500, and Value Line have exceptionally large kurtosis. These extremes result from the stock market crash in 1987. Kurtosis can be greatly affected by a single outlier, even in a sample of 2500.

Table II
SUMMARY STATISTICS OF DAILY FUTURES RETURNS

Commodity	Sample Size	Mean	Variance	Skewness	Relative Kurtosis
Corn	2524	−0.015	1.433	0.038	1.899[a]
Coffee	2510	0.018	3.104	−0.279[a]	3.771[a]
Oats	2522	−0.002	2.673	0.103[a]	1.093[a]
Soybean	2523	−0.024	1.943	−0.131[a]	1.317[a]
Soybean meal	2524	−0.017	2.190	0.005	1.740[a]
Wheat (Chi)	2524	−0.011	1.874	0.722[a]	13.289[a]
Wheat (KC)	2494	−0.000	1.580	0.615[a]	29.084[a]
Copper	2519	0.017	3.207	−0.260[a]	2.994[a]
Gold (NY)	2521	−0.016	1.350	−0.016	3.469[a]
Palladium	2496	0.017	4.532	−0.093[a]	1.742[a]
Platinum	2508	−0.011	4.528	−0.112[a]	0.947[a]
Silver	2514	−0.041	4.893	−0.193[a]	1.485[a]
NYSE	1353	0.021	2.286	−6.280[a]	170.806[a]
S&P 500	1669	0.038	2.316	−5.720[a]	156.713[a]
Value Line	1350	0.003	1.623	−4.819[a]	72.804[a]

[a]Denotes rejection of the null hypothesis of a normal distribution.

RESULTS

Brock's Residual Test

Table III shows the estimated correlation dimensions for the raw and rescaled data (standardized residuals). If time series data are stochastic, the estimated dimensions should have full dimension; that is, equal or very close to the embedding dimensions. Only silver has an estimated dimension distinctly lower than the embedding dimension. This can be evidence of a deterministic structure. However, silver does not pass Brock's residual test. The estimates for the rescaled data for silver are larger.

Estimated dimensions detect some contracts which seem to pass the residual test. However, they tend to have full dimensions; that is, those estimated dimensions are close to the embedding dimensions implying no evidence of deterministic structure. To confirm deterministic chaos, diagnostics should meet the saturation condition as well as Brock's residual test. That is, beyond some embedding dimension, estimated correlation dimensions for both raw and residual data should be the same and be stable with embedding dimension. None of the series that pass the residual test also satisfy the saturation condition.[2] Correlation dimensions and embedding dimensions increase together.

Brock's residual test shows no evidence of low-dimension deterministic chaos for the futures data. However, formal hypothesis tests are not possible with Brock's residual test. Also, higher dimensions might be needed to find the saturation point.

[2]Higher levels of embedding dimensions have been considered in past research. The data might meet the saturation condition at higher embedding dimensions. Thus, the results only provide a test of low-dimension deterministic chaos.

Table III
CORRELATION DIMENSION ESTIMATES[a]

Commodity	Original Data			Rescaled Data		
	$m = 4$	$m = 6$	$m = 8$	$m = 4$	$m = 6$	$m = 8$
Corn	3.48	4.93	6.11	3.78	5.52	7.19
Coffee	3.68	5.49	7.13	3.68	5.43	7.39
Oats	3.53	5.09	6.80	3.77	5.72	7.39
Soybean	3.63	5.33	6.90	3.74	5.42	7.12
Soybean meal	3.57	5.18	6.52	3.64	5.44	6.88
Wheat (Chi)	3.73	5.45	7.02	3.72	5.44	7.02
Wheat (KC)	3.49	4.94	6.52	3.59	5.31	6.79
Copper	3.63	5.22	5.92	3.71	5.62	7.88
Gold (NY)	3.52	5.35	6.90	3.58	5.44	7.01
Palladium	3.69	5.45	7.02	3.62	5.20	6.91
Platinum	3.78	5.59	7.13	3.73	5.36	6.86
Silver	3.69	4.63	2.19	3.69	5.01	5.84
NYSE	3.63	5.44	7.16	3.58	5.53	7.41
S&P 500	3.60	5.62	7.09	3.66	5.44	6.89
Value Line	3.68	5.44	7.29	3.71	5.52	7.60

[a]For each embedding dimension, eleven adjacent values of epsilon, 0.9^n, $n = 1, 2, \ldots, 11$ are used. The median is reported as the estimate of the correlation dimension.

Estimated Results of the GARCH Process

Table IV reports estimates and test results of the GARCH model. First, estimated coefficients for the ARCH term, α and the GARCH term, β are positive and significant at the 5% level for all commodities. The conditional standard deviation is not significant in the means equation of any commodity. This may mean that traders are risk neutral or that the conditional standard deviation is a poor measure of risk.

The null hypothesis of no linear dependence is rejected only for silver and platinum. This is less autocorrelation than found in past studies such as Taylor (1986). Mean returns differ from day to day only for coffee. The results, however, show volatility does differ by day of the week. Including the holiday effect, only three series have no significant differences in variances. Thus, different variability of returns for each day of the week seems to be a general phenomenon in futures markets. Specifically, most Mondays and all holidays have significantly higher variances, and most Wednesdays have lower variances.

The type of day of the week effects found are not truly an anomaly. The fact that variances are higher on Monday and following holidays favors the calendar time hypothesis over the trading time hypothesis.

The maturity effect is significant in six cases. Moreover, the effect is less than 1% of the variance. This implies that studies which use nearby price series do not need to be concerned about differences in maturity. The maturity effect might be more important when differences in maturity are greater.

Nine cases reveal significant seasonal patterns in price volatility. The results agree with Kenyon et al. (1987) who found that the price volatility of corn, soybeans, and wheat are affected by the season of the year.

Table IV
ESTIMATED PARAMETERS AND STATISTICS OF THE GARCH(1, 1) PROCESS

	Wheat (Chi)	Wheat (KC)	Soybean	Soybean meal	Oats
Mean					
Intercept	−0.025	−0.116	−0.057	−0.055	−0.043
	(−0.26)	(−1.77)	(−0.64)	(−0.58)	(−0.40)
D (Monday)	−0.041	−0.058	−0.064	−0.170[a]	−0.121
	(−0.54)	(−0.93)	(−0.84)	(−2.04)	(−1.40)
D (Tuesday)	−0.028	0.024	0.037	−0.060	−0.096
	(−0.40)	(0.43)	(0.53)	(−0.75)	(−1.11)
D (Wednesday)	0.118	0.172	0.138[a]	0.095	−0.004
	(1.71)	(2.82)	(2.00)	(1.28)	(−0.05)
D (Thursday)	−0.120	−0.043	0.012	0.005	−0.184[a]
	(−1.75)	(−0.73)	(0.18)	(0.07)	(−2.11)
Std. dev.	0.021	0.102	−0.007	0.035	0.080
	(0.26)	(1.88)	(−0.10)	(0.50)	(1.24)
Variance					
Intercept	0.133	0.228[a]	−0.075	0.003	0.175
	(1.78)	(5.97)	(−0.99)	(0.03)	(1.79)
Alpha	0.084[a]	0.157[a]	0.063[a]	0.067[a]	0.067[a]
	(10.26)	(15.46)	(7.48)	(8.46)	(8.17)
Beta	0.881[a]	0.810[a]	0.916[a]	0.912[a]	0.928[a]
	(73.28)	(80.97)	(90.91)	(94.33)	(109.7)
D (Monday)	0.270[a]	−0.074	0.464[a]	0.435[a]	−0.205
	(2.18)	(−1.06)	(3.60)	(3.23)	(−1.24)
D (Tuesday)	−0.449[a]	−0.437[a]	−0.368[a]	−0.352[a]	−0.141
	(−3.72)	(−9.14)	(−3.20)	(−2.95)	(−0.97)

D (Wednesday)	−0.127	−0.117[a]	0.085	−0.010	−0.162
	(−1.19)	(−2.50)	(0.85)	(−0.08)	(−1.19)
D (Thursday)	−0.121	−0.239[a]	0.215	−0.040	−0.214
	(−0.91)	(−3.87)	(1.71)	(−0.297)	(−1.42)
D (holiday)	0.594[a]	0.231[a]	0.356[a]	0.399[a]	0.150[a]
	(5.42)	(4.91)	(3.73)	(4.13)	(1.33)
Maturity	−0.001	0.000	0.008[a]	0.006[a]	−0.007
	(−0.39)	(0.23)	(2.85)	(2.61)	(−1.90)
Autocorrelation(10)[b]	1.31	1.27	1.17	0.92	0.99
M (day-of-the-week)[b]	4.56	3.54	1.17	2.66	0.95
V (day-of-the-week)[b]	15.96[a]	72.26[a]	54.79[a]	13.59[a]	0.81[a]
V (seasonality)[b]	15.72[a]	78.18[a]	15.40[a]	22.32[a]	3.26
Ljung–Box(15)[c]					
$e(t)/h(t)$	4.80	13.06	7.57	10.01	9.36
$e(t)^2/h(t)^2$	16.25	2.62	15.90	12.60	8.05
BDS tests ($\varepsilon = \sigma$)					
Raw data					
dimensions = 3	2.02[a]	3.14[a]	3.24[a]	3.55[a]	3.33[a]
dimensions = 6	7.92[a]	12.69[a]	14.23[a]	14.70[a]	15.79[a]
dimensions = 9	18.20[a]	29.53[a]	36.86[a]	36.35[a]	47.44[a]
Standardized residuals					
dimensions = 3	−0.09	0.18	−0.23	0.33	0.27
dimensions = 6	−0.78	−0.20	−0.44	0.58	0.77
dimensions = 9	−1.33	0.19	−0.09	1.28	1.04

Table IV (continued)

	Corn	Coffee	Copper	Gold	Silver
Mean					
Intercept	−0.005	0.165	−0.031	−0.055	−0.148
	(−0.08)	(1.95)	(−0.31)	(−0.58)	(−0.89)
D (Monday)	0.019	−0.151	−0.284[a]	−0.102	−0.392[a]
	(0.30)	(−1.90)	(−3.19)	(−1.21)	(−3.13)
D (Tuesday)	−0.026	−0.119	−0.096	0.003	−0.135
	(−0.43)	(−1.50)	(−1.01)	(0.04)	(−1.13)
D (Wednesday)	0.090	−0.065	0.114	0.060	0.038
	(1.52)	(−0.87)	(1.29)	(0.73)	(0.32)
D (Thursday)	−0.076	0.006	0.046	−0.043	−0.138
	(−1.31)	(0.08)	(0.08)	(−0.54)	(−1.18)
Std. dev.	−0.013	−0.048	0.037	0.014	0.142
	(−0.20)	(−0.81)	(0.66)	(0.25)	(1.81)
Variance					
Intercept	0.093[a]	0.034	0.193[a]	0.384[a]	0.254
	(2.22)	(0.45)	(2.32)	(5.13)	(1.12)
Alpha	0.088[a]	0.119[a]	0.043[a]	0.077[a]	0.101[a]
	(7.94)	(12.42)	(8.28)	(11.28)	(9.99)
Beta	0.883[a]	0.864[a]	0.952[a]	0.900	0.855[a]
	(62.00)	(84.86)	(172.6)	(117.0)	(66.67)
D (Monday)	0.186[a]	0.250	−0.096	−0.141	1.107[a]
	(2.33)	(1.91)	(−0.76)	(−1.02)	(2.92)
D (Tuesday)	−0.292[a]	−0.046	−0.168	−0.992	−1.208[a]
	(−4.06)	(−0.36)	(−1.11)	(−8.51)	(−3.65)

D (Wednesday)	−0.084	−0.031	−0.572[a]	−0.202[a]	−0.106
	(−1.39)	(−0.26)	(−4.89)	(−2.43)	(−0.38)
D (Thursday)	−0.140	−0.116	−0.176	−0.484[a]	−0.091
	(−1.77)	(−0.80)	(−1.19)	(−4.41)	(0.29)
D (holiday)	0.283[a]	0.390[a]	0.556[a]	0.946[a]	2.139[a]
	(4.14)	(3.00)	(5.05)	(9.37)	(6.13)
Maturity	0.001	−0.002	0.001	0.002	−0.018[a]
	(0.72)	(−0.56)	(0.53)	(0.95)	(−2.11)
Autocorrelation(10)[b]	1.20	1.27	1.81	1.63	3.78
M (day-of-the-week)[b]	2.15	19.00[a]	3.60	0.82	3.57
V (day-of-the-week)[b]	20.12[a]	3.08	8.51	26.35[a]	25.53[a]
V (seasonality)[b]	16.58[a]	10.88[a]	21.91[a]	4.98	14.49[a]
Ljung–Box(15)[c]					
$e(t)/h(t)$	6.14	9.31	2.69	15.82	21.13
$e(t)^2/h(t)^2$	9.57	31.68[a]	11.86	13.39	12.03
BDS tests ($\varepsilon = \sigma$)					
Raw data					
dimensions = 3	3.99[a]	5.44[a]	3.83[a]	4.13[a]	3.99[a]
dimensions = 6	18.97[a]	24.37[a]	16.74[a]	16.29[a]	16.32[a]
dimensions = 9	52.67[a]	66.00[a]	41.42[a]	39.05[a]	40.73[a]
Standardized residuals					
dimensions = 3	0.08	0.40	0.26	0.60	0.56
dimensions = 6	1.32[a]	0.89	0.73	1.47[a]	1.04
dimensions = 9	2.65[a]	1.02	0.80	2.06[a]	1.63[a]

Table IV (continued)

	Pall.	Plat.	NYSE	S&P 500	Value Line
Mean					
Intercept	0.073	−0.105	0.019	−0.085	0.163
	(0.57)	(−0.55)	(0.16)	(−0.87)	(1.81)
D (Monday)	−0.084	−0.146	−0.009	0.007	−0.015
	(−0.75)	(−1.13)	(−0.11)	(0.09)	(−0.17)
D (Tuesday)	0.075	−0.01	0.027	0.018	0.019
	(0.74)	(−0.01)	(0.32)	(0.24)	(0.24)
D (Wednesday)	0.146	0.007	−0.003	−0.003	−0.037
	(1.44)	(0.05)	(−0.04)	(−0.04)	(−0.51)
D (Thursday)	−0.076	−0.125	−0.029	−0.023	0.089
	(−0.073)	(−1.01)	(−0.34)	(−0.30)	(1.09)
Std. dev.	−0.038	0.067	0.050	0.155	−0.148
	(−0.61)	(0.78)	(0.45)	(1.75)	(−1.52)
Variance					
Intercept	0.093	0.650[a]	−0.053	−0.084	−0.108
	(0.56)	(3.05)	(−0.70)	(−1.31)	(−1.62)
Alpha	0.141[a]	0.064[a]	0.200[a]	0.148[a]	0.146[a]
	(11.25)	(6.65)	(13.89)	(18.31)	(16.61)
Beta	0.821[a]	0.909[a]	0.712[a]	0.822[a]	0.774[a]
	(68.47)	(66.61)	(24.16)	(57.90)	(36.67)
D (Monday)	0.786[a]	−0.044	0.304[a]	0.266[a]	0.468[a]
	(2.58)	(−0.12)	(2.38)	(2.36)	(4.08)
D (Tuesday)	−1.02[a]	−1.885[a]	0.222	0.251[a]	0.183
	(−4.03)	(−5.36)	(1.92)	(2.28)	(1.87)

D (Wednesday)	0.210	−0.385	0.070	−0.001	0.090
	(1.06)	(−1.39)	(0.65)	(−0.09)	(0.87)
D (Thursday)	0.180	−0.405	0.430[a]	0.277[a]	0.428[a]
	(0.77)	(−1.10)	(3.12)	(2.20)	(3.19)
D (holiday)	1.267[a]	1.200[a]	0.688[a]	0.442[a]	0.764[a]
	(4.29)	(3.93)	(3.62)	(3.37)	(4.67)
Maturity	0.006	−0.009	−0.011[a]	−0.007[a]	−0.013[a]
	(0.99)	(−1.23)	(−2.21)	(−2.25)	(−4.06)
Autocorrelation(10)[b]	0.44	1.85[a]	0.40	0.76	0.27
M (day-of-the-week)[b]	0.04	0.36	0.04	0.39	0.70
V (day-of-the-week)[b]	55.97[a]	43.72[a]	32.78[a]	22.83[a]	45.84[a]
V (seasonality)[b]	11.10[a]	6.20	5.52	5.61	10.37
Ljung–Box(15)[c]					
$e(t)/h(t)$	12.95	8.80	11.40	8.89	8.39
$e(t)^2/h(t)^2$	26.44	19.87	8.27	5.58	17.63
BDS tests ($\varepsilon = \sigma$)					
Raw data					
dimensions = 3	5.54[a]	3.39[a]	0.76	1.34	0.29
dimensions = 6	23.54[a]	12.94[a]	2.82[a]	5.55[a]	1.11
dimensions = 9	59.63[a]	29.11[a]	5.80[a]	12.67[a]	2.19[a]
Standardized residuals					
dimensions = 3	1.49[a]	0.72	−1.07	−1.00	0.46
dimensions = 6	3.80[a]	1.66[a]	−3.06[a]	−2.99[a]	2.56[a]
dimensions = 9	4.70[a]	2.63[a]	−4.17[a]	−4.25[a]	8.20[a]

[a]The null hypothesis of a normal distribution is rejected.

[b]The test statistics are Wald F-statistics.

[c]Test statistics are asymptotically distributed as chi-squared. Significance is based on five degrees of freedom for $e(t)/h(t)$ and thirteen degrees of freedom for $\hat{e}(t)^2/h(t)^2$ under the null hypothesis of no autocorrelation.

Model Validation

This study assumes that the standardized residuals of the GARCH model are Gaussian white noise. Second-order dependence in standardized residuals, $\hat{e}_t/\hat{h}_t$ is detected only for coffee. The results imply that the lag structure of the conditional variance is correctly identified in most cases.

In every case, the BDS test for the raw data (Table IV) rejects the null hypothesis of i.i.d. Such a finding is consistent with deterministic chaos. The GARCH model, however, removes considerable serial dependence in the raw data. The BDS test statistics for the rescaled data indicate significant dependence for eight contracts. In two of these cases, the BDS statistic is negative, which is not consistent with deterministic chaos. Results for about half of the commodities are consistent with deterministic chaos.[3] Neither the Ljung–Box nor the BDS test for the rescaled data reject the null hypothesis of independence for six cases. A higher order GARCH process may be appropriate when the Ljung–Box test is rejected.

Table V contains Kolmogorov–Smirnov tests of normality and goodness-of-fit tests for the rescaled data from the GARCH model. Estimated skewness for the rescaled data from the GARCH model is significant in 10 of 15 cases compared to 12 of 15 for the raw data. One explanation for skewness is asymmetry in price movements. Traders generally believe that prices fall faster than they rise which would result in negative skewness. However, soybean meal and the two wheat futures have positive skewness so asymmetry does not seem to be a complete explanation of the skewness. The GARCH model's inability to remove skewness is no surprise, because the model can capture skewness only through the exogenous factors. In contrast, the GARCH model reduces leptokurtosis considerably. The most dramatic reduction in kurtosis is for the NYSE index futures from 170.8 to 10.1. The other two stock indices, S&P 500 and Value Line show similar reductions. Only oats passes the Kolmogorov–Smirnov test of fit.

Thus, the GARCH models do not model precisely the distributions of daily futures price changes under consideration in this study. However, the models do remove most of the dependence and reduce the observed leptokurtosis considerably. The hypothesis tests in Table IV are asymptotically valid via the central limit theorem if the variance is finite, even though the residuals are still leptokurtic, but are not asymptotically valid if unexplained heteroskedasticity or other nonlinearity remains.

CONCLUSIONS

The daily price changes considered in this study are largely linearly independent, but nonlinearly dependent. All of the estimated GARCH terms are significant at the 1% level, which implies variances of price changes are autocorrelated. The BDS test supports serial dependence in daily price changes. The BDS test rejects the null hypothesis of i.i.d. for all the original data. Moreover, no distribution is normal. They are slightly skewed and have fat tails.

[3]In his study of stock prices, Hsieh (1991) argues that remaining nonlinearity is in the variance and not in the mean. This may not be true for futures prices. Past research [Irwin and Brorsen (1985); Lukac and Brorsen (1989)] shows that trend-following technical trading systems yield positive gross returns (research is not conclusive about whether returns net of transaction costs are positive). Little linear correlation is found in the mean here. Thus, technical analysis yielding positive gross returns suggests possible nonlinear dynamics in the mean.

Table V
NORMALITY AND GOODNESS-OF-FIT TESTS WITH STANDARDIZED GARCH RESIDUALS

	Skewness	Kurtosis	D_{max}
Corn	0.065	0.888	0.018[a]
Coffee	−0.120[a]	1.003[a]	0.033[a]
Oats	−0.059	0.473[a]	0.016
Soybean	−0.017	0.496[a]	0.021[a]
Soybean meal	0.162[a]	1.038[a]	0.034[a]
Wheat (Chi)	0.141[a]	1.032[a]	0.029[a]
Wheat (KC)	1.838[a]	27.349[a]	0.056[a]
Copper	−0.297[a]	1.885[a]	0.030[a]
Gold (NY)	0.026	2.093[a]	0.056[a]
Palladium	0.026	1.071[a]	0.037[a]
Platinum	−0.155[a]	0.386[a]	0.024[a]
Silver	0.113[a]	1.352[a]	0.031[a]
NYSE	−1.082[a]	10.132[a]	0.064[a]
S&P 500	−1.058[a]	11.329[a]	0.057[a]
Value Line	−0.748[a]	4.427[a]	0.057[a]

[a]The null hypothesis of a normal distribution is rejected.

The hypothesis of low-dimension deterministic chaos is not supported by Brock's residual test. Estimated dimensions for rescaled data are either larger than those for the raw data, or estimates for both series are increasing with embedding dimensions. This result favors the stochastic approach when determining the return generating process. But, the results using the preferred BDS test statistic are mixed. The GARCH residuals are not i.i.d. for about half of the commodities, which is consistent with deterministic chaos.

The GARCH(1, 1) process is not a perfect model of the data. Kolmogorov–Smirnov test of fit rejects the GARCH(1, 1) process for all cases. Results suggest a higher order GARCH process may be appropriate for some price series. However, the model provides great improvements. Nonlinear dependence in the raw data and leptokurtosis are reduced. For the price series for which the GARCH model is able to remove dependence, the hypothesis tests for market anomalies are asymptotically valid even though the distribution is not normal provided the variance of the error term is finite and the model is specified correctly.

The most prominent market anomaly is day-of-the-week effects on variance. The variance is larger on Mondays and after holidays. This offers some partial support for the calendar-time hypothesis. Several commodities, especially agricultural commodities, exhibit seasonality in variance. A few commodities show significant autocorrelation, day-of-the-week in mean, or maturity effects on variance. In no case is the variance of the futures prices significant in the mean equation.

The results provide strong support for the existence of conditional heteroskedasticity, but they also show the GARCH model is not totally adequate. The results do not provide strong support for or against deterministic chaos. Results for about half of the commodities examined are consistent with deterministic chaos.

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