The Impact of Delivery Options on Futures Prices: A Survey

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INTRODUCTION

Futures contracts often possess options as to what, when, where, and how much of the underlying commodity is delivered. Because of their potential impact on futures prices, these delivery options (known as the quality, timing, location, and quantity options, respectively) have attracted considerable attention from academicians and practitioners alike. The trader who controls these options is the short (or seller), not the long (or buyer). Thus, if they are valuable, it is the short who benefits, and the futures price should be bid down to compensate the long for the additional delivery risk.

To illustrate the type of delivery options available, consider the Chicago Board of Trade (CBOT) Treasury bond futures contract. This contract is the most actively traded contract in the United States. It contains four well-known options that give the short considerable flexibility regarding delivery. These consist of a quality option and three timing options known as the accrued interest, wild card, and end-of-month options.

The quality option allows the short to deliver any Treasury bond with at least 15 years remaining until maturity or its first call date. The accrued interest option allows the short to deliver on any business day of the delivery month. If the coupon rate for the cheapest-to-deliver bond differs significantly from prevailing short-term interest rates, the short's profits could depend on whether delivery occurs early or late in the month. The wild card option allows the short to announce delivery as late as 8 PM Chicago time, based on an invoice price established immediately after futures trading ends at 2 PM. Since the spot market for Treasury bonds stays open after 2 PM, the short can use information that arrives between 2 and 8 PM in deciding when to deliver. The end-of-month option arises because the last trading day for an expiring contract is the 8th-to-last business day of the delivery month. Contracts open after that date must be delivered on one of the last 7 business days. Since the settlement price for the last trading day determines invoice prices for that day and the 7 remaining business days, the short can observe spot prices over this interval and choose when to deliver.[1]

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[1]For more details regarding the Treasury bond futures contract and its delivery process, see Chicago Board of Trade (1980, 1986, 1987).

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Because of their potential impact on futures prices, delivery options can affect the hedging performance and ultimate success of a futures contract. Johnston and McConnell (1989) provide striking evidence of this for the Government National Mortgage Association (GNMA) Collateralized Depository Receipt (CDR) contract at the Chicago Board of Trade. In the late 1970s and early 1980s this contract was actively traded. However, when mortgage rates fell sharply after 1982, high-coupon mortgages had a tendency to be repaid early. Quality option values increased dramatically, often exceeding 5% of the futures price and climbing as high as 19%. This led to a marked drop in the contract's hedging performance, and by 1987 trading volume for the GNMA CDR contract was virtually nonexistent.

Delivery options can also lead to spurious conclusions involving market efficiency. Any test of market efficiency depends on the model of market equilibrium used. Because futures pricing models usually fail to incorporate delivery options, apparent evidence of market efficiency might simply reflect the inadequacy of the pricing model. Alternatively, a trader who believes he has identified an arbitrage opportunity could see his arbitrage profits vanish if he fails to take delivery options into account.[2]

Similarly, conclusions regarding price discovery and risk premia could depend on underlying assumptions regarding delivery options. For instance, Fama and French (1987) examine these topics using a regression methodology that effectively ignores timing and quality options. They assume that futures contracts expire on the first trading day of the delivery month. They then use the price of the expiring futures contract on that date to proxy for the corresponding spot price. If timing and quality options have significant value, however, ignoring them could affect the regression results and their interpretation substantially.

As awareness of delivery options and their potential impact has increased, so has the volume of research involving such options. This literature has grown increasingly complex, reflecting recent advances in option pricing theory and empirical methods. Nevertheless, disagreement still remains over the value and impact of delivery options. It therefore seems appropriate to step back and assess the current state of affairs.

Accordingly, this article surveys the theoretical and empirical research involving delivery options, with emphasis on the quality and timing options in the CBOT Treasury bond futures contract. This article does not provide new findings. The goal is to objectively appraise the existing literature which will assist those working (or desiring to work) in this field by providing background material, summarizing known results, and suggesting directions for future research. Not every relevant article is reviewed in detail, but an extensive list of references is included for readers wishing to pursue this topic further.

The article is organized as follows: The next section discusses the valuation and estimation of quality and timing options. The third section presents theoretical and empirical results regarding their impact on future prices, and the fourth section summarizes research findings to date and suggests possible avenues for future study. For convenience, the Appendix provides capsule summaries of the theoretical and empirical research on delivery options.

[2]Readers familiar with the efficient markets literature might recall that far more research has been done on the Treasury bill contract than on the Treasury bond contract. This is no accident. The Treasury bill contract contains no delivery options, making it relatively easy to price. To price Treasury bond futures, most studies ignore one or more of the available delivery options. (Gay and Manaster (1991) is a notable exception, however.) For a comprehensive discussion of interest rate futures market efficiency, see pp. 361–370 of Kolb (1990). For an example of how failure to incorporate delivery options in soybean futures pricing models can lead to apparent market inefficiency, see Kamara (1990).

THE ANALYTICAL FRAMEWORK

This section discusses several methods used in the valuation and estimation of delivery options. Each has its advantages and disadvantages and opinions differ as to which method is best. One reason for the lack of concensus is that unlike most options, delivery options are not traded in an organized market. Consequently, one cannot simply compare theoretical delivery option prices to actual market prices. One must resort to less direct methods for judging the performance of a particular valuation formula.

Two major assumptions are made to simplify the analysis—capital markets are perfect and futures prices equal forward prices. The first assumption allows one to ignore taxes, transaction costs, short selling restrictions, and other market frictions. The second assumption allows one to price futures contracts as if they are forward contracts, thus ignoring the daily resettlement feature possessed by futures contracts.[3]

Without loss of generality, this article focuses entirely on quality and timing options.[4] Unfortunately, these two options are usually intertwined in a given contract. If a contract contains one of these options, it generally contains the other. Empirically, one can usually observe only the joint effect of these options, not their individual effects. The inability to separate the individual effects of these two options is a fundamental problem, and none of the methods discussed here provides a completely satisfactory solution.

Valuation of the Quality Option

If the quality option (or any other delivery option) has value, then its presence must lower the equilibrium futures price from what it would be otherwise. This is easily shown by considering two futures contracts, identical in every respect except that the first contract requires delivery of a particular grade of the underlying commodity while the second allows multiple delivery grades. Suppose that the first contract trades at the price F_1 and the second contract trades at the price F_2. Then $F_1 \geq F_2$ and equality holds if, and only if, the quality option has no value.

To prove this claim, suppose that $F_1 < F_2$. Then one can obtain an arbitrage profit by going long in the first contract and short in the second contract. This strategy (which is outlined in Table I) requires no initial investment, and there are no cash flows until contract expiration. To settle the first contract at expiration, one pays F_1 and receives the particular grade specified by the first contract. One can simultaneously deliver that grade and receive F_2 to settle the second contract. Thus, one has locked in the arbitrage profit $F_2 - F_1$ at expiration, which implies that the initial assumption $F_1 < F_2$ must be false.

[3]For theoretical analyses relating daily resettlement to the futures-forward price differential, see Cox, Ingersoll, and Ross (1981), Jarrow and Oldfield (1981), Richard and Sundaresan (1981), and French (1982). For empirical analyses see Cornell and Reinganum (1981), French (1983), and Elton, Gruber, and Rentzler (1984). The empirical evidence generally suggests that the impact of daily resettlement on this price differential is both statistically and economically insignificant. Although French (1983) finds a statistically significant difference between futures and forward prices for silver and copper, the theoretical models developed in Cox, Ingersoll, and Ross (1981), Richard and Sundaresan (1981), and French (1982) do not help to explain these differences. In addition, although Kamara (1988) finds a significant price differential for Treasury bills, he concludes that differences in market trading structures explain this differential, not daily resettlement.

[4]Clearly, the location option is an interesting and important topic for agricultural futures. It is not analyzed here because it can be viewed as a type of quality option. Suppose that a futures contract calls for the delivery of one particular grade of wheat, but allows the delivery point to vary. Different delivery points lead to variation in spot prices and storage or transportation costs which can be modeled by defining different qualities of wheat, one for each eligible delivery point. As for the quantity option, various authors [Cox, Ingersoll, and Ross (1981, p. 345), Gay and Manaster (1984, p. 354)] have noted that it will have no effect on futures prices given the assumption of perfect capital markets. One can always deliver the standard amount by adjusting the number of contracts accordingly.

Table 1
THE REDUCTION IN FUTURES PRICES DUE TO THE PRESENCE OF THE QUALITY OPTION[a]

Strategy	Cash Flow at Initiation	Cash Flow at Expiration
Buy a futures contract that requires delivery of the *i*th grade of the underlying commodity	0	$S_i(T) - F_1(t,T)$
Sell a futures contract that allows delivery of any grade of the underlying commodity	0	$F_2(t,T) - \min_{1 \le j \le n}\{S_j(T)\}$
	0	$[F_2(t,T) - F_1(t,T)] + [S_i(T) - \min_{1 \le j \le n}\{S_j(T)\}]$[b]

[a]This table assumes that capital markets are perfect and that futures and forward prices are equal. It also ignores institutional details such as conversion factors, margins, etc. The futures contracts are identical in every respect except that the first contract requires delivery of a particular grade (the ith grade) of the underlying commodity while the second contract allows multiple delivery grades (grades 1, . . . , n). Futures prices at time t are denoted by $F_1(t,T)$ and $F_2(t,T)$, respectively, for contracts expiring at time T. The spot price of the jth grade of the underlying commodity at time T is given by $S_j(T)$.

[b]The second component of this payoff, i.e., the value at time T_1 of an option to deliver any deliverable grade rather than the ith deliverable grade, must have nonnegative value. Because the strategy costs nothing to initiate, the first component of this payoff cannot exceed zero or an arbitrage opportunity would exist. Hence, $F_1(t,T) \ge F_2(t,T)$.

It remains to show that $F_1 = F_2$ if, and only if, the option is worthless. Reconsider the preceding strategy. The option is worthless if, and only if, the grade required by the first contract always is the optimal one to deliver for the second contract. That implies that the option is worthless if, and only if, the strategy always yields the fixed payoff $F_2 - F_1$ at expiration. However, the strategy requires no initial investment and yields no other cash flows. If an investment strategy costs nothing to initiate and yields a single fixed payoff, that payoff must be zero. Therefore, $F_1 = F_2$ if, and only if, the option is worthless.

The quality option has value if, and only if, its presence lowers the equilibrium futures price from what it would be otherwise. It remains to determine how valuable this option really is. Clearly, if the option is valuable, its presence should lower the corresponding futures price. But by what amount?

Consider a futures contract that contains a quality option, but no timing option. The contract is initiated at time t and calls for delivery at time T. Assume that n grades of the underlying commodity are eligible for delivery. Furthermore, assume that the size and timing of payouts associated with the underlying commodity over the interval $[t, T]$ are known at time t.[5] The following notation is used:

$S_i(t)$ = the spot price of the ith deliverable grade at time t,
$F(t, T)$ = the futures price at time t for a contract expiring at time T that requires the short to deliver at time T,
$R(t, T)$ = the nonannualized risk-free rate of return from time t to time T,
$P_i(t, T)$ = the value at time t of all payouts associated with the ith grade from time t to time T,
$V_i(t, T)$ = the value at time t of an exchange option, which allows the short to deliver any deliverable grade rather than the ith deliverable grade at time T.

Consider the investment strategy initiated at time t which is outlined in Table II. There are four components:[6]

1. Pay $S_i(t)$ to buy the ith deliverable grade,
2. Take a short futures position to deliver at time T for the price $F(t, T)$,
3. Receive $V_i(t, T)$ for writing the corresponding exchange option contract,
4. Borrow $F(t, T)/[1 + R(t, T)] + P_i(t, T)$ at the risk-free rate with repayment at time T.

What are the cash flows associated with this strategy? There are four components to the cash flow at expiration. There is an inflow of $S_i(T) + P_i(t, T)[1 + R(t, T)]$ from selling the ith grade of the commodity. There is an inflow of $F(t, T) - S_i(T) + V_i(T, T)$ from delivering the cheapest grade of commodity to settle the futures contract. There is an outflow of $V_i(T, T)$ from covering the exchange option position. There is an outflow

[5]These payouts can be positive, such as coupon payments for Treasury bonds and dividends for stock indexes. Or they can be negative, such as storage costs for agricultural commodities. Sometimes the underlying commodity is held for the physical services it offers, not just its investment potential. For instance, a wheat processor might hold wheat to avoid plant shutdowns due to wheat shortages. This incremental value, which is separate from its pure investment value, is called a convenience value. Convenience values are not incorporated in this analysis. For a detailed discussion of convenience values and their impact on futures prices, see pp. 88–95 of Siegel and Siegel (1990).

[6]The sophisticated reader undoubtedly realizes that certain technical details are omitted. The actual strategy and notation can be more complex. With Treasury bond futures, for instance, one must include conversion factors in the analysis. Such details are omitted because they are secondary in importance and yield little insight. It is straightforward to modify the analysis given here to take them into account.

Table II
A FUTURES PRICING EQUATION THAT INCORPORATES THE QUALITY OPTION[a]

Strategy	Cash Flow at Initiation	Cash Flow at Expiration
Pay $S_i(t)$ to buy the ith deliverable grade	$-S_i(t)$	$S_i(T) + P_i(t,T)[1 + R(t,T)]$
Take a short futures position at time t	0	$F(t, T) - S_i(T) + V_i(T,T)$
Write the corresponding exchange option contract	$V_i(t,T)$	$-V_i(T,T)$
Borrow the amount $F(t,T)/[1 + R(t,T)] + P_i(t,T)$ at the risk-free rate	$F(t,T)/[1 + R(t,T)] + P_i(t,T)$	$-F(t,T) - P_i(t,T)[1 + R(t,T)]$
	$-S_i(t) + V_i(t,T) + F(t,T)/[1 + R(t,T)] + P_i(t,T)$[b]	0

[a]This table assumes capital markets are perfect, futures and forward prices are equal, and interest rates are deterministic. It also ignores institutional details such as converson factors, margins, etc. The futures contract allows multiple delivery grades (grades 1, . . . ,n) of the underlying commodity to be delivered. The futures price at time t is $F(t,T)$ for a futures contract expiring at time T. $S_i(T)$ denotes the spote price of the jth grade of the underlying commodity at time T. $R(t,T)$ denotes the nonannualized risk-free rate of return from time t to time T. $P_i(t,T)$ is the value at time t of all payouts associated with the ith deliverable grade from time t to time T. $V_i(t,T)$ denotes the value at time t of an exchange option, which allows the short to deliver any deliverable grade rather than the ith deliverable grade at time T.

[b]This initial cash flow must equal zero or an arbitrage opportunity would result. Therefore, $F(t,T) = [S_i(t) - P_i(t,T) - V_i(t,T)][1 + R(t,T)]$, and eq. (2) holds as desired.

of $F(t, T) + P_i(t, T)[1 + R(t, T)]$ from repaying the loan. Hence, the total cash flow at time T is zero.

The above exhibits an investment strategy that yields a zero payoff at time T that is known with certainty at time t. To prevent arbitrage, the initial outlay for this strategy at time t must equal zero. Because the futures contract costs nothing to initiate, there are three nonzero components to the total cash flow at time t. There is an outflow of $S_i(t)$ from buying the ith deliverable grade. There are cash inflows of $V_i(t, T)$ from writing the option contract and $F(t, T)/[1 + R(t, T)] + P_i(t, T)$ from borrowing. Summing these cash flows and equating to zero yields

$$-S_i(t) + V_i(t, T) + F(t, T)/[1 + R(t, T)] + P_i(t, T) = 0 \tag{1}$$

which in turn gives the futures pricing equation

$$F(t, T) = [S_i(t) - P_i(t, T) - V_i(t, T)][1 + R(t, T)] \tag{2}$$

As noted earlier, the presence of delivery options lowers the equilibrium futures price. If there is only one deliverable grade, i.e., $n = 1 = i$, then $V_i(t, T)$ equals zero and eq. (2) reduces to the familiar cost of carry model for futures prices:[7]

$$F(t, T) = [S(t) - P(t, T)][1 + R(t, T)] \tag{3}$$

Estimation of the Quality Option

The preceding analysis suggests several methods for pricing or estimating the quality option. Perhaps the most obvious approach is to use an exchange option pricing formula such as the one developed by Margrabe (1978). The Margrabe formula assumes that there are two assets, but higher-dimensional extensions of the formula exist.[8] This approach has been used by various researchers, including Gay and Manaster (1984), Boyle (1989), and Hemler (1990), to estimate quality option values numerically. One drawback with this approach, however, is that it assumes the relevant asset prices follow geometric Brownian motion. This is done for tractability; it allows one to calculate closed-form solutions easily. But from a theoretical standpoint, geometric Brownian motion is less desirable, especially to examine the quality option in the Treasury bond futures contract.[9]

Another method that has been used is to calculate payoffs from exercising the quality option at contract expiration. These payoffs can be simulated payoffs as in Kane and Marcus (1986a) or they can be historical payoffs based on actual market data as in Hegde (1990a) and Hemler (1990). This method has the advantage of not depending on any particular option valuation formula. Unfortunately, although this method sheds light on the option value at expiration, it cannot price the option prior to expiration.

A third method is to calculate an implied option value $V_i(t, T)$ by using eq. (2)—one solves for $V_i(t, T)$ in terms of the other observable variables. This allows one to estimate the value of the option prior to expiration without invoking a sophisticated model such as the Margrabe (1978) exchange option pricing formula. The approach has been used by Hegde (1988) and Hemler (1990) in the context of Treasury bond futures. A disadvantage with this method, however, is that the estimate is a residual. That is, the estimate is

[7]Both here and later in the article, the subscripts are dropped from $P_i(t, T)$, $S_i(t)$, and $V_i(t, T)$ when there is only one deliverable grade.

[8]For details, see Margrabe (1982), Chowdhry (1986), Johnson (1987), Hemler (1988), and Cheng (1990).

[9]The quality option in the Treasury bond futures contract is a type of interest rate option. But the use of geometric brownian motion is problematic for interest rate options. Interest rates exhibit mean-reversion; geometric brownian motion does not. Moreover, bond price volatility must approach zero as the bond approaches maturity because the terminal value of the bond is known with certainty. If bond prices follow geometric brownian motion, however, then bond price volatility is constant.

"backed out" of the equation in terms of the other variables. This implies that any misspecification in eq. (2) is transmitted to the quality option estimate. For example, since eq. (2) ignores the timing option, quality option estimates obtained using this method could actually represent the joint effect of quality and timing options.

Yet another method for estimating the quality option is to calculate payoffs obtained from a "switching strategy." Suppose that one continually switches from one delivery grade to another depending on which grade is cheapest. One generates a stream of cash flows which can be viewed as the profits from a switching strategy. This method has been used for Treasury bond futures by Barnhill and Seale (1988a, 1988b) and Hegde (1990a). Although it has simplicity as an advantage, this method cannot price the option prior to expiration. Moreover, while measurement error is always worrisome, it is especially so here. One poor quote can yield two apparent profit opportunities that do not really exist, leading to an upward bias in estimated payoffs. For example, one might switch because of a bad quote, then switch back again later when quotes are back in line. The two "profits" obtained from switching would not actually exist.

Although other methods have been used, the above are the most representative. Variations are clearly possible. For instance, in the case of Treasury bond futures, different bond pricing models can be used, which in turn leads to differences in exchange option estimates. But estimates are usually based on one of the methods just described. For convenience, they shall be referred to as the exchange option, terminal payoff, implied value, and switching option methods, respectively.

Valuation of the Timing Option

For simplicity, assume that there is but one grade of the underlying commodity eligible for delivery. Also assume that instead of a fixed delivery date T, there is a delivery interval $[T_1, T_2]$. Then depending on one's viewpoint, one either has an option to deliver late or an option to deliver early. The objective is to determine how the value of these options depends on factors such as spot prices, the risk-free rate, and the payouts associated with the underlying commodity.

Begin with the option to deliver late. Consider two futures contracts that are identical in every respect except that the first requires delivery at time T_1 and the second allows delivery anytime during the interval $[T_1, T_2]$. For $t \leq T_1$, let F_1 and F_2 represent prices at time t for the first and second contracts, respectively.

It is clear that $F_1 \geq F_2$. If not, one could obtain an arbitrage profit by going long in the first contract and short in the second contract. This strategy (which is outlined in Table III) requires no initial investment, and there are no cash flows until time T_1. At time T_1 one can settle the first contract by paying F_1 and receiving the underlying commodity. One can simultaneously deliver the commodity and receive F_2 to settle the second contract. Thus, one has locked in an arbitrage profit $F_2 - F_1$ at time T_1, which implies that the initial assumption $F_1 < F_2$ must be false.

When does the equality $F_1 = F_2$ hold? One special case occurs when there are no payouts associated with the underlying commodity and there is but one delivery grade.[10] Under these assumptions it is optimal to deliver as early as possible with the second contract. The intuition is that postponing delivery involves a trade-off between receiving payouts associated with the underlying commodity and foregoing interest income one would receive from investing the futures price at the risk-free rate. The greater the payouts, the more likely that postponing delivery is optimal for the short. When there are

[10]This case is analyzed by Boyle (1989) who also considers the case where there are multiple delivery grades.

Table III
THE REDUCTION IN FUTURES PRICES DUE TO THE PRESENCE OF THE OPTION TO POSTPONE DELIVERY[a]

Strategy	Cash Flow at Initiation	Cash Flow from Settling Contracts at Start of Delivery Period
Buy a futures contract that requires delivery at the start of the delivery period	0	$S(T_1) - F_1(t, T_1)$
Sell a futures contract that allows delivery anytime during the delivery period	0	$F_2(t, [T_1, T_2]) - S(T_1)$
	0	$F_2(t, [T_1, T_2]) - F_1(t, T_1)$[b]

[a]This table assumes that capital markets are perfect and that futures and forward prices are equal. It also ignores institutional details such as conversion factors, margins, etc. The futures contracts are identical in every respect except that the first contract requires delivery at the start of the delivery period $[T_1, T_2]$ while the second contract allows delivery anytime during the delivery period $[T_1, T_2]$. Futures prices at time t are denoted by $F_1(t, T_1)$ and $F_2(t, [T_1, T_2])$ for the respective contracts. The spot price of the underlying commodity at time T_1 is given by $S(T_1)$.

[b]Because this strategy cost nothing to initiate, the payoff $F_2(t, [T_1, T_2]) - F_1(t, T_1)$ cannot exceed zero or an arbitrage opportunity would exist. Hence, $F_1(t, T_1) \geq F_2(t, [T_1, T_2])$ as desired.

no payouts, one gains nothing by postponing. One only loses the interest that could be received by investing the futures price earlier rather than later. So the option to postpone delivery is worthless and $F_1 = F_2$. In general, however, it is difficult to characterize when this equality holds.

Reconsider the strategy of going long in the first contract and short in the second. Assume that one settles the second contract at time $T \in [T_1, T_2]$. (One does not have any choice with the first contract—it must be settled at time T_1.) Table IV summarizes the cash flows associated with this strategy. The second contract yields a cash flow of $F_2 - S(T)$ at time T. The first contract yields a cash flow of $S(T_1) - F_1$ at time T_1. However, at time T_1 one can buy the commodity for $S(T_1)$ and borrow F_1 at the risk-free rate to obtain

$$S(T) + P(T_1, T)[1 + R(T_1, T)] - F_1[1 + R(T_1, T)] \tag{4}$$

at time T. When one adds the payoff $F_2 - S(T)$ from the second contract, the total payoff at time T from the strategy equals

$$F_2 - [F_1 - P(T_1, T)][1 + R(T_1, T)] \tag{5}$$

Since this is known with certainty at time T_1, it is equivalent to receiving

$$F_2/[1 + R(T_1, T)] - [F_1 - P(T_1, T)] \tag{6}$$

at time T_1. Therefore, the optimal strategy at time T_1 is to maximize the payoff in (6) over $T \in [T_1, T_2]$.[11] Since the strategy costs nothing to implement, the maximum payoff cannot exceed zero, i.e.,

$$\max_{T \in [T_1, T_2]} \{F_2/[1 + R(T_1, T)] - [F_1 - P(T_1, T)]\} \leq 0 \tag{7}$$

[11]For simplicity, it is assumed that the payoff is a continuous function of T, so that a maximum exists.

Table IV
A FUTURES PRICING EQUATION THAT INCORPORATES THE OPTION TO POSTPONE DELIVERY[a]

Strategy	Cash Flow at Initiation	Cash Flow at Expiration of First Contract	Cash Flow at Time of Delivery T for Second Contract
Buy a futures contract that requires delivery at the start of the delivery period	0	$S(T_1) - F_1(t, T_1)$	
Sell a futures contract that allows delivery anytime during the delivery period	0	0	$F_2(t, [T_1, T_2]) - S(T)$
Buy underlying commodity upon delivering first futures contract		$-S(T_1)$	$S(T) + P(T_1, T)[1 + R(T_1, T)]$
Borrow $F_1(t, T_1)$ at risk-free rate upon delivering first contract		$F_1(t, T_1)$	$-F_1(t, T_1)[1 + R(T_1, T)]$
	0	0	$F_2(t, [T_1, T_2]) - [F_1(t, T_1) - P(T_1, T)][1 + R(T_1, T)]$[b]

[a]This table assumes capital markets are perfect, futures and forward prices are equal, and interest rates are deterministic. It also ignores institutional details such as conversion factors, margins, etc. The futures contracts are identical in every respect except that the first contract requires delivery at the start of the delivery period $[T_1, T_2]$ while the second allows delivery anytime during the period $[T_1, T_2]$. Futures prices at time t are denoted by $F_1(t, T_1)$ and $F_2(t, [T_1, T_2])$ for the respective contracts. $S(T)$ denotes the spot price of the underlying commodity at time T. $R(t, T)$ denotes the nonannualized risk-free rate of return from time t to time T. $P(t, T)$ is the value at time t of all payouts associated with the underlying commodity from time t to time T.

[b]Since this is known with certainty at time T_1, it is equivalent to receiving $F_2(t, [T_1, T_2])/[1 + R(T_1, T)] - [F_1(t, T_1) - P(T_1, T)]$ at time T_1. The optimal strategy at time T_1 is to maximize this last payoff over $T \in [T_1, T_2]$. Since the strategy costs nothing to initiate, the maximum payoff cannot exceed zero, i.e., $\max_{T \in [T_1, T_2]}\{F_2(t, [T_1, T_2])/[1 + R(T_1, T)] - [F_1(t, T_1) - P(T_1, T)]\} \le 0$, and (7) holds as desired.

Table V
THE REDUCTION IN FUTURES PRICES DUE TO THE PERESENCE OF THE OPTION TO DELIVER EARLY[a]

Strategy	Cash Flow at Initiation	Cash Flow from Settling Contracts at End of Delivery Period
Buy a futures contract that requires delivery at the end of the delivery period	0	$S(T_2) - F_1(t, T_2)$
Sell a futures contract that allows delivery anytime during the delivery period	0	$F_2(t, [T_1, T_2]) - S(T_2)$
	0	$F_2(t, [T_1, T_2]) - F_1(t, T_2)$[b]

[a]This table assumes that capital markets are perfect and that futures and forward prices are equal. It also ignores institutional details such as conversion factors, margins, etc. The futures contracts are identical in every respect except that the first contract requires delivery at the end of the delivery period $[T_1, T_2]$ while the second contract allows delivery anytime during the delivery period $[T_1, T_2]$. Futures prices at time t are denoted by $F_1(t, T_2)$ and $F_2(t, [T_1, T_2])$ for the respective contracts. The spot price of the underlying commodity at time T_2 is given by $S(T_2)$.

[b]Because this strategy costs nothing to initiate, the payoff $F_2(t, [T_1, T_2]) - F_1(t, T_2)$ cannot exceed zero or an arbitrage opportunity would exist. Hence, $F_1(t, T_2) \geq F_2(t, [T_1, T_2])$ as desired.

Inequality (7) gives insight as to when postponing delivery is optimal. If there are no payouts on the underlying commodity, then $P(T_1, T) = 0$ for all $T \in [T_1, T_2]$ and (7) implies $F_1 \geq F_2$. More generally, one can transform a bound on $P(T_1, T)$ to a bound on $F_1 - F_2$. The smaller the payout $P(T_1, T)$, the smaller the price differential $F_1 - F_2$.[12]

Focus now on the option to deliver early. Consider two futures contracts that are identical in every respect except that the first requires delivery at time T_2 and the second allows delivery anytime during the interval $[T_1, T_2]$. For $t \leq T_1$, let F_1 and F_2 represent prices at time t for the first and second contracts, respectively. Then $F_1 \geq F_2$, or else one could obtain an arbitrage profit by going long in the first contract and short in the second contract. Table V gives the cash flows associated with this strategy. At time T_2 one can settle the first contract by paying F_1 and receiving the underlying commodity. One can simultaneously deliver the commodity and receive F_2 to settle the second contract. This yields the arbitrage profit $F_2 - F_1$ at time T_2, which implies that the assumption $F_1 < F_2$ must be false. Thus, $F_1 \geq F_2$ as desired.

When is F_1 strictly greater than F_2? As in the case of the option to postpone delivery, it is difficult to specify necessary and sufficient conditions. But certain cases are easy to analyze and lend insight, such as when there are no payouts on the underlying commodity. When $P(T_1, T)$ is identically zero for all $T \in [T_1, T_2]$, it is always optimal to deliver as early as possible. By postponing delivery until after T_1, one receives the fixed price F_2 later rather than sooner. One gains nothing from holding the commodity.

[12]A more precisely stated result is the following: If there exists a $\delta > 0$ such that $P(T_1, T) \leq \delta$ for all $T \in [T_1, T_2]$, then there exists a $T^* \in [T_1, T_2]$ such that $F_1 - F_2 \leq \delta[1 + R(T_1, T^*)]$. The proof is straightforward.

Thus, $F_1 = S(t)[1 + R(t, T_2)]$ and $F_2 = S(t)[1 + R(t, T_1)]$, so that $F_1 > F_2$ whenever $T_1 < T_2$.

To summarize, the value of timing options allowing the short to deliver early or late depends on a trade-off between payouts on the underlying commodity and interest rates. The greater the payouts, the more likely that postponing delivery is optimal for the short. Thus, the greater the payouts, the larger the value of the option to postpone and the smaller the value of the option to deliver early. In the extreme case that no payouts are associated with the underlying commodity and only one grade of the commodity is deliverable, the option to deliver late must be worthless and the option to delivery early must be valuable.

Estimation of the Timing Option

Although the preceding theoretical analysis of timing options is simple, the estimation of their value based on market data is considerably more difficult. In the marketplace one can observe only the joint effect of all available delivery options. This makes it hard to determine how much of the joint value is due to a timing option and how much is due to a quality option. Individual effects are entangled. To illustrate, consider the Treasury bond futures contract. For a given delivery month, more than 30 bond issues have been eligible for delivery and as many as 19 issues have been delivered. So the quality option clearly has the potential to be of significant value. To make matters worse, there is not one, but three, timing options (the accrued interest, wild card, and end-of-month options) present. Suppose one wants to determine the incremental effect of the wild card option given the other three delivery options. It is then necessary to compare the current contract to a contract identical in all respects except that the short can no longer wait until 8 PM to declare delivery. Instead, he must declare delivery when the invoice price is established immediately after futures trading ends at 2 PM.[13]

To be more precise, when researchers use market data to measure the impact of delivery options, they are measuring a function $f(t, n, [T_1, T_2])$ with three arguments: t (the time when the measurement is taken), n (the number of grades eligible for delivery), and $[T_1, T_2]$ (the time period during which delivery can occur). When researchers talk about a quality option, they generally are talking about $f(t, n, [T_1, T_2])$ given some fixed delivery interval $[T_1, T_2]$. In fact, when focusing on the quality option, researchers have usually assumed $T_1 = T_2$. That is, delivery can only occur on a single day. When focusing on timing options, researchers have generally held n, the number of deliverable grades, constant. Unfortunately, because different authors have made different assumptions regarding n and $[T_1, T_2]$, definitions of quality and timing options are not uniform throughout the literature. Care must be exercised when comparing results across studies.

One can use a similar approach to handle more complex situations. Reconsider the problem of estimating the incremental value of the wild card option in the Treasury bond futures contracts. The short can decide to deliver on any business day between 8 PM and 8 PM. Define the following three sets of time:

$S_1 =$ the union over all business days of the time intervals from 8 AM to 2 PM

$S_2 =$ the union over all business days of the time intervals from 2 PM to 8 PM

$S =$ the union over all business days of the time intervals from 8 AM to 8 PM

[13]Cohen (1990) comes the closest to actually performing an incremental analysis of this wild card option. He obtains an upper bound for its value and presents simulation evidence that the value is less than 0.1 percentage points of par.

With the Treasury bond contract, the short can decide to deliver at any time $T \in S$. If there is no wild card option available, the short can deliver at any time $T \in S_1$. Hence, $f(t, n, S) - f(t, n, S_1)$ is the incremental value of the wild card option. One could analyze the accrued interest and end-of-month options similarly by decomposing S_1 into two parts, one for the last 7 business days and one for the rest of the month.[14]

Once a definition for the timing option is decided upon, there is still the question of how to gauge its importance. Most empirical timing option studies have used one of three approaches. The most common approach calculates payoffs from some trading strategy over a time period up to and including the expiration date. Examples of this approach include Gay and Manaster (1986) and Kane and Marcus (1986b). Such an analysis cannot value the option prior to expiration, however. To actually value the option prior to expiration, one must resort to option pricing techniques, such as the numerical option pricing method utilized by Boyle (1989). Peck and Williams (1990) use yet another approach. Using logistic regression analysis, they relate the timing of deliveries throughout the delivery month to economic fundamentals. Such fundamentals include the nearby futures spread (the difference in futures prices for the expiring futures contract and the next contract), the basis (the difference between the spot price and the futures price for the expiring contract), and the level of interest rates.

THEORETICAL AND EMPIRICAL RESULTS

Representative research findings are divided for discussion into three periods. The early work of Kilcollin (1982) and Garbade and Silber (1983) is considered first. Then the pivotal studies of Gay and Manaster (1984, 1986) and Kane and Marcus (1986a, 1986b) are discussed and, finally, more recent findings are reported.

Kilcollin (1982) and Garbade and Silber (1983)

Kilcollin (1982) considers two methods for determining the premium or discount associated with different grades of the commodity underlying a futures contract. These methods (known as difference systems) are based on security yields. Under either system, nonpar securities are deliverable at the futures price multiplied by a factor. The methods differ in terms of the factor used. With the variable rate or yield maintenance approach, the factor is the price of the delivered security relative to the par security where both are discounted at the yield of the par security selling at the futures price. With the fixed rate or factor pricing approach, the factor is the price of the delivered security relative to the par security where both are discounted at the yield of the par security selling at par.

Kilcollin characterizes financial futures market equilibrium with yield-based difference systems. He finds that both difference systems provide optimal delivery securities and yield financial futures prices that systematically understate the cash market price of the par security. When he compares the two systems historically for GNMA futures, he finds that factor pricing leads to lower-coupon longer-maturity optimal delivery securities and less backwardation than does yield maintenance. Finally, he concludes that such biases are unavoidable; there is no ideal system that can eliminate these biases and simultaneously guard against market failures.

While they also examine futures contracts for commodities with multiple grades, Garbade and Silber (1983) focus on the use of such contracts for hedging. They develop a

[14]This decomposition is debatable. It is inconsistent with the usual definition of the accrued interest option, which is generally considered to be the short's right to choose any business day of the delivery month to deliver. Things are redefined here slightly so that the total value from all three timing options is broken into three mutually exclusive and exhaustive components.

simple model showing that if multiple delivery grades exist, then a hedged position in at least one must possess some unhedgeable risk. Then they illustrate the existence of such residual risk using Monte Carlo simulation of long cash-short futures positions. When they examine how risk exposure is related to the difference system, they find no rationale for setting contract premiums and discounts equal to actual commercial premiums and discounts.

Gay and Manaster (1984, 1986) and Kane and Marcus (1986a, 1986b)

Gay and Manaster (1984) are also aware of the difference systems used with futures contracts for commodities with multiple grades. However, unlike the previous authors, Gay and Manaster are primarily interested in the impact of the quality option on futures prices. They value the quality option in CBOT wheat futures contracts by using the Margrabe (1978) exchange option formula mentioned in the second section. This allows them to value the option prior to expiration, not just calculate payoffs at expiration. They conclude that this option does have a significant impact on futures prices, lowering them by approximately 2.2% on average over the life of the contract.

Using a different approach from Gay and Manaster (1984), Kane and Marcus calculate average simulated payoffs from exercising the option at expiration; they do not value the option prior to expiration.[15] Based on a 3-month horizon, they find that the value of the quality option ranges from 1.39 to 4.60 percentage points of par for four delivery months from 1981–1983. These are average values based on 10,000 Monte Carlo simulations using historically estimated term structures from the period 1978–1982, estimated volatilities for those term structures, and the actual sets of delivery-grade bonds available during this period.

Both the Gay and Manaster (1986) and Kane and Marcus (1986b) papers deal with timing options in the Treasury bond futures contract. The former examines the wild card, end-of-month, and accrued interest options; the latter examines only the wild card option. Both studies discuss optimal trading strategies for the short to exploit these options. In addition, Kane and Marcus present a valuation model for the wild card option and compute option value estimates.

Gay and Manaster find that their optimal delivery policy produces profits that are positive and statistically significant. That is, futures prices are "too high" in the sense that the short can earn profits by exercising his delivery options appropriately. When they compare their optimal delivery policy with the actual policies observed in the market, they find that traders appear to depart substantially from the optimal strategy.

Kane and Marcus find that the wild card option is worth approximately 0.2 percentage points of par at the start of the delivery month; it then falls to zero in value at the end of the month. They also derive an optimal trading policy which differs from that of Gay and Manaster. Unlike Gay and Manaster, they do not provide evidence as to how well their proposed strategy works and how much it differs (if at all) from the actual delivery policy followed by traders.[16]

[15] Benninga and Smirlock (1985) also study this particular quality option, but do not price it explicitly. Instead, they provide regression evidence that delivery options affect futures prices, where they use price sensitivity to proxy for the value of the option.

[16] As noted earlier in Footnote 13, Cohen (1990) also estimates the value of the wild card option and finds it to be less than 0.1 percentage points of par. LaBarge (1989) replicates the Kane and Marcus (1986b) study with actual bond data. The estimates vary considerably by time period, but are generally much smaller than those of Kane and Marcus. Arak and Goodman (1987) value the wild card option using the Black and Scholes (1973) model; they also obtain much smaller values than Kane and Marcus.

More Recent Studies

Hegde (1988, 1990a), LaBarge (1988), and Hemler (1990) have all studied the quality option in Treasury bond futures contracts. These papers overlap in certain respects. Hegde and Hemler study the time period 1977–1986. Both authors estimate option values using the terminal payoff and implied value methods described earlier. Hegde also uses a switching option approach, while Hemler also uses an exchange option approach involving an extension of the Margrabe (1978) formula to more then two assets. LaBarge examines the 1977–1983 period using the implied value method.

The estimates of all three authors are quite similar; each obtains significantly smaller quality option estimates than to Kane and Marcus (1986a). Recall that for a 3-month horizon, Kane and Marcus find that the value of the quality option ranges from 1.39 to 4.60 percentage points of par for 4 delivery months from 1981–1983. On the other hand, Hemler (1990) finds that the median and mean payoffs are approximately 0.15 and 0.30 percentage points of par, respectively, measured relative to the issue that was cheapest to deliver 3 months earlier. This represents less than 0.5% of the corresponding futures price. Similarly, when computing estimates from the implied value method for a 3-month horizon, Hemler obtains median and mean values of approximately 0.24 and 0.13 percentage points of par, respectively. Although the exchange option estimates obtained by Hemler are somewhat larger, they still are considerably less than those of Kane and Marcus.

Several factors could explain why Kane and Marcus obtain higher quality option estimates than do Hegde and Hemler. The former use simulated payoffs; the latter use historical payoffs. The former average 10,000 outcomes per delivery month; the latter have one realization available per delivery month. The former must estimate their yield curve dynamics over the extremely volatile period 1978–1982; the latter do not use estimated term structures. The former estimate values over a period of relatively high interest rate volatility when, ceteris paribus, the option values should be higher; the latter estimate values over the entire period 1977–1986.

Barnhill (1990), Barnhill and Seale (1988a, 1988b), and Broadie and Sundaresan (1988) all consider the switching method. For instance, Barnhill and Seale (1988a) consider a dynamic trading strategy in which Treasury bond traders holding long cash/short futures positions continually switch their cash position to hold the cheapest-to-deliver bond issue. They examine the distribution of switching profits as a function of transaction costs and hurdle rates. They find that strategies designed to take moderately sized switching profits whenever they occur are likely to yield better results than strategies designed to capture larger but less frequent profits. As would be expected, they also find that the value of the switching option is positively related to interest rate volatility and negatively related to transaction costs.

A theoretical article by Boyle (1989) presents a clear treatment of both the quality and timing options. He shows the link between the quality option and exchange options on two or more assets. Then he considers the timing option under various conditions. In addition, he discusses the interaction between the quality and timing options when there are two or more deliverable grades and provides simulated option values based upon the theory of order statistics. Using fairly restrictive assumptions, he estimates the value of the quality option prior to expiration given as many as 50 deliverable assets.

Livingston (1987a) also discusses the theory of delivery options. He argues that in a perfect market with an infinite number of delivery grades and without marking-to-market, a continuously adjusted hedge exists that will drive the value of the delivery options toward zero. Any value associated with these options stems from market imperfections.

This conclusion conflicts with most of the work in this area and has generated some controversy.

Barnhill (1988), Kane and Marcus (1988), and Carr and Jarrow (1990) have all criticized Livingston for a subtle flaw in his reasoning. At issue is the feasibility of the dynamic hedging strategy Livingston proposed. Barnhill argues that if such a strategy is possible, futures prices will never attain a stable equilibrium. Kane and Marcus argue that the hedging strategy fails because it ignores a basic fact regarding diffusion processes: if a sample path equals a particular value at some time t_0, it must equal that value an infinite number of times in any given time interval containing t_0. Carr and Jarrow make a related observation, noting that the quality option cannot be priced by arbitrage when the continuous price process is of bounded variation, and has positive value otherwise.

The impact of delivery options on hedging effectiveness is an issue of great importance. Johnston and McConnell (1989) argue that the large value of the quality option led to a significant drop in the hedging effectiveness of the GNMA CDR contract which ultimately led to the demise of the contract. Garbade and Silber (1983) show that if multiple delivery grades exist, then a hedged position in at least one must possess some unhedgeable risk. Kamara and Siegel (1987) provide both theoretical and empirical results regarding optimal hedging. They show that one cannot simply use conversion factors for hedge ratios, even though both academics and practitioners do so. Such a strategy is not optimal when delivery risk exists. They then derive optimal hedging strategies when there are one or two deliverable grades. Furthermore, they compare the effectiveness of optimal hedging strategies with full hedge and no-hedge strategies involving CBOT wheat futures contracts. They conclude that the use of optimal strategies would significantly increase hedging effectiveness, resulting in significantly lower variances but similar means.

On the other hand, Hegde (1990b) finds that the quality option has little adverse effect on the performance of optimal hedging strategies with Treasury bond futures contracts. Kamara and Siegel consider variance-minimizing hedges while Hegde considers optimal hedges that attempt to maximize portfolio returns per unit of risk. Hegde uses both ex post and ex ante tests and finds considerably less effect than that indicated by variance-minimizing hedging strategies.

CONCLUDING REMARKS

One must be cautious drawing conclusions from research on delivery options. There is, of course, the usual problem that different authors study different contracts using different methods over different time periods. But there are also other, more subtle difficulties.

For example, Gay and Manaster (1984) and Kane and Marcus (1986a) provide commonly cited estimated values of the quality option. But although one is tempted to compare their estimates, such a comparison would be misleading. The problem is not simply that Gay and Manaster use wheat futures while Kane and Marcus use Treasury bond futures. The "option values" provided in the two articles are fundamentally different. Gay and Manaster provide values of the option prior to expiration; Kane and Marcus provide average values at the time of expiration. Alternatively, in the terminology of the second section, Gay and Manaster use an exchange option approach and Kane and Marcus use a terminal payoff approach. Although the two types of estimates can be related by some (unknown) discount rate, the estimates themselves are not directly comparable.

Other measurement problems arise. As noted, the presence of several delivery options in the same contract can make it hard to disentangle the incremental value of a particular delivery option. Moreover, quality option values must be measured relative to a particular grade, and they vary considerably depending on the benchmark chosen. Consider two deliverable grades for a contract where the first grade is always cheaper to deliver than the second. Then the option to exchange the first for the second must be worthless, while the option to exchange the second for the first must always be "in-the-money." Hence, because different authors measure the quality option relative to different grades, their estimates often cannot be easily compared.[17]

Nevertheless, the following general conclusions can be made:

1. Researchers generally estimate the quality option to be worth less (often considerably less) than 1 to 2% of the futures price 3 months prior to delivery. Exceptions include Johnston and McConnell (1989), who obtain values ranging as high as 19% for GNMA CDR futures, and Kane and Marcus (1986a), who obtain values (at expiration) exceeding 4% for Treasury bond futures.
2. Theoretically, the value of the timing option to deliver any business day during the delivery month depends on a tradeoff between holding the underlying commodity to receive any associated payouts and delivering the commodity to receive a futures price which could then be invested at the risk-free rate. The greater the payout associated with the commodity relative to the risk-free rate, the more likely it is that postponing delivery is optimal for the short. Empirically, few researchers have actually valued timing options.[18] Instead, most have examined various trading strategies and found that the short could profitably exploit timing options using such strategies.
3. Delivery options can impair the hedging effectiveness of futures contracts. Failure to incorporate delivery options in futures pricing models can lead to nonoptimal hedge ratios. Moreover, the presence of delivery options can prevent futures prices from tracking spot prices closely. For the Treasury bond contract the quality option is relatively small, and there is little impact on hedging effectiveness. On the other hand, hedging effectiveness for the GNMA CDR contract dropped dramatically when the quality option increased significantly in value which, in turn, contributed to the demise of the contract.
4. Failure to incorporate delivery options in futures pricing models can lead to incorrect inferences about futures pricing and market inefficiency. In the case of soybean futures, Kamara (1990) finds that this "inefficiency" disappears when the equilibrium pricing model allows for delivery options.

Although much has already been done, considerable scope for future research remains. Possibilities include the following:

1. The measurement of individual delivery options could be improved. Researchers have been able to measure the total effect of having several delivery options, but they generally have not been able to separate the total effect into incremental amounts resulting from individual options. One way to tackle this problem is

[17]The large discrepancy between quality option values reported for the GNMA CDR and Treasury bond futures contracts can be partially attributed to the choice of benchmark. Johnston and McConnell (1989) measure quality option values for the GNMA CDR contract using current coupon mortgage securities. This is natural given their focus on the hedging effectiveness of the contract. But it also leads to larger quality option estimates than would occur if cheapest-to-deliver mortgage securities are used as the benchmark. In the Treasury bond literature, cheapest-to-deliver issues are a standard benchmark, and that yields smaller quality option estimates.

[18]Exceptions include Arak and Goodman (1987), who use the Black-Scholes (1973) model, and Broadie and Sundaresan (1988), who provide examples based on a binomial model of interest rate movements. As noted in Section II, Peck and Williams (1990) use logistic regression to relate the timing of deliveries throughout the delivery month to economic fundamentals.

to focus on contracts that contain only one type of delivery options. There are numerous contracts at U.S. exchanges that contain quality and timing options simultaneously. However, the Japanese long-term government bond futures contract traded on the Tokyo Stock Exchange contains a quality option, but no timing option. So one could use market data for this contract to obtain quality option estimates that are uncontaminated by timing options.[19]

2. Although it is clear from casual inspection that the short delivers at various times throughout the delivery month, surprisingly little work has been done to explain observed patterns regarding the timing of delivery. Silk (1988) and Peck and Williams (1990) have examined this problem for agricultural futures contracts, but much more could be done. For example, consider the Treasury bond futures contract where the timing of delivery depends on a trade-off between payouts on the underlying commodity and interest rates. The greater the payouts, the more likely that postponing delivery is optimal for the short. That suggests that the timing of delivery depends on a rate differential, namely, the risk-free rate less the coupon rate for the cheapest-to-deliver issue. There is no study that rigorously tests for such a relation, even though several authors [for instance, Broadie and Sundaresan (1988) and Hemler (1988)] provide descriptive evidence regarding the timing of delivery observed historically in the Treasury bond futures market.
3. Except for an unpublished report by the MidAmerica Institute for Public Policy Research (1991) that uses exchange options to analyze location options little work has been done on location options. The choice of delivery points is of critical importance for agricultural futures, relating to issues such as market manipulation, convergence of cash and futures prices, and hedging effectiveness.[20] [See, for instance, the references in Silk (1988).] Reexamining these topics from the perspective of financial economics could provide new insight, however.
4. If better Treasury bond spot data become available, it would be useful to reexamine research involving trading strategies in the Treasury bond spot and futures markets. At present researchers generally use daily bid and ask spot price quotations from the Federal Reserve Bank of New York. Such quotes are less than ideal for studying trading strategies since they do not represent synchronized transaction prices. It would be preferable to have intraday spot data of the same high quality as that currently available for stocks. For example, one can obtain transaction data for all stocks on the NYSE from the Institute for the Study of Security Markets. But comparable spot market data are not publicly available for Treasury bonds. Even if intraday Treasury bond spot market data are obtained from a vendor such as Telerate Systems, the spot price quotes do not correspond necessarily to actual transactions.

The analysis of delivery options continues to challenge our ingenuity. Although recent research has been fruitful, unanswered questions remain. Solving them would

[19]There is work in progress by one of the authors (Hemler) and Naoki Kishimoto along these lines. Although other contracts could be used, the Japanese long-term government bond contract is a natural choice because of the widespread interest in Japanese financial markets and the contract's trading volume, which is one of the largest in the world.

[20]For instance, in 1989 there was a highly publicized case of alleged manipulation in the soybean futures market by Ferruzzi S.A. Moreover, the decline of Chicago and Toledo as grain marketing centers has raised questions regarding the price performance of grain futures contracts. Even if cash and futures prices converge as well as before, Chicago and Toledo cash prices are decreasingly relevant for many traders. The hedging value of CBOT grain futures contracts could possibly increase by having delivery points that are more compatible with existing trading patterns.

yield valuable information regarding the pricing of futures contracts. This in turn could yield improved tests of market efficiency, better evaluation of trading strategies, and more effective hedging procedures. These are the research areas that appear most promising—not the valuation of delivery options per se, but gauging the impact of delivery options on other, more fundamental issues in futures markets.

Appendix

CAPSULE SUMMARIES OF RESEARCH ON DELIVERY OPTIONS IN FUTURES CONTRACTS

Article[a]	Major Topics Discussed[b]	Brief Summary
Arak, Goodman, and Ross (1986)	DB, H	The authors show how the cheapest-to-deliver issue stems from biases in both the conversion factors and the cash market. Using simulations they examine the effect of yield changes and relative price changes on the cheapest-to-deliver issue. They also discuss implications for hedging and trading strategies.
Arak and Goodman (1987)	EOM, T, WC	The authors examine wild card and end-of-month options in Treasury bond futures contracts. Their numerical estimates show that the average value of the options is less than 0.3 percentage points of par during 1984–1986.
Barnhill (1988)	Q	The author argues that the Livingston (1987a) proof that the quality option has no value must be flawed, or else futures prices will never reach a stable equilibrium.
Barnhill (1990)	Q, SW	The author estimates potential profits from quality and switching options in Treasury bond futures contracts. He finds that the switching option is the more valuable of the two options. Marking-to-market has a significant effect on the switching option.
Barnhill and Seale (1988a)	SW	The authors examine the switching option in Treasury bond futures contracts. They examine switching profits as a function of transaction costs and hurdle rates. They find that the option value is positively related to interest rate volatility.

APPENDIX (continued)

Article[a]	Major Topics Discussed[b]	Brief Summary
Barnhill and Seale (1988b)	Q, SW	The authors develop a Monte Carlo model for valuing quality and switching options in Treasury bond futures under perfect and noisy market conditions. They conclude that bond price noise is much more important than term structure shifts in creating value for these options.
Benninga and Smirlock (1985)	Q	The authors use regression analysis to show that the quality option affects futures prices. They do not value the option. Instead, they use price sensitivity as a proxy for the value.
Boyle (1989)	Q, T	The author analyzes quality and timing options and their interaction. He gives theoretical results along with numerical estimates and examples for up to 50 deliverable assets. Estimates of the quality option range from 1–11%.
Broadie and Sundaresan (1988)	EOM, Q, T	The authors examine quality and timing options in Treasury bond and note futures contracts. They provide pricing principles and a model consistent with their empirical evidence on actual delivery strategies and discounts in futures prices.
Carr (1988)	Q	The author develops factor models for pricing Treasury bond futures contracts, and uses them to analyze the quality option available to the short. He assumes one or two factors; instrumental variables for these factors are the short and long rates of interest. Some simulation results are provided.
Cheng (1990)	Q	The author derives a valuation formula for two-asset options in the context of a model which incorporates stochastic bond prices. Two examples of multi-asset options (including the delivery option in the Treasury bond futures contract) are priced in closed form using this material.

APPENDIX (continued)

Article[a]	Major Topics Discussed[b]	Brief Summary
Cohen (1990)	T, WC	The author studies the wild card option in Treasury bond futures contracts. He obtains an upper bound for its value, which he estimates as less than 0.1 percentage points of par.
Garbade and Silber (1983)	HE	The authors focus on hedging using futures contracts that allow multiple delivery grades. They show hedged positions possess unhedgeable risk, which they illustrate via Monte Carlo simulation.
Gay and Manaster (1984)	Q	The authors use exchange options to price the quality option in wheat futures contracts. They estimate the option value as being about 1–2% of the corresponding futures prices.
Gay and Manaster (1986)	AI, EOM, T, WC	The authors derive the optimal delivery policy for Treasury bond futures contracts. They show that it produces profits that are positive and statistically significant. They find that the actual delivery policies of market participants differ substantially from the optimal strategy. They also argue that the accrued interest option has no value.
Gay and Manaster (1991)	AI, EOM, Q, T, WC	The authors develop an analytical framework for the valuation of futures prices in the presence of delivery options with particular application to the Treasury bond futures contract. Unlike previous models, their model explicitly and simultaneously accounts for the quality, wild card, accrued interest, and end-of-month options.
Hegde (1987)	DB	The author documents coupon and maturity characteristics of the cheapest-to-deliver bond during the delivery month and prior to delivery as well as weekly changes in the cheapest-to-deliver issue.
Hegde (1988)	AI, EOM, Q, T, WC	The author estimates the composite value of the quality, wild card, and end-of-month options implicit in the Treasury bond futures contract. He finds that the

APPENDIX (continued)

Article[a]	Major Topics Discussed[b]	Brief Summary
Hegde (1988) *(cont.)*	AI, EOM, Q, T, WC	average value of these options over the last quarter of the nearby contract is less than $500 per contract (alternatively, less than 0.5% of the average futures price).
Hegde (1989)	AI, EOM, Q, T	The author provides ex ante values of the quality and timing delivery options in the Treasury bond futures contract. He also provides evidence to show that conversion factors are not useful proxies for hedge ratios, even though practitioners sometimes regard them as such.
Hegde (1990a)	Q	The author develops three empirical estimates of the value of the quality delivery option implicit in the Treasury bond futures contract. Three months prior to delivery the average values of these estimates are $464, $329, and $2,075, respectively. The differences appear to be largely due to nonsynchronous spot-futures data.
Hegde (1990b)	HE	The author finds that although the quality option reduces the effectiveness of variance-minimizing hedges, it has little impact on utility-optimal hedges.
Hegde (1992)	Q, T	The author estimates the value of the timing and quality options in the Treasury Bond futures contract after the last day of trading. He finds that the timing option is nearly worthless, the quality option is worth about $150 per contract, and that actual deliveries are consistent with the optimal exercise of delivery options.
Hemler (1990)	Q	The author studies the quality delivery option in Treasury bond futures contracts. To value the option he uses exchange options, historical payoffs, and implied values. He concludes that the value of this option is quite small compared to the estimates previously reported by Kane and Marcus (1986a).

APPENDIX (continued)

Article[a]	Major Topics Discussed[b]	Brief Summary
Johnston and McConnell (1989)	HE	The authors analyze the hedging effectiveness of the GNMA CDR contract. They conclude that the quality option grew dramatically in value in the mid-1908's, leading to a significant drop in hedging effectiveness and the demise of the contract.
Jones (1985)	DB	The author criticizes the Kane and Marcus (1984) article for ignoring marking-to-market and argues that the bond futures price is determined by the deliverable bonds with the highest and lowest durations.
Kamara (1990)	ME	The author shows how the failure to incorporate delivery options in soybean futures prices can lead to apparent market inefficiency.
Kamara and Siegel (1987)	HE	The authors provide theoretical and empirical results regarding optimal hedging with wheat futures when there are multiple delivery grades. Hedges are more effective after accounting for delivery options.
Kane and Marcus (1984)	DB	The authors examine how conversion factor risk can affect variability of futures returns and create profitable trading opportunities. They argue that a slight change in the settlement procedure could substantially reduce conversion factor risk.
Kane and Marcus (1986a)	Q	The authors value the quality option in Treasury bond futures contracts by calculating simulated payoffs at expiration. They obtain values of 1.39 to 4.60 percentage points of par 3 months prior to delivery.
Kane and Marcus (1986b)	T, WC	The authors examine the wild card option in Treasury bond futures markets. Their numerical estimates show that the option is worth about 0.2 percentage

APPENDIX (continued)

Article[a]	Major Topics Discussed[b]	Brief Summary
Kane and Marcus (1986b) *(cont.)*	T, WC	points of par at the start of the delivery month and then falls to zero at the end of the month.
Kane and Marcus (1988)	Q	The authors disagree with the Livingston (1987a) conclusion that the delivery option has no value. They argue that the hedging strategy proposed by Livingston fails because of a basic fact regarding diffusion processes.
Kilcollin (1982)	DB	The author investigates different rules for setting premiums and discounts associated with various delivery grades. He finds that current systems effectively limit deliverable supply in the futures markets and lead to futures prices that understate cash prices. He also concludes that there is no perfect delivery adjustment procedure.
Koenigsberg (1990)	Q, T	The author discusses the Salomon Brothers delivery option model for Treasury bond and note contracts. The model is a term-structure-based model that prices quality and timing options simultaneously.
LaBarge (1988)	AI, EOM, Q, T, WC	The author estimates the total value of delivery options in the Treasury bond futures contract. She calculates implied values over the period of 1977–1983 and concludes that they are less than 0.1% of par.
LaBarge (1989)	T, WC	The author applies the same methodology as Kane and Marcus (1986b), but uses historical data. She concludes that in the vast majority of cases the wild card option has considerably less value than reported by Kane and Marcus.
Livingston (1984)	DB	The author examines the impact of coupon levels and time to maturity on the cheapest deliverable bond for the

APPENDIX (continued)

Article[a]	Major Topics Discussed[b]	Brief Summary
Livingston (1984) *(cont.)*	DB	Treasury bond futures contract. He also examines shortage of supply and finds it can have significant impact.
Livingston (1987a)	Q	The author argues that in a perfect market with an infinite number of delivery grades and without marking-to-market, a continuously adjusted hedge will drive the value of the delivery option in a forward contract to zero.
Livingston (1987b)	DB	The author derives the impact of coupon level upon the cheapest-to-deliver issue in the Treasury bond futures market for various term structures holding time to maturity constant.
Meisner and Labuszewski (1984)	DB	The authors examine sources of delivery bias in the Treasury bond futures market. They conclude that the conversion factor invoicing system favors low coupon, long term bonds when yields exceed 8%. But this bias can be offset by cash market biases.
MidAmerica Institute (1991)	L	This report has several basic conclusions regarding the grain futures delivery process. It finds that the process has worked well over the past 6 years. No radically different substitute mechanisms with superior attributes to the one now in use are apparent. However, a change to an "economic par" delivery system including a delivery point on the Mississippi River in addition to the existing points of Chicago and Toledo might improve hedging effectiveness significantly.
Peck and Williams (1990)	T	The authors find that the timing of delivery on certain agricultural futures varies with net carrying charges. The greater the carrying charges, the earlier the

APPENDIX (continued)

Article[a]	Major Topics Discussed[b]	Brief Summary
Peck and Williams (1990) *(cont.)*	T	delivery. They find that the value of the timing option for five commodities is 0.1–10.0% 3 to 4 months prior to delivery.
Silk (1988)	L, Q, T	The author studies delivery options in soybean, corn, and wheat futures. He finds that the timing of deliveries is a function of the net cost of carry. A surprising result is that 8–19% of peak open contracts result in delivery.

[a]Detailed citations are given in the references.

[b]To help the reader find studies of interest, we use abbreviations to indicate various topics related to delivery options. These topics and their corresponding abbreviations are: quality option (Q), timing option (T), wild card option (WC), accrued interest option (AI), end-of-month option (EOM), switching option (SW), location option (L), market efficiency (ME), hedging effectiveness (HE), and delivery biases (DB). Quite often it is difficult to classify exactly which topics pertain to a particular article. The topics overlap to some degree and opinions can vary as to what a particular article actually documents. Consequently, readers should only consider the topic listings to be a rough guide.

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