Efficient Use of Information, Convergence Adjustments, and Regression Estimates of Hedge Ratios

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INTRODUCTION

Organized futures markets are traditionally thought of as vehicles for hedging and minimizing risk since they allow investors to transfer the risk of price changes to speculators. To use these markets, the hedger first needs to determine how many futures contracts he should sell for each unit of spot commodity on which he bears price risk. Johnson (1960) formulated the hedger's problem using portfolio theory and derived the optimal variance-minimizing hedge.[1] The optimal hedge ratio formula in the portfolio model resembles the Ordinary Least Squares (OLS) estimator of the slope in a simple regression framework. Hence, a convenient and usual method of estimating the hedge ratio is through an OLS regression. The change in the spot price is regressed against the change in the futures price, and the estimated slope coefficient is taken to be the hedge ratio.[2] This article examines the ability of the traditional regression framework to address two concerns articulated in the literature: the potential information inefficiency of the traditional regression procedure and the lack of implicit incorporation of spot–futures convergence into its estimation methodology.

The information inefficiency problem was studied early on by Bell and Krasker (1986). They showed (Proposition 1 in their article) that if the expected futures price change depends on the information set, then the traditional regression method (TR, henceforth)

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[1]Although the variance-minimizing hedge ratio has been modified to take into account return considerations, even these more complicated hedge ratio expressions can often be written as the sum of two terms, one of which is the variance-minimizing hedge ratio. See, for example, Kahl (1983) and Adler and Detemple (1988).

[2]This method is used by many researchers, for example, Ederington (1979); Hill and Schneeweis (1981); and Witt, Schroeder, and Hayenga (1987). In practice, other forms of the regression specification are used, such as the return formulation [Brown (1985)], but this does not affect the tenor of the argument presented here.

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will yield a biased estimate of the hedge ratio.[3] For example, if the futures price is a biased estimate of the spot price at maturity, this bias must go to zero at maturity. Nevertheless, the expected futures price change at each point in time before maturity could, and most likely does, depend on new information generated at that point in time. Hilliard (1984) proposed to correct for the estimation bias by regressing the unexpected change in the spot price against the unexpected change in the spot price. Bell and Krasker (1986, Proposition 2) showed that this procedure will yield the correct result provided the hedge ratio itself is independent of the information set. Myers and Thompson (1987) proposed a single equation framework to replace the multiple equation estimation required by Hilliard's method. However, both of these researchers assumed that a single hedge ratio could be estimated independent of the time to maturity of the futures contract. On the other hand, Castelino (1990) argues that it is reasonable to expect the hedge ratio to depend on when the hedge is lifted.[4] Similarly, the hedge ratio may depend on the duration of the hedge also. This would be the case, for example, if there are quality options imbedded in the futures contract, as shown by Kamara and Siegel (1987).[5]

Castelino (1990) also argues that the regression procedure does not produce a hedge ratio of unity when the hedge is lifted at maturity as it should, given spot–futures convergence. He concludes that the regression procedure is inappropriate if spot and futures prices are expected to converge at maturity. Previous research using the regression procedure [including Hilliard (1984) and Myers and Thompson (1987)] does not adjust for spot–futures convergence.

This article considers a modification of the Myers and Thompson procedure taking into account the possibility of spot–futures convergence and the dependence of the hedge ratio on hedge duration and the time left to maturity.[6] Instead of including a large number of lagged spot and futures price changes, as suggested by Myers and Thompson, Fama and French's (1987) argument that the current basis should have power to predict changes in the spot and futures price is used. This implies a regression of the spot price change on the futures price change and the current (or beginning) basis.[7]

Three agricultural commodities—soybeans, wheat, and corn—are used to empirically demonstrate this procedure. After estimation of the hedge ratios, two sets of portfolios are formed using out-of-sample data: one set using the hedge ratios estimated from the TR approach and another using the basis-corrected (BC henceforth) approach. The variances of the unpredictable components of the dollar returns are then compared for both approaches.[8] The basis-corrected approach produces hedged portfolio returns that

[3]Similarly Myers and Thompson (1987) point out that the optimal hedge ratio should take account of relevant conditioning information. However, they claim that the TR procedure leads to biased estimates even if the expected *spot* price change depends on the information set. Intuitively, this is false by thinking of an errors-in-the-variables framework, where only errors in the independent variables lead to bias.

[4]Hilliard (1984) adjusts for this to some extent by hedging with a portfolio of spot and futures positions. This article rejects that approach as being too expensive and concentrates on hedging with a single futures contract.

[5]Ederington (1979) finds that the hedge ratios for two-week hedges are different from the hedge ratios for four-week hedges.

[6]In contrast to Castelino's method, this modified regression method also works if convergence is not guaranteed, which is often the case in agricultural commodities, where there are quality and timing options.

[7]Fama and French themselves are only interested in the power of the basis to predict spot price changes. However, their argument can be applied equally well to futures price changes.

[8]Obviously, only unpredictable variation over time is relevant for an economic agent. Deterministic variation over time of the hedged portfolio returns can be avoided, if desired, by creating zero-value hedges as described below. However, agents' desires to eliminate such predictable variation over time cannot be attributed to risk aversion.

often have significantly smaller variances, although the pattern is not uniform for all commodities.

The procedure may be of interest for hedgers who wish to minimize the variance over time of their hedged portfolio returns. This can be done by taking an appropriate position in the T-bill markets. The size of this position is a byproduct of the procedure, as explained below.

THEORY

Consider an economic agent at time t holding one unit of the spot commodity. At time $t + \tau$, the change in the value of the holdings will be $P_{t+\tau} - P_t$, where P_t is the spot price at time t. If the position is hedged by selling b units of a futures contract maturing at T, the change in value of the holdings will be $R \equiv (P_{t+\tau} - P_t) - b(f_{t+\tau} - f_t)$ where f_t is the futures price at time t. The question is how does one optimally choose b, the hedge ratio?

One must first decide on the appropriate objective function of the hedger. Assuming that variance is the appropriate measure of risk, two alternatives present themselves: the hedger desires to minimize the unconditional variance of R (which is equivalent to minimizing the time variation in R); or, the hedger desires to minimize the variance of R, conditional on all available information. The appropriate objective to use for purposes of hedge ratio estimation is the second one because if any part of the return R is known when the hedge is initiated, then the time variation of that part is irrelevant for this problem. For example, if P_t follows a random walk with drift, then part of the change in the value of the hedger's position over time will be predictable. This predictable component is irrelevant for the hedger's problem, because he could always offset this movement with probability one by taking an appropriate position in a τ-period bond. Hence, even if minimization of the time variation of R is the ultimate objective of the hedger, it can be done in two steps: (1) choosing the hedge ratio to minimize the conditional variance of R, and (2) taking an appropriate position in τ-period bonds. This will be explained in greater detail, once the model is fleshed out.

Assuming that the agent chooses to minimize Var(R), the variance of R conditional on all available information at time t, the objective function is differentiated with respect to b, the hedge ratio. Solving for the optimal b:

$$b = \text{Cov}(P_{t+\tau}, f_{t+\tau})/\text{Var}_t(f_{t+\tau}) \tag{1}$$

where $\text{Cov}(P_{t+\tau}, f_{t+\tau})$ stands for the covariance between the unpredictable components of the spot and futures prices and $\text{Var}(f_{t+\tau})$ stands for the variance of the unpredictable component of the futures price. As proved in Bell and Krasker (1986), one can estimate (1) as the slope coefficient in the regression of $P_{t+\tau} - E_t(P_{t+\tau})$ on $f_{t+\tau} - E_t(f_{t+\tau})$:

$$P_{t+\tau} - E_t(P_{t+\tau}) = a_1 + b[f_{t+\tau} - E_t(f_{t+\tau})] + \epsilon_1 \tag{2}$$

However, to run this regression, measures of $E_t(P_{t+\tau})$ and $E_t(f_{t+\tau})$ are needed. A complete expectational model would require an equilibrium pricing model as well as a model of expectations formation. Alternatively, one could take an empirical approach and simply relate the price expectations empirically to the variables. Fama and French (1987) argue that the current basis (defined as $f_t - P_t$) should have substantial power to predict the spot price at maturity and they show that this is in fact true for many commodities. To the extent that there are persistent shocks in the spot price series (which should be true for all storable commodities) this also implies that the current basis has information about all future spot prices. Consequently:

$$E(f_{t+\tau}) = f_t + \delta_1(f_t - P_t) \tag{3a}$$

Fama and French's methodology could be used also to show that the current basis has information about the futures price. Hence[9]:

$$E(P_{t+\tau}) = P_t + \delta_2(f_t - P_t) \tag{3b}$$

It can be shown that the estimate of b obtained from estimating the system of eqs. (2) and (3) is equivalent to the estimate of b_{BC} in the regression equation:

$$P_{t+\tau} - P_t = a_1 + b_{BC}(f_{t+\tau} - f_t) + c(f_t - P_t) + \epsilon_1 \tag{4}$$

where $c = \delta_2 - b_{BC}\delta_1$, where the subscript BC of b indicates that this is the basis corrected hedge ratio estimate. On the other hand, the traditional hedge ratio, b_{TR}, is estimated from the following regression equation:

$$P_{t+\tau} - P_t = a_2 + b_{TR}(f_{t+\tau} - f_t) + \epsilon_2 \tag{5}$$

Under what circumstances will b_{BC} differ from b_{TR}? Two conditions are necessary. First, as mentioned above, the expected futures price change must depend on the information set. In the context of this model, this means that the expected futures price change must be a function of the current basis, i.e., δ_1 must be different from zero in eq. (3a). Second, δ_2 must be nonzero in equation (3b); or, equivalently, c must be different from zero in regression eq. (4) [Theil (1971), p. 549]. If these conditions are satisfied, it can be said not only that the basis corrected hedge ratio estimate should be different from the traditional estimate, but that b_{BC} should be significantly greater than b_{TR}. This is because the hedge ratio no longer needs to reflect the variation in the beginning basis.

There is another difference between eqs. (4) and (5). It is easy to see that eq. (4) is consistent with spot–futures convergence at maturity. Consider what happens to eq. (4) if the hedge is lifted at maturity, i.e., if $t + \tau = T$. In this case:

$$P_T - P_t = a_1 + b(f_T - f_t) + c(f_t - P_t).$$

If there is spot–futures convergence at maturity, this equation becomes an identity when all the coefficients are set to one. Clearly this would minimize the sum of squared errors. Hence, the OLS procedure is guaranteed to yield a hedge ratio of unity in this case, as desired. On the other hand, the procedure remains valid even if spot–futures convergence is not guaranteed, because of the existence of quantity, quality and timing options. Equation (5), however, does not guarantee a hedge ratio of unity in the case described above.

The next section discusses the data and presents the estimation results.

EMPIRICAL RESULTS

A. Data Description

Daily data on wheat, corn, and soybean prices in cents per bushel for the period January, 1978 to December, 1988 are from the USDA. Details on the data are presented in

[9]Of course, the size of δ_1 and δ_2 depends on the amount of time left before maturity when the hedge is lifted and on the duration of the hedge. For example, if it is assumed that the beginning basis goes linearly over time to zero, then $\delta_1(t_1, t_2) = (t_1/t_1 + t_2) \times \delta(t_1 + t_2, 0)$, where the first argument to $\delta(.\,,.)$ is the hedge duration and the second argument is the amount of time left before maturity when the hedge is lifted.

Table I.[10] Futures settlement price data on wheat, corn, and soybean contracts for the same period are from the Chicago Board of Trade. Corn and wheat have five delivery months every year—March, May, July, September, and December—giving 55 contracts each. Soybeans have six delivery months per year—January, March, May, July, September, and November—or 65 contracts in all. The January, 1978 contract is excluded because only delivery month data are available for that contract.

Table I
DETAILS OF AGRICULTURAL SPOT PRICE DATA (ALL PRICES ARE BIDS FOR DELIVERY AT CHICAGO)

Trade Level	Spot/Forward	Delivery Date	Delivery Location	Subperiod
No. 2 Yellow Corn				
Unspecified	Cash price at close of trading		Rail car (hopper)	1/78–3/82
Unspecified	To arrive	≤15 days	Truck and rail	4/82–6/82
Unspecified	To arrive	≤30 days	Truck and rail	7/82
Processors and merchandisers	To arrive	≤15 days	Truck and rail	8/82–12/88
No. 2 Soft Red Wheat				
Terminals, mills, processors, and merchandisers	To arrive	≤30 days	Truck and rail	1/83–12/86
Terminals, mills, processors, and merchandisers	To arrive	≤30 days	Truck and rail	1/87–2/88
Terminals, mills, and merchandisers	To arrive	≤30 days	Truck and rail	3/88–12/88
No. 1 Yellow Soybeans				
Unspecified	Cash price at close of trading		Rail car	1/78–3/82
Unspecified	To arrive	≤15 days	Truck and rail	4/82–7/82
Processors and merchandisers	To arrive	≤15 days	Truck and rail	8/82–12/86
Terminals, mills, processors, and merchandisers	To arrive	≤15 days	Truck and rail	1/87–11/87
Terminals, mills, processors, and merchandisers	To arrive	≤15 days	Truck and rail	12/87–12/88

[10]Unfortunately, the specifications of the prices are not exactly consistent from one subperiod to another, although there is no clear evidence of any obvious discontinuities. Data are available for most days in the sample. In most cases, where there are missing data, it is for isolated dates. In these cases, prices are reconstructed through linear interpolation. However, data on wheat prices are missing for two periods. These prices are not interpolated.

Hedge ratio estimates for corn are derived from 35 contracts from March, 1978 to September, 1984. Hedge portfolios are formed using these hedge ratios and returns are computed for the remaining 20 contracts. For wheat, the same procedure is followed, except that the December, 1980 and the September, 1982 contracts are deleted from the estimation period because clean spot price data are not available. The sample size used for estimation, therefore, is 33. For soybeans, the contracts used for the estimation are from March, 1978 to July, 1985 and the hedging strategy is implemented for the remaining 20 contracts.

B. Variance of Hedged Portfolios for TR and BC Hedge Ratios

The results are used to compare the performance of two different hedge estimation methods: the traditional regression technique, which regresses the spot price change on the futures price change (TR); and the basis corrected (BC) method, which regresses the spot price change on the futures price change *and* the beginning basis. To recapitulate, the hedge ratios are estimated from the following regressions:

$$P_{t+\tau} - P_t = a_1 + b_{BC}(f_{t+\tau} - f_t) + c(f_t - P_t) \tag{4}$$

$$P_{t+\tau} - P_t = a_2 + b_{TR}(f_{t+\tau} - f_t) \tag{5}$$

Now the appropriate criterion must be selected to compare the performance of the traditional and basis corrected hedge ratio estimation methods. If there are two alternative objective functions—(i) minimizing the time variance of hedged portfolio returns, and (ii) minimizing the variance of the unpredictable portion of the hedged portfolio return—should eqs. (4) and (5) be compared on the basis of both objective functions? After all, eqs. (4) and (5) are only alternative estimation methods. In other words, should one compare the variances of $P_{t+\tau} - P_t - b_i(f_{t+\tau} - f_t)$, $i = b_{BC}, b_{TR}$ on the basis of the first objective function and the variances of $P_{t+\tau} - P_t - b_i(f_{t+\tau} - f_t) - c(f_t - P_t)$, $i = b_{BC}, b_{TR}$ on the basis of the second? The latter comparison is appropriate independent of the hedger's objective function.

Suppose the hedger is interested in minimizing the time variation in R, i.e., the unconditional variance of R. Given an estimated hedged ratio of b, the return of the hedger on the hedged portfolio can be written as:

$$\begin{aligned} R &= P_{t+\tau} - P_t - b(f_{t+\tau} - f_t) \\ &= [P_{t+\tau} - E_tP_{t+\tau} - b(f_{t+\tau} - E_tf_{t+\tau})] + [E_tP_{t+\tau} - P_t - b(E_tf_{t+\tau} - f_t)] \end{aligned}$$

The first component of R is clearly uncorrelated by definition with the second component of R. Hence, one can write the unconditional variance of R as equal to the unconditional variance of the first component plus the unconditional variance of the second component. Using eqs. (3a) and (3b), one can write the second component as $(\delta_2 - b\delta_1)(f_t - P_t)$. Now, if the hedger sells τ-period discount bonds with a face value of $c(f_t - P_t) = (\delta_2 - b\delta_1)(f_t - P_t)$, he will decrease his $t + \tau$ date cash flow by exactly $(\delta_2 - b\delta_1)(F_t - P_t)$, and the second component disappears entirely. In this case, the conditional variance of R is the same as that of the unconditional variance. Hence, he would choose b to minimize the conditional variance of R. Furthermore, any other method of choosing b can only do as well as the above method, never better. The appropriate quantities to compare, therefore, in evaluating the traditional hedge ratio estimate b_{TR} versus the basis corrected hedge ratio estimate b_{BC}, are:

$$UR_{BC} = P_{t+\tau} - P_t - b_{BC}(f_{t+\tau} - f_t) - c(f_t - P_t)$$

and

$$UR_{TR} = P_{t+\tau} - P_t - b_{TR}(f_{t+\tau} - f_t) - c(f_t - P_t)$$

where c is as estimated from eq. (4).

Hedges of varying durations (τ = 2, 4, and 6 weeks) are constructed with the hedge being lifted at different time periods before the maturity of the contract (denoted by ζ =2, 4, 6, and 8 weeks). The hedge ratio estimation is done separately for different values of τ and ζ, since the theory does not, a priori, require the hedge ratios to be the same over different values of τ and ζ, as argued earlier. The hedge ratio estimates are given in Table II. For wheat regressions, one can reject the hypothesis that the hedge ratios b_{TR} and b_{BC} are equal for the (τ, ζ) combinations (4,2), (4,8), (6,4), (6,6), and

Table IIa
HEDGE RATIO ESTIMATION FOR CORN

	ζ	$a_{\tau,\zeta}$	$b_{\tau,\zeta}$	$c_{\tau,\zeta}$	R^2	ζ	$a_{\tau,\zeta}$	$b_{\tau,\zeta}$	$c_{\tau,\zeta}$	R^2
Estimates for hedge duration = 2 weeks										
TR	2	1.39	0.82		0.78	6	2.81	0.77		0.60
		(0.78)	(0.07)				(1.08)	(0.11)		
BC	2	1.35	0.82	0.02	0.77	6	2.52	0.78	0.07	0.60
		(0.80)	(0.08)	(0.07)			(1.14)	(0.11)	(0.09)	
TR	4	−0.66	1.00		0.76	8	0.55	0.87		0.57
		(1.02)	(0.10)				(1.10)	(0.13)		
BC	4	−0.85	1.01	0.17	0.79	8	−0.45	0.87	0.22	0.68
		(0.97)	(0.09)	(0.07)			(0.99)	(0.11)	(0.06)	
Estimates for hedge duration = 4 weeks										
TR	2	0.54	0.86		0.74	6	3.33	0.92		0.63
		(1.44)	(0.09)				(1.70)	(0.12)		
BC	2	0.25	0.88	0.24	0.77	6	1.92	0.93	0.32	0.70
		(1.36)	(0.08)	(0.11)			(1.58)	(0.11)	(0.10)	
TR	4	2.01	0.91		0.87	8	0.52	0.87		0.63
		(1.09)	(0.06)				(1.56)	(0.12)		
BC	4	1.46	0.93	0.14	0.88	8	−0.93	0.97	0.36	0.80
		(1.11)	(0.06)	(0.08)			(1.17)	(0.09)	(0.07)	
Estimates for hedge duration = 6 weeks										
TR	2	3.37	0.80		0.84	6	3.28	0.87		0.62
		(1.34)	(0.06)				(1.97)	(0.12)		
BC	2	2.68	0.82	0.18	0.84	6	1.51	0.95	0.42	0.77
		(1.36)	(0.06)	(0.11)			(1.59)	(0.09)	(0.09)	
TR	4	2.65	0.95		0.79	8	1.85	0.88		0.47
		(1.73)	(0.08)				(2.43)	(0.16)		
BC	4	1.00	0.97	0.37	0.85	8	−1.08	1.04	0.54	0.79
		(1.52)	(0.07)	(0.10)			(1.57)	(0.10)	(0.08)	

Notes: (1) Standard errors in parentheses. (2) Sample sizes are: corn, 35 contracts, soybeans, 45 contracts, and wheat, 33 contracts. (3) The regression models are:

TR: $P_{t+\tau} - P_t = a_{\tau,\zeta} + b_{\tau,\zeta}(f_{t+\tau} - f_t) + \epsilon_{\tau,\zeta}$

BC: $P_{t+\tau} - P_t = a_{\tau,\zeta} + b_{\tau,\zeta}(f_{t+\tau} - f_t) + c_{\tau,\zeta}(f_t - P_t) + \epsilon_{\tau,\zeta}$

Table IIb
HEDGE RATIO ESTIMATION FOR WHEAT

	ζ	$a_{\tau,\zeta}$	$b_{\tau,\zeta}$	$c_{\tau,\zeta}$	R^2	ζ	$a_{\tau,\zeta}$	$b_{\tau,\zeta}$	$c_{\tau,\zeta}$	R^2
Estimates for hedge duration = 2 weeks										
TR	2	0.88	0.83		0.86	6	1.29	1.06		0.82
		(1.00)	(0.06)				(1.33)	(0.09)		
BC	2	−0.26	0.89	0.16	0.88	6	0.03	1.06	0.17	0.85
		(1.03)	(0.06)	(0.06)			(1.31)	(0.08)	(0.06)	
TR	4	0.10	0.94		0.85	8	3.05	0.81		0.86
		(1.20)	(0.07)				(1.00)	(0.06)		
BC	4	−0.77	0.96	0.15	0.86	8	2.35	0.85	0.07	0.86
		(1.20)	(0.07)	(0.07)			(1.12)	(0.06)	(0.05)	
Estimates for hedge duration = 4 weeks										
TR	2	0.72	0.80		0.84	6	4.07	0.77		0.76
		(1.38)	(0.06)				(1.60)	(0.08)		
BC	2	−0.93	0.88[a]	0.26	0.88	6	1.93	0.86	0.22	0.80
		(1.30)	(0.06)	(0.08)			(1.62)	(0.08)	(0.08)	
TR	4	1.40	0.96		0.83	8	3.04	0.79		0.88
		(1.98)	(0.08)				(1.45)	(0.05)		
BC	4	−0.91	0.99	0.32	0.88	8	1.53	0.87[a]	0.17	0.90
		(1.80)	(0.07)	(0.09)			(1.44)	(0.06)	(0.07)	
Estimates for hedge duration = 6 weeks										
TR	2	1.70	0.89		0.81	6	3.43	0.70		0.78
		(2.13)	(0.08)				(1.80)	(0.07)		
BC	2	−1.60	0.97	0.44	0.90	6	1.14	0.82[a]	0.27	0.85
		(1.71)	(0.06)	(0.09)			(1.61)	(0.06)	(0.07)	
TR	4	3.90	0.80		0.77	8	4.28	0.76		0.83
		(2.17)	(0.08)				(1.96)	(0.06)		
BC	4	0.54	0.91[a]	0.36	0.84	8	1.86	0.89[a]	0.26	0.87
		(2.01)	(0.07)	(0.09)			(1.94)	(0.07)	(0.09)	

[a] The basis-corrected hedge ratio is significantly higher than the corresponding traditional hedge ratio at a 5% level of significance.

(6, 8) using a one-tailed, t-test at the 5% level of significance. For all other commodities, it is not possible to reject the hypothesis of equality of hedge ratios.[11]

As an auxiliary check, the statistical significance of the slope coefficient of the beginning basis in the basis-corrected regression is examined. See in Table II that this slope coefficient is significant in 30 of the 36 regressions, the exceptions being for the (τ, ζ) pairs (2, 2), (2, 6), (4, 4), and (6, 2) for corn; (2, 8) for wheat; and (2, 2) for soybeans. Furthermore, if one regresses $f_{t+\tau} - f_t$, the futures price change, on the beginning basis, a significant relationship is found for wheat for the (τ, ζ) pairs (2, 2), (2, 8), (4, 2), (4, 6), (4, 8), (6, 4), (6, 6), and (6, 8). However, the hypothesis of no relationship for corn and soybeans cannot be rejected.[12] If these sample relationships actually hold in the

[11] Actually for corn, the variance of the traditional hedged portfolio return is substantially *lower* than that of the basis-corrected hedged portfolio return for the (τ, ζ) pairs (4, 8) and (6, 8). However, this is simply a chance occurrence; in fact, b_{BC} is greater than b_{TR}, as expected.

[12] Regressions not reported.

Table IIc
HEDGE RATIO ESTIMATION FOR SOYBEANS

	ζ	$a_{\tau,\zeta}$	$b_{\tau,\zeta}$	$c_{\tau,\zeta}$	R^2	ζ	$a_{\tau,\zeta}$	$b_{\tau,\zeta}$	$c_{\tau,\zeta}$	R^2
Estimates for hedge duration = 2 weeks										
TR	2	2.59	0.94		0.99	6	3.85	0.93		0.89
		(0.79)	(0.02)				(1.57)	(0.05)		
BC	2	2.20	0.94	0.04	0.99	6	0.64	0.92	0.21	0.91
		(1.01)	(0.02)	(0.06)			(1.80)	(0.05)	(0.07)	
TR	4	2.16	1.00		0.87	8	1.59	0.98		0.95
		(1.63)	(0.06)				(1.06)	(0.03)		
BC	4	−2.65	1.03	0.42	0.95	8	−0.26	0.96	0.10	0.96
		(1.15)	(0.04)	(0.05)			(1.33)	(0.03)	(0.05)	
Estimates for hedge duration = 4 weeks										
TR	2	3.98	0.92		0.96	6	5.25	0.92		0.93
		(1.69)	(0.03)				(1.82)	(0.04)		
BC	2	−0.70	0.96	0.42	0.98	6	−0.44	0.88	0.33	0.95
		(1.38)	(0.02)	(0.06)			(1.90)	(0.03)	(0.07)	
TR	4	5.60	0.97		0.91	8	2.81	0.99		0.95
		(1.96)	(0.05)				(1.73)	(0.04)		
BC	4	−2.34	0.97	0.50	0.97	8	−0.56	0.98	0.18	0.95
		(1.44)	(0.03)	(0.05)			(2.15)	(0.03)	(0.07)	
Estimates for hedge duration = 6 weeks										
TR	2	7.72	0.90		0.94	6	6.58	0.93		0.95
		(2.32)	(0.03)				(1.80)	(0.03)		
BC	2	−0.94	0.94	0.57	0.97	6	1.12	0.92	0.29	0.97
		(1.89)	(0.02)	(0.07)			(2.01)	(0.03)	(0.07)	
TR	4	7.18	0.99		0.93	8	3.14	1.02		0.89
		(2.17)	(0.04)				(2.94)	(0.05)		
BC	4	−2.72	0.96	0.56	0.98	8	−5.68	1.00	0.46	0.95
		(1.53)	(0.02)	(0.05)			(2.28)	(0.04)	(0.06)	

population, one would expect the basis-corrected method to have an impact on the hedge ratios at least for those (τ, ζ) pairs, where both the necessary conditions discussed under "Theory" hold; i.e., for the (τ, ζ) pairs (2, 2), (4, 2), (4, 6), (4, 8), (6, 4), (6, 6), and (6, 8) for wheat. In fact, as described above, the hypothesis of no change in the hedge ratio is rejected for five out of these seven cases.

The standard deviations of the unexpected components of the hedged portfolio returns are given in Table III, for hedges of varying durations and times to maturity. A t-test suggested by Morgan (1939) for the difference between the two variances in a bivariate normal population is used to test the null hypothesis that the variances of UR_{TR} and UR_{BC} are equal.[13] The null hypothesis of no difference between Var(UR_{TR}) and Var(UR_{BC}) is rejected in nine cases in favor of the alternate hypothesis that Var(UR_{TR}) is greater than

[13]Morgan's test of equal variances for bivariate normal contemporaneously correlated series A and B, given that they are each independent over time is:

$$t = (\sigma_A^2 - \sigma_B^2)/\text{sqrt}[(1 - r_{AB}^2)\sigma_A^2\sigma_B^2] \times \text{sqrt}(n - 2)/2$$

This statistic is distributed as a t with $n - 2$ degrees of freedom.

Table III
STANDARD DEVIATION OF DOLLAR PAYOFFS

τ	2 Weeks				4 Weeks				6 Weeks			
ζ	2	4	6	8	2	4	6	8	2	4	6	8
Corn												
UH	13.35	8.62	10.39	11.97	18.10	13.08	16.53	22.81	20.94	20.03	23.53	25.49
TR	5.30	3.28	5.45	5.24	6.33	6.12	6.20	6.39	8.04	5.58	8.96	8.45
BC	5.30	3.30	5.44	5.24	6.38	6.17	6.18	8.11	8.07	5.55	9.17	12.10
Soybeans												
UH	31.54	26.58	22.34	20.33	41.83	44.05	38.16	24.27	58.29	58.62	28.86	44.89
TR	6.01	6.57	6.25[a]	10.81	8.49	6.75	9.07	7.53[a]	8.65	8.95	7.08	11.81[a]
BC	6.02	6.64	6.16	10.80	8.90	6.80	9.17	7.47	8.57	9.64	6.96	11.14
Wheat												
UH	12.25	23.75	15.42	14.29	31.20	33.17	20.80	15.24	39.22	32.57	18.89	24.11
TR	5.52	14.33[a]	12.79[a]	8.21	14.80[a]	16.16	10.72	11.04[a]	16.58	16.80	10.58[a]	15.57[a]
BC	5.80	14.20	12.78	8.05	14.08	16.03	10.17	10.67	16.25	16.15	9.85	14.73

Notes: Prices are in cents per bushel. Each portfolio consists of one bushel and a short position in the appropriate number of futures contracts. UH: unhedged portfolio; TR: hedged portfolio using traditional hedge ratio; BC: hedged portfolio using basis-corrected hedge ratio.

[a] TR portfolio variance is significantly higher than the corresponding BC portfolio variance at the 5% significance level.

Var(UR_{BC}). This is true for the (τ, ζ) pairs (2,4), (2,6), (4,2), (4,8), (6,6), and (6,8) for wheat and for the pairs (2,6), (4,8), and (6,8) for soybeans. For all other hedge durations and times to maturity, the null hypothesis of equal variances is not rejected.

This supports the view that the traditional hedge ratio estimation method is, in fact, inefficient and does not use all the available information. However, the improvement in variance is clearly not uniform. For example, the hypothesis of no improvement is not rejected for any of the corn hedges. Even for the wheat and soybean hedges, the effects are somewhat more pronounced for hedges lifted far away from maturity, than for hedges lifted close to maturity. Replication of these tests on other commodities may provide further evidence regarding a common underlying pattern.

CONCLUSION

The hedge ratio estimate produced by the traditional regression method is the ratio of the unconditional covariance between the changes in the spot and futures prices and the unconditional variance of the futures price changes. The theoretically correct quantities to use are the conditional variance and covariance. The traditional method is criticized because it does not adjust for convergence of the spot and futures prices at maturity. This article presents a simple method of achieving both objectives, using a basis-correction methodology. The basis corrected estimation method can be used also to construct zero-value hedges that minimize the time variation in hedged portfolio returns.

Hedge ratios are estimated both under the traditional method and the basis-corrected method. Hedged portfolios are formed on a separate sample and the variances of the unexpected components of the two hedged portfolios are compared. The basis-correction methodology produces significantly smaller hedged portfolio return variances in many cases. However, the improvement is not similar across the board. For example, there seemed to be no effect on corn hedges at all. For wheat and soybean hedges, a weak pattern is detected, in that the effects are more visible for hedges lifted far from maturity.

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