Does the S&P 500 Futures Mispricing Series Exhibit Nonlinear Dependence across Time?

Ravi Vaidyanathan
Tim Krehbiel

INTRODUCTION

The S&P 500 futures contract provides unique advantages for institutional investors who wish to hedge diversified holdings and for those who choose to speculate on broad market movements. For such purposes the index futures contract and the underlying stock basket are perfect substitutes and the pricing of the futures contract should reflect this fact. Empirical analysis of the S&P 500 futures contract by Modest and Sundaresan (1983); Cornell and French (1983); Figlewski (1984a, b); Merrick (1987a, b); Arditti, Ayadin, Mattu, and Rigsbee (1986); and MacKinlay and Ramaswamy (1987) document significant and sustained deviation of the index futures price from its theoretical value. Extending these findings, Arditti et al. (1986), Merrick (1989), and Finnerty and Park (1988) show that index arbitrage could have been profitable through these periods. Yadav and Pope (1990) corroborate these results by finding evidence of mispricing in the U.K. Financial Times Stock Exchange (FTSE-100) stock index futures contract traded on the London International Financial Futures Exchange. Mispricing was also observed to be extensive during the period of the 1987 stock market crash and computerized stock index arbitrage was believed by many to have destabilized the stock market.[1]

Such findings of sustained mispricing and the availability of a "free lunch" by way of risk-free arbitrage trades are not consistent with the hypothesis of market efficiency. Following the stock market crash of 1987, policymakers have focused upon the cash and futures market linkages and many commissions have been appointed to examine the events surrounding the crash period.[2] Academic interest has been drawn also to the mispricing anomaly, and there have been attempts to statistically

[1]Mr. Charles Schwab in an article in *The Wall Street Journal,* April 25, 1989, states: "When program trading is roiling the water, there is no safe haven. At such times the futures market drives the stock market . . . " "To regain the confidence of individual investors, policy makers and the security industry need to take the lead in corralling the destruction aspects of program trading."

[2]Mackay (1988) summarizes the recommendation of the many commissions set up to investigate the crash.

Ravi Vaidyanathan is an Assistant Professor of Finance at Northeastern State University.

Tim Krehbiel is an Assistant Professor in the Finance Department at Oklahoma State University.

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analyze the model and process generating the index futures mispricing.[3]

The purpose of this study is to test for the existence of nonlinear time dependence in the S&P 500 futures mispricing series. The presence of nonlinear structure in the mispricing series would be consistent with a deterministic, as opposed to a stochastic explanation, for the mispricing anomaly. Nonlinearities are also a necessary condition for the existence of deterministic chaos. The observed irregular fluctuations in index futures mispricing levels and the steep discounts observed during the crash period are typical of a system displaying chaotic dynamics. This study also identifies some aspects of market microstructure that can generate chaotic dynamics in the mispricing series.

THEORETICAL BACKGROUND

Index futures mispricing is defined as the difference between the market price of the index futures contract and its theoretical value:

$$F(t) = S(t) \cdot \exp(r - d)(T - t) \tag{1}$$

The right-hand side of eq. (1) expresses the fair (theoretical) futures price as the cost of carrying the index basket of stocks to maturity. Here $F(t)$ is the price of the index futures contract at time t. $S(t)$ is the level of the underlying index at time t. r represents the risk-free interest rate and d is the dividend rate on the underlying basket of stock. Both rates are assumed constant for the duration of the futures contract. T is the maturity date of the futures contract.

In a continuous efficient market, and in the absence of transaction costs, there should be no deviation of the index futures price from its theoretical value. Any deviation would result in a profitable, risk-free, arbitrage opportunity. For instance, if $F(t)$ is greater than its theoretical value, the appropriate arbitrage action would be to sell the index futures contract and buy the underlying basket of stocks. The stock purchase is financed by borrowing at the risk-free rate. The riskless arbitrage profit would then amount to $F(t) - S(t)\exp(r - d)(T - t)$.

The opposite arbitrage action of purchasing the index futures contract and selling the underlying index basket of stock, would occur if the index futures price is below its theoretical value. In this case, the sale proceeds are assumed to be invested at the risk-free rate, thereby providing an arbitrage profit of $S(t)\exp(r - d)(T - t) - F(t)$.

Introducing transactions costs into the model provides bounds within which arbitrage would not be profitable. Since such costs depend upon the size of the arbitrage trade, the practice has been to express the index futures mispricing and the associated transaction costs as a percentage of the index value. If $M_{t,T}$ denotes the percentage mispricing at time t for the contract expiring in time period T, then its value is given by;

$$M_{t,T} = \frac{F_{t,T} - S(t)\exp(r - d)(T - t)}{S(t)}$$

[3]MacKinlay and Ramaswamy (1988), Brennan and Schwartz (1990), and Yadav and Pope (1990) represent some of this work.

The corresponding transaction costs bounds would be determined by

$$|M_{t,T}| = (T_s + I_s + T_f + I_f)$$

where:

T_s = percentage round trip stock commission,
T_f = percentage round trip futures commission,
I_s = percentage market impact in stocks, and
I_f = percentage market impact in futures.

The market impact costs should be negligible in an efficiently functioning market since the demand for both the stock basket and the related futures contract would be perfectly elastic at the full information price.

EMPIRICAL FINDINGS ON INDEX FUTURES MISPRICING

The existence of transactions costs would imply that mispricing within transaction cost bounds would still be consistent with market efficiency. However, index futures mispricing in excess of transactions cost bounds have been observed throughout the history of the S&P 500 futures contract. In an early study, using data for the June and December 1982 contracts, Modest and Sundaresan (1983) conclude that the futures price violates the theoretical bounds if market participants were assumed to have full use of short sale proceeds. Even so, as Figlewski (1984a) notes, well-diversified institutional investors need not resort to short sales to take advantage of profitable arbitrage opportunities. Evidence of mispricing for different time periods has also been documented by other researchers including Figlewski (1984a, b), Merrick (1987a, b), and Finnerty and Park (1988). In the most recent and comprehensive study yet of the extent of mispricing in the S&P 500 futures contract, MacKinlay and Ramaswamy (1988) compute intra-day mispricing levels at 15-minute intervals. The price histories studied cover the period from 1983 through 1987. Transaction cost boundaries are conservatively estimated at +0.6% and −0.6% of the cash index. Their results show that over the 16 contracts examined, the average mispricing amounted to 0.12% of the index value, with a high for the December 1986 contract of 0.78%. The authors record that out of 26,070 observations over all contracts there were 3149 upper-bound transactions cost violations and 602 lower-bound transactions cost violations. Therefore, on average, the transaction cost bounds were violated 14.3% of the time.

A number of reasons have been put forward to explain the mispricing puzzle. Almost all of these can be seen as variants of the transaction cost argument. That is, in some way, transactions costs are greater, or that index arbitrage is riskier than is assumed by the cost-of-carry model. These include: the beneficial tax timing option that is available only to the spot position and not the futures position [Cornell and French (1983)]; the real world stochasticity of dividend and interest rates in violation of the constant dividend and interest rate assumptions of the pricing model [Cox, Ingersoll, and Ross (1981)]; and the difficulty of tracking the S&P 500 index with only a subset of stocks [Figlewski (1984a)].

In conflict with these explanations is the observation by Rubinstein (1987) that arbitrageurs comprise large institutional investors with broad-based portfolio holdings. Institutional investors are not affected by tax considerations nor are they constrained to track the index with only a subset of stocks. An empirical study by

Cornell (1985) designed to follow up on the Cornell and French (1983) study failed to find supporting evidence for the tax argument.

If the stochasticity of interest rates is a factor in the mispricing of the index futures contract, then the same extent of mispricing should be observed in the case of other financial instruments that have futures contracts traded. An investigation of mispricing in the foreign exchange futures contract by Cornell and Reinganum (1981) does not support this explanation. Simulation of the impact of stochastic interest rates and marking to market on equilibrium stock index futures price by Modest (1984) shows that this effect would be minimal. These studies suggest that the observed mispricing is related to market features specific to the S&P 500 index futures contract.

In reality, stocks do not trade continuously and the stock index value often reflects stale prices. The existence of stale prices or nonsynchronous trading can cause autocorrelations in the index prices. As Harris (1989) points out, "If the nonsynchronous trading problem is ignored, spurious conclusions about volatility, market efficiency, and the relation between the futures and the cash market can be obtained and *arbitrage opportunities can be falsely identified*" (p. 78) (emphasis added). However, an empirical investigation by Harris (1989) of the October 1987 S&P 500 stock futures basis shows that "portfolio returns are autocorrelated even after the effects of nonsynchronous trading are explicitly removed" (p. 78). MacKinlay and Ramaswamy (1988) come to a similar conclusion using intraday prices for the period September 1983 through June 1987.

Yadav and Pope (1990) argue that transactions costs may well be lower than is commonly estimated. In an efficient market, it would be the transaction costs of the lowest cost trader that would be binding and not those of a marginal operator with the highest cost. It should be recognized also that arbitrageurs have the option to reverse their positions even prior to expiration if the mispricing level overshoots and changes sign. Such overshootings have the potential to increase arbitrage profits. The only cost of an early unwinding is the market impact cost of closing the futures position. The market impact cost should be negligible in an efficient market. MacKinlay and Ramaswamy (1988) show that such overshootings are common in the history of the contract. This factor could induce arbitrage trades at levels of mispricing within transactions cost bounds.

MISPRICING AND THE STOCK MARKET CRASH

The stock market crash of 1987, followed by the minibreak of 1989, graphically underscore the importance of index futures mispricing in the overall functioning of the market. It also brings the issue of index arbitrage into the focus of the public and the policymakers. On both occasions, the S&P 500 futures contract was observed to trade at a considerable discount, not only to the theoretical value but also to the S&P 500 index itself. Such a phenomenon should be a near impossibility in an efficient market. Specifically, for such a large discount to occur, given the cost-of-carry pricing relation, the expected dividend earnings from holding the index basket of stocks should exceed the interest cost, thereby enabling a negative cost of carry for the arbitrage transaction. The stock market crash and the resulting public outcry against computer-based index arbitrage, led to a number of studies[4] that focused upon the nexus between index arbitrage and stock market volatility. The

[4]See Mackay (1988).

results from these studies document the association between severe market volatility and the S&P 500 futures mispricing. The formulation of a regulatory framework to stabilize markets would be improved by analysis, understanding, and modeling of index futures mispricing.

MODELING INDEX FUTURES MISPRICING: RANDOMNESS VERSUS DETERMINISM

In a recent article, Stoll and Whaley (1990) demonstrate that, if the cost-of-carry pricing model holds, there should be no evidence of either linear of nonlinear serial dependence in the mispricing series. The path of mispricing observations should be random. The empirical observations to date however, contrast sharply with the theoretical extensions.

The MacKinlay and Ramaswamy (1988) study takes the first step toward examining the statistical characteristics of the mispricing series. Some of their observations relevant to this research can be summarized as follows:

1. The mispricing series display sharp reversals with a tendency for the mispricing levels to stay above or below zero for considerable periods of time.
2. The series is found to be extremely autocorrelated. In this context, MacKinlay and Ramaswamy (1988) suggest that the option to unwind arbitrage positions prematurely (prior to maturity) brings about the observed path dependence in the data.
3. The mispricing series does not appear to fluctuate randomly around zero.

MacKinlay and Ramaswamy (1988) conclude their study by recommending that future research should aim at modeling the arbitrage process, and the mispricing series.

In a recent study, using the same data, Brennan and Schwartz (1990) use a Brownian Bridge process to represent the evolving arbitrage opportunity (mispricing) to determine optimal arbitrage strategies for a trader. The authors note, however, that the Markov (path-independent) nature of the Brownian Bridge would not be in accordance with the observed path dependence in the mispricing series. In their conclusion, they observe that the ". . . real challenge remains to *endogenize* [emphasis added] the stochastic behavior of the simple arbitrage opportunity given the nature of transaction costs and the structure of the market" (pp. 22). An endogenous explanation would require a deterministic as opposed to a stochastic view of the intermarket linkages and the arbitrage process. As Yadav and Pope (1990) point out, "It is difficult to select a stochastic process to model 'mispricing' . . . when the empirical evidence by MacKinlay and Ramaswamy (1987) indicates that the mispricing series is path dependent" (p. 577).

THE HYPOTHESIS OF NONLINEAR DEPENDENCE

As stated, the objective of this research is to test for the presence of nonlinear dependencies in the S&P 500 mispricing series. Findings of nonlinear, low dimensional structure in the data series have some significant implications for the modeling of the mispricing series.

1. Nonlinear dependence is a necessary condition for the existence of deterministic chaos which is commonly interpreted to be the random appearing trajectory of

a purely deterministic system [Savit (1988)]. The observed irregular fluctuations of the index futures mispricing series suggest a chaotic (deterministic) explanation to the dynamic behavior of index futures mispricing.
2. A low-order (deterministic) dimensional reading, as opposed to a large or infinite (random) dimensional series, would suggest that the irregular fluctuations in the mispricing series have a deterministic (structural) explanation. A finding of a large or infinite dimension on the other hand would imply that the dynamic behavior of the mispricing series is caused by very many factors, and the appropriate modeling approach would be to determine the stochastic process that best explains its dynamic features.
3. Nonlinearities are a necessary condition for catastrophes. A catastrophe in this context, according to Rahn (1981), represents a sudden change in the state of a system in response to a smooth change in the parameters of the independent variables. An example would be a sudden change from a bull market to a bear market (stock market crash) that cannot be associated with discontinuous changes in any of the relevant environmental parameters. It is precisely this kind of turning point that modelers find difficult to forecast and, as Forrester (1987) points out, may well be a characteristic of nonlinear systems.

The case for the existence of nonlinear dependencies in the index futures mispricing series can be made by using a mix of empirical findings and observations of market microstructure. There have been a series of research studies that document the evidence of nonlinear dependence in economic and financial data. Brock (1988) surveys some of this work. Scheinkman and LeBaron (1989) examine a time history of daily returns on the value-weighted portfolio provided by CRSP and conclude that the portfolio returns in the sample exhibit nonlinear dependence across time. Blank (1990) finds strong evidence of nonlinear dependence and deterministic chaos in the S&P 500 futures price series.

Chaotic fluctuations could arise in the mispricing series due to the widely differing microstructure of the futures market and the stock market. Arbitrage serves to couple these two very different markets. The index futures market and the stock market differ functionally due to their methods of trading: an open outcry system in the Chicago Mercantile Exchange as compared to the specialist system that operates in the New York Stock Exchange. Information is quickly and fully reflected in futures market prices even if it involves large discontinuous jumps. In contrast, the specialist system is designed to cushion shocks, slow down price changes, and maintain continuity in price movements.[5] Stoll and Whaley (1990) conclude that the S&P 500 futures price leads the index by as much as ten minutes. It would appear that price discovery takes place in the futures market and the information is carried via arbitrage to the stock market. Because of the difference in the microstructure of the two markets, an arbitrageur, while responding to a mispricing, is likely to face a delay in putting through the stock market leg of the arbitrage. The immediate response in the mispricing level would be only partial, reflecting the change in the futures price alone. With more than one arbitrageur in the market, the partial response in the mispricing level may induce further arbitrage activity and could actually result in the overshooting of the arbitrage bounds.

[5]Report of the Committee on Market Volatility and Investor Confidence (1990): New York Stock Exchange Publication.

The existence of the uptick rule in the stock market and its absence in the futures market is another microstructure difference which may induce a delay in the response of the stock market price to arbitrage. The existence of the rule is cited by Edwards (1988) as one factor that exacerbated the market crash. The uptick rule restricts short sales on a stock below the price at which the last sale was effected. The purpose of this rule is to prevent panic selling in a declining market. It also serves as a deterrent against any bear raids.[6] In the context of futures mispricing, the uptick rule effectively delays the stock market leg of an arbitrage trade in an asymmetric manner: the time delay will be greater when the index futures is trading at a discount to the spot index, because the appropriate arbitrage response in this case requires the purchase of the futures contract in the CME and the short sale of the underlying basket of stocks in the NYSE. The delays that arise in executing the stock market leg of an arbitrage trade, for both of the above reasons, are capable of causing chaotic fluctuations and instabilities in the observed mispricing series.

The observation made above, that response delays within coupled nonlinear systems can cause chaotic fluctuations in observed time histories of system variables, is well established in system dynamics literature. Rasmussen and Mosekilde (1988) use system simulation to show how chaotic behavior can arise in a firm that allocates its resources between production and market depending upon its order backlog/inventory of finished goods. Faced with a backlog of orders, the company would shift its resources to production. If the inventory build up becomes excessive, the company would transfer resources to marketing. Nonlinear responses in this model are coupled with delays in generating production/sales to achieve a desired inventory. This results in irregular cyclical variation in sales. When select system parameters such as the customer defect rate are changed, the entire system shifts into chaos. The arbitrageurs response to perceived mispricing is analogous to the foregoing description of a typical firm's response to unsatisfactory levels of inventory. In both cases, a perfectly rational response by participants when channeled through a system characterized by nonlinearities and adjustment delays can cause very complex dynamical behavior.

Sterman (1989) investigates the capital investment decisions of experimental subjects operating in a simple multiplier accelerator (nonlinear) economy. Here, too, the time delay between capital investment and generation of production capacity generates instabilities and chaos in system levels.

Using a Kaldor-type business cycle model, Lorenz (1987) shows that the coupling of two regularly oscillating economic sectors can lead to irregular chaotic behavior in the entire system. This compares to the coupling of the dissimilar futures and cash markets by arbitrage activity.

Modeling of predator–prey relationships by Pool (1989) and Swart (1990) provide the clearest picture of the route to nonlinearities and chaotic fluctuations in the population level of the two species. There are equilibrating forces at work in nature that maintain population levels. An overly large predator population would decimate the numbers of prey. The shortage of prey would in time bring down the predator population enabling the prey to flourish once again. Surprisingly, however, the population levels of the predator and prey oscillate continuously and never reach a long-term equilibrium. The different gestation periods of the two species introduces a differential delay in the population adjustment mechanism causing continuous,

[6]Report of the Committee on Market Volatility and Investor Confidence (1990).

irregular cycles in the populations. The differential trading delays in the futures and the stock markets is analogous to the differential gestation periods of the predator–prey populations.

These studies identify two characteristics common to many systems that can occur together and bring about chaotic dynamics in the time values of the system variables:

1. When two or more dissimilar systems characterized by nonlinear relationships among variables are coupled through some form of feedback linkage.
2. When there are time delays in adjusting to system changes.

The market for the S&P 500 futures contract and the stock market are structurally dissimilar systems and arbitrage activity links the two. This fact, in combination with system delays in putting through arbitrage trades, could cause chaotic fluctuations in the mispricing series. An empirical verification of nonlinear dependence and a low-order dimension in the mispricing series would indicate that endogenous factors and system microstructure are one likely source of observed futures mispricing. Nonlinearities are a necessary condition for the generation of chaotic dynamics.

METHODOLOGY: TESTING FOR NONLINEAR SERIAL DEPENDENCE

This study follows the testing procedure first laid out by Brock, Dechert, and Scheinkman (BDS) (1986). The test relies on two concepts of dynamical systems: attractors and dimension. This section begins with a review of these relevant concepts.

The Concept of an Attractor

An attractor is a set of points toward which the dynamical path of a system converges. An attractor is a useful concept because it can be visualized as a geometric form. A formal definition of an attractor is found in Ramsey (1990):

"An attractor is a compact set A, such that the limit set of the orbit $\{x_n\}$ or $\{x_t\}$, as n or $t \rightarrow \infty$ is A, for almost all initial conditions within a neighborhood of A." (p. 125)

An attractor can have different forms: a single point, a limit or periodic orbit, a torus (doughnut-shaped attractor), or a string (chaotic) attractor. The form of the attractor, according to Radzicki (1990), can be used to classify dynamical systems:

1. Linear dynamical systems have restricted time paths. The attractor for linear dynamical systems is always a single point.
2. Nonlinear dynamical systems correspond to all four forms of attractors of which the strange attractor along is associated with deterministic chaos. Nonlinearity is therefore a necessary but not a sufficient condition for chaos. Dynamical systems, represented by torus-shaped attractors, display time paths that are aperiodic as in the case of chaotic attractors. However, sensitive dependence on initial conditions are characteristic only of chaotic attractors.

The Concept of Dimension

The aim of modeling is to represent the given data set with only a few dimensions or degrees of freedom. In mathematical terms, a single point is said to have dimension zero; a line has dimension one; a square has two dimensions; and a solid is three dimensional. A random series, by definition, cannot be modeled with a finite number of variables. It is said to have an infinite number of dimensions.

The most popular method of determining the dimension of a data series is the correlation dimension attributed to Grassberger and Procaccia (1983).

$$C_m(L) = \lim_{N\to\infty}\left[\frac{1}{N(N-1)}\right]\sum_{i\neq j}^{n}(L, Y_i, Y_j)$$

C_m is the correlation dimension, and is a measure of the number of points in embedded space m whose distance between each other is less than a predetermined length L. N is the number of observations in the sample. Y_i and Y_j are vector-valued observations and $(L, Y_i, Y_j) = (1, \text{ if } |Y_i - Y_j| < L;\ 0, \text{ otherwise})$.

One difficulty with this procedure is that the number of available observations decreases with each increase in embedding dimension. For instance, with an embedding dimension of 10, a continuous set of ten original observations would denote a single point in ten dimensions. This would mean that a data series as large as 1000 observations would provide only 100 data points in ten dimensions. It is important, therefore, to have a sufficiently large data set to estimate the Grassberger-Proccacia correlation dimension.

The dimension of the dynamical system is determined by first estimating the slope of the regression line of $\log C_m(L)$ versus $\log(L)$ for each embedded dimension m. The value of the slope where it stabilizes as the embedding dimension is increased, giving the estimate of the correlation dimension. If the data are generated by a random process, then the slope would continually increase with the embedding dimension m. This fact is made use of by Brock and Dechert (1986) to develop a test for serial independence in a data series.[7]

THE BROCK-DECHERT-SCHEINKMAN (BDS) PROCEDURE AND STATISTIC

If the data series is independently, identically distributed (iid), then the dimension d would never stabilize at any embedding dimension m, but would scale upward as m is increased. The test statistic for independence, based upon the correlation dimension, is then given by:

$$B(m, L, N) = N^{0.5}[C_m(L) - C_1(L)^m]$$

The statistic $B(m, L, N)$ converges to a normal distribution with zero mean and variance V. The variance V can also be consistently estimated from the sample data as $V(m, L, N)$. Dividing the statistic by the estimate of the standard deviation gives:

$$W(m, L, N) = B(m, L, N)/V(m, L, N)^{0.5}$$

The W statistic converges to a normal distribution with unit variance, i.e., $N(0, 1)$, which implies that inference based upon the standard normal distribution is possible. Rejecting the null hypothesis provides evidence of serial dependence in the data.

The alternate hypothesis can imply either linear of nonlinear dependence in the data series. To eliminate the possibility of linear dependence, the methodology used [Scheinkman and LeBaron (1989)] is to first fit an autoregressive (AR) process to the data and test the residuals from such a fit using the BDS statistic. A rejection by the BDS test applied to the filtered data would then support the hypothesis of nonlinear serial dependence in the mispricing series.

[7]For a derivation of the statistic and its properties see Brock, Dechert, and Scheinkman (1986).

Testing Procedure

1. Take the individual mispricing series (Y_t) and compute the BDS statistic for various embedding dimensions.[8] Determine whether the null hypothesis of iid innovations is rejected in favor of the alternate hypothesis of serial dependence in the data series.
2. For the same raw series (Y_t) fit the best linear autoregressive model to the data. Denote the number of lags as L.
3. Filter the raw series using the autoregressive filter of lag (L) and extract the residuals. Designate filtered residuals so obtained as U_t.
4. Compute the BDS statistic for the filtered residuals U_t. Rejection of the null hypothesis would support the alternative of nonlinear serial dependence in the mispricing series.
5. As a control measure, scramble the residuals U_t to eliminate any nonlinear structure. Denote the scrambled series S_t. Compute the BDS statistic for S_t. The null hypothesis of iid innovations should fail to be rejected in this case.
6. Nonlinear stochastic processes, such as the ARCH process, are capable of explaining much of the nonlinearity in economic time series. To account for this possibility, fit an ARCH model to the mispricing series. Use the standardized residuals from the estimation to test for dependence using the BDS statistic. If the ARCH process accounts for the nonlinear dependence in the data then the BDS test should fail to reject the null hypothesis of independence in the residuals. In a study of the nonlinear dependence in stock return series, Scheinkman and LeBaron (1989) apply the ARCH process, described in Engle (1982), as a filter for the stock return series. Following Scheinkman and LeBaron (1989) the order of the ARCH process used to produce standardized residuals is chosen using the Akaike Information Criteria (AIC). Application of the AIC to the June 1987 raw mispricing series leads to the selection of 1 as the optimal order for the ARCH process. The standardized residuals, $\epsilon_t = (y_t - m_t) * (v_t)^{-1/2}$, are obtained from maximum likelihood estimation of the ARCH model described by (1)–(3).

$$(y_t | y_{t-1}) \sim N(m_t, v_t) \tag{1}$$

$$m_t = \alpha_0 + \alpha_1 y_{t-1} + v_t \tag{2}$$

$$v_t = v + \delta(y_{t-1} - m_{t-1})^2 \tag{3}$$

7. For the raw series, regress the $Ln\ Cm(L)$ against the $Ln(L)$ for each embedding dimension (m). Denote the value of the slope of each regression line as d. Plot the slope d against the embedding dimension m, and determine whether the slope stabilizes (becomes horizontal) at any point. The value of the slope at this point is the correlation dimension of the data series. If the series is random (infinite or large dimension) the slope would never stabilize at any point but would instead scale upward with every increase in m. On the other hand, the finding of a low-order correlation dimension in combination with a finding of nonlinearity is evidence supporting the hypothesis of chaotic structure in the mispricing series. The combined March and June 1987 mispricing series is

[8]This study uses a computer algorithm to determine the correlation integral and the BDS statistic for every embedding dimension. The software was provided by Dr. William Brock of the University of Wisconsin (Madison). His assistance is gratefully acknowledged.

used to generate a sufficiently long and continuous series of observations for estimating the correlation dimension.

DATA

The data for this study are from and is used with the permission of MacKinlay and Ramaswamy (1987, 1988). Each data series represents the futures mispricing for the S&P 500 futures contract which is computed as the difference between the S&P 500 index futures price and the theoretical forward price. It is expressed as a percentage of the index value. The data series starts with the September 1983 S&P 500 contract and follows the December, March, June, September cycle with the last series relating to the June 1987 S&P 500 contract. Each of the series has approximately 1600 observations. The stock index quotes are time stamped approximately one minute apart while the futures are time stamped transaction data. MacKinlay and Ramaswamy (1987, 1988) use the futures quotes at 15-minute intervals to compute the mispricing series. The cost-of-carry is computed using the daily dividend yield of the value-weighted index of all NYSE stocks supplied by the Center for Research in Security Prices as a proxy for the dividend yield on the S&P 500. The daily interest rate on certificates of deposit is the remaining input used to compute the theoretical forward price and the mispricing series.

The futures mispricing is computed through the entire calendar period using futures prices from the nearest contract at any point. The nearest contract is also the most frequently traded contract. Observations for the June 1984 mispricing series for example, use prices from the commencement of trading on March 16, 1986, the day after expiry of the March 1984 contract. This makes it possible to link the mispricing observations into a single, continuous data series stretching across contract periods.

RESULTS

The testing procedure described in the Methodology section is performed on the June 1987 contract, which is the most recent data series available.

Table I gives the results of the BDS test for independence. The lengths used for the BDS test are defined as various proportions of the standard deviation of the series. The results of the test for independence are reported for embedding dimensions 1 through 4. The null hypothesis of an independent, identically distributed times series can be rejected at the 5% level if the W statistic value exceeds 1.96. As can be observed from the last column of Table I, the null is rejected in every case which supports the alternate hypothesis of serial dependence in the mispricing innovations.

To eliminate the possibility of linear dependence in the mispricing series, the procedure adopted is to fit the best AR process to the data. The AR order is determined using Parzen's CAT criterion. For the June 1987 mispricing series, the CAT criterion finds an AR order of 10 to be the best linear fit to the data.

Table II gives the AR values and the corresponding lags. The residuals obtained using this AR filter are used for the second stage of the testing.

Table III shows the results of the BDS test for the filtered residuals using the linear filter. Even though the value of the W statistic in every case is lower than the raw series, the null hypothesis of iid innovations is clearly rejected, supporting the alternate hypothesis of nonlinear dependence in the mispricing series.

As a further control, the filtered residuals are scrambled to eliminate any deterministic structure. The BDS test should fail to reject the null hypothesis of a random iid

Table I
BDS STATISTICS FOR THE JUNE 1987 MISPRICING SERIES

Length (in Std. Dev.)	Embedding Dimension	BDS $[B(m, L, N)]$	Std. Dev. (BDS)	W Statistic (BDS/SD)
0.50	2.00	1.395	0.030	46.49
0.50	3.00	1.262	0.023	54.93
0.50	4.00	0.851	0.013	64.52
0.40	2.00	0.979	0.020	48.76
0.40	3.00	0.711	0.012	58.00
0.40	4.00	0.388	0.006	69.10
0.30	2.00	0.628	0.012	51.05
0.30	3.00	0.356	0.006	61.57
0.30	4.00	0.151	0.002	74.19
0.20	2.00	0.296	0.006	53.41
0.20	3.00	0.113	0.002	65.67
0.20	4.00	0.033	0.000	82.43

Table II
AUTOREGRESSIVE LAGS AND COEFFICIENTS

Lags	1	2	3	4	5
Coeff.	−0.3489	−0.0885	−0.1186	−0.0305	−0.0595
Lags	6	7	8	9	10
Coeff.	−0.0555	−0.0302	−0.0237	−0.0379	−0.0711

series. This proves to be the case as evidenced by the W statistics for the scrambled series shown in Table IV. As can be seen, the null hypothesis of iid innovations is not rejected in all of the runs at different lengths and embedding dimensions. In addition, the wide difference in the value of the W statistic for the filtered residuals series, as compared to the scrambled residuals, further supports the hypothesis of nonlinear structure in the mispricing series.

Table V shows the results of applying the BDS test on the residuals from the best-fitting ARCH process. Rejection by the BDS statistic would indicate that the mispricing series cannot be modeled as a nonlinear stochastic ARCH process and would support the possibility of a deterministic/chaotic explanation for the data. As can be seen in Table V, the null hypothesis of independence is rejected in three out of five cases, implying that the ARCH process used cannot account for all the nonlinearity in the data series.

IDENTIFYING THE CORRELATION DIMENSION

A finding of low-order dimension would be strong evidence in favor of a chaotic (deterministic) explanation for the persistence of index futures mispricing. The objective here is to identify the Grassberger-Proccacia correlation dimension by searching across 15 embedded dimensions. The combined March and June 1987 mispricing series is used for this purpose.

Table III
BDS STATISTIC FOR FILTERED RESIDUALS

Length (in Std. Dev.)	Embedding Dimension	BDS $B(m, L, N)$	Std. Dev.	W Statistic (BDS/SD)
0.50	2.00	0.3225	0.0585	5.51
0.50	3.00	0.3462	0.0534	6.49
0.50	4.00	0.2519	0.0365	6.89
0.40	2.00	0.2074	0.0421	4.92
0.40	3.00	0.1743	0.0311	5.61
0.40	4.00	0.0987	0.0172	5.74
0.30	2.00	0.1214	0.0260	4.66
0.30	3.00	0.0761	0.0146	5.23
0.30	4.00	0.0322	0.0061	5.27
0.20	2.00	0.0581	0.0130	4.48
0.20	3.00	0.0283	0.0050	5.67
0.20	4.00	0.0100	0.0014	6.91
0.10	2.00	0.0179	0.0035	5.10
0.10	3.00	0.0050	0.0007	7.24
0.10	4.00	0.0010	0.0001	9.33

Table IV
BDS STATISTIC FOR SCRAMBLED RESIDUALS

Length (in Std. Dev.)	Embedding Dimension	BDS $[B(m, L, N)]$	Std. Dev.	W Statistic (BDS/SD)
0.5	2.0	−0.005722	0.061033	−0.0938
0.5	3.0	0.003723	0.057704	0.0645
0.5	4.0	0.015724	0.040942	0.3841
0.4	2.0	−0.002134	0.041781	−0.0511
0.4	3.0	0.000642	0.030756	0.0209
0.4	4.0	0.004803	0.016997	0.2826
0.3	2.0	0.005448	0.026521	0.2054
0.3	3.0	0.003634	0.015016	0.2420
0.3	4.0	0.001582	0.006385	0.2478
0.2	2.0	0.003674	0.012521	0.2934
0.2	3.0	0.003254	0.004707	0.6913
0.2	4.0	0.001980	0.001329	1.4890

Table VI shows the results of the regression of $Ln(Cm)$ against $Ln(L)$ for 15 embedding dimensions. The X coefficients are the slope of the regression line which provides an excellent fit as evidenced by the regression coefficient (R^2), which in all cases is greater than 0.9. The value of the X coefficient (slope of regression line) where it stabilizes is the correlation dimension of the mispricing series. As can be observed from Table VI, the slope coefficient increases for initial embedded dimensions and then stabilizes around the value of 6 even as the embedded dimension is increased.

Table V
BDS STATISTIC FOR RESIDUALS FROM ARCH FILTER

Length (in Std. Dev.)	Embedding Dimension	BDS	SD	W Statistic BDS/SD
0.5	2.00	0.012456	0.043062	0.29
0.5	3.00	0.033067	0.035284	0.94
0.5	4.00	0.053446	0.021691	2.46
0.5	5.00	0.045943	0.01168	3.93
0.5	6.00	0.032889	0.0058218	5.65
0.4	2.00	0.0030502	0.031101	0.10
0.4	3.00	0.018341	0.021021	0.87
0.4	4.00	0.024707	0.010662	2.32
0.4	5.00	0.016253	0.0047368	3.43
0.4	6.00	0.010008	0.0019483	5.14
0.3	2.00	0.0023077	0.018121	0.13
0.3	3.00	0.0070085	0.0090912	0.77
0.3	4.00	0.0091386	0.0034234	2.67
0.3	5.00	0.0037672	0.0011293	3.34
0.3	6.00	0.0017429	0.00034493	5.05

Table VI
ESTIMATING THE CORRELATION DIMENSION: JUNE 1987 MISPRICING SERIES

Emb. Dim.	1	2	3	4	5
Slope Coeff.	0.97	1.86	2.73	3.62	4.58
Emb. Dim.	6	7	8	9	10
Slope Coeff.	5.23	5.63	6.02	6.30	6.50
Emb. Dim.	11	12	13	14	15
Slope Ceoff.	6.58	6.66	6.66	6.73	6.96

Figure 1 shows a plot of the slope coefficient of the regression equations against the embedding dimension. The 45° line represents the theoretical plot of a random process which is shown for comparison. As stated earlier, for a random process the slope coefficient would increase with an increase in the embedding dimension. The slope coefficient of a deterministic process, on the other hand, would stabilize at the value of the correlation dimension. For the combined mispricing series, the slope coefficient stabilizes at around a value of 6 which is consistent with a low-order nonlinear deterministic process.

The value of the correlation dimension obtained here has to be viewed with caution. Empirical estimates are sensitive to sample size and the presence of noise in the data set. Results from Ramsey and Yuan (1989) indicate that dimension estimates are biased downward for random noise and biased upward for attractors. As Sayers (1990) states, " . . . a correlation dimension estimate of 0.214 could imply an actual correlation dimension value of as high as 1.68" (p. 185). The emphasis of this study is merely to examine whether a deterministic explanation exists for the behavior of index futures mispricing. The finding of a low-order correlation dimension is

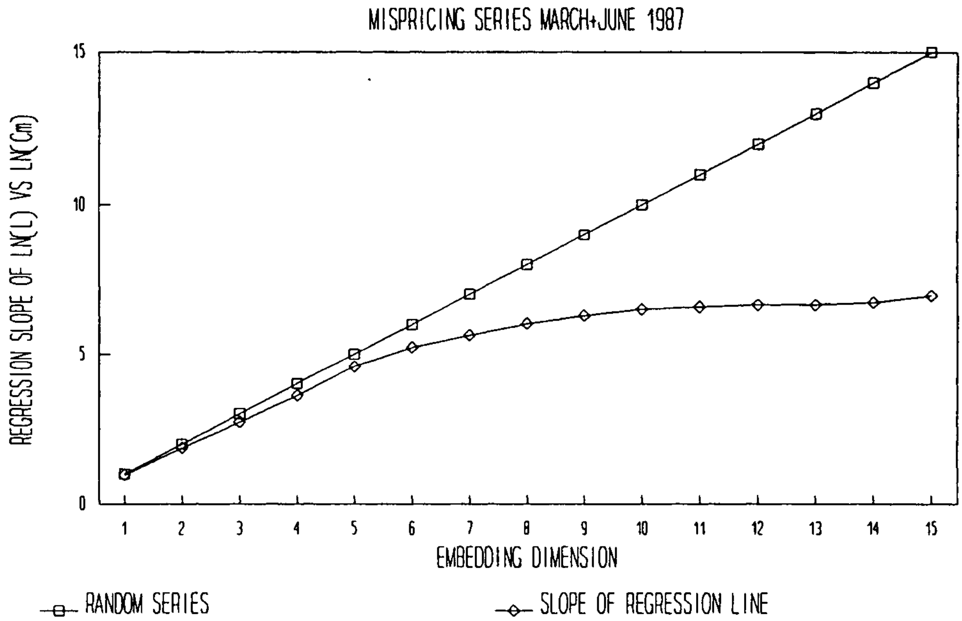

Figure 1. Correlation dimension.

sufficient to support the hypothesis of nonlinear dependence in the mispricing series. Determination of the precise value of the correlation dimension is not critical to the arguments made in this article.

SUMMARY AND CONCLUSIONS

There have been many findings in recent years of regularities and long-term serial dependence in financial time series. Such findings contradict the assumption of market efficiency. The science of nonlinear dynamics can provide some insight into the nature and form of such regularities. In particular, it can be demonstrated that a time history of a nonlinear function (F) of the form,

$$Y(t) = F[Y(t-1), Y(t-2), \ldots, Y(t-i)]$$

can appear quite random even though it is completely deterministic.[9] It has been hypothesized that the apparent randomness of economic and financial series may actually be caused by nonlinear relationships among the variables. This may help explain the tantalizing findings of regularities in financial data.

Researchers have identified nonlinearities and deterministic chaos in financial time series. However, a finding of low-order dimension and nonlinear structure tell us nothing about the precise nonlinear functional form or even the general class of nonlinear relations among the variables under investigation. Nonetheless, the empirical finding of chaotic dynamics in the mispricing series is of considerable value even without the specification of the precise nonlinear mathematical relationships. *This is so because market efficiency should imply the absence of any deterministic structure whatsoever.* This study finds evidence of low-order determinism and

[9]May (1976) illustrates this using select nonlinear functions. So do Baumol and Benhabib (1990).

nonlinearities in the S&P 500 mispricing series. In addition, it identifies some structural issues such as the differential delays in trading the index basket in the futures market and the cash market which, when combined with nonlinear adjustment processes within the system, can generate sustained and irregular fluctuations in mispricing levels. Such wide fluctuations can potentially destabilize the market. A rigorous examination of the dynamics of the arbitrage process in a nonlinear environment with adjustment delays is necessary to validate this hypothesis.

An ongoing research project uses the system dynamics simulation methodology to represent the arbitrage linkage between the two markets. Trading delays are then introduced into the system and its effect on the path of index futures mispricing is analyzed. Preliminary simulation results indicate that introducing differential delays in an otherwise equilibrating system causes sustained fluctuations in mispricing levels. Increasing the strength of arbitrage in this system only serves to increase the fluctuations. This result seems counterintuitive but is not altogether surprising in the context of system dynamics.

If structural differences are indeed a potential cause of market instabilities, then the appropriate policy action should not rely upon different uptick rules and different restrictions on program trading for the cash and futures markets. These only serve to lengthen the differential trading delay in the futures and the cash market. The emphasis of regulatory effort should instead be on reducing the structural differences between the two markets by speeding up trading in the stock market as opposed to slowing it down. These conclusions are in line with the free market position on market regulation and can be used to validate this policy stance.

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