Optimal Weights and International Portfolio Hedging with U.S. Dollar Index Futures: An Empirical Investigation

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INTRODUCTION

The recent explosion in overseas equity and bond portfolios has created a need for hedging in several currencies. One alternative to using multiple forward contracts is to use a currency index futures contract such as the U.S. Dollar Index (USDX). The USDX is a weighted geometric average of the foreign exchange rates (FX) of ten major currencies. Its base value of 100 was set in March 1973. Specifically, the index value, I, in period t is given by the following (see Table I for details):

$$I_t = 100 \prod_{i=1}^{10} [B_i / S_{i,t}]^{w_i} \tag{1}$$

where:

- B_i = base exchange rate for currency i, set in March 1973, expressed in U.S. currency units per unit of foreign currency (American terms).
- $S_{i,t}$ = current prices of currency i in period t, also expressed in American terms.
- w_i = weight assigned to each currency in the index.

The USDX is distinct from other indices because:

(a) it is a geometric rather than an arithmetic average;
(b) the current prices are stated in the denominator; and
(c) the weights assigned are chosen to reflect the relative importance of each country's trade.

These features not only distinguish the USDX from other indices, but also affect the valuation of its derivative security, the USDX futures. USDX futures have been trading on the Financial Instrument Exchange (FINEX), a division of the New York Cotton Exchange, since November 20, 1985.

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Table I
COMPOSITION OF THE U.S. DOLLAR INDEX

Country/Currency	Base Exchange Rate[a]	Index Weights
Germany/deutschemark	35.5480	20.8%
Japan/yen	0.3819	13.6%
France/franc	22.1910	13.1%
United Kingdom/pound	247.2400	11.9%
Canada/dollar	100.3300	9.1%
Italy/lira	0.1760	9.0%
Netherlands/guilder	34.8340	8.3%
Belgium/franc	2.5377	6.4%
Sweden/krona	22.5820	4.2%
Switzerland/franc	31.0840	3.6%

[a]U.S. cents per unit of foreign currency. The base period is as of March 1973.

U.S. DOLLAR INDEX FUTURES CONTRACT SPECIFICATION

USDX futures:
Contract size $500 times the U.S. Dollar Index (beginning with the September 1992 contracts, this will become 1000 times the Index).

Trading hours:
8:20 AM to 3:00 PM New York time, except on the last trading day, when trading ceases at 10:00 AM.

Contract months:
March, June, September, and December.

Ticker symbol:
DX.

Price quotation:
The U.S. Dollar Index is quoted as a percentage of its value as of March 1973, calculated to two decimal places (e.g., 123.55).

Minimum price fluctuation:
The minimum price fluctuation, or tick, is 0.01 of a U.S. Dollar Index point, which is equivalent to $5.00 per futures contract.

Daily limit on price movement:
None.

Last day of trading:
The last day of trading is the third Wednesday of the expiring contract month.

Settlement procedures:
Contracts held to expiration are subject to cash settlement, based on the settlement value of the U.S. Dollar Index at 10:00 AM (New York time) on the last day of trading for an expiring contract. The U.S. Dollar Index settlement value is computed by Reuters, Ltd., in accordance with the New York Cotton Exchange regulations.

Prior articles have studied the effects of the first two features on the estimation of the USDX futures and show that they cause an upward bias. Redfield (1986) shows that this upward "drift" factor, which he calls the *volatility premium,* results from the use of the current prices in the denominator of the index. An advantage of this feature is that the index behaves as if it is quoted in European terms. Thus, the index increases (falls) when the dollar, expressed in American terms, appreciates (depreciates). A disadvantage, however, is that a $1 + x$ rise in currency rates leads to a $1/(1 - x)$ rise in the index, due to Jensen's inequality. Eytan, Harpaz, and Krull (EHK, 1988) show that the drift also arises as a consequence of the index being a geometric average as opposed to arithmetic average. To model this drift, one has to make some assumptions on the currencies' stochastic price changes. EHK provide one such model by using Merton's (1973) option pricing methodology. Assuming foreign exchange rates were lognormally distributed, they show that the drift, δ, can be modeled as follows:

$$\delta = \frac{1}{2}\left[\sum_{i=1}^{10} w_i \sigma_i^2 + \sum_{i=1}^{10}\sum_{j=1}^{10} w_i w_j \sigma_i \sigma_j \rho_{ij}\right] \tag{2}$$

where w_i = the weight of currency i in the index, σ_i = standard deviation of currency i, and ρ_{ij} = correlation between currency i and j. The drift factor is not only a function of the variance–covariance matrix but also of the weights used in the index.

The goals of this article are twofold. The first is to assess whether the current index provides a reasonable hedge for an international portfolio of equity assets which are weighted according to market capitalizations. The second is to assess whether an index futures contract that is weighted according to market capitalizations can provide a better hedge than the existing contract. This is examined by constructing a hypothetical futures contract using a set of market weights.

IMPACTS OF WEIGHTS ON INDICES

Most indices are influenced by the weights used in their construction. For example, price-weighted indices like the Dow Jones Industrial Average (DJIA) or the Major Market Index (MMI) use absolute prices when they are estimated. Consequently, all stocks with equal prices have equal weights. One disadvantage is that the effect of a percentage change in the price of a stock in the index will be sensitive to the initial price of the stock. Another disadvantage is that changes in the prices of thinly traded stocks have the same weight as those that are heavily traded. This disadvantage also exists with an equally weighted index, such as the Value Line Index, which uses percentage price changes instead of absolute price changes. An additional disadvantage of an equally weighted index is that it is difficult to mimic, because a replicating portfolio must be continuously rebalanced when there is a price change of any stock included in the index [see Siegel and Siegel (1990, chap. 4) for some illustrative examples].

On the other hand, value-weighted indices, such as Standard & Poor's S&P 500, weight each stock by its relative market capitalization. This method of constructing an index gives more weight to firms with larger market capitalizations. It is also easy to mimic such an index because there is no need to rebalance a properly weighted portfolio.

In the case of the USDX, the weights assigned reflect the multilateral trading volume of the ten countries in the index. Such a weighting scheme is common in most of the dollar indices, including Morgan Guaranty's 15-country nominal and real indices, the Atlanta Fed's 18-country, and the Chicago Fed's 16-country indices.

The use of trade weights reflects the intended purpose of these indices, to summarize the impact of various exchange rates on trade balances and vice versa. The precise weights assigned to the index appear to be unimportant. Hervey and Strauss (1987) examined 12 trade-weighted indices and concluded that, for the most part, neither the difference in weights nor the inclusion nor omission of certain countries materially affected their values.[1]

Two major users of the USDX futures contracts (hereafter referred to as the USDX), apart from speculators, are corporate and portfolio managers, who use the contracts as hedging instruments. Since the current weights of the USDX are based on world trade patterns in the early 1970s, they have little relationship to either the outstanding stock of currencies or current market capitalizations.[2] Intuitively, the optimal weights of a futures contract should equal those of the underlying cash portfolio.

Ott (1987) has recognized the importance of capital flows in the formulation of a dollar index. He notes that the demand for dollars is a function of trade flows *and* capital flows. He demonstrates that capital flows-weighted indices do not behave differently from trade flows-weighted indices. The motivation and analysis of this study are similar in nature. One question addressed is whether portfolio managers are better off with a futures contract that has weights similar to those of their cash portfolios. To show this, one can test whether a futures contract with weights similar to cash portfolios provides a better hedge for currency risk than the USDX. If this is the case, then it may be advisable for FINEX to change the weights of the USDX, or to introduce a new contract to satisfy the hedging needs of portfolio managers.

HEDGING STRATEGIES AND HYPOTHESIS

Modern portfolio theory, applied internationally, states that the optimal world market portfolio should be comprised of assets weighted according to their market capitalizations. A portfolio manager holding such a portfolio is subject to currency risk, which can be hedged by using currency futures.[3] However, as reported in the FINEX Report (1988), such a manager does incur the following problems, which are minimized if currency index futures are used:

(a) high transaction costs for setting up hedges in several currencies;
(b) illiquid markets for some of the minor currencies in the index; and
(c) higher basis risk between the cash and futures positions when dealing with several currencies.

Two international indices currently popular in the market are the Morgan Stanley/ Capital International's Europe, Australia and Far East Equity Index (EAFE) and the Salomon Brother's Non-U.S. Dollar World Government Bond Index (WGBI). Japanese assets comprise more than 50% of both indices. However, no futures contracts on these indices currently exist. The only currency index futures contract traded is the USDX, but it assigns a weight of only 13.6% for the Japanese yen.[4] Recognizing this imbalance, a FINEX Report (1988) recommends hedging

[1]See also Hunter (1990) who shows that the effects of inflation is minimal on the Atlanta Fed's Index.

[2]Specifically, the Federal Reserve Board's Index is based on a multilateral weighting scheme as opposed to a bilateral one. See Rosenweig (1986) for a review and problems between the two approaches.

[3]See Black (1989) for his derivation of a universal hedging formula that can satisfy holders of any combination of foreign assets.

[4]FINEX also offers a European Currency unit (ECU) futures which has not been very successful.

an international portfolio by shorting yen currency futures (50%) and going long on the USDX (50%).

One purpose of this study is to examine whether the USDX serves as an effective hedging tool for equity managers with market-weighted international portfolios. A manager is assumed to hold a portfolio of stocks from the ten countries in the USDX. This portfolio is weighted according to the ten countries' stock market capitalizations. To hedge currency risk a manager can:

1. Hedge the market-weighted cash portfolio with a trade-weighted USDX futures contract (currently trading on FINEX).
2. Hedge a market-weighted cast portfolio with a trade-weighted futures contract and adjust for the difference in weights by buying/selling currency futures.

Another purpose of this study is to simulate the hedging ability of a market-weighted futures contract. If a market-weighted futures contract is available, an additional option is available:

3. Hedge a market-weighted cash portfolio with a market-weighted futures contract (a hypothetical contract).

The payoffs of these strategies for each day, i.e., from day t to $t + 1$, are given in eqs. (A1)–(A3) in the Appendix.

When hedging, the optimal strategy for a portfolio manager is to minimize variance [as in Ederington (1979)]. This strategy is applied in this study. It is clear that implementing strategy 2 will incur substantially higher transactions costs than strategy 1 (for the reasons cited earlier regarding the use of several currency futures). Therefore, the analysis is restricted to strategies 1 and 3. This should determine whether the USDX is superior to a market-weighted index. Moreover, if strategy 3 is superior to strategy 1, it must also be superior to strategy 2.

To test empirically which index provides the lower variance portfolio, a portfolio of currencies is constructed to replicate the holdings of a portfolio of stocks. This portfolio is then hedged with two futures contracts, one based on the existing trade-weighted index (USDX) and the other on a hypothetical market-weighted index, which shall be called the USDM. An examination of the portfolio variances should indicate which index futures provides a better hedge. The two hedged portfolios are defined as follows:

1. Portfolio P_1: consists of a portfolio of ten currencies weighted according to market capitalizations, but hedged with the actual trade-weighted index futures (USDX).
2. Portfolio P_2: consists of a portfolio of ten currencies weighted according to market capitalizations and hedged with a (hypothetical) market-weighted index futures contract (USDM).

The terms are defined as follows:

R_{sm} = the average daily percentage return of a spot portfolio of currencies weighted by market capitalizations, m_i. The weights are specified in Table II.

R_{fa} = the average daily percentage return of the trade-weighted or *actual* USDX futures contract, currently trading on FINEX.

Table II
WEIGHTS CURRENTLY USED IN THE USDX (TRADE-WEIGHTED) AND THE USDM (MARKET CAPITALIZATION WEIGHTS)

Currency	Old Weights (Trade)	New Weights (Market)[a]
BP	11.9%	15.35%
DM	20.8%	4.97%
JY	13.6%	60.96%
FF	13.1%	3.61%
CD	9.1%	4.97%
IL	9.0%	2.71%
FL	8.3%	1.96%
BF	6.4%	0.95%
SK	4.2%	1.58%
SF	3.6%	2.94%

[a]The market weights are based on December 31, 1987 capitalizations of the ten countries. See Table III for abbreviations.

R_{fm} = the average daily percentage return of a market-weighted futures contract, constructed by using weights, m_i (USDM).

$\sigma^2_{sm}, \sigma^2_{fa}, \sigma^2_{fm}$ = the respective variances of the above three returns.

Finally:

$R_1 = R_{sm} + R_{fa}$, the return of portfolio P_1. The signs of R_{sm} and R_{fa} will generally be opposite.

$R_2 = R_{sm} + R_{fm}$, the return of portfolio P_2.

σ^2_1, σ^2_2 = variances of the above two returns.

The daily returns (R_1, R_2) and the variances (σ^2_1, σ^2_2) of the hedged portfolios are the focus of examination. The hedged portfolio that provides the lowest variance is considered the best alternative. Specifically, the following hypotheses are tested: $\mathbf{H}_0$: $\sigma^2_2 = \sigma^2_1$ and $\mathbf{H}_1$: $\sigma^2_2 < \sigma^2_1$. The alternative hypothesis states that the variance of the daily returns of P_2 is less than P_1. If the null is rejected, it suggests that portfolio managers holding market-weighted spot portfolios are better off hedging with a similarly weighted currency index futures contract (USDM) rather than the existing trade-weighted futures contract (USDX). This implies that any well-diversified portfolio is optimally hedged with this market-weighted futures contract.[5]

Finally, the hedging effectiveness of portfolios P_1 and P_2 is examined. Specifically, the reduction in risk relative to the cash portfolio is estimated. Risk reduction is measured as the percentage difference in the variances between the hedged and unhedged (cash) portfolio, relative to the unhedged portfolio variance, i.e., $(\sigma^2_{sm} - \sigma^2_j)/\sigma^2_{sm}$, where $j = 1, 2$. If the existing USDX futures contract reduces risk less than the constructed USDM futures contract, there would appear to be strong grounds to recommend the introduction of a market-weighted futures contract to assist portfolio managers.

[5]Any well-diversified portfolio is near perfectly correlated with the market portfolio, by definition.

DATA AND METHODOLOGY

The data for this study consists of daily USDX spot and future prices as well as the individual currency rates. The data is from FINEX. There is a slight discrepancy regarding the timing of the data. The USDX spot and futures values are from 2:40 PM, the close of the USDX market.[6] The individual daily spot and forward prices are recorded at 1:00 PM. However, since the variances of the time-series movements are of greater concern, this discrepancy should not affect the results materially.[7] Table III summarizes the data used in the model. The mean and standard deviation of the spot and the forward rates do not exhibit significant differences. The minimum and maximum values suggest that both rates move across wide range during the period of study. The Shapiro–Wilks tests reject the null hypothesis of normality for all the currencies. This requires the use of nonparametric tests to estimate the significance of the test statistics on the variances of the average daily returns.

The period of study begins on November 20, 1985, when the USDX began trading, and extends until September 6, 1988. This period is divided into 11 subperiods, corresponding to the trading periods of the nearby futures contracts. The first USDX futures contract has a maturity of 120 days. The contracts traded thereafter average the normal 90 days. The usable trading days range from a high of 73 to a low of 48 days per subperiod, as listed in Table IV. Finally, the 1987 market capitalizations are used to construct the market-weighted portfolio and the USDM futures contract. This data are obtained from Black (1989, Table 11). Since his data also includes the U.S. capitalization, the U.S. share is excluded and the weights are recomputed. A summary of the new weights is provided in Table II.[8]

The development of an appropriate unbiased USDM futures contract presents a few problems. The futures price of the USDM has to be approximated by using market weights and substituting f_i for s_i in eq. (1). However, two problems have to be addressed before computing a reasonable estimate of the price. The first deals with the fixed 90-day maturity of the daily forward rates provided by FINEX. The index computation requires that the maturity of the forward rates declines toward zero as the futures contract approaches its expiration date. The forward rates are adjusted by using a simple extrapolation technique, i.e., by assuming a linear relationship between the spot and forward rates. Specifically,

$$f_t = s_t\left(1 + x \cdot \frac{dtm}{90}\right) \quad \text{where} \quad x = \frac{f_t}{s_t} - 1 \quad \text{and} \quad dtm = \text{days to maturity.}$$

The second problem deals with the inverse geometric nature of the index, which is shown by Eytan, Harpaz, and Krull (1988) to create a upward bias. To correct for this bias, an adjustment factor is estimated from the actual futures price on day t. A forward index, $I_{w,t}$, is constructed first using forward prices and the *existing* USDX trade weights for each day, t. This is followed by dividing the actual price prevailing

[6]Trading now continues until 3 PM each day.

[7]The data set, supplied by FINEX, is identical to the set used in Harpaz, Krull, and Yagil (1990). The spot index at 2:40 PM combined with the interest rate differential from 1 PM is used to create a forward index at 2:40 PM. Since the weighted average interest rate differential is quite stable, the time difference should not bias the results. However, the standard error would be increased, reducing the possibility of rejecting the null hypothesis.

[8]Black obtained this data from *Activities and Statistics: 1987 Report* by the Federation Internationale de Bourses de Valeurs. It is possible to use market capitalizations for each of the four years, 1985–1988. Since the goal of this article is to determine whether the weights chosen in 1973 are inadequate, a direct comparison is made by choosing fixed weights from a more recent period.

Table III

SUMMARY STATISTICS OF THE USDX SPOT, USDX FUTURES, AND THE SPOT AND FORWARD RATES OF 10 CURRENCIES IN THE INDEX

Item	N	Mean	Std. Dev.	Min.	Max.	W Normal[a]
USDX	659	102.1583	10.41476	85.42	128.54	0.912093[a]
USDX futures	659	102.4205	10.49739	85.66	128.86	0.913527[a]
SBF	659	0.025186	0.002751	0.018965	0.030193	0.923586[a]
FBF	659	0.025183	0.002776	0.018900	0.030226	0.923139[a]
SCD	659	0.752663	0.034407	0.6944	0.83612	0.886797[a]
FCD	659	0.751248	0.034365	0.693190	0.834200	0.889862[a]
SFF	659	0.158378	0.01414	0.125549	0.187617	0.941580[a]
FFF	659	0.158072	0.014227	0.124537	0.186869	0.940906[a]
SIL	659	0.000732	0.000066	0.000567	0.000857	0.927404[a]
FIL	659	0.000728	0.000067	0.000550	0.000850	0.924456[a]
SJY	659	0.00672	0.000851	0.004889	0.008251	0.940561[a]
FJY	659	0.006739	0.000860	0.004896	0.008306	0.941558[a]
SFL	659	0.463673	0.055217	0.22604	0.56243	0.933245[a]
FFL	659	0.464750	0.055552	0.226145	0.565699	0.934220[a]
SSK	659	0.152341	0.01168	0.126103	0.173611	0.934965[a]
FSK	659	0.151881	0.011760	0.124856	0.172994	0.936096[a]
SSF	659	0.631043	0.075681	0.4673	0.785238	0.935739[a]
FSF	659	0.633666	0.076366	0.475690	0.792328	0.936342[a]
SBP	659	1.605914	0.143568	1.373	1.8963	0.913377[a]
FBP	659	1.600929	0.144830	0.368937	1.894333	0.913759[a]
SDM	659	0.522535	0.060801	0.3829	0.636537	0.922673[a]
FDM	659	0.524406	0.061235	0.388717	0.641858	0.922922[a]

Forward Rates are of varying maturity. All rates quoted in \$/FC where FC = foreign currency. Period: November 20, 1985 to September 6, 1988. W: Normal = Shapiro–Wilks test for normality.

SBF = spot Belgian francs; FBF = forward Belgian francs etc.; CD = Canadian dollar; FF = French franc; IL = Italian lira; JY = Japanese yen; FL = Netherlands guilder; SK = Swedish krona; SF = Swiss franc; BP = British pound; DM = German deutschemark.

[a]Null hypothesis rejected at the 5% level.

on day t, $I_{f,t}$, with the constructed index to obtain an adjustment factor, A_t. That is, $A_t = I_{f,t}/I_{w,t}$. A forward index $I_{m,t}$ is then constructed using the market weights. The estimate of $I_{m,t}$ is then multiplied by A_t. The assumption made here is that the random error term affecting the futures prices of both indices will be similar. This is reasonable because, a priori, there is no reason to believe that the daily random components would be different for an alternative set of weights. In this case, by assigning a similar random component to the newly created index, the focus is on the difference in weights only.[9]

[9]Another way of adjusting the bias would be to estimate the drift as specified in Harpaz, Krull, and Yagil (1990). The problem with this method is that the random component associated with daily changes in prices will not be captured. The exclusion of a daily random component will result in a high correlation between the constructed portfolio and the constructed index. By including a random component obtained from the actual futures contract, and assuming that the markets are reasonably efficient, the comparison of the constructed portfolio with a portfolio using the actual futures is more appropriate.

Table IV
DATES OF THE SUBPERIODS USED IN THIS STUDY: EACH SUBPERIOD CORRESPONDS TO THE NEARBY FUTURES UNTIL ITS EXPIRATION DATE

Period	Commencement Date	Expiration Date	Sample Size (N)
1	11-20-85	03-19-86	73
2	03-20-86	06-18-86	59
3	06-19-86	09-17-86	57
4	09-18-86	12-17-86	62
5	12-18-86	03-18-87	59
6	03-19-87	06-17-87	61
7	06-18-87	09-16-87	60
8	09-17-87	12-16-87	64
9	12-18-87	03-16-88	60
10	03-17-88	06-15-88	56
11	06-17-88	09-06-88	48

RESULTS

Table V provides a summary of the returns of the spot, futures, and hedged portfolios. The total amount of currencies held in each portfolio on a given day t is given by $\gamma_i = (I_t m_i 500)/s_{i,t}$, where I_t is the value of nearby futures contract on day t, m_i are the market weights, and $s_{i,t}$ is the spot price of currency i. This portfolio is rebalanced every day. The daily percentage change in the value of the spot portfolio ($R_{sm,t} = (\Sigma\gamma_{i,t+1} - \Sigma\gamma_{i,t})/\Sigma\gamma_{i,t}$) is computed, along with the percentage change in the value of the USDX and USDM futures contract ($R_{fa,t}, R_{fm,t}$). The sum, $R_{1,t} = R_{sm,t} + R_{fa,t}$, is the daily return for portfolio P_1, and $R_{2,t} = R_{sm,t} + R_{fm,t}$ for portfolio P_2. The average returns are estimated for each of the 11 trading periods as well as for the entire period as a whole. The variances for each period are reported below the returns.

The daily returns of the spot portfolio (R_{sm}) are positive for the first eight periods (except for period 4) and negative thereafter, a reflection of the declining dollar in the first part of the study. The dollar began to strengthen after reaching the lowest point of its decline in the first quarter of 1988 ($1.892 against the BP on May 13, 1988 and $0.63 against the DM on December 31, 1987). The daily average return during this period is approximately 0.0005 or 0.05%. The returns of the USDX futures contract (R_{fa}) are the opposite, negative for the first eight subperiods (except for period 4) and positive thereafter. The correlation between these two returns averages about −0.66, suggesting a high but not perfect negative correlation. The average daily returns for the eleven trading periods is −0.05%. The net return of the hedged position (R_1) averages 0.005%, and is not significantly different from zero for each of the eleven subperiods. The variances of the average daily returns of the hedged portfolio, (σ_1^2), are lower than the variances of the average daily spot or futures returns (except for subperiod 3). Subperiod 3 is an exception because the correlation between the spot and futures markets is substantially lower than the other subperiods, −0.46. Overall, however, it appears that the existing futures contract would provide a reasonable hedge against a portfolio of currencies whose weights differ significantly from the futures contract.

Table V
AVERAGE DAILY RETURNS *IN PERCENTAGES* AND VARIANCES OF THE UNHEDGED SPOT PORTFOLIOS (OF CURRENCIES), WEIGHTED ACCORDING MARKET CAPITALIZATIONS (R_{sm}, σ^2_{sm}) AND OF RETURNS AND VARIANCES OF FUTURES POSITIONS WEIGHTED WITH ACTUAL (R_{fa}, σ^2_{fa}) AND MARKET WEIGHTS (R_{fm}, σ^2_{fm})

Subperiod	R_{sm} / σ^2_{sm}	R_{fa} / σ^2_{fa}	R_{fm} / σ^2_{fm}	R_1 / σ^2_1	R_2 / σ^2_2	N[a]
1	0.16	−0.15	−0.16	0.006	0.001	72
	0.36	0.40	0.54	0.26	0.29	
2	0.06	−0.01	−0.05	0.047	0.010	58
	0.64	0.73	0.76	0.54	0.48	
3	0.10	−0.12	−0.11	−0.015	−0.000	56
	0.28	0.38	0.38	0.43	0.37	
4	−0.07	0.03	0.08	−0.039	0.005	61
	0.21	0.31	0.33	0.16	0.14	
5	0.14	−0.15	−0.14	−0.014	−0.002	58
	0.38	0.51	0.45	0.23	0.30	
6	0.05	−0.02	−0.05	0.034	0.001	60
	0.35	0.25	0.36	0.19	0.15	
7	0.01	−0.02	−0.01	−0.009	0.000	59
	0.35	0.27	0.35	0.29	0.16	
8	0.17	−0.15	−0.17	0.023	0.004	63
	0.44	0.40	0.44	0.35	0.34	
9	−0.00	0.01	0.00	0.011	0.001	59
	0.61	0.60	0.80	0.17	0.16	
10	−0.00	0.03	−0.01	0.031	−0.006	55
	0.22	0.28	0.33	0.16	0.13	
11	−0.13	0.11	0.14	−0.019	0.006	47
	0.49	0.44	0.58	0.27	0.30	
Total period	0.05	−0.05	−0.05	0.005	0.002	648
	0.39	0.42	0.49	0.27	0.25	

[a]N = total trading days in each of the 11 subperiods.

The returns of the USDM are listed in the third column (R_{fm}). The average return is lower than the USDX contract (−0.05%) but so is the variance (0.48). However, the average daily return of the hedged portfolio (R_2) is 0.002%, with none of the returns in the 11 periods significantly different from zero. The variances, σ_2^2, are lower (than σ_1^2) for eight of 11 subperiods. The variance for the whole period of study is 0.27 for P_1 versus 0.25 for P_2, despite the fact that the variance of the USDX used in P_1 is lower than the variance of the USDM used in P_2. The reason for this is the higher negative correlation between the returns of the spot portfolio (R_{sm}) and the USDM (−0.71), as compared to the spot portfolio and the USDX (−0.66). This suggests that the weights are an important factor.

Table VI presents the results of the tests of the differences in variances among the pair of returns. The standard F-test using the estimates of σ_1^2/σ_2^2 shows that only during subperiod 3 is the variance of the net returns using the USDX higher than the returns using the USDM. However, the standard F-test requires the assumption of independence among the two groups of samples. Since the returns of both strategies are correlated, a test is performed that corrects for this bias. The test statistic is given by

$$\rho_{12} = \frac{(F - 1)}{(F + 1)^2 - 4\Phi^2 F}$$

where $F = \sigma_1^2/\sigma_2^2$ and Φ^2 is the correlation between the returns. The test statistic is tested using the tables for significance of correlation coefficients.[10] If ρ_{12} is positively significant, it indicates $\sigma_1^2 > \sigma_2^2$. The results show significance for three periods; periods 3 and 7 indicate that the variance of the returns using the USDX is larger, while period 5 indicates that the variance with the USDM is larger. Thus, it cannot be concluded positively that the variances are lower when using the market-weighted index.

Table VI also shows the percent reduction in risk of the hedged portfolio versus the unhedged portfolio. Reduction in risk is measured as the difference between the variance of the unhedged portfolio less the variance of the hedged portfolio, measured as a percentage of the unhedged portfolio. Except for one case, it is evident that hedging with either contract reduces risk, in some cases by as much as 73.77%. The mean reduction in risk is 24.54% and 29.67% for P_1 and P_2 but is 32.36% and 38.05%, respectively, if the third subperiod is omitted. When the difference between the two strategies is evaluated, seven of the 11 subperiods show a larger reduction in risk when the USDM is used (last column), but the differences are not statistically significant.

In summary, it appears that the USDX provides a reasonable currency hedge for an internationally diversified portfolio. This suggests that a manager holding assets denominated in several currencies will find the USDX appropriate for hedging, whatever the weights in his portfolio. The creation of a market-weighted index could reduce risks further, but it is not clear how much savings it could provide. Since the study uses a hypothetically constructed futures contract, there is always the question of its accuracy, even with the inclusion of a random component. As Table V shows, the USDM variance in each subperiod is higher than the USDX variance in each corresponding subperiod.

Since the USDM is less diversified than the USDX, due to the Japanese yen's large weight in the USDM, the findings of higher variances for the USDM are reasonable. However, it is possible that the variances of the USDM are overestimated. This may be the result of the inclusion of random components derived from the USDX. If this is the case, the USDM may provide a better hedge than these results suggest.

CONCLUSIONS

This article examines whether the weights used in the construction of the U.S. Dollar Index, which parallel the trade-weighted index set by the Federal Reserve Board in 1973, are appropriate for portfolio managers who use this instrument to hedge their foreign currency exposures.

A cash portfolio comprised of ten currencies, weighted according to market capitalizations, is hedged with the USDX futures contract. The results show that the existing futures contract does provide a reasonable hedge against the portfolio of

[10]See Snedecor and Cochran (1967, chap. 7), for details.

Table VI
TESTS OF DIFFERENCES OF VARIANCES AND PERCENTAGE REDUCTION IN RISK FOR HEDGE PORTFOLIOS P_1 AND P_2

Subperiod	σ_1^2/σ_2^2 ρ_{12}^{a}	Portfolio P_1	Portfolio P_2	Difference P_2-P_1
1	0.90	27.78%	19.44%	−8.34%
	0.097			
2	1.13	15.63%	25.00%	9.37%
	0.146			
3	1.16	−53.57%	−54.17%	−0.60%
	0.268[b]			
4	1.14	23.81%	33.33%	9.52%
	0.106			
5	0.77	39.47%	21.05%	−18.42%
	0.279[b]			
6	1.27	45.71%	57.14%	11.43%
	0.238			
7	1.81[b]	6.45%	48.39%	41.94%
	0.570[b]			
8	1.03	20.46%	22.73%	2.27%
	0.032			
9	1.06	72.13%	73.77%	1.64%
	0.059			
10	1.23	27.27%	40.91%	13.64%
	0.256			
11	0.90	44.90%	38.78%	−6.12%
	0.110			

σ_1^2/σ_2^2 Is the one-tail F-test for equality of variances, i.e., $\sigma_1^2 > \sigma_2^2$. ρ_{12} is a similar test except it allows returns to be correlated. Reduction is measured as $(\sigma_s^2 - \sigma_j^2)/\sigma_s^2$. A negative term in the last column would imply an increase in risk for P_2.

$F_{60,60} = 1.43$ (upper tail) and 0.70 (lower tail, for periods 1, 5, and 11) at the 5% level.

[a]See Table A11 in Snedecor and Cochran (1967) for significance levels.

[b]Significant at the 5% level.

currencies. An alternative futures contract, weighted according to the equity market-capitalizations of the ten countries in the index, is then constructed with 1987 as the base year. The hypothesis tested is whether a futures contract with weights similar to the cash portfolio provide a better hedge. The results show that, on average, the variances are lower when using a market-weighted futures contract. However, for the majority of the trading periods, the difference is not statistically significant. Thus, it is not clear that FINEX should consider the introduction of a new market-weighted currency index futures contract.

However, the results have to be interpreted with care because the market-weighted futures index is only simulated. The variance of this hypothetical index is higher than the existing index. Consequently, the results would be much stronger if the variance of the new contract is similar to the existing contract. In addition, December 31, 1987 is chosen as the base date for selecting market weights. The hedging results might be superior if the weights are adjusted as relative market capitalizations change.

Optimally, the USDM weights would exactly match the underlying portfolio's weights on the settlement day of the USDM. This would be similar in construction to the S&P 500 futures contract. This is a topic for further research.

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Appendix

The U.S. Dollar Index, specified in eq. (1), can also be written as:

$$I = 100 \prod B_i^{w_i} \prod \left(\frac{1}{S_i} \right)^{w_i}$$

The existing weights, w_i, are listed in Table II. Using those weights and rearranging, one obtains:

$$I = 12.7506 e^{\Sigma w_i \ln \left(\frac{1}{s_i} \right)}$$

If one denotes $A = 12.7506$ and substitutes f_i, the forward rates, for s_i, the change in the value of the futures contract on the USDX can be specified as follows:

$$A e^{\sum w_i (\ln f_{i,t} - \ln f_{i,t+1})}$$

Similarly, if the market weights, m_i, are substituted for the existing weights, w_i, then the change in the value of the futures contract is given by

$$A^* e^{\sum m_i(\ln f_{i,t} - \ln f_{i,t+1})}$$

where

$$A^* = 100 \prod B_i^{m_i}, \qquad A^* \neq A$$

The payoffs for the three strategies specified earlier are given by the following:

$$\sum_{i=1}^{10} (S_{i,t+1} - S_{i,t})[Im_i/S_{i,t}] - Ae^{\sum w_i[\ln f_{i,t} - \ln f_{i,t+1}]} = e_{t+1,w} \tag{A1}$$

$$\sum_{i=1}^{10} (S_{i,t+1} - S_{i,t})[Im_i/S_{i,t}] - Ae^{\sum_{i=1}^{10} w_i[\ln f_{i,t} - \ln f_{i,t+1}]} + \sum_{i=1}^{10} \gamma_i(m_i - w_i)(f_{i,t+1} - f_{i,t}) = e_{t+1,c} \tag{A2}$$

$$\sum_{i=1}^{10} (S_{i,t+1} - S_{i,t})[Im_i/S_{i,t}] - A^* e^{\sum m_i[\ln f_{i,t} - \ln f_{i,t+1}]} = e_{t+1,m} \tag{A3}$$

where $\frac{Im_i}{S_{i,t}}$ is the value of the portfolio of the ith country, measured in the ith currency, $m_i \neq w_i$ is the capitalization weight of the ith country in the world portfolio of ten countries, and γ_i is the amount of foreign currency futures in the portfolio. Note the sign of the gamma term is positive. This is because supplementing a *long* USDX futures requires going *short* in the foreign currency futures, since the former is an inverse index. The first term of all three equations shows the change in the value of the portfolio when the spot rates change from t to $t + 1$. The second term shows the change in the value of the USDX or USDM. The third term in eq. (A2) shows the change in the value of the position in currency futures. If the portfolio hedged with USDX futures is corrected with $m_i - w_i$ currency futures to adjust the difference in weights, then the additional impact on the change in the value of the hedged portfolio is given by:

$$\sum_{i=1}^{10} \gamma_i(m_i - w_i)(f_{i,t+1} - f_{i,t})$$

Ignoring the drift term, which is common to all three strategies, if the arbitrage is perfect, e_{t+1} will not be significantly different from zero.