Hedging with Synthetics, Foreign-Exchange Forwards, and the Export Decision

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INTRODUCTION

Exchange rates of the major industrial countries have exhibited great volatility in the last decade [see Krugman (1989) and Franke (1991)]. International firms, therefore, are increasingly using hedging instruments such as currency forwards and options to protect against their exchange rate risk. Exchange rate risk also affects international trade as reported by Thursby and Thursby (1985) and Cushman (1988). The aim of this article is to study the interaction between exchange rate risk and the export and hedging decision of an international firm.

The financial hedging technique on which this investigation focuses is the so-called synthetic forward contract. This contract is a portfolio of European currency put and call options such that a currency forward is duplicated. The possibility of constructing synthetic forward contracts is of great practical relevance for international firms, because these contracts are just as effective as real forward contracts with regard to the "Separation property." This property means that the export decision does not depend on the firm's risk behavior or expectations. It follows that access to such hedging opportunities promotes international trade.

To illustrate the use of synthetic currency forwards, assume that a domestic exporting firm receives a payment in foreign currency at a future date, in yen. If the firm hedges against exchange rate risk in the forward market, it is committed to receive a certain amount of domestic currency. Because of the put-call parity theorem [see Stoll (1969)] this situation can also be achieved by purchasing yen put options

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and writing yen call options on the underlying currency, both with equal maturity and striking prices. If the yen depreciates and the exchange rate falls below the striking price, the exporter will exercise the put option; whereas, the buyer of the call option will then let the call option expire. Alternatively, if the exchange rate does not reach the striking price, the firm will let the put option expire; whereas, the buyer of the call option will then exercise the call option, implying that the firm is committed to deliver the currency.

This currency option portfolio implies that, like a currency forward contract, a specified amount of one currency will be delivered at a fixed future date and price. Hence, the synthetic forward contract protects the firm against the risk of adverse movements in exchange rates for which the firm pays the put price. The contract also eliminates the possibility of gaining a profit from favorable movements for which the firm receives the call price [see Cox and Rubinstein (1985)].

The main rationale of this study is as follows. If exporting firms are risk-averse, and if there are no hedging markets available, then an increase of the exchange rate risk results in a reduction of its export volume. The benchmark being the so-called *certainty (equivalent) case*, which implies that the random spot exchange rate of the next period $\tilde{e}_1$ is replaced by its mean $\bar{e}_1$ [see Leland (1972)]. The optimal export under a certain exchange rate $\bar{e}_1$ will then be larger than the optimal export under a random exchange rate with expectation $\bar{e}_1$. Hence, the question is, how can currency put and call options be used as hedging instruments by international firms to undo the inverse effect of uncertainty upon the export volume?

THE EXPORT DECISION IN THE ABSENCE OF HEDGING MARKETS

Consider a competitive exporting firm under exchange rate risk. Production and exports give rise to a deterministic cost function $C(x)$ denominated in domestic currency, where x represents export volume. It is assumed that the function C is strictly convex, increasing and twice differentiable, and that the firm always produces a positive amount.

The export decision is made at date 0 and output will be exported and sold at the given foreign currency price p, that yields risky revenue in domestic currency at date 1. Using time subscripts for random variables only, the random profit from exports can be written [Sandmo (1971)][1]:

$$\tilde{\Pi}_1 = \tilde{e}_1 px - C(x)$$

where $\tilde{e}_1$ denotes the random spot rate of foreign exchange at date 1. The exchange rate is defined in domestic currency per unit of foreign currency.

The firm is risk-averse with a von Neumann–Morgenstern utility function, U, and maximizes the expected utility of its profits in domestic currency. Hence, if there are no hedging markets, then the firm's decision problem reads:

$$\max_x E\{U(\tilde{\Pi}_1)\} \tag{1}$$

where E is the expectations operator. Let C' denote marginal cost. Then the interior solution requires that

$$E\{U'(\tilde{\Pi}_1)[\tilde{e}_1 p - C'(x)]\} = 0 \tag{2}$$

[1]Throughout this article it is assumed, for convenience, that interest rates are zero.

To explore the impact of risky exchange rates, eq. (2) is used. Since Π_1 increases in e_1 and U' is a decreasing function, $\text{Cov}\{\tilde{e}_1, U'(\tilde{\Pi}_1)\} < 0$. Therefore, eq. (2) implies:

$$\bar{e}_1 p - C'(x) > 0 \tag{3}$$

where $\bar{e}_1$ is the expected spot exchange rate at date 1, $\bar{e}_1 \equiv E\{\tilde{e}_1\}$.[2] Eq. (3) means that the risk-averse exporting firm equates marginal cost with something less than the expected marginal revenue. Then, from eq. (3) and the optimality condition for the certainty case, one can state:

Proposition 1 (*Effect of Uncertainty*). *If the spot rate of foreign exchange is risky, then the firm's optimal export is lower than its optimal export in the certainty (equivalent) case.*

Proof: Let x_c denote the optimal export level when $\bar{e}_1$ is the certain spot exchange rate, i.e., $\bar{e}_1 p - C'(x_c) = 0$. Since $C'' > 0$ it follows from eq. (3) that, under the uncertain spot rate $\tilde{e}_1$ with mean $\bar{e}_1$, one obtains $x_c > x$.

Introducing exchange rate risk causes the risk-averse firm to reduce its export volume to deal with the uncertainty of the exchange rate. Suppose that exchange rate risk is measured by the volatility of the exchange rate; then, without hedging markets, the firm's optimal export is inversly related to this volatility, other things being equal. Hence, a risk-averse firm has an incentive to take into account hedging opportunities.

INTRODUCING SYNTHETIC FORWARDS

The Export Decision

The financial hedging instrument which underlies the following investigation is the option contract. Such contracts provide the holder with the right to sell (put) or buy (call) the underlying asset at a prefixed striking price and expiration date. Consider a currency option market that offers put and call options for every desired striking price, k. For every k there exists an associated put and call price, p_o and c_o, respectively.

Purchasing a put option gives the exporting firm the right to sell the contracted currency at expiration date 1, which the firm will do if the striking price exceeds the prevailing spot rate. Otherwise, the firm will just let the put option expire. For this alternative the firm must pay the put price. If the firm finances the purchase of the *put* option contract by writing an appropriate *call* option contract with the same striking price, the firm will have to deliver the currency if the put is not exercised. For this service the firm receives the call price.

Hence, the put price is the maximum amount the firm can lose from the put option contract, and the call price is the maximum amount the firm can gain from the call option contract.[3] The hedge of the firm is self-financing if and only if the put price and the call price coincide.

[2]Note that eq. (2) can be written as $\bar{e}_1 p - C'(x) = -p\text{Cov}\{\tilde{e}_1, U'(\tilde{\Pi}_1)\}/E\{U'(\tilde{\Pi}_1)\}$, since $E\{U'(\tilde{\Pi}_1)\} > 0$.

[3]If, incidentally, the exchange rate at date 1 equals the striking price, the firm sells the foreign currency in the spot market. Note that transaction costs, margin requirements, and taxes are not included in the calculations.

Suppose these definitions:

$z_p \equiv$ the number of currency units of the European put option contract;
$z_c \equiv$ the number of currency units of the European call option contract;
$e_1 \equiv$ the realized exchange rate at date 1;
$(k - e_1)^+ \equiv \max\{0, k - e_1\}$; and
$(e_1 - k)^+ \equiv \max\{0, e_1 - k\}$.

The firm purchases (writes) put options if z_p is positive (negative). On the other hand, the firm purchases (writes) call options if z_c is negative (positive). Thereby the hedging variables, z_p and z_c, are defined in *foreign* currency units.

Then the decision problem of the international firm is given by:

$$\max_{x, z_p, z_c} E\{U(\tilde{\Pi}_1)\} \tag{4}$$

where:

$$\tilde{\Pi}_1 = \tilde{e}_1 px - C(x) + z_p[(k - \tilde{e}_1)^+ - p_o] + z_c[c_o - (\tilde{e}_1 - k)^+]$$

The first order conditions are:

$$E\{U'(\tilde{\Pi}_1)[\tilde{e}_1 p - C'(x)]\} = 0 \tag{5}$$

$$E\{U'(\tilde{\Pi}_1)[(k - \tilde{e}_1)^+ - p_o]\} = 0 \tag{6}$$

$$E\{U'(\tilde{\Pi}_1)[c_o - (\tilde{e}_1 - k)^+]\} = 0 \tag{7}$$

From these conditions, a *Separation* result for currency options is proved.

Proposition 2 (*Separation*). *If currency put and call options are available, the firm's optimal export,* x_o, *satisfies*

$$C'(x_o) = (k + c_o - p_o)p \tag{8}$$

The optimal export can be determined independently of the utility function and of the probability distribution of the random spot exchange rate.

Proof: To simplify calculations, the standardization marginal utility $\hat{U}'(\tilde{\Pi}_1) \equiv U'(\tilde{\Pi}_1)/E\{U'(\tilde{\Pi}_1)\}$ is introduced. Note that $k - \tilde{e}_1 = (k - \tilde{e}_1)^+ - (\tilde{e}_1 - k)^+$. Rewrite the first order conditions (5)–(7) using standardized marginal utility which is always possible because $E\{U'(\tilde{\Pi}_1)\}$ is positive. Then, adding the modified eqs. (6) and (7), it follows after some easy manipulations that $E\{\hat{U}'(\tilde{\Pi}_1)\tilde{e}_1\} = k + c_o - p_o$. Combining this result with $E\{\hat{U}'(\tilde{\Pi}_1)\tilde{e}_1\}p = C'(x)$ from the modified eq. (5) implies the claim.

The Separation result with respect to an export decision is well-known under forward markets [see, e.g., Danthine (1978); Holthausen (1979); Katz and Paroush (1979); Kawai and Zilcha (1986); Paroush and Wolf (1989); Broll and Zilcha (1991); and Zilcha and Eldor (1991)]. The result from above follows because a forward contract hedge is implicitly considered. The hedging is realized by building a portfolio of currency put and call options such that a *synthetic* forward contract [see Cox and Rubinstein (1985)] is obtained.

The Hedging Decision

Consider now the optimal hedging policy of the international firm. The result is summarized in the following.

Proposition 3 (*Hedging*). (*A*) *The firm completely hedges its risky export revenue in the options market via a synthetic forward contract if and only if the risk premium in the put price is equal to the risk premium in the call price.* (*B*) *If the put option risk premium is higher* (*lower*) *than the call option risk premium, then the firm will over* (*under*) *hedge its risky export revenue.*

The option price may be different from the actuarial fair price. The risk premium represents this deviation. The premium may be due to the risk of nonperformance of the option contracts or to changes in the interest rates. Also, the deviation may arise from execution risk in constructing the synthetic position.

Proof: In the following, parts (A) and (B) of the proposition are separately proved.

(i) Let $\lambda_p \equiv E\{(k - \tilde{e}_1)^+\} - p_o$ denote the risk premium in the put price, and let $\lambda_c \equiv E\{(\tilde{e}_1 - k)^+\} - c_o$ denote the risk premium in the call price. From eq. (5) we get $C'(x)/p = E\{\hat{U}'(\tilde{\Pi}_1)\tilde{e}_1\} = E\{\tilde{e}_1\} + \text{Cov}\{\tilde{e}_1, \hat{U}'(\tilde{\Pi}_1)\}$. Combining this results with eq. (8), one deduces $c_o - p_o + k - E\{\tilde{e}_1\} = \text{Cov}\{\tilde{e}_1, \hat{U}'(\tilde{\Pi}_1)\}$. Using the fact that $k - E\{\tilde{e}_1\} = E\{(k - \tilde{e}_1)^+\} - E\{(\tilde{e}_1 - k)^+\}$ and applying the risk premium notation, one obtains:

$$\lambda_p - \lambda_c = \text{Cov}\{\tilde{e}_1, \hat{U}'(\tilde{\Pi}_1)\} \tag{9}$$

The optimal hedging policy has to be such that the covariance between the (standardized) marginal utility and the exchange rate is equal to the difference in the risk premia.

Given the synthetic forward contract, $z_p = z_c$ must hold in the optimum.[4] Then, the profit function may be written as:

$$\tilde{\Pi}_1 = -C(x) + z_p(k + c_o - p_o) + (px - z_p)\tilde{e}_1 \tag{10}$$

Combining eqs. (9) and (10) it follows that $\lambda_p = \lambda_c$ implies and is implied by a zero covariance. This covariance is zero for all probability beliefs if and only if $\tilde{\Pi}_1$ is independent of $\tilde{e}_1$. Therefore, a vanishing covariance implies and is implied by a full hedge of the export revenue, i.e., $px = z_p$. This proves part (A) of the claim.

(ii) Consider eq. (10) in its general form, i.e., $\tilde{\Pi}_1 = a + b\tilde{e}_1$. Then with the strictly decreasing function of $\tilde{\Pi}_1$, $U'(\tilde{\Pi}_1)$, it follows that $\text{sgn}(\text{Cov}\{\tilde{e}_1, U'(\tilde{\Pi}_1)\}) = -\text{sgn}(b) = \text{sgn}(\text{Cov}\{\tilde{e}_1, \hat{U}'(\tilde{\Pi}_1)\})$. This continuity argument proves part (B) of the claim.

Proposition 3 shows that it is of great economic importance for the hedging policy of the firm, whether of not the risk premia in the call and put prices differ. So the "full-hedging" theorem will be violated if (and only if) a difference in risk premia exists.

Note that the condition of zero-risk premia in the options markets, i.e., "fair" option prices are sufficient but not necessary for a full hedge to be optimal. Furthermore, if in both markets the prevailing risk premia are positive, then *over*hedge

[4]Suppose $z_p \neq z_c$. Then the firm's profit is a nonlinear function of the exchange rate, i.e., $\tilde{\Pi}_1 = -C(x) + z_p(k + c_o - p_o) + (px - z_p)\tilde{e}_1 + (z_c - z_p)[c_o - (\tilde{e}_1 - k)^+]$. Therefore, hedging with *forwards*, be they synthetic or real, cannot be optimal [see Franke, Stapleton, and Subrahmanyam (1991)].

(i.e., $px < z_p$) is still possible if $\lambda_p > \lambda_c$.[5] Intuition suggests that, from the viewpoint of the exporting firm, a higher risk premium in the put price makes the put more attractive; whereas, a higher risk premium in the call price makes the call less attractive. Hence, a speculative position of an overhedge occurs if the advantage (disadvantage) of a positive (negative) risk premium in the put price is greater (smaller) than the disadvantage (advantage) of a positive (negative) risk premium in the call. The argument for an underhedge position (i.e., $px > z_p$) follows analogously.[6] Hence, only the relative magnitude of the risk premia with respect to the put and call prices is important to determine the optimal hedging policy of the firm. Note that the put and call prices may be identical although the implied risk premia are not.

SUMMARY

The financial hedging instrument underlying this investigation is the currency option contract. Suppose that there are no arbitrage opportunities in the hedging markets. Then selling a newly written real forward contract would be equivalent to a portfolio consisting of one purchased European put option on the underlying currency and one written European call option on the underlying currency, both with expiration at date 1 equal to the delivery date of the forward contract and both with a common striking price equal to the forward rate. Since forward markets imply Separation, it follows that this property also holds in option markets if the traded options allow for constructing a synthetic forward contract. Furthermore, full hedging of the export revenue by such synthetic forwards occurs if and only if the risk premia implied in the currency put and call options' prices are identical.

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[5]This is different from the result in real forward markets [see Benninga, Eldor, and Zilcha (1984, 1985); Broll and Wahl (1991)]. Here a positive risk premium in the forward price implies an underhedge position.

[6]Only the ambiguous case of $\text{sgn}(\lambda_p) = \text{sgn}(\lambda_c)$ is considered. The sign of $\lambda_p - \lambda_c$ is unambiguous if $\text{sgn}(\lambda_p) = -\text{sgn}(\lambda_c)$.

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