Futures Prices Are not Stable-Paretian Distributed

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INTRODUCTION

The (unconditional) distribution of stock and futures prices has been considered by many researchers [e.g., see review in Badrinath and Chatterjee (1988)]. One major theory asserts that these prices follow the stable (Pareto–Levy) distribution. This article uses a newly developed statistical methodology [Lau, Lau, and Wingender (1990)] to show conclusively that the stable distribution is unsuitable as a general model for futures price changes.

BRIEF LITERATURE REVIEW

The characteristic function of a stable distribution is

$$\log \phi(t) = i\delta t - c|t|^{\alpha} \cdot [1 + i\gamma(t/|t|) \tan(\pi\alpha/2)] \quad \text{for } 0 < \alpha \leq 2 \tag{1}$$

c and δ are the scale and location parameters, respectively. Parameter γ measures asymmetry, $-1 \leq \gamma \leq 1$. Parameter α controls tail thickness, smaller α gives thicker tails. If $\alpha \leq 1$, the distribution's mean is undefined; if $1 < \alpha \leq 2$, the distribution has a finite mean but infinite second- and higher-order moments.

Mandelbrot (1963) and Fama (1965) claimed that prices of many financial instruments should be modeled by stable distributions with $1 < \alpha \leq 2$. Since then numerous studies have presented evidence for and against this claim. Most of the studies consider only stock prices [e.g., see review in Akgiray and Booth (1988)]. For futures prices, Cornew, Town, and Crowson (1984) show that stable distributions provide a better fit than the normal distribution for various types of commodities. Similarly, So (1987) claims that the stable distribution is suitable for most currency futures and spot rates. In contrast, Hudson, Leuthold, and Sarassoro (1987) and Hall, Brorsen, and Irwin (1989) use the "stability-under-addition" test to show that futures prices do not appear to be stable distributed. Note that all preceding four

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futures studies consider only the hypothesis of identically distributed stable-Paretian price changes.

This study explains why a test stronger than the stability-under-addition test is needed to resolve the controversy of whether futures prices are stable distributed. Further, such a test is presented, applied and analyzed.

A CRITIQUE OF THE "STABILITY-UNDER-ADDITION" TEST

Review of the "Stability-Under-Addition" Test

Fama and Roll (1971) propose the following "stability-under-addition" method for testing whether a given series is stable distributed:

If an n-element series $\{X\} = \{x_1, x_2, \ldots x_n\}$ is stable distributed with a given α-value [see eq. (1)], then if one forms an m-element series $\{Y\} = \{y_1, y_2, \ldots y_m\}$ by summing up k consecutive elements in {X}; i.e., $m = n/k$ and:

$$y_1 = \sum_{i=1}^{k} x_i \qquad y_2 = \sum_{i=k+1}^{2k} x_i \qquad y_3 = \sum_{i=2k+1}^{3k} x_i \quad \text{etc.} \tag{2}$$

then $\{Y\}$ should have the same α-value. However, if {X} is not stable distributed, then $\{Y\}$ will have increasingly larger α-values for increasingly larger k-values [see eq. (2)] used to construct $\{Y\}$.

Applying this test to futures prices of various commodities, both Hudson et al. (1987) and Hall et al. (1989) conclude that, for a large percentage of these commodities, the α-values of their $\{Y\}$-series increase with k. Therefore, they conclude that the stable model may not be generally valid.

However, two points, as explained below, can be used to defend the stable model against "unfavorable" results from the stability-under-addition test.

Defense Using Fielitz and Rozelle's (1983) Results

First, Fielitz and Rozelle (1983) show that [see Fielitz and Rozelle (1983, Tables 8 and 9)] if series $\{X\}$ is formed by mixtures of stable distributions with different c-and γ-values [see eq. (1)], the estimated α-values from the $\{Y\}$-series will also increase with k [k defined in eq. (2)]. In other words, the results observed by Hudson et al. (1987) and Hall et al. (1989) can be explained by the alternative hypothesis that the futures prices are generated by mixtures of stable distributions. In fact, Fielitz and Rozelle (1983, p. 34) conclude that their "simulations indicate that the more general nonnormal *stable* mixtures model may be preferred (over the normal mixture model)." This conclusion is not addressed by Hudson et al. (1987).

Defense Based on the Sampling Error of α-Estimates

The second defense for the stable distribution is the fact that α-estimates are affected by sampling errors, and one cannot be sure whether an observed change (or trend) in α-estimates is merely a random fluctuation or due to actual changes in the α-parameter. Table I presents examples of the Koutrouvelis method's sampling error properties as reported by Koutrouvelis (1980).

For example, the last row of Table I means that, from a stable population with $\alpha = 1.3$ and $\gamma = 0$, a sample of $n = 200$ observations is drawn from it, and α is estimated from this sample using the Koutrouvelis method. This procedure is repeated

Table I
EXTRACTS FROM KOUTROUVELIS (1980, TABLE 3): STATISTICAL CHARACTERISTICS OF α-ESTIMATES FROM KOUTROUVELIS' METHOD

Population (α, γ)	# of Observations Used to Estimate an α-Value	# of α-Estimates	Statistics of α-Estimates Max.	Min.	SD
(1.9, 0)	1600	50	1.96	1.82	0.037
(1.9, 0)	200	160	2.00	1.70	0.083
(1.7, 0)	1600	50	1.79	1.59	0.042
(1.7, 0)	200	160	1.95	1.45	0.107
(1.5, 0)	1600	50	1.58	1.39	0.041
(1.5, 0)	200	160	1.81	1.16	0.119
(1.3, 0)	1600	50	1.38	1.19	0.044
(1.3, 0)	200	160	1.65	1.04	0.130

160 times. The values of the resultant 160 α-estimates range from 1.04–1.65, and the estimates' S.D. is 0.130. For smaller n, the range of probable α-estimate will be larger. Also, compared with the more recent and sophisticated Koutrouvelis (1980) method, the older Fama and Roll (1971) method of estimating α is less reliable, and hence, will produce a wider range of α-estimates [Akgiray and Lamoureux (1989)].

Consider now the extracts shown in Table II from Hall et al.'s (1989) "best" results (i.e., their Table 4). Table II shows only one example (gold) that purports to reject the stable hypothesis, and one example (copper) that does not. For gold, although Table II indicates that the initial number of observations is 2511, when $k = 4$, 10, 20, or 30, the effective sample size is reduced to, respectively, 627, 251, 125, or 83.

Referring back to Table I, it is apparent that gold's various α-estimates, shown in Table II, could have been generated from a stable population with, say, $\alpha = 1.55$. In other words, the variation of gold's α-estimates (including the fact that the α-estimate reaches 2 when $k = 30$) can be explained as sampling errors from a genuine stable population. Similarly, Copper's α-estimates shown in Table II could have been generated from a stable population with, say, $\alpha = 1.5$. Also, under the stability-under-addition logic, Copper's α-estimates seem to indicate that copper's futures are clearly stable distributed.

The Need to Specify a Significance Level in Hypothesis Testing

The sampling error factor discussed above reinforces the basic statistical concept that, in testing a hypothesis, one should have a test that can reject the hypothesis at a specified significance level (denoted in this study by "AL" instead of the conventional "α" symbol). However, this concept is not (and cannot be) implemented in the stability-under-addition test. To implement this concept, it is necessary to know the

Table II
EXTRACTS FROM TABLE IV IN HALL ET AL. (1989)
ESTIMATE OF α-VALUE AT DIFFERENT k

Commodity	# of Observations in $\{X\}$-Series	k-Value 1	2	4	10	20	30
Gold	2511	1.45	1.66	1.90	1.50	1.49	2.00
Copper	6236	1.49	1.44	1.40	1.55	1.58	1.36

statistical characteristics of the sampling distributions of α-estimates obtained from stable distributions with: (i) various (α, γ)-parameters; (ii) at different sample sizes, n; and (iii) using various methods [e.g., those by Fama and Roll (1971) or Koutrouvelis (1980)]. Of course, this information is currently unavailable and is also very difficult to compile. In contrast, the method described below enables one to test (and reject) the stable hypothesis at a specified AL.

BRIEF SUMMARY OF THE LAU ET AL. (1990) METHOD FOR TESTING THE STABLE HYPOTHESIS

Basic Definitions and Properties

For a random variable x, let μ_x be its expected value, $\mu_k(x) = E(x - \mu_x)^k$ be its kth central moment, $\beta_1(x) = \mu_3(x)/\mu_2(x)^{1.5}$, $\beta_2(x) = \mu_4(x)/\mu_2(x)^2$, $\beta_4(x) = \mu_6(x)/\mu_2(x)^3$. β_1 and β_2 are, respectively, the standardized skewness and kurtosis indicators for finite μ_k distributions. The sample estimates of $\mu_i(x)$ and $\beta_i(x)$ are, respectively, m_i and b_i. To compute these estimates with n sample observations x_is ($i = 1$ to n), set

$$m_1(x) = \Sigma x_i/n, \qquad m_k(x) = \Sigma(x_i - m_1)^k/n \quad \text{for } k \geq 2 \tag{3}$$

$$b_2(x) = m_4(x)/[m_2(x)]^2, \qquad b_4(x) = m_6(x)/[m_2(x)]^3 \tag{4}$$

When the identity of the random variable is obvious, the qualifier "x" is omitted.

A normal distribution has $\beta_1 = 0$ and $\beta_2 = 3$. Distributions with $\beta_2 > 3$ have "thick tails" and are called leptokurtic.

Brief Description of the Lau et al. Test

A new method developed by Lau et al. (1990) for testing whether a reasonably large sample of (say) futures price changes comes from a stable population is based on the following simple principle: If a population has finite higher-order moments, the b_2 and b_4 computed with a sample of size n from this population should approach the population's finite β_2 and β_4 when n is large. On the other hand, if the population is stable distributed, and hence has infinite higher moments (therefore, infinite β_2 and β_4), the b_2 and b_4 computed from large samples will tend to increase without bound as n increases. Therefore, if the b_2 and b_4 of a sample of future prices are much smaller than the expected magnitudes of b_2 and b_4 from an equal-sized sample of a stable population, one can conclude that the sample of future prices does not come from a stable population.

For example, from a stable population with $\alpha = 1.7$, a sample of $n = 5000$ observations are computer simulated and a pair of (b_2, b_4) is computed (say, $b_2 = 46$ and $b_4 = 8772$). This process is repeated 500 times. From the resultant 500 (b_2, b_4) values, the statistical characteristics of b_2 and b_4 obtained for 5000-observation samples from a stable population with $\alpha = 1.7$ are shown in Table III.

Lau et al. (1990) show that the above characteristics are applicable for stable populations of $\alpha = 1.7$ with any skewness. Therefore, if a sample of 5000 observations of a commodity's futures price changes has a fitted $\alpha = 1.7$, and if these observations have one of the following characteristics: (a) a $b_2 = 14$ (say) which is lower than the 1st percentile of b_2 (equal to 30 from Table III) from a stable population, or (b) a $b_4 = 1070$ (say) which is lower than the 1st percentile of b_4 (equal to 3278 from Table III) from a stable population, then, one can reject, at $AL = 0.01$, the null hypothesis (H_0) that futures price changes are stable distributed, regardless of whether

Table III
MEAN, SD, AND LOWER PERCENTILES OF b_2 AND b_4 ESTIMATED FROM SAMPLES WITH n = 5000 FROM A STABLE (α = 1.7) POPULATION

			Lower Percentiles			
Statistic	**Mean**	**SD**	**1st**	**2nd**	**5th**	**10th**
b_2	598	880	30	35	46	67
b_4	1,520,861	3,873,438	3278	4561	8772	14768

the population is skewed. If both characteristics (a) and (b) are observed, H_0 is rejected at an $AL < 0.01$, but not at AL $= 0.01 \times 0.01$, since b_2 and b_4 are closely correlated.

Applicability of the Lau et al. Test to Dependent Variables

Fama–Roll's stability-under-addition test is for a series with independent elements. However, as shown in Fielitz (1983) and Hall et al. (1989), the test can be easily adapted for a dependent series by simply randomizing the series. Note that the series' (unconditional) distribution (the issue considered in this and earlier related studies) is not affected by the ordering of the series' individual elements. In using the Lau et al. test with dependent series, even the randomizing step necessary to Fama–Roll's test can be eliminated, since the values of b_2 and b_4 of a series are not affected by the ordering of the individual elements (in contrast, the fitted α-value in Fama–Roll's test *is* affected by the ordering). In other words, if the Lau et al. test concludes that a series' distribution is not stable-Paretian, this conclusion holds, regardless of whether the series' elements are dependent.

DATA AND PROCEDURES

Let P_t be the price of a financial instrument at day t (e.g., the closing price of a futures contract). Define:

$$\text{“price change” as } PC_t = P_t - P_{t-1} \tag{5a}$$

$$\text{“price ratio” as } PR_t = P_t/P_{t-1} \tag{5b}$$

$$\text{“log of price ratio” as } LPR_t = \mathrm{Log}(PR_t) \tag{5c}$$

Mandelbrot's (1963) theoretical arguments for stable-distributed price changes are based on the *arithmetic* price difference (i.e., PC_t), but practically all previous empirical studies supporting the stable hypothesis for futures prices [e.g., Cornew, Town, and Crowson (1984); So (1987)] consider LPR_t. However, it is far from obvious (and nobody has shown) how Mandelbrot's arguments based on the arithmetic price differences can also be valid for price ratios (i.e., PR_t or LPR_t). For more conclusive results, this empirical study tests the validity fo the stable hypothesis for both PC_t and LPR_t.

Futures prices for 13 commodities are from the Center for the Study of Futures Markets (CSFM) at Columbia University. Following the procedure of Cornew et al. (1984) and Hall et al. (1989), for each commodity and each contract, the values of PC_ts are compiled for consecutive trading days until approximately two weeks before the contract's delivery date. For each commodity, the first 5000 PC_ts generated by the CSFM data are collected. Although more than 5000 PC_ts can be obtained

Table IV
VALUES OF ESTIMATED α, b_2, AND b_4 PC_tS FOR 13 FUTURES (n = 5000)

Commodity	α-Estimate	b_2	b_4
1. Treasury bill, 91-day (IMM)	1.615	6.71	103.9
2. Treasury bonds (DayNight)	0.443	6.14	68.1
3. Swiss franc (IMM)	1.362	8.27	252.7
4. Japanese yen (IMM)	0.524	28.35	5395.6
5. British pound (IMM)	0.880	11.74	801.3
6. Copper (COMEX)	1.682	3.84	31.3
7. Silver (COMEX)	1.677	4.54	50.0
8. Gold (COMEX)	1.634	6.08	88.9
9. Corn (CBOT)	1.609	8.82	240.7
10. Soybeans (CBOT)	1.294	13.68	316.1
11. Wheat (CBOT)	1.722	7.07	136.9
12. Live cattle (MIDAM)	1.676	14.19	1708.0
13. Live hogs (MIDAM)	1.641	10.25	885.4

Table V
ESTIMATED LOWER PERCENTILES OF STABLE DISTRIBUTIONS' b_2 AND b_4 WHEN n = 5000 (BASED ON 500 SIMULATIONS PER α-VALUE)

	α = 1.5		α = 1.6		α = 1.7		α = 1.8		α = 1.9	
Percentile	b_2	b_4	b_2	b_4	b_2	b_4	b_2	b_4	b_2	b_4
1st	79	15,814	51	7,539	30	3,278	14.7	925	5.8	145
2nd	88	21,934	59	10,681	35	4,561	17.1	1,229	6.4	178
5th	121	37,560	79	20,017	46	8,772	22.5	2,529	7.7	337
10th	175	74,489	112	37,104	67	14,768	30.4	4,524	9.6	552

from the CSFM data for most commodities, it will be apparent later that the sample size of 5000 is more than adequate for the purpose.

For each commodity, the Koutrouvelis (1980) method is used to estimate the α-value of the 5000 PC_ts; Akgiray and Lamoureux (1989) show that the Koutrouvelis method is the best among several alternatives. The values of b_2 and b_4 [see eq. (4)] are also computed for these PC_ts. The results are shown in Table IV. The Lau et al. (1990) Table III is represented here as Table V. This table gives the lower percentiles of the sampling distributions of b_2 and b_4 computed with 5000 observations from stable populations.

RESULTS: PRESENTATION AND INTERPRETATION

Interpreting Table IV for PC_ts Distribution

Table IV shows that the 5000 PC_t observations for commodity 1 (IMM 91-day Treasury bill) has a fitted α of 1.615. Table V has entries for only α = 1.6 and 1.7 , but not for α = 1.615. Table V also shows that the lower percentiles of stable-distribution's b_2 and b_4 decrease as α increases. Therefore, to be conservative, commodity 1's b_2 and b_4 are compared with the entries for α = 1.7 in Table V. Since commodity 1's b_2 (= 6.74) and b_4 (104.4) are both considerably smaller than their corresponding 1st-percentile values (30 and 3278) in Table V, the stable distribution null hypothesis for commodity 1's PC_t can be rejected at $AL < 0.01$.

Consider now commodity 3 (IMM Swiss Franc), with $\alpha = 1.362$. Table V has no entries for $\alpha = 1.3$ or 1.4, but since the percentiles in Table V increase with decreasing α, the entries of $\alpha = 1.5$ in Table V can be used to benefit the position of the stable distribution null hypothesis. Even with this benefit, commodity 3's sample b_2 and b_4 again reject this null hypothesis at $AL < 0.01$ by using the entries for $\alpha = 1.5$ in Table V.

Using the above approach, it can be verified that the stable hypothesis can be rejected at $AL < 0.01$ for all the 13 commodities shown in Table IV. In contrast, recall from Table II that the Hall et al. (1989) results indicate that, for example, copper's (commodity 6) futures are stable distributed. The four groupings of commodities shown in Table IV include all major categories of futures markets.

Explanation for Cases with $\alpha < 1$

Table IV shows that commodities 2, 4, and 5 have $\alpha < 1$. Although the entries for $\alpha = 1.5$ in Table V (as in commodity 3) can be used to reject the stable hypothesis with a wide margin, one might question whether the appearance of cases with $\alpha < 1$ implies something wrong with the α-fitting procedure. Note that a stable distribution with $\alpha < 1$ has an infinite mean, and earlier advocates of the stable distribution suggest (though without clear justifications) that financial variables such as futures price changes come from populations with $\alpha > 1$. It might be argued that if one is willing to believe that these populations have infinite variances, there is no reason why one should not also believe that these populations have infinite means. Nevertheless, those empirical PC_t distributions with $\alpha < 1$ are examined closely.

Table VI shows the frequency distributions of the PC_ts that generate the results in Table IV for commodities 2 (Treasury bonds, with $\alpha = 0.443$) and 7 (silver, with $\alpha = 1.677$). The frequency distribution for commodity 7 is unimodal with tails stretching out at both ends. This is the expected form for a stable distribution. In contrast, commodity 2's frequency distribution exhibits an upturn at the end of the right-hand tail which is a distinctive nonstable characteristic. Other distributions with fitted α-values less than 1 are found to have similar nonstable characteristics.

Table VI
FREQUENCY DISTRIBUTIONS OF PC_tS OF COMMODITIES 2 AND 7

Commodity 2 (Treasury Bonds)		Commodity 7 (Silver)	
Interval	**Frequency**	**Interval**	**Frequency**
> −312.0 to −258.7	1	> −230.0 to −192.6	1
> −258.7 to −205.4	2	> −192.6 to −155.2	0
> −205.4 to −152.1	52	> −155.2 to −117.8	0
> −152.1 to −98.8	94	> −117.8 to −80.4	76
> −98.8 to −45.5	648	> −80.4 to −43.0	351
> −45.5 to 7.8	3054	> −43.0 to −5.6	1497
> 7.8 to 61.1	512	> −5.6 to 31.8	2369
> 61.1 to 114.4	565	> 31.8 to 69.2	556
> 114.4 to 167.7	9	> 69.2 to 106.6	146
> 167.7 to 221.0	63	> 106.6 to 144.0	4

Since Koutrouvelis' procedure is designed to estimate stable's α for a distribution that is *actually* stable (so are other α-estimation procedures), one can now understand why the procedure produces unrealistic estimates when it is forced to produce an α-estimate for a distribution that is clearly nonstable. Hall et al. (1989) also report (without explanation) a few α-estimates less than 1 with the Fama–Roll (1971) α-estimation procedure.

Distributions of Log of Price Ratios

Table VII is the counterpart of Table IV for the variable LPR_t [see eq. (5c)]. The same conclusion is reached: the stable hypothesis is rejected for all commodities at a very low AL.

Price Limits as a Possible Factor Limiting the Magnitudes of b_2 and b_4

While it is the purpose of this study to test *whether* empirical futures price changes are actually stable-Paretian, the fact that many commodities have imposed price-change limits may explain *why* the b_2s and b_4s of these price changes do not attain the large magnitudes expected from stable-Paretian populations. Therefore, if the stable-Paretian model is convenient, one might attempt to salvage it by conjecturing that futures price changes *could have been* stable-Paretian in a modified world that has no imposed price-change limits. However, it is shown later that the stable-Paretian model is unnecessarily troublesome compared to other currently available alternatives and is not worth salvaging. Incidentally, Akgiray and Booth (1988) and Lau et al. (1990) have shown that stock prices that have no price-change limits are not stable-Paretain.

CONSIDERATION OF NONSTATIONARY PARAMETERS

The conclusions in the preceding section are based on the implicit assumption that the PC_ts (or LPR_ts) are identically (though not necessarily independently) distrib-

Table VII
VALUES OF ESTIMATED α, b_2, AND b_4 OF LOG(P_t/P_{t-1})s FOR 13 FUTURES (n = 5000)

Commodity	α-Estimate	b_2	b_4
1. Treasury bill, 91-day (IMM)	1.617	6.67	102.4
2. Treasury bonds (DayNight)	0.442	8.06	116.9
3. Swiss franc (IMM)	1.382	8.78	273.9
4. Japanese yen (IMM)	0.549	27.61	5195.5
5. British pound (IMM)	0.817	12.12	535.6
6. Copper (COMEX)	1.825	3.37	18.3
7. Silver (COMEX)	1.748	4.08	37.5
8. Gold (COMEX)	1.610	5.86	68.2
9. Corn (CBOT)	1.675	7.57	158.9
10. Soybeans (CBOT)	1.366	11.27	225.5
11. Wheat (CBOT)	1.812	5.83	102.9
12. Live cattle (MIDAM)	1.645	12.54	1260.6
13. Live hogs (MIDAM)	1.686	9.93	769.1

uted, since Koutrouvelis' α-estimation procedure assumes a homogeneous population, and the b_2/b_4 characteristics tabulated in Table V are for homogeneous stable populations. However, it is suggested [e.g., see Samuelson (1965) and Doukas and Rahman (1986)] that the PC_ts and LPR_ts come from a mixture of different stable distributions due to one or more of the following reasons: (i) economic and trading environments go through fundamental changes during the long time span required to generate 5000 observations; (ii) prices become more volatile as a contract's maturity approaches; (iii) price changes over two consecutive calendar days are different from price changes over two consecutive trading days separated by a weekend or holiday. Therefore, the applicability of the Lau et al. (1990) procedure when the PC_ts and LPR_ts have nonstationary parameters is considered now.

The Lau et al. Experiment

Lau et al. (1990) generate 100 theoretical PC_t series with 5000 elements per series. Each series is composed of 10 subseries, with 500 stable-distributed elements per subseries. The parameters δ, c, α and γ [see eq. (1)] in each subseries of each series are generated randomly, and the ranges of these parameters are much wider than what can be reasonably expected from empirical distributions of stock prices. They find that the b_2s and b_4s obtained from these simulated stable series with heterogeneous parameters are still very large and correspond closely to those depicted in Table V, but not as small as those observed in empirical price-change distributions (as in Tables IV and VII). Therefore, the small b_2s and b_4s observed from empirical PC_t distributions are still effective indicators for rejecting the type of *exaggerated* parameter heterogeneity considered in the Lau et al. experiment.

Stable Populations with Heterogeneous Scale Parameters

A reviewer has suggested that "a stable distribution with $\alpha = 1.9$ and a time-varying scale parameter remains a viable model" for the empirical data depicted in Tables IV and VII. To systematically study the effect of mixing two stable populations with the same α of 1.9 but with different scale parameters, consider now two stable populations, A and B, with identical $\delta\,(= 0)$, $\gamma\,(= 0)$, and $\alpha\,(= 1.9)$. The following procedure is used to generate PC_t series: Let populations A and B have c (scale parameter) $= 1$ and K, respectively, and let the "mixing proportion" be $(1 - p)$ and p. That is, for each PC_t series, $5000(1 - p)$ elements are generated from population A with $\alpha = 1.9$ and $c = 1$, and $5000p$ elements are generated from population B with $\alpha = 1.9$ and $c = K$.

For each pair of (p, K), 200 PC_t series (each with 5000 elements) are generated. For each series, its b_2 and b_4 are computed and its α is estimated by the Koutrouvelis method. From the 200 sets of values of b_2s, b_4s, and α-estimates, the α-estimates' mean and standard deviation are computed, as well as the 10th, 30th, and 50th percentiles of the b_2s and the b_4s. Thus, Table VIII shows that, with $p = 0.01$ and $K = 3$, the average of the 200 α-estimates is 1.87; the 10th, 30th, and 50th percentiles of the b_2s are 10.4, 21.6, and 16,132, respectively; and the corresponding percentiles of the b_4s are 573, 3307, and 16,132. Table VIII also gives the corresponding values for other (p, K) combinations. For each (p, K) combination, only the average of the α-estimates is given, because it is found that the standard deviation is about 0.02 for all combinations; i.e., within each (p, K) combination the variation of the α-estimates is small, as would be expected since the "sample size" here is 5000. Note that the high α-value of 1.9 is chosen here to give the stable-distribution null

Table VIII
CHARACTERISTICS OF b_2S, b_4S AND α-ESTIMATES FROM MIXTURES OF TWO STABLE α = 1.9 SUBPOPULATIONS

	K = 3			K = 10			K = 50		
	α	b_2	b_4	α	b_2	b_4	α	b_2	b_4
$p = 0.01$	1.87	10.4	573	1.78	53.5	7,730	1.78	214	65,899
		21.6	3307		72.7	14,964		254	100,035
		53.9	16,132		92.8	25,554		293	137,821
$p = 0.1$	1.68	13.3	691	1.23	29.4	1,740	1.13	35.0	2,210
		22.8	2,438		36.0	3,406		42.3	4,413
		41.3[a]	10,185		49.8	8,770		58.7	11,536
$p = 0.3$	1.50	13.3	696	0.75	16.0	734	0.51	16.8	795
		21.8	2,914		26.4	3,500		27.5	3,636
		41.5	9,691[a]		47.8	12,002		49.7	12,262
$p = 0.5$	1.52	10.7	474[a]	0.68	11.6	560	0.15	11.8	578
		19.2	1,982[a]		20.0	2,194		20.4	2,274
		45.7	14,419		52.8	16,613		53.9	17,253
$p = 0.7$	1.67	10.0	524	1.26	10.6	553	1.19	10.7	561
		21.7	3,276		22.2	2,890		22.4	2,968
		44.1	11,820		45.9	11,665		46.3	11,785
$p = 0.9$	1.84	9.5	488	1.81	9.7	502	1.80	9.8	504
		21.1	3,823		21.3	3,950		21.4	3,962
		44.7	10,841		45.0	11,179		45.1	11,212
$p = 0.99$	1.89	8.8[a]	476	1.89	8.8	478	1.89	8.8	478
		20.9[a]	3,655		20.9	3,665		20.9	3,666
		56.2	19,357		56.3	19,393		56.3	19,396

[a] Indicates lowest percentile value among all (p, K) combinations.

hypothesis a favorable position, since a higher α gives lower expected b_2s and b_4s (as depicted in Table V), thus making it more difficult to reject a stable-population null hypothesis.

The most glaring phenomenon depicted in Table VIII is that the estimated α of a large sample from a mixture of populations can be much smaller than the true α of the component populations. In the extreme case, where $p = 0.5$ and $K = 50$, even though the two component populations have $\alpha = 1.9$, the average estimated α of samples from the mixture is only 0.15 (the actual range of estimated α values from the 200 series is from 0.13 to 0.16). However, for the purpose of this study, a more relevant and important observation is that, for any (p, K) combination, the b_2s and b_4s of the 5000-element samples are still very large. For example, b_2s 10th percentile is as large as 214 when $p = 0.01$ and $K = 50$ but, even in the smallest case, it is 8.8 when $p = 0.99$ and $K = 3$.

Consider now commodity 1 (Treasury bill in Table VII) with its α-estimate of 1.617 and $b_2 = 6.67$. The 1st-percentile b_2 entry for $\alpha = 1.7$ in Table V was used earlier to reject the identically distributed stable hypothesis. At this point, however, one might argue that the low estimated α of 1.617 may arise from the following model:

Hypothesis X: The commodity's PC_ts come from a mixture of two stable populations with different scale parameters but identical $\alpha = 1.9$.

However, under Model X, the b_2s 10th percentile should still be *at least* 8.8 (if $p = 0.99$) but could be much higher (say, when $p = 0.01$ and $K = 50$). Therefore, Hypothesis X can be rejected very comfortably at $AL = 0.1$, no matter what (p, K) combination a Hypothesis X proponent wants to assume.

In Table VIII, for each set of percentiles (say, the 10th percentiles of b_2), the *lowest* percentile value among all (p, K) combinations is identified with an "a". Noting now that the lowest 10th percentile for b_4 in Table VIII is 474 (when $p = 0.5$ and $K = 3$), the preceding logic can be used to reject Hypothesis X at $AL = 0.1$ for 9 of the 13 commodities in Table IV (i.e., commodities 1, 2, 3, 6, 7, 8, 9, 10, and 11). Of the remaining four commodities, since the lowest 30th percentile is 20.9 for b_2 (at $p = 0.99$ in Table VIII), and is 1982 for b_4 (at $p = 0.5$ and $K = 3$), the null hypothesis can be rejected very comfortably at $AL = 0.3$ for three of them (commodities 5, 12, and 13). Therefore, the null Hypothesis X cannot be rejected for only one commodity (4). However, so far, the null Hypothesis X is considered as a model for each *individual* commodity. Next, the null Hypothesis X is considered as a general model for *all* commodities.

It is well known that, setting up a certain hypothesis as a null hypothesis puts it at a great advantage over the alternative hypothesis. Therefore, in setting up the null hypothesis, one should have ascertained from other considerations that the null hypothesis is more likely to be true than the alternative hypothesis. Table VIII illustrates that nine of the 13 commodities have either their b_2s or b_4s (most have both) below the *lowest* 10th-percentile b_2 and b_4 and three of the remaining four commodities have either their b_2s or b_4s below the *lowest* 30th-percentile b_2 and b_4. It should be obvious that Model X is not a suitable general model for all commodities. Therefore, Hypothesis X should not be given the status of a null hypothesis. Since commodity 4's b_2 and b_4 are smaller then the *lowest* 50th-percentile b_2 and b_4 (see Table VIII) there is no reason to assume that commodity 4's PC_ts are stable distributed.

It is impossible to test all the possible variations of mixtures of stable distributions that one can propose as possible models for the PC_ts depicted in Table IV. However, the experiments described in this section illustrate a simple principle: if the population is composed of several different stable subpopulations, the sample b_2 and b_4 will still be as large as those shown in Table V (or even larger), but not as small as those observed in Tables IV and VII.[1]

CONCLUSION

This study shows conclusively that the stable distribution (with or without dependence and/or parameter stationarity) is unsuitable as a *general* model for futures prices. Although one or two commodities have price changes that are not totally

[1]On a related factor, Lau et al. (1990) have shown that the b_2/b_4 test is robust against special events such as the stock market's 1987 "Black Monday" effect, which may be considered as a form of parameter heterogeneity of the generating distributions.

inconsistent with a model of stable distribution with nonstationary parameters, it is shown below that simpler alternative models are available for these cases. The high b_2 values shown in Tables VI and VII do confirm the well-known fact that prices of financial instruments are typically nonnormal.

Many models have been proposed to explain the nonnormal and eptokurtic nature of futures prices. These models include, among others: the stable-Paretian, the mixture of normals or exponentials, time series, four-parameter general distribution functions, etc. Besides the stable-Paretian model, all other alternatives make use of finite-moment distributions that have explicit density functions and much more tractable fitting and computational procedures. In contrast, the stable-Paretian model requires the queer notion of infinite variance and higher moments, has no explicit density function, and is generally difficult to handle. The earlier literature advocating stable-Paretian models concentrates on the version of i. i. d. stable-Paretian variables because this version requires the determination of only four parameters. However, if a mixture of stables has to be used, then not only is there no developed procedure for fitting empirical data to a mixture of stable-Paretian distributions, but the model would involve more parameters than (say) a mixture-of-normals model. Since this study shows only very weak evidence for the mixture-of-stables hypothesis (and then only for a small minority of the commodities considered), there appears to be very little reason to attempt to salvage the stable-Paretian models.

The belief in stable-distributed stock and futures prices has spurred a considerable amount of effort to develop supporting investment decision models for stable-Paretian markets [e.g., Samuelson (1967); Ohlson (1975); Chamberlain et al. (1990), to name but a few]. It is hoped that the conclusions of this study will help to spur research in other directions.

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