Constructing Accurate Cash Settlement Indices: The Role of Index Specifications

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INTRODUCTION

While cash settlement provides a means for avoiding the two fundamental problems periodically associated with physical delivery: the existence of extensive delivery costs and the possibility of a maturity problem,[1] its use does not guarantee the futures contract will be immune to all conditions which might lead to a convergence problem. By design, cash settlement forces convergence of the cash price to the futures price. However, it may fail to generate the convergence of these prices to the true commercial value of the underlying commodity or instrument.[2] The deciding factor is simply the precision with which the cash settlement index serves as a measure of commercial value. Since a contract's hedging performance depends primarily on the convergence of prices to commercial value, the importance of designing a precise cash settlement index is clear.

The feasibility of cash settlement depends on the availability of reliable cash price information. The premise for questioning cash price reliability is perhaps strongest when the futures exchange is forced to rely on external pricing agents to obtain that information. As discussed by Paul (1985), there are basically two techniques used to obtain price information. One of the techniques requires "no special machinery." For example, the computation of stock indexes largely depends on (closing) cash price information that is available to the public; therefore, the exchange is not con-

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[1]The nonconvergence that results from either a squeeze or dumping, whether stemming from intentional manipulation or accidental congestions, is referred to by Anderson (1984) as a maturity problem.

[2]For a discussion on the convergence of cash and futures prices to commercial value, see Garbade and Silber (1983).

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strained to rely on private sources for the necessary information.[3] On the other hand, the settlement of some contracts requires the use of special machinery for purposes of obtaining cash price data, meaning the exchange must rely on external agents[4] to report what is essentially private information. For instance, using the settlement procedure of the Eurodollar futures contract as an example, Paul suggests the possibility of "inaccurate and biased [cash price] reporting." It is at least conceivable that the agents which report cash price information to futures exchanges have an incentive to submit a cash price different than the true or observed cash price. This purposeful manipulation of cash price information, and the resultant distortion of the settlement index, is referred to as *intentional bias.*

In addition to the Eurodollar futures contract traded at the International Monetary Market, other futures contracts call for the reporting of cash price information. Examples include the Municipal Bond Index, traded at the Chicago Board of Trade, and two stock indexes: the S&P 500 and the Value Line Index traded at the Chicago Mercantile Exchange and the Kansas Board of Trade, respectively. The Eurodollar contract is based on Eurodollar CDs, whose rates are privately negotiated. The other examples are based either partially or entirely on assets traded over-the-counter. In these cases, the exchange must solicit external pricing agents for reports on cash market prices prior to computing the settlement index.

In addition to intentional bias, there are two other basic sources of index inaccuracy, sampling and nonsampling error. The cause and effect of both are well known.

The purpose of this work is to provide an analysis of contract design features which might be used to mitigate all sources of index inaccuracy. However, the bulk of the analysis is directed at the problem of intentional bias. The decision to intentionally manipulate cash price information depends on the reporting agent's expected costs and benefits of doing so. Intentional bias may be influenced also by strategic considerations. For example, a reporting agent may send false price information based on his expectations of the cash price information that is submitted by other reporting agents.

This article analyzes intentional bias through the use of a cash price reporting game and provides a procedure for constructing (i.e., computing) the optimal settlement index vis-à-vis promoting truthful reporting. Procedures for dealing with sampling and nonsampling errors are analyzed via the use of a Monte Carlo study. The best overall procedure for minimizing index error, irrespective of its source, is discussed in the conclusion.

A CASH PRICE REPORTING MODEL

Preliminaries

The specification of a cash settlement index requires certain (contract design) decisions regarding its composition and computation. These decisions, as well as others made by the exchange, affect the accuracy of the index. This section investigates how the exchange's selection of an aggregation procedure, the number of price re-

[3]Even if the exchange uses an external agent to get cash price data, and if the data is publicly available, the exchange has a ready means of verifying its accuracy. Having ease of verification is essentially equivalent to having direct access.

[4]Either cash market brokers or dealers can serve in this capacity. In the case of the VLI contract, the KCBT uses a subsidiary corporation to compute the index value.

ports to solicit, and use of a cash price auditing scheme[5] influence the likelihood and severity of intentional bias. (Hereafter, these three decisions are referred to collectively as the index provisions.) A brief description of the procedure used to compute the Chicago Board of Trade's (CBOT) Municipal Bond Index is given as an example of a contract that depends on the reporting of perceived cash prices by external agents.

An Example: Computation of the Municipal Bond Index[6]

The Municipal Bond Index futures contract, as implied by its name, is based on an underlying cash price index. The cash prices included in the computation of the index belong to 40 different municipal bonds that are actively traded, general obligation and revenue bonds. To be included in the index each bond must satisfy certain criteria. To price each bond and to maintain "objectivity" on a daily basis, the exchange requests and receives price indications[7] from six independent brokers. Using independent brokers, described by the CBOT as "firms [that] do not take positions in either the cash or futures markets" (and are thus perceived as being objective), is a simple device used to promote index precision. These brokers act as agents who only broker bonds among municipal bond dealers. Each pricing broker submits to the exchange an evaluation of the market's bid price (expressed as a percentage of face value) for $100,000 face value of each bond issue included in the index.[8] Hence, each pricing broker submits 40 price indications; equivalently, the exchange obtains six cash price reports for every bond. For each bond, the highest and lowest reported prices are dropped, and the simple average of the remaining four is determined giving a symmetrically truncated average.[9] This average is divided by the bond's conversion factor, yielding the individual bond's converted price. The simple mean of the 40 converted prices is multiplied by a coefficient (used to maintain index continuity when the composition of the index changes) and then rounded to the nearest 1/32 of a point. The resultant figure is the daily Municipal Bond Index. To determine the value of the futures contract, the index is multiplied by $1000.

[5]The general purpose of any cash price auditing scheme is to monitor, in some fashion, the quality of reported prices. The aim of an audit is to verify through a secondary and, ideally, perfectly independent source the reliability of cash prices submitted to the exchange. In practice, objective confirmation of cash price reports may be difficult to obtain since the total number of souces for that sort of information may be quite limited.

[6]This example is drawn from material available in the CBOT publications listed in the Bibliography.

[7]Rather than solicit price indications, the exchange might request the most recent transaction price or the latest bid or offer or both. In general, transaction prices tend to be the most accurate type of price information. Furthermore, the competitiveness of the pricing mechanism certainly affects the overall quality of the price information it generates. See Garbade and Silber (1983) and Jones (1982) for discussions on the relative quality of different types of price information, as well as the influence of different pricing mechanisms on price accuracy. For purposes of controlling inaccuracy, both the type and source (meaning the generating mechanism) of price information need to be considered when the exchange decides the composition of a settlement index.

[8]Pricing brokers are asked to report only the current market bid price. It is left to the market to determine whether the bond was priced to its maturity or first call date, or somewhere between the two.

[9]Truncation is yet another practice with which exchanges attempt to reduce the influence of possibly inaccurate price reporting. The usual argument for truncation stems from the notion that extreme price reports are presumably the ones likely to be subject to some sort of contamination, bias, or error.

It may seem that the incentive for brokers to send intentionally biased quotes when, in total, 240 indications are collected is negligible; but consider, for example, the pricing procedure for a single bond. Suppose two of six brokers arrive at identical price indications and decide, somehow, and for whatever reason,[10] to report price indications that are intentionally biased upward 1%. The resultant expected distortion in the final value of the futures contract is:

$$[(1/4) \times (CF) \times (1/40) \times (COE)] \times \$1000 \qquad (1)$$

More specifically, if the conversion factor, CF, and coefficient, COE, equal 1.0 and 0.9363, respectively,[11] then the value of a single futures contract, after rounding, is biased upward approximately $5.84. Since the sample size of price indications for each bond is small, when a single biased indication is included in index computations, it carries considerable influence. If, instead, the two indications are distorted by 1/32 of a percentage point, which is practically imperceptable, the resulting error in the value of the contract is $0.18. By most any standard this is a small amount of bias in terms of the dollar value. Nonetheless, it is worth remembering that it results from only two brokers manipulating their price reports on a single bond and by only 1/32 of a percentage point. Alternatively, suppose the two brokers in question distort all of their price reports by +1/32 of a percentage point. The value of the futures contract is now biased upward by approximately $7.20. And perhaps more importantly, with the individual distortions being small, it would be difficult for the exchange to detect the dishonest brokers. Furthermore, the futures contract can be distorted with daily frequency so that, over the life of a contract, the total distortion in value is potentially significant even when the distortion of individual daily reports is small in number and in amount.

Kyle (1984) suggests the possibility that the venue of manipulative behavior may simply change as a result of exchanges using cash settlement. The example in this study clearly supports his contention. Potential aside, whether a broker intentionally misreports price information depends on a number of things. It depends on whether the broker has any self-interest in the outcome of the settlement index. Besides possible benefits, the pricing broker must anticipate any expected costs from misreporting.

Price Reporting in a Bayesian Game Framework

The representative pricing broker, hereafter referred to as the *i*th broker, is engaged in two distinct activities. The *i*th broker's first task is simply one of observing current market conditions, signals, information (including the various types of price information), etc. Given the broker's expertise and the type of price information requested by the exchange, the broker arrives at an observation price, P_0^i. It is assumed that the observation price is arrived at objectively, meaning its determination is not in any way influenced by the broker's private interest. P_0^i, especially if it is a price indication, is conditioned by the broker's expertise and opinion of prevailing

[10]It is not necessary for the two brokers to explicitly cooperate and decide to bias their reported price information. One broker could intentionally and another broker mistakenly send biased reports.

[11]For simplicity it is assumed the index is composed of bond issues that, in terms of yield, are symmetric about the hypothetical par grade bond—which would yield 8%. In this case the representative bond would have a conversion factor close to 1. Between June 11, 1985 and June 15, 1989 the coefficient ranges between 0.9817 and 0.8910. The coefficient used in this example is the average of these two numbers.

market conditions which do not diminish objectivity per se. Assuming the observation price is assessed strictly on a consideration of market forces, it can be described as being objective and, thus, free from manipulation.[12]

The broker's second task is to report a price to the exchange. This is the ith broker's reported price, P_R^i, and this price is arrived at by way of the broker's reporting decision. It is conceivable that this decision is affected by the broker's private interest in the outcome of the settlement index. It is also possible that the ith broker's reporting decision may hinge on the expected price reports submitted by other pricing brokers. Whenever the price reporting decision is distorted, that is, when $P_0^i \neq P_R^i$, the reported price reflects other than purely market forces—the disrupting forces being brought to bear by the broker's strategic self-interest.

The broker's self-interest can be posited in several ways, but can be immediately categorized as either direct of indirect. (However, the later is far more likely to be the case.) Should the broker have positions in either the cash or futures markets his interest is direct. Having options is another example of direct interest. However, if the exchange specifies the use of pricing agents that broker in the cash market, then, because brokers take no market positions of any type by definition, at least on the surface, the problem of direct self-interest can be ignored.[13]

Nonetheless, it is possible the reporting broker's interest is indirect. For example, the broker may have an incentive to offer a biased price report contingent on outstanding arrangements previously made with third parties who do have a direct interest in the futures index outcome. In this case, the reporting broker acts as an agent for both the exchange and other principals. Here there is a myriad of possibilities and each can be more easily obscured because of its indirect nature. (The difficulty for any pricing broker is that their status makes them a real target of, for lack of a better term, special interests.) For all practical purposes, the distinction between direct and indirect self-interest is immaterial; the key is, regardless of which type, that the broker's self-interest provides the grounds for intentionally biased reporting.

The ith broker's reporting decision, besides being influenced by P_0^i and self-interest, if any, might be altered by strategic considerations. It is plausible that brokers condition their price reports to reflect their idea of what prices other brokers are likely to report. For example, if all brokers realize that the exchange drops the extreme price reports, the broker interested in submitting a biased price does not want to distort his reported price to the extent that it becomes one of the extremes. Surely, if the broker could anticipate, in some fashion, the price reports of other brokers, then the maximal amount of distortion without becoming an extreme could likewise be anticipated. In part, a broker's ability to determine the expected price reports of other brokers depends not only on the exchange's aggregation procedure, but also on the expectation of the other broker's self-interest since it is self-interest that forms the basis for any dishonest reporting.

Another factor to consider is the possibility that sending manipulated price reports is costly if its detection subjects brokers to some penalty. To make these features of the broker's reporting decision explicit, it is best analyzed within the

[12]It is a minor consideration, but the price may be exposed to some degree of market making behavior. If so, it is assumed to be nonexcessive.

[13]Such reliance may in any case be questionable. It is possible that since the pricing broker operates professionally in the cash market, a private position in the futures market could go largely undetected.

context of a price reporting game played among all pricing brokers and constrained by the index specifications. With self-interest providing a basis for biased reporting, and because brokers are most likely to be uncertain about the self-interests of other pricing brokers, it is appropriate to construct the price reporting game within the Bayesian framework.

The Model of a Price Reporting Game

Each pricing broker (player) is characterized by an information set. The *ith* broker's information set contains two elements: the observation price, P_0^i, and a variable that indicates self-interest, S_i.

Assume the true cash price (being equivalent to the true commercial value), P_T, is a discrete random variable taking, with equal probability, one of three values: P_L, P_M, or P_H. Let P represent the set of possible cash prices, $P = \{P_L, P_M, P_H\}$. Further, suppose the difference between the "low" and "middle" prices, P_L and P_M, respectively, is the same as between the middle and "high" prices, P_M and P_H. This difference is somewhat arbitrary, but should be constrained by realism. Although this terminology is not used in descriptions of the cash market, it is reasonable to suggest the difference between, say, P_L and P_M equals one tick, whatever its size; similarly, for P_M and P_H. The broker's observation price is taken from the set, P.

Let S_i indicate the ith broker's self-interest. For each i, assume $S_i \in \Sigma = \{-\alpha, 0, \alpha\}$, with $\alpha > 0$. For simplicity, each element of Σ is taken to be equally likely. Measuring self-interest is problematic, especially if it is of the indirect sort. Because there is no qualitative difference between direct and indirect self-interest, it is perhaps easiest to simply think of S_i as the ith pricing broker's position in the futures market.[14] Hence, if $S_i = -\alpha$, the ith broker is short α futures contracts. Since the value of the futures position is determined by the outcome of the settlement index, the connection betwen the pricing broker's possibly distorted cash price report and self-interest is clear. For completeness, α is bound by the exchange's net or gross position limit. It is assumed that each broker knows only their own self-interest, and is uncertain of the self-interests of the other reporting brokers. Further, suppose all brokers know the distribution of Σ.

The ith broker's information set is given by (S_i, P_0^i), with $S_i \in \Sigma$ and $P_0^i \in P$. Both elements of the set, from the exchange's point of view, are random variables. The information set provides a heuristic device for categorizing the brokers by type. However, even though the ith broker's self-interest is established a priori, and since it is also determined at the broker's discretion, S_i provides an equivalent means of indicating player type. [To be sure, there are three types of pricing brokers, those that are long α contracts, those that are short α contracts, and those without a position in the futures market.] To simplify matters, symmetry among reporting brokers is assumed.

Furthermore, it is assumed that the exchange institutes and advances some sort of auditing scheme designed to check, if not preclude, possible distortions in reported

[14]Alternatively, in the case of indirect self-interest, S_i might measure the futures market position of the third party(ies) that the broker has agreed to represent. As the model stands, the broker's self-interest (i.e., futures position) is essentially treated as a given. A referee suggested making it an additional decision variable. It is felt that such an extension would strengthen the incentive for misreporting. Nevertheless, the Bayesian game analysis becomes more complicated and is left for future research.

prices.[15] Since intentionally biased reporting might be detected as a result of an audit,[16] each broker must consider its possible repercussions. There are a number of ways the repercussions of dishonest reporting might be modeled. This study suggests that brokers caught reporting biased prices incur, primarily, a loss in reputation and thus its associated cost. This suggestion is motivated, in part, by the specifications of the CBOT's municipal bond contract. (See the referenced CBOT publications.)

In general, the exchange is interested in soliciting price information only from "high"-volume brokers. On the positive side, such information tends to be less stale. Yet the number of high-volume brokers can be quite limited, which is an obvious drawback. The CBOT publishes the list of municipal bond brokers used for price information. Whether by design or not, this strategy does signal the identity of high volume brokers and provides them with a slight competitive advantage relative to unlisted brokers.

When an exchange establishes the dishonesty of a broker, the realistic response for the exchange is to cease using information from that broker. Hence, that broker no longer receives free advertising as a high-volume dealer. The possibility of being dropped from the exchange's list implies a possible loss of reputation, which would result (expectedly) in some cost. Also, exchanges may provide remuneration to pricing brokers for their reporting services. Of course, the larger the remuneration the greater the expected cost of being dropped as a pricing broker.

In summary, the main intent of a cash price auditing scheme is to improve index accuracy by monitoring for possible reporting "errors." Depending on the design of the scheme, its use may ultimately result in the termination of some brokers from being used as reliable pricing agents. When such an auditing scheme is in place, the decision by an ith broker to submit purposefully altered prices is subject to some expected cost. To capture this notion of expected cost, the following function is constructed.

For the ith reporting broker:

$$
\begin{aligned}
C(P_0^i, P_R^i) &= C_0 = 0 \quad \text{if} \quad P_0^i = P_R^i \\
&= C_1 \quad \text{if} \quad \begin{cases} P_0^i = P_M \quad \text{and} \quad P_R^i = P_H \\ P_0^i = P_H \quad \text{or} \quad P_L \quad \text{and} \quad P_R^i = P_M \\ P_0^i = P_M \quad \text{and} \quad P_R^i = P_L \end{cases} \\
&= C_2 \quad \text{if} \quad \begin{cases} P_0^i = P_L \quad \text{and} \quad P_R^i = P_H \\ P_0^i = P_H \quad \text{and} \quad P_R^i = P_L \end{cases}
\end{aligned}
$$

[15]At least one exchange currently performs a rather informal audit. The procedure amounts to performing a double-check on price reports that appear "out-of-line." For those price reports that appear to be significantly different than the others, the exchange calls the broker in question in an attempt to obtain some verification of the initial report. This is a rather curious monitoring scheme. While it is at least capable of detecting possible errors, regardless of their source, it always provides dishonest brokers with an out. For example, a broker could easily claim a mistake was made somewhere in the communication process. But most importantly, it indicates small errors, embodied in reports that do not appear to be out-of-line, intentional, or otherwise, go largely undetected.

[16]Indeed, detecting slight errors is no small task, and differentiating the source of an error (once detected) is essentially impossible. Equivalently, it would be most difficult to prove a pricing broker is actually sending intentionally biased prices. But brokers that consistently send what appear to be biased reports will sooner be suspect. The larger the pool of brokers that might be called upon to send reports, the more particular the exchange can be. We thank Dale Michaels for describing some of the challenges and difficulties involved in monitoring cash price reports.

with $0 = C_0 < C_1 < C_2$. Having assumed symmetry, each broker is subject to the same expected cost function. This function shows if the broker reports honestly then the expected cost is zero. However, when the broker decides to send a purposefully distorted price report, the expected cost is positive and proportional to the size of the bias.

Having discussed basic premises, it is now possible to establish the ith broker's actual reporting decision. The ith broker's objective function needs to be specified. Inspection of the objective function(s) reveals that each pricing broker's reporting decision is directly affected by the exchange's index specifications. Brokers take the index specifications as given. The number of pricing brokers which submit price reports is equal to $2k + 1, k > 0$. Three estimation procedures are analyzed: the median (MED), the symmetrically truncated average (SYM), and the simple mean (SIM). The MED procedure establishes the median[17] of reported prices as the index value. The SYM procedure determines the index value by dropping the highest and lowest reported prices and simply averaging the remaining prices.[18] The SIM procedure is self-explanatory.[19]

Without loss of generality, assume the ith broker's self-interest is positive, $S_i = \alpha$. The broker's price reporting objective functions, respective to the exchange's aggregation procedure, are as follows:

$$\{[(2^k/3^{2k}) \cdot (2k!/k!k!)] \cdot \beta \cdot S_i \cdot (P_R^i - P_0^i)\} - C_j \tag{MED 1}$$

with $k > 0, \beta > 0$, and $j = 0, 1, 2$. The β signifies the possibility that the exchange modifies reported prices by the use of a conversion factor and/or adjustment coefficient. In the case where cash settlement is based on an index, it may also reflect the value of some constant used to convert the index value to the value of trading units.[20]

$$\{[(3^{2k} - 2^{2k} - 1)/(3^{2k}(2k - 1))] \cdot \beta \cdot S_i \cdot (P_R^i - P_0^i)\} - C_j, j = 0, 1, 2 \tag{SYM 1}$$

$$\{[1/(2k + 1)] \cdot \beta \cdot S_i \cdot (P_R^i - P_0^i)\} - C_j, \qquad j = 0, 1, 2, \tag{SIM 1}$$

For simplicity, hereafter, β is assumed to equal 1. The objective function measures the broker's expected net benefit from each individual price reporting decision. The term in $[\cdot]$ brackets is, for lack of a better name, a weighted probability. It is the product of: (i) the probability that a single price report will directly affect the index value; and (ii) the weight attached to those prices that are used for index calculation.[21]

Consider eq. (SYM 1). With symmetric single truncation the probability any one price report is directly used for index computation is $[(3^{2k} - 2^{2k} - 1)/3^{2k}]$. The amount of distortion, if any, in the ith broker's price report is $(P_R^i - P_0^i)$, and the weight attached to that distortion is $(1/2k - 1)$. The two other estimation procedures are polar cases. The probability that any report is the median is $(2^k/3^{2k})$

[17]Because the sample size is always odd, the median is always equivalent to at least one of the broker's reported prices.

[18]For a general analysis of the symmetric double truncation procedure, which of course, is the aggregation procedure used to settle the IMM Eurodollar contract [see Cita and Lien (1990)].

[19]It is implicitly assumed that the composition of the settlement index requires the pricing of only a single asset.

[20]For example, the Municipal Bond futures trading unit is $1000 times the value of the Municipal Bond Index.

[21]The calculations that give rise to the weighted probabilities (along with supporting proofs) are presented in Cita and Lien (1990), which is available upon request.

$(2k!/k!k!)$, but the median report, biased or otherwise, carries with it a weight of 1. With the SIM procedure, it is certain that all price reports will affect the index computation and with a weight of only $(1/2k + 1)$. The MED procedure provides the greatest weight to distorted reports but the smallest chance that they will get through; i.e., affect the index value. The opposite is true for the SIM procedure. In terms of both probability and weight, the SYM procedure falls in between.

For each of the estimation procedures, the selection of k affects the probability of manipulation occurring or the weight attached to any distorted price or both. Obviously, increasing the size of k reduces the magnitude of both.

Each broker's reporting decision is clearly conditioned by the index specifications selected by the exchange. The estimation procedure dictates which of the three objective functions the pricing brokers will work with. Again, the number of price reports solicited by the exchange influences the likelihood of and severity of any distorted price reports. The decision to impose an official auditing scheme subjects dishonesty to some expected cost.[22]

Ignoring strategic possibilities, given the index specifications and an information set, (P_0^i, S_i), the ith broker chooses $P_R^i \in P$ to maximize expected net benefit. The appropriate subscript for the cost function, j, depends on the reporting decision, or equivalently, the reporting distortion $(P_R^i - P_0^i)$, being contemplated by the broker. It also depends on P_0^i. For example, if $S_i = \alpha$ and $P_0^i = P_M$, the broker will contemplate reporting either P_M or P_H. In the first case, an honest report is being contemplated which implies $C(\cdot) = C_0 = 0$. In the latter case, a report distorted one "tick" is being considered. This requires consideration of the expected cost, $C(\cdot) = C_1$.

It is assumed that each broker knows the amount of their self-interest, but is uncertain of the self-interest of others. Hence, since brokers are symmetric, except for their self-interest, they are in a position to anticipate the price reports of other brokers. That is, each broker forms expectations of what prices other brokers will report to the exchange. These expectations are based, in turn, on what each broker expects the other brokers' self-interest to be. For example, if the representative broker expects, say, broker j to be short in the futures market, i.e., $S_j = -\alpha$, then the representative broker's expectation will be that broker j may submit prices biased downward. Thus, expected price reports are contingent on the expected self-interest of the other brokers.[23] Anticipating price reports in this way affects the price ultimately reported by the ith broker. Since brokers have incomplete information on each others' self-interest, and because of the influence of self-interest on expected reporting decisions, the ith broker's reporting decision is framed within the context of an incomplete, asymmetric information game.

Bayesian–Nash Equilibria

For a game of this type, the appropriate solution concept is a symmetric Bayesian–Nash (B–N) equilibrium. In a B–N equilibrium each broker correctly anticipates the price reported by each of the other brokers. More specifically, the B–N solution

[22]An anonymous referee has suggested that, in the absence of a direct monitoring system, there exists the possibility of indirect auditing where pricing brokers would essentially police one another. This might work in a setting where reporting brokers have competing self-interest and, thus, reporting motives. A real problem might occur when interests are complementary and the number of pricing brokers is small.

[23]In addition to expected self-interest, anticipated price reports are also contingent on expectations of observed prices. Since the results of this study are qualitatively unperturbed, this additional complication is ignored. It receives full attention in Cita and Lien (1990).

consists of a set of type-contingent (i.e., self-interest-contingent) functions or strategies, one function per broker. Each of these functions maximizes the broker's expected net benefits given the amount of self-interest and the type-contingent price decisions of all other brokers. By this set of functions, each broker is able to compute the equilibrium price reports of the other brokers depending on their self-interest. The B–N solution for the ith broker is a rule, $\Pi_i(\cdot)$, that maps from the information set to the reported price: $\Pi_i(S_i, P_0^i) \rightarrow P_R^i$, it is the broker's optimal decision rule. In this framework, the function $\Pi_i(\cdot)$ can be described as the ith broker's reporting function. Under certain conditions, truthful reporting is obtained in equilibrium, while other conditions prompt dishonesty in equilibrium. Both sets of conditions are reviewed below.

Bayesian Solutions: Truthful Reporting

There are five cases, distinguished by information sets, where reporting is honest in equilibrium. The five cases are:

$$\text{(i)} \quad \Pi_i(\alpha, P_H) = P_H$$

$$\text{(ii)} \quad \Pi_i(0, P_H) = P_H$$

$$\text{(iii)} \quad \Pi_i(-\alpha, P_L) = P_L$$

$$\text{(iv)} \quad \Pi_i(0, P_L) = P_L$$

$$\text{(v)} \quad \Pi_i(0, P_M) = P_M$$

Independent of the other broker's decisions, whenever the broker is indifferent about the outcome of the index, i.e., when $S_i = 0$, then to avoid the expected cost due to biased reporting, the broker will always report the observation price. Also, if the broker observes an extreme price, either P_L or P_H, that complements their self-interest, the broker will always report that price. In each of the above cases the observed price is the same as the price reported, $\Pi_i(S_i, P_0) = P_R = P_0$. Neither the aggregation procedure nor the sample size affect the honesty of reporting.

Bayesian Solutions: Possibly Biased Reporting

There are four cases where, in equilibrium, the broker may submit biased prices. But, most importantly, in each of these cases, the incentive to do so is sensitive to the estimation procedure used by the exchange. (A numerical example of this sensitivity is provided.)

The four cases where dishonest reporting might be submitted are:

$$\text{Case I:} \quad \Pi_i(\alpha, P_M) = P_M \quad \text{or} \quad P_H$$

$$\text{Case II:} \quad \Pi_i(-\alpha, P_M) = P_M \quad or \quad P_L$$

$$\text{Case III:} \quad \Pi_i(\alpha, P_L) = P_L \quad \text{or} \quad P_M \quad \text{or} \quad P_H$$

$$\text{Case IV:} \quad \Pi_i(-\alpha, P_H) = P_L \quad or \quad P_M \quad or \quad P_H$$

The amount and direction of possible distortion is a function of the broker's information set. Though honesty may prevail in each of the above cases, if bias occurs, in cases I and II, it will be at most one "tick" in absolute value. In cases III and IV, again if it occurs, the bias may be either one or two "ticks" in absolute value. Only when an extreme price is observed is the latter a possibility, and only then will the greater expected cost, C_2, of sending a highly distorted price, need to be considered. Again, given P_0^i, index specifications, and the correctly expected price reports of the other brokers, the ith broker's objective is to report that price, distorted zero, one, or two ticks, that maximizes the expected net benefit. An equilibrium is established when each broker reports the price that each of the other brokers expects to be reported, while at the same time, each broker makes an optimal reporting decision given the behavior (i.e., correctly anticipated reports) of all other brokers.

These cases also reveal that, in the absence of an auditing scheme (in which case $C(\cdot) = 0$ by default),[24] brokers will always send biased price reports, and always biased as much as possible. This is true regardless of the estimation procedure used by the exchange. Hence, the use of a truncated average does not, by itself, prevent dishonest reporting. The proof of this is made clear upon checking eq. (SYM 1). Paul's (1985) suggestion that dropping the extreme price reports, as an alternative to auditing, "probably instills confidence in the accuracy of the cash settlement price" does not appear to be well justified. Truncation in the absence of auditing does not remove the incentive to purposefully distort price reporting. The importance of using an auditing scheme that can somehow impose, in all likelihood, a cost on dishonest reporting is self-evident.

Case I. $P_0^i = P_M, S_i = \alpha$:
When this case obtains, the ith broker's objective function, respective of the aggregation procedures, is:

$$[2^k/3^{2k} \cdot 2k!/k!k!] \cdot \alpha(P_R^i - P_M) - C_j, j = 0, 1 \qquad \text{(MED 2)}$$

$$[3^{2k} - 2^{2k} - 1/3^{2k}(2k - 1)] \cdot \alpha(P_R^i - P_M) - C_j, j = 0, 1 \qquad \text{(STP 2)}$$

$$[1/2k + 1] \cdot \alpha(P_R^i - P_M) - C_j, j = 0, 1 \qquad \text{(SIM 2)}$$

Knowing the aggregation proccedure specified by the exchange and taking all other brokers' expected price reports as given, the broker will report P_M or P_H, depending on which generates the greatest expected net benefit. Clearly, if $P_R^i = P_M$, then with $C_j = C_0 = 0$ and the expected net benefit is equal to zero, honesty yields no net gain or loss. On the other hand, if $P_R^i = P_H$, then $C_j = C_1 > 0$, and the expected net benefit can be, in addition to zero, positive or negative. The decision to send a price report that is honest, P_M, or biased, P_H, depends on which generates the greatest expected net benefit. If brokers expect biased price reporting to generate any positive net benefit, in equilibrium, a convergence problem will result—futures prices will not converge to the actual cash market price of the underlying instrument.

Case II. $P_0^i = P_M, S_i = -\alpha$:
This case is essentially identical to the previous one, except that the broker's self-interest is in the opposite direction. Consequently, the ith broker is interested in a reporting price that, if distorted, is biased downward.

In the remaining cases, the ith broker observes one of the extreme prices, P_H or P_L, rather than the middle price, P_M.

[24]Without auditing, distorted price reporting will not be uncovered. The absence of any informal or indirect auditing is also assumed. Certainly, if biased reporting goes undetected, it can have no expected costly consequences.

Case III. $P_0^i = P_L, S_i = \alpha$:
In spirit, this case is identical to case I. The only difference is that the broker can feasibly distort the price report upward by two ticks instead of just a single tick. As previously noted, the optimal distortion (if any) depends on which yields the highest expected net benefit.

Case IV. $P_0^i = P_H, S_i = -\alpha$.
This case is similar to case III, except the broker contemplates sending price reports that are possibly downward biased.

A Numerical Example: Comparing Aggregation Procedures

To show how the different settlement procedures influence the incentive to report dishonest prices, assume the number of reporting brokers is five,[25] i.e., $k = 2$. Further, suppose the ith broker's information set is (α, P_0^i). By using eqs. (MED 2), (SYM 2), and (SIM 2), simple calculations show that the SIM procedure leaves all brokers with the smallest gross payoff from misreporting and, consequently, it provides them with the smallest net incentive for sending dishonest price reports. The MED procedure facilitates the greatest incentive to manipulate the settlement price. The SYM method falls in between. Taking the auditing scheme as given, if the exchange's objective is to select an aggregation method that minimizes the gross benefit for distorting prices reports, the implication is clear.

In this example, for cases I–IV, it is possible that truthful reporting is obtained in a B–N equilibrium under the SIM procedure, but not under the other procedures. This would occur, without loss of generality. Suppose $S_i = \alpha$, when each of the following equations[26] hold:

$$0.200\ \alpha\ (P_H - P_M) < C_1 < 0.263\ \alpha\ (P_H - P_M) \tag{A}$$

$$0.200\ \alpha\ (P_M - P_L) < C_1 < 0.263\ \alpha\ (P_M - P_L) \quad \text{and} \tag{B}$$

$$0.200\ \alpha\ (P_H - P_L) < C_2 < 0.263\ \alpha\ (P_H - P_L) \tag{C}$$

The left-hand term of each equation shows the gross benefit received for submitting a price report distorted by the amount in parentheses if the exchange is using the SIM procedure; the same applies for the right-hand terms, except they show the gross benefits possible under the SYM procedure.[27]

Depending on the values of C_1 and C_2, it is possible that the SIM procedure will deter biased reporting while the other procedures will not.

This example shows the exchange's selection of an auditing method, that is, its indirect "choice" of C_1 and C_2 determines in part which, if any, of the aggregation procedures supports honest reporting. Yet, if the expected costs of being caught reporting distorted price information is high enough, the selection of an aggregation method for its ability to reduce the incentive to misreport becomes a nondecision.

[25]Though it would be ideal to set $k = 5/2$ to match the current procedure used to settle the Municipal Bond contract, results are not obtainable. However, by setting $k = 2$ or 3, qualitatively equivalent results are obtained; the implication for the case where six price reports are solicited is clear.

[26]Put $k = 2$, and use eq. (MED 1), (SYM 1), and (SIM 1) with $j = 1$ and 2, and $P_0^i = P_L, P_M$, and P_H.

[27]The respected gross benefits possible under MED procedures are the largest among the three estimation procedures considered.

However, this may be rather expensive for the exchange to achieve. Its not likely exchanges would impose excessive penalties. Therefore, if price monitoring is to be the sole means of countering dishonest reporting, it must be stringent. Certainly, auditing schemes designed to detect even small intentional biases would be expensive. Because the selection of the auditing and aggregation schemes is complementary, if the exchange's objective is to make decisions which maximize its net return from investing in an auditing system, the decisions will need to be made jointly. In this regard, the argument for using the SIM estimator is strengthened. [When the broker observes an extreme price (either P_L or P_H), some of the results obtained for this example are upset when the size of k and the size of the sample space are allowed to change. For analysis of a more general model—one permitting a larger sample space of observable prices and fuller consideration of the influence of extreme price observations—see Cita and Lien (1990).]

In conclusion, the incentive for each broker to submit an intentionally biased price report is certainly influenced by the exchange's selection of index provisions. As the number of price submissions increases, the incentive for purposely altering price information is reduced. Of the three estimation procedures analyzed, the SIM offers the smallest incentive for intentional bias. The absence of a formal auditing system, especially when the number of pricing brokers is small, suggests the real possibility of biased reporting with near impunity—the only concern of the broker being to report prices that do not appear as being too out-of-line. Not failing this, it appears that intentional bias could easily affect the outcome of the settlement index. Though detecting all errors is impossible, the more errors an auditing system is designed to catch the greater its cost. Since the cost of using any one estimation procedure is essentially the same, the exchange should use the estimator that provides the smallest possible incentive for biased reporting.

INDUCING HONEST PRICE REPORTING: THE OPTIMAL ESTIMATOR

In this section, the estimator that minimizes the expected gain from biased reporting is identified. Again, the analysis pertains to the pricing of a single underlying asset, and the Municipal Bond contract is used as an example. The exchange's task is to determine how to aggregate the price reports on a single bond issue such that the incentive to submit an intentionally distorted price is minimized.

Assume the true commercial value of the asset, P_T, is drawn from the set, P, as defined above. Furthermore, suppose five pricing brokers submit information to the exchange. Let q_i denote the price quote of the ith broker, $i = 1, \ldots, 5$. Also, let r_i denote the order statistic of $\{q_i, \ldots, q_5\}$ so that:

$$\min\{q_1, \ldots, q_5\} = r_1 \leq r_2 \leq r_3 \leq r_4 \leq r_5 = \max\{q_1, \ldots, q_5\}$$

The exchange's problem is one of selecting an estimator that will minimize the representative broker's expected (gross) gain from sending distorted prices. Equivalently, the exchange's objective is to choose a weighting scheme, $\{w_i\}$, $i = 1, \ldots, 5$, to be used in the computation of the settlement index (or converted price), denoted P^s, subject to the constraints: (i) $P^s = \Sigma_{i=1}^{5} w_i r_{6-i}$; and (ii) $w_i \geq 0$, $\forall i$. Implicitly, it is being assumed that the exchange is selecting from the class of linear estimators.

Note that the estimators considered above are simply special cases of the linear estimator. More specifically, the MED procedure imposes $w_1 = w_2 = w_4 = w_5 = 0$ and $w_3 = 1$; the SYM procedure satisfies $w_1 = w_5 = 0$, $w_2 = w_3 = w_4 = 1/3$; and finally, the SIM requires $w_i = 1/5$, $\forall i$. In the spirit of the MED and SYM procedures, an additional constraint is assumed: (iii) $w_1 \leq w_2 \leq w_3 \geq w_4 \geq w_5$.

Thus, the exchange might want to consider placing more weight on the middle price reports since it is conceivable they are less likely to be affected by various biases.

Before further refining the exchange's estimation selection problem, it is necessary to determine the ith broker's expected gross gain from submitting dishonest price reports.

First, the only cases of interest are those that might support dishonest reporting in equilibrium. These cases have already been identified, i.e., recall cases I–IV. Because any of these cases can occur in equilibrium, the exchange must determine for each case the expected gross gain that the ith broker can obtain via dishonest reporting.

Expected Gross Gains from Intentional Bias

Case I. The representative broker's information set is (α, P_M), implying contemplation of reporting either P_M or P_H. Depending on the possible price reports submitted by the four other brokers—there are 15 possible combinations to consider—the representative broker can determine the expected gross gain from biasing the price report by one tick as compared to reporting honestly. The ith broker's total expected gross gain from a distorted price report is:

$$(1/3)^4(P_H - P_M)[16w_1 + 32w_2 + 24w_3 + 8w_4 + w_5] \tag{1A}$$

(Details for this and the following cases are provided in Appendix 1.)

Case II. the equivalent expression in this case, where the broker's information set is given by $(-\alpha, P_M)$, is:

$$(1/3)^4(P_M - P_L)[16w_5 + 32w_4 + 24w_3 + 8w_2 + w_1] \tag{2A}$$

Cases III and IV are more involved since the broker must consider sending each of the possible prices. Thus, the expected gain from sending prices distorted one or two ticks must be determined.

Case III. The broker's information set is (α, P_L). If the broker reports a price that is distorted two ticks, i.e., P_H, the expected gain is:

$$\begin{aligned}&(1/3)^4(P_H - P_M)[16w_1 + 32w_2 + 24w_3 + 8w_4 + w_5]\\ &\quad + (1/3)^4(P_M - P_L)[w_1 + 8w_2 + 24w_3 + 32w_4 + 16w_5]\end{aligned} \tag{3A}$$

The expected gain from sending $P_R^i = P_M$ is:

$$(1/3)^4(P_M - P_L)[16w_1 + 32w_2 + 24w_3 + 8w_4 + w_5] \tag{3B}$$

For case III, the representative broker's expected gain from misreporting is given by the summation of eqs. (3A) and (3B).

Case IV. The expected gains from manipulative price reporting with the information set $(-\alpha, P_H)$ are given by:

$$\begin{aligned}&(1/3)^4(P_M - P_L)[16w_5 + 32w_4 + 24w_3 + 8w_2 + w_1]\\ &\quad + (1/3)^4(P_H - P_M)[w_5 + 8w_4 + 24w_3 + 32w_2 + 16w_1]\end{aligned} \tag{4A}$$

$$(1/3)^4(P_H - P_M)[16w_5 + 32w_4 + 24w_3 + 8w_2 + w_1] \tag{4B}$$

Eq. (4A) is obtained when $P_R^i = P_L$, and eq. (4B) holds when $P_R^i = P_M$.

Over cases I–IV six different price distortions are possible. (Each is indexed by an information set and the size of the distortion.) Each is equally likely from the exchange's point of view, since observation prices and broker self-interests are each equally likely. Also, recall that the difference between P_H and P_M is the same as that between P_M and P_L.[28] Thus, if the exchange is interested in minimizing the incentive for intentionally biased reporting, the exchange's problem is to select $\{w_1\}$ to minimize:

$$[17w_1 + 40w_2 + 48w_3 + 40w_4 + 17w_5]$$

subject to:

$$w_1 \leq w_2 \leq w_3 \geq w_4 \geq w_5$$

and

$$w_1 + w_2 + w_3 + w_4 + w_5 = 1$$

The solution to this problem is: $w_1 = w_2 = w_3 = w_4 = w_5 = 1/5$. The simple average is best in terms of promoting the truthful reporting of each broker. The solution to this particular problem is unique.

Results for more general cases can be obtained by extending the above analysis. Since the coefficients of the arguments in the objective function are symmetric, the following equalities must hold: $w_1 = w_5$ and $w_2 = w_4$. Denote the coefficient for w_1 as c_1, the coefficient for w_2 as c_2, and so on. Then let $a_1 = c_1 + c_5$, or using the symmetry argument, $a_1 = 2c_1$, $a_2 = 2c_2$, and $a_3 = c_3$. Whenever $4a_3 \geq a_1 + a_2$ and $3a_1 \leq 2a_2 + 2a_3$, then the SIM procedure is the best linear weighting procedure. If $2a_3 \geq a_2$ and $3a_1 \geq 2a_3 + 2a_2$, then the SYM is best and, if $2a_3 \leq a_2$ and $4a_3 \leq a_1 + a_2$, then the MED procedure is superior. The best procedure for estimating the settlement price in regard to promoting honest price reporting behavior is sensitive to the number of reports submitted by pricing brokers and the number of prices in the set P, since both of these parameters influence the relative size of a, b, and c. [see Cita and Lien (1990)]. In conclusion, when P contains three prices and when five brokers offer price information to the exchange, in general, the SIM procedure is best at encouraging honest reporting (see Appendix 2).

NONSAMPLING ERROR—A MONTE CARLO ANALYSIS

The problem of nonsampling error or unintentionally biased reporting can occur regardless of any incentive a broker has for rendering distorted prices. But, like the incentive to give intentionally biased reports, the potential severity of unintentional bias is influenced by the exchange's selection of an aggregation method. To access the ability of different aggregation methods to reduce index inaccuracy caused by unintentional bias, several different Monte Carlo simulations are used. Mean square error (MSE) criteria are used to measure accuracy.

Given the presence of market noise, the ith broker will derive price observation, P_0^i, from some probability function of cash prices. Ideally, all brokers derive their observed prices from the same density function, say $f(\cdot)$, with (a) the expectation of observed prices, $E_f(P_0^i)$, equaling the true cash price, P_T; and (b) the variance of the distribution, $\sigma_f(P_0^i)$, being finite. In this situation, there will be no unintentionally biased reporting and, if brokers are prone or otherwise induced to give honest

[28]This condition is invoked for reasons of simplicity, the conclusions hold even if $(P_H - P_M) \neq (P_M - P_L)$.

price indications, the settlement index will not be biased (but will be subject to some amount of sampling error).

When exchanges rely on samples of reported prices, there are a number of different sources of unintentional bias. Sampling may be asynchronous; therefore, observations obtained at even slightly different moments in time may introduce some contamination. The ability of brokers to read the market accurately influences the bias of reported prices—inexperienced brokers being the concern here. Other types of measurement error can also lead to unintended bias. The very act of communicating price information admits the possibility of mistakes being made.

To allow for unintentionally biased reporting, assume brokers may draw observations from densities $g(\cdot)$ or $h(\cdot)$. Observations drawn from these densities are said to be contaminated. These densities are assumed to differ from $f(\cdot)$ only in terms of the location parameter, and all have the same variance. The dissimilarities in the distributions are characterized as follows: $E_f(P_0^i) = P_T$

$$E_g(P_0^i) = E_f(P_0^i) + n\sigma_f(P_0^i) \tag{A}$$

$$E_h(P_0^i) = E_f(P_0^i) - n\sigma_f(P_0^i) \tag{B}$$

where $E_g(\cdot)$ and $E_h(\cdot)$ denote the expectation of P_0 given $g(\cdot)$ and $h(\cdot)$, respectively. The second term of each equation gives the magnitude of contamination or drift, with n being the number of standard deviations or drift, $n > 0$. Further, it is supposed that intentionally biased reporting is nonexistent; all biased reporting is due only to drift, either positive or negative.

For each Monte Carlo experiment, a particular functional form for $f(\cdot)$ is specified. Again, $f(\cdot)$ gives the true distribution of cash prices. Four functional forms of $f(\cdot)$ are considered:

(i) Standard normal: $f(P_0^i - P_T) = (1/2\pi)^{1/2} \exp(-(P_0^i - P_T)^2/2)$
(ii) Uniform: $f(P_0^i - P_T) = 1, \quad -0.5 \leq P_0 \leq 0.5$
(iii) Double exponential: $f(P_0^i - P_T) = (1/2) \exp(-|P_0^i - P_T|)$

and

(iv) Stable:

$$\begin{aligned} f(P_0^i - P_T) &= \exp[-|P_0^i - P_T|^\alpha \exp(-(1/2)\pi i\beta k(\alpha) \cdot \operatorname{sgn}(P_0^i - P_T))], \\ &\quad 0 < \alpha \leq 2, \quad \alpha \neq 1 \\ &= \exp[-|P_0^i - P_T|(1 + (2/\pi)i\beta \ln|P_0^i - P_T| \cdot \operatorname{sgn}(P_0^i - P_T))], \\ &\quad \alpha \neq 1 \end{aligned}$$

where $k(\alpha) = 1 - |1 - \alpha|$, and $-1 < \beta < 1$.

Other than the stable distribution, all are symmetric with mean and median equal to zero and standard deviations of: (i) 1.0; (ii) $0.5/3^{1/2}$; and (iii) $2^{1/2}$, respectively.

The stable distribution has two additional parameters denoted by α and β. The so-called characteristic exponent value is given by α, while β is its skewness parameter value. There is some empirical evidence to suggest that cash and futures price series are fit reasonably well by stable distributions [Cornew, Town, and Crowson (1984)]. Two different parameterizations of this distribution are considered. One is representative of commodity prices with $\alpha = 1.5$ and $\beta = 0$, the other is more characteristic of prices for financial instruments with $\alpha = 1.5$ and $\beta = 0.5$. In either case, σ_f does not exist. For the stable distribution,[29] the contaminated distributions $g(\cdot)$ and $h(\cdot)$ are established by setting $\sigma_f(\cdot) = 1.0$.

For each Monte Carlo experiment, 10,000 random samples taken from $f(\cdot)$ are generated. In an effort to provide further analysis of the aggregation procedure used to calculate the CBOT's Municipal Bond Index, samples are grouped into sets of five. Each set represents the exchange's collection of five price reports. Given the total sample size, 2000 sets of reported prices (hereafter, price sets) are generated. Next, some degree of contamination is introduced into each of the price sets. This can be done in a number of ways. For example, one possibility is to assume two brokers out of each price set "draw" from distributions other than $f(\cdot)$, one drawing from $g(\cdot)$ and the other from $h(\cdot)$. This is accomplished by shifting the draw of one broker according to eq. (A), the other according to eq. (B); for one broker some amount of contamination (unintentional bias) is added and for the other some is subtracted, the amount controlled by the size of n. Another possibility is that out of each price set, a pair of brokers draw from $g(\cdot)$ and a different pair draw from $h(\cdot)$. In both of these examples, the type of contamination is usually described as symmetric. Other possibilities involve asymmetric contamination. Examples of this type of contamination are when one or two brokers draw from either $g(\cdot)$ or $h(\cdot)$; all of the drawings to be contaminated are shifted either right or left. Both sorts of contamination are analyzed.

Finally, for each experiment, the amount of drift, i.e., the size of n, must be specified. A series of experiments are conducted starting with $n = 0$, no contamination, and then change n successively over the values 1/2, 1, 3/2, and 2. For each value of n greater than zero, experiments involving symmetric and asymmetric contamination are considered.

Once price sets have been drawn and the requisite contamination introduced, both in type and quantity, for each price set three settlement prices are calculated, one for each of the aggregation procedures. For each aggregation procedure this generates 2000 estimates of the settlement price. Determination of the MSEs is straightforward. By taking the ratio of MSEs a clear comparison of the aggregation methods is readily availed (Table I). Ratio A is calculated by dividing the MSE of the MED estimator by the SIM estimator. Ratio B is similarly calculated, except the MSE of the SYM estimator is used in the numerator. When ratio A exceeds 1.0 it indicates the SIM outperforms the MED procedure, while, if ratio B exceeds 1.0, it shows that the SIM outperforms the SYM procedure. For a particular experiment, if ratio A exceeds ratio B, the SYM outperforms the MED procedure.

The results show that when unintentional bias is asymmetric no single aggregation procedure is best for all cash price distributions analyzed. The determination of which procedure is best is sensitive to both the amount of drift or contamination and the underlying distribution of cash prices. However, the SIM usually performs better than the others, and the SYM is always better than the MED procedure. Of particular interest are the following cases of asymmetric drift. If cash prices are distributed normally and the amount of drift is excessive, i.e., $n = 2$, then the SYM procedure is best. If $n = 3/2$, the SIM and SYM procedures are nearly equivalent. Should prices exhibit a stable distribution and, if unintentional bias is asymmetric and sizeable, the SIM and SYM procedures are nearly equivalent and both are bet-

[29]To obtain samples from a stable distribution, the algorithm presented in Chambers, Mallows, and Stuck (1976), noting the corrections in Panton (1989), is utilized. The algorithm provides a nonlinear transformation of two independent uniform random variables into one stable random variable.

Table I
MONTE CARLO RESULTS

			Asymmetric[a]				Symmetric[b]			
Dist.	**Ratio[c]**	**Zero[d]**	**1/2**	**1**	**3/2**	**2**	**1/2**	**1**	**3/2**	**2**
Normal	A	1.412	1.416	1.389	1.298	1.142	1.451	1.625	1.864	2.078
	B	1.144	1.137	1.103	1.033	0.921	1.165	1.208	1.294	1.395
Uniform	A	2.112	1.712	1.073	0.595	0.366	2.161	2.835	2.843	2.844
	B	1.478	1.191	0.742	0.413	0.256	1.364	1.599	1.612	1.614
D. exp.	A	0.901	0.997	0.957	0.944	0.825	1.026	1.250	1.464	1.526
	B	0.812	0.883	0.872	0.855	0.802	0.853	1.011	1.182	1.316
Stable ($\alpha = 1.5$, $\beta = 0.0$)	A	1.598	1.555	1.495	1.421	1.320	1.570	1.639	1.771	1.954
	B	1.227	1.218	1.181	1.121	1.037	1.220	1.223	1.259	1.331
Stable ($\alpha = 1.5$, $\beta = 0.5$)	A	1.611	1.613	1.559	1.493	1.400	1.601	1.615	1.719	1.861
	B	1.213	1.218	1.201	1.155	1.067	1.205	1.211	1.230	1.298

[a]One price report is drawn from distribution $g(\cdot)$, four are drawn from $f(\cdot)$. The numbers below indicate the number of standard deviations of drift.

[b]One price report is drawn from each distribution, $g(\cdot)$ and $h(\cdot)$, the reports are drawn from $f(\cdot)$. Again, the numbers below indicate the number of standard deviations of drift.

[c]Ratio A is the MSE of the MED estimator relative to the MSE of the SIM estimator. Similarly, ratio B is the MSE of the SYM estimator relative to the MSE of the SIM estimator.

[d]This column shows the results when there is no contamination.

ter than the MED procedure. If unintentional bias is symmetric, the SIM procedure is clearly best, outperforming the other two in every case. With symmetric contamination, again, the SYM is almost always better than the MED procedure.

In conclusion, it appears the SIM estimator usually handles unintentional bias better than others, basically because it utilizes the greatest amount of sample information. However, if the exchange expects unintentional bias to be asymmetric and rather sizeable in magnitude (which might be characterized as a situation where the exchange expects it to be a minor problem) the SYM procedure is usually best.[30]

Sampling Error

With an interest in receiving reliable price information, exchanges seek pricing agents that deal (as brokers) in market settings where there is significant trading volume. For some assets it may be difficult to find a large number of possible agents that fit this requirement. Certainly the CBOT's recent move to increase the number of pricing brokers for the Municipal Bond contract to six is motivated primarily to reduce sampling error. It also reduces the incentive for and effect of any intentionally biased reporting. Also, if the basic price reporting system is without any biases or flaws of its own, increasing the sample size will tend to reduce the influence of nonsampling error—the expectation being that the additional price reports are not subject to contamination.

CONCLUSION

All cash settlement contracts are susceptible to sampling and nonsampling errors, while those that depend on external pricing agents are also subject to the possibility of intentional bias. The proper specification of settlement procedures plays an important role in keeping the various sources of index error in check. Perhaps the most important specification is that of a price monitoring scheme coupled with some sort of penalty for submitting low quality price information. An auditing scheme provides a means of catching all types of errors, but also provides a real deterrent for dishonest reporting. Even with a well-designed price monitoring scheme, small errors can go undetected. This highlights the importance of selecting the best possible price aggregation procedure. Compared with all other linear estimators, the simple mean, in many settings, provides reporting agents with the least incentive to report biased prices. The same can be said about its ability to minimize the influence of nonsampling error. It is true that the SIM procedure is not the best procedure in general. Theoretically, the use of symmetric truncation does not appear to be wrong prima facia. However, for the practical examples considered in this article, it does appear that the SIM estimator is superior, both in terms of its ability to mitigate intentional bias and nonsampling error. Thus, for most applications, the simple mean is the best overall estimator.

Appendix 1

In case I, the ith broker's information set is taken to be (α, P_M) implying the broker's reporting decision is between P_M and P_H. Since the broker's gain from misreporting

[30]Qualitatively similar conclusions are obtained with an identical Monte Carlo study of the IMM Eurodollar settlement procedure. The results here extend the Lien (1989) Monte Carlo study by showing the superiority, in many instances, of the SIM procedure relative to either the SYM or MED procedures. Results are available upon request.

is contingent on the price reports submitted by the four other pricing brokers, the ith broker must assess the expected gain from misreporting under each of the possible combinations of price reports submitted by the other brokers. For cases I–IV, the price reports of the four other brokers take 15 different combinations. (For simplicity assume $\alpha = 1$.)

Combination 1. None of the other brokers report P_M or P_H. The probability of this combination occurring is: $(1/3)^4$. If the ith broker reports truthfully, the settlement index is:

$$w_1 P_M + (1 - w_1)P_L$$

However, if a distorted price is reported the settlement index is:

$$w_1 P_H + (1 - w_1)P_L$$

The gain from misreporting is: $w_1(P_H - P_M)$. Thus, the expected gain is: $(1/3)^4 w_1 (P_H - P_M)$.

Combination 2. Three brokers report P_L, and one reports P_H or P_M. The probability of this combination occurring is: $8(1/3)^4$. If the ith broker reports truthfully the settlement index is:

$$(w_3 + w_4 + w_5)P_L + w_2 P_M + w_1 P_H$$

or

$$(w_3 + w_4 + w_5)P_L + (w_1 + w_2)P_M$$

However, if a distorted price is reported, the settlement index is:

$$(w_3 + w_4 + w_5)P_L + (w_1 + w_2)P_H$$

or

$$(w_3 + w_4 + w_5)P_L + w_2 P_M + w_1 P_H$$

The gain from misreporting is: $w_2(P_H - P_M)$ or $w_1(P_H - P_M)$. Thus, the expected gain is: $4(1/3)^4 w_2(P_H - P_M) + 4(1/3)^4 w_1(P_H - P_M)$.

By proceeding in a similar fashion through the remaining 13 combinations it is possible to determine the expected gain under each. Eq. (1A) is simply the summation of expected gains over all price reporting combinations. The derivation of equation (2A) is essentially identical to that of equation (1A).

Cases III and IV require the representative broker to determine the expected gains from sending reports biased either one or two ticks. Eq. (3A) is obtained by using the same procedure used to derive (1A), the same holds for eq. (3B). Eqs. (4A) and (4B) follow accordingly.

Appendix 2

Using the simplified notation, the exchange's problem can be rewritten as: choose w_1, w_2, and w_3 to

$$\text{Min}[a_1 w_1 + a_2 w_2 + a_3 w_3]$$

subject to

$$\text{(i)}\ 2w_1 + 2w_2 + w_3 = 1$$

and

$$\text{(ii)}\ w_1 \leq w_2 \leq w_3$$

Note that the three aggregation procedures of interest—SIM, MED, and SYM—are the only candidates for solutions to this problem. Simple calculations show that the SIM procedure solves the above problem if:

$$4a_3 \geq a_1 + a_2$$

and

$$3a_1 \leq 2a_2 + 2a_3$$

The SYM estimator provides the solution if:

$$2a_3 \geq a_2$$

and

$$3a_1 \geq 2a_3 + 2a_2$$

Finally, the MED procedure solves the problem if:

$$2a_3 \leq a_2$$

and

$$4a_3 \leq a_1 + a_2$$

For the problem at hand, $a_l = 37$, $a_2 = 80$, and $a_3 = 48$. With these values, the conditions necessary for the SIM procedure to provide a solution to the exchange's problem are satisfied.

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