Evidence of Chaos in Commodity Futures Prices

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INTRODUCTION

The analysis and modeling of commodity futures price behavior continues to be an important issue. Much of this research centers on the following issues (Kamara, 1982): (1) Forecasting performance—is the futures price an unbiased predictor of the spot price at maturity, or are there explainable systematic biases? (2) Stochastic or structural character of futures prices—can they be explained by some linear or nonlinear model, or some form of random walk? (3) Distribution of futures prices—do they follow a normal distribution, if not, what other form? (4) Equilibrium pricing of futures contracts within the capital market—how are they related to the prices of other risky assets?

This study is concerned with the second issue; specifically, whether there exists nonlinear dynamic structure and, in particular, chaotic structure in the behavior of futures prices.[1] Chaotic dynamic systems are capable of generating a rich variety of time series patterns, and such time series are often random-appearing. Yet, if such structure can be shown to exist, the implication would be that the empirical validity of simple, early versions of the efficient markets hypothesis [e.g., Fama (1970)] which imply a random walk for asset prices is called into question.[2] This would indicate that future research should be directed toward the difficult task of identifying the specific form of the underlying price structure. When that is accomplished, a

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[1]Chaos refers to deterministic dynamic behavior which is bounded and neither periodic nor asymptotically periodic. Some nonlinear dynamic equations generate chaos, while some generate periodic behavior (repeating sequences). Accessible introductions to the mathematics of chaos are to be found in Savit (1988), Gabisch (1987), Kelsey (1988), Baumol and Quandt (1985), Baumol and Benhabib (1989), and Frank and Stengos (1988a).

[2]Van der Ploeg (1986) presents a model in which asset prices can evolve according to the logistic equation, which admits chaos, implying that rates of return would not be white noise. Lucas (1978) gives another model with structure in rates of return.

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better understanding of the economic behavior underlying the markets, and the potential for improved short-term (though not long-term) predictions will result.[3]

The search for nonlinear structure in financial and economic time series has become widespread in the past few years. Of particular note in the context of commodity prices are the works of Frank and Stengos (1989), who find evidence of nonlinear structure in gold and silver rates of return, and of Blank (1990), who finds nonlinear structure in soybean futures prices. Other examples include Scheinkman and LeBaron (1989a) and Hinich and Patterson (1985) on stock returns; Blank (1990) on stock futures prices; Barnett and Chen (1988) and DeCoster and Mitchell (1991b) on monetary aggregates and components; Hsieh (1989) on exchange rates; and Frank and Stengos (1988b), Frank, Gencay, and Stengos (1988), Brock and Sayers (1988), and Scheinkman and LeBaron (1989b) on macroeconomic data. These studies collectively suggest that chaotic dynamics may be of considerable importance in financial and economic data.

A technique commonly used in these articles is the correlation dimension technique of Grassberger and Procaccia (1983). The purpose of the present article is to use the correlation dimension technique to search for chaotic structure in daily futures price data for each of four commodities: sugar, coffee, silver, and copper. This study finds that, for all four futures price series, there is strong evidence of nonlinear structure, and possibly, chaos. The results are not explained by the alternative hypothesis of an ARCH model. As pointed out by Barnett and Hinich (forthcoming), this evidence can be interpreted as a failure to reject the null hypothesis of chaos in the data. It should be noted that all the data series contain in excess of 4000 observations (far more than in most previous correlation dimension studies in economics). Thus, while there is some question as to the effectiveness of this technique in very small samples [Ramsey, Sayers, and Rothman (1988)], the use of the technique is strongly justified by the sizes of this study's data sets. The concern in DeCoster and Mitchell (1991a) about possible insufficient ability to detect chaos in samples of the present size is not important in the present context.

MODELING FUTURES PRICES

The idea that futures prices might follow a random walk is initially found in Working (1958), who postulates that the continuous flow of many different kinds of information into the market causes frequent price changes which might be nearly random. Still this model allows for some gradualness of price changes, and thus, for some degree of a very short-term predictability. Samuelson (1965) further developed this model by postulating that futures prices follow a Martingale process. That is, futures prices, insofar as they constitute unbiased estimates of future spot prices, can be described by a stochastic process in which the expected price in the next period equals the current price.

Most of the empirical tests of the random walk and Martingale processes search for the existence of serial correlation and trends, since both processes require price changes to be independent. In practice, even when systematic dependencies in price changes are detected, it is very difficult to determine whether their magnitude and

[3] Long-term prediction of a chaotic system is impossible even if the form and parameterization of the system are perfectly known. This is because chaotic systems have sensitive dependence on initial conditions, meaning that minute errors in the observation of the state of the system when predictions are made get translated into increasingly large errors in the predicted state of the system as the predictions go farther into the future. However, short-term prediction remains feasible.

frequency are sufficient to violate the random walk model. Previous findings in this regard are well known and need not be mentioned except briefly. Trend deviations from random walk were first discovered for wheat and corn by Houthakker (1961), and for soybeans by Smidt (1965). Their results based on filter or trading rules are questioned by Cargill and Rausser (1975). Rocca (1969), Labys and Granger (1970), and Cargill and Rausser (1972) in performing spectral analysis of more than 20 futures price series find some evidence for a modified random walk process, mostly resembling the Martingale process. Such studies test for the existence of a linear model; yet it is possible that some form of nonlinear model might underlie the price fluctuations in question. Stevenson and Bear (1970) and Leuthold (1972) employing filter rules confirm positive and negative price dependence to cast doubt on the validity of the random walk model.

Another approach of interest deals with the testing of the weak form and semi-strong form efficient market models. Even though the Martingale model may be rejected in empirical tests of futures price behavior, this does not constitute a rejection of the efficient market model underlying that behavior. A market can be said to use information efficiently if no way exists to use available information to increase expected wealth by frequent trading. This implies that trading rules are worthless and price information cannot indicate the best opportunities to buy and sell. Regarding the efficient market hypothesis, it can be said to be true if the risk-adjusted return net of all costs from the best trading rule is not more than the comparable figure, when assets are traded infrequently [Jensen (1978); Taylor (1986)]. The interpretation of this hypothesis in a commodity futures price context is sometimes confusing: (1) trading rules do not replicate real trading possibilities; (2) it is difficult to find benchmarks for return comparisons; (3) insufficient information is available about the distribution of filter returns to confirm the significance of a distribution; and (4) retrospective optimization of the parameters or structure of a filter is dubious [see Taylor (1986)]. Danthine (1977) finds confusion in linking Martingale processes and efficiency in commodity markets for other reasons. In particular, the possibility of shortages may lead to above normal expected profits, and a diminishing marginal rate of transformation over time may cause a severe correlation of price changes, even in an efficient market.

Guimaraes, Kingsman, and Taylor (1989) attempt to reappraise the efficiency hypothesis. Taylor's (1980) approach evaluates a futures price model which resembles a random walk yet includes a price-trend term for which formal stochastic processes are conjectural. This model, used on tests of ten commodity futures contracts traded in London, identifies a "price drift," which can be associated with the weak form of the efficient market model. Gross (1988) is concerned also about whether futures prices fully incorporate all publicly available information at the time of contracting. Based on a study of LME futures prices for copper and aluminum, he finds that the efficiency hypothesis cannot be rejected for these metals. Gupta and Mayer (1981), in performing efficiency tests for sugar, cocoa, and coffee prices as well as tin prices, also confirm the efficiency hypothesis. Both tests involve the more rigorous semi-strong form of the hypothesis. Among studies which have difficulty in confirming the hypothesis, MacDonald and Taylor (1988), also employing LME futures prices, confirm the hypothesis for copper and lead but not for zinc. They employ a more rigorous vector autoregressive methodology using appropriate stationarity-inducing transformations. Brorsen et al (1984) can not confirm the hypothesis for cotton futures prices representing contract funding in New York.

While most of the above studies are based on linear models, a more recent approach to futures price modeling considers the possibility that other deviations from random processes may exist. That approach involves the use of nonlinear equations generating chaotic behavior. Interestingly, the application of the concept of chaos to commodity futures price behavior can be found in an earlier work by Mandelbrot.

Drawing upon Houthakker's (1961) analysis of cotton prices, Mandelbrot (1963a and b) developed a model of price behavior by replacing Gaussian probability laws with those termed "stable Paretian." His approach represented an attempt to discover orderly behavior within what appeared to be a random series of price fluctuations. It is in this context that Frank and Stengos (1989) investigated the Martingale hypothesis using an approach of Sims (1984) as well as a chaos-based approach. Although Frank and Stengos were not able to reject the Martingale hypothesis in a series of standard econometric tests involving daily and weekly silver and gold prices, they did provide correlation dimension-based evidence of the presence of nonlinear structure. Note that such structure can be consistent with efficient markets. This article follows Frank and Stengos in investigating whether nonlinear structure exists in futures price data for four commodities.

THE CORRELATION DIMENSION TECHNIQUE

The correlation dimension technique was originally developed by the physicists Grassberger and Procaccia (1983).[4] The technique is intended to detect the presence of chaotic structure in data by embedding overlapping subsequences of the data in m-space for various embedding dimensions, m. Purely random data are infinite-dimensional, and thus will "fill" a region of m-space for any finite m. On the other hand, data generated by a deterministic system will have a finite dimension, which will be no greater than the number of independent state variables in the system and which may be a noninteger. Thus, for sufficiently large m, one will be able to detect structure in deterministic data.

Specifically, suppose a deterministic system $y_t = f(y_{t-1})$ where y is an n-dimensional vector of (possibly unobserved) state variables. Suppose further a time series of a scalar variable, x, generated by $x_t = g(y_t)$ for some observer function, g. Then Takens' (1980) embedding theorem states that $m \geq 2n + 1$ is a sufficiently large embedding dimension to detect the structure. Since n is not known, an increasing sequence of m values must be tried.

For any value of m, one computes the correlation dimension of a stationary series $\{x_t\}$ by first computing the correlation integral $C(\varepsilon, m)$ for various values of a critical distance, ε. $C(\varepsilon, m)$ is defined to be the fraction of pairs of m-dimensional points $(x_i x_{i+1} \ldots x_{i+m-1})$, $(x_j x_{j+1} \ldots x_{j+m-1})$ whose distance from each other is no greater than ε. The distance is measured according to the commonly used sup norm, defined as:

$$\|(x_i x_{i+1} \ldots x_{i+m-1}), (x_j x_{j+1} \ldots x_{j+m-1})\| \equiv \max_{k\varepsilon[0, m-1]}\{|x_{i+k} - x_{j+k}|\}$$

One computes the correlation integral for a variety of ε equally spaced on a log scale, so that $C(\varepsilon, m)$ ranges from near zero to unity.

The correlation dimension for given m is the elasticity of $C(\varepsilon, m)$ with respect to ε. Thus, one needs to run a linear regression of $\log C(\varepsilon, m)$ on $\log \varepsilon$ with an intercept. To prepare for this regression, one first plots $\log C(\varepsilon, m)$ against $\log \varepsilon$, and

[4]See also Brock (1986) and Barnett and Chen (1988) for descriptions of this technique.

visually inspects the plot. While, in principle, the plot should be linear throughout, in practice, the presence of noise in the data is likely to result in a separate and relatively steep region at low values of ε; and finiteness of the data set may cause a choppy appearance in the plot as well as curvature at high values of ε. Therefore, one must judgmentally determine the linear range over which to run the regression. The slope estimate from this regression is the correlation dimension estimate for this value of m.

This procedure is repeated for a sequence of increasing values of m. If the data are purely random, then in infinite samples, the correlation dimension will equal m for all m. In practice, with a finite data set, the correlation dimension estimate may be substantially below m and may rise with m at less than a one-for-one rate [Frank and Stengos (1988b, 1989); Ramsey and Yuan (1987)]. If the data are deterministic, the slope estimates should "saturate" at some m, not rising any more as m is further increased; this saturation value of the slope is the correlation dimension estimate for the unobserved structure which generates the data. In practice, the slope estimates may never completely stop rising. Thus, the process of determining whether saturation has occurred, and structure has been detected, can, in general, be rather judgmental.[5] However, in all the cases reported in this study, the evidence that structure is detected is strong. The slope estimates are usually well under one-third of the respective m values used to generate them, and they rise only slowly with m.

The data for which this procedure is conducted must be stationary. Since the raw data, as described below, are futures prices, the data are rendered stationary by taking the first difference of the logs of the price data. Thus, the transformed data are rates of return.

To make sure that the detected structure is nonlinear rather than linear, Brock's (1986) residual test is employed. The stationary data are delinearized by replacing the data with residuals from an autoregression of the data. If nonlinear structure is present, this procedure should, in principle, make no difference for the estimated correlation dimensions. However, in practice [Brock and Sayers (1988, p. 84)] the correlation dimension estimates rise with the number of terms in the autoregression. Therefore, this procedure should be used cautiously. The following procedure is utilized. Simple autocorrelations of rates of return are computed for lags 1–24. Then, an autoregression is run for the rate of return including any lags for which the simple autocorrelation is significant at the 1% level. The residuals from this autoregression serve as the AR-transformed data. Thus, the correlation dimension technique is performed for each commodity price series on both the rate of return data and on the AR-transformed data.

The shuffle test is also appropriate, given the tendency of finite sets of both noise and structured data to depart from idealized behavior in their correlation dimension estimates. The shuffle test of Scheinkman and LeBaron (1989a) tests the hypothesis that a given correlation dimension estimate can come from a data series of noise with the same data distribution as that exhibited by the actual data. For the AR residuals of each of the four rate of return series, 20 artificial noise series with the same length and distribution as the AR residuals are created by sampling with replacement from those residuals. Then the correlation dimension is estimated for

[5]Note that the slope estimate for given m can be quite sensitive to the choice of the linear range over which to run the regression. Thus numerical estimates of the correlation dimension must be regarded with caution. However, the judgment of whether saturation has occurred tends to be insensitive to the choice of linear range.

each artificial series, at the previously determined saturation embedding dimension and also at the highest embedding dimension (40). If at least 95% of these correlation dimension estimates for noise are above the corresponding estimates for the unshuffled data, the hypothesis that the unshuffled data are noise is rejected and the alternative hypothesis of nonlinear structure is accepted.

Finally, financial market data frequently appear to have a time-varying variance, which could produce correlation dimension estimates suggestive of deterministic nonlinear structure even if there are no deterministic components in the data. A time-varying variance could be generated by either a (stochastic) ARCH process or a deterministic process. Therefore, an ARCH filter is used to see if it is possible to rule out an ARCH process as an alternative explanation of the data. The same AR structure is used as before, and an ARCH(10) test is employed. Correlation dimension estimates are calculated for the standardized ARCH residuals and shuffle tests are conducted for each series at the saturation embedding dimension and at the highest embedding dimension.

DATA AND RESULTS

Daily data series of near futures prices for sugar, silver, copper, and coffee are used. These series have, respectively, 4462, 5297, 5308, and 4087 observations beginning between January 1968 and October 1972 and ending in March 1989. Further details appear in the Appendix.

As previously indicated, the data are transformed into first differences of logs of futures prices. The AR-transformed data, derived as described above, involve AR regressions with lags at 1 and 21 for sugar; at 3 and 15 for silver; at 2, 3, and 6 for copper; and at 1, 16, and 17 for coffee.

The correlation integral graphs, computed for embedding dimension values 4–40 by increments of 4, all look very similar to each other. For the non–AR-transformed data, a typical correlation integral graph, that of copper, is shown in Figure 1 for embedding dimensions 28–40 by increments of 4. Figure 2 shows a blow-up of the nearly linear region for ε values 0.020258 through 0.027296. The curves appear close to parallel, suggesting saturation.

Table I shows the correlation dimension estimates for the four rate of return series for embedding dimensions 4–40 by increments of 4. The correlation dimensions are always far below the corresponding embedding dimensions, and never rise above 11. The most striking results are for coffee. The correlation dimension is always below 6, strongly suggesting nonlinear structure with a relatively low state dimension. The other three series show strong evidence of structure as well.

The information provided by the level of the correlation dimension estimates is confirmed by the data in Table II, which shows how fast these estimates rise with the embedding dimension. This rate of rise is measured by $H \equiv \Delta$ (correlation dimension)/Δm, where the increment Δm is always 4. By the time an embedding dimension of 20 is reached, the H values for the four series are 0.16, 0.25, 0.21, and 0.13, and for higher embedding dimensions they drop even lower. In each case, these are much closer to the theoretical value of zero for pure structured, noiseless data than to the theoretical value of unity for structureless noise; thus, these results effectively approximate saturation, implying the presence of noisy structure. So the results for both the level and the saturation of the correlation dimension support a hypothesis of structure in the data. Note again that any inference of the specific saturation embedding dimension would necessarily be very imprecise; the point is simply that there is strong evidence for saturation and thus for the presence of structure.

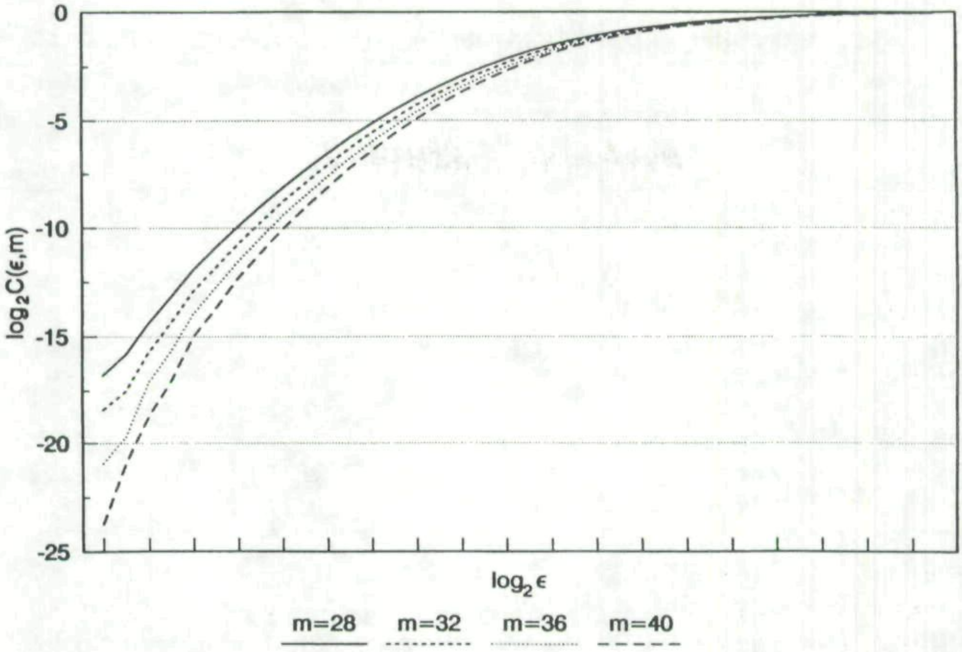

Figure 1
Correlation integral, copper.

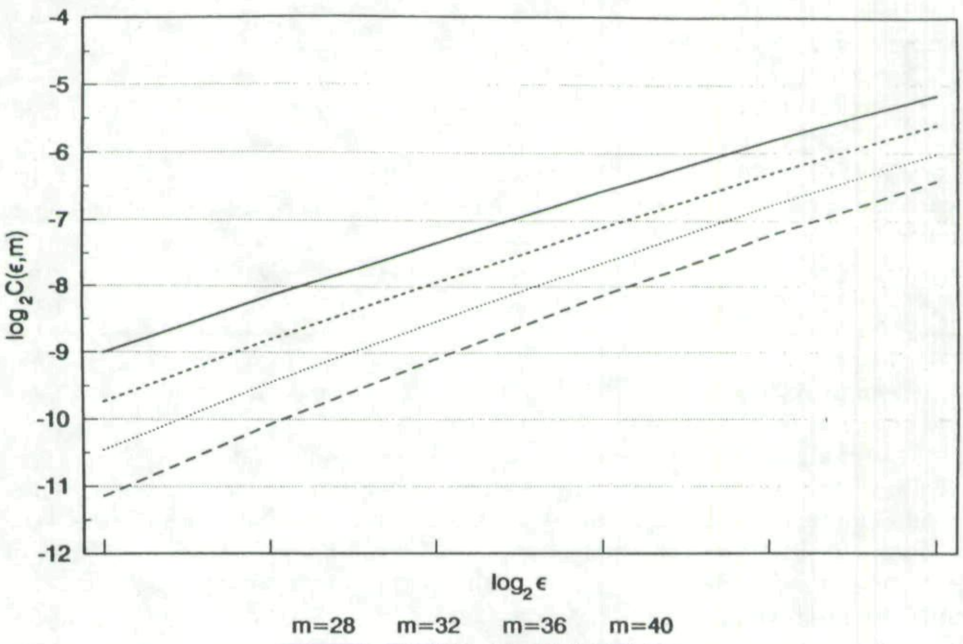

Figure 2
Linear region, correlation integral, copper.

To make sure that this structure is not linear, the residual test as described above is performed. Correlation dimension estimates computed for the autoregressive residuals are reported in Table III. The results in Table III are very close to those in Table I for the non–AR-transformed data, thereby supporting the notion that the

Table I
CORRELATION DIMENSION ESTIMATES FOR DAILY RATE OF RETURN DATA

	Correlation Dimension			
m	Silver ($N = 5297$)	Copper ($N = 5308$)	Sugar ($N = 4462$)	Coffee ($N = 4087$)
4	1.65	2.16	1.80	1.51
8	2.73	3.84	3.14	2.46
12	3.58	5.23	4.18	3.23
16	4.28	6.39	5.06	3.84
20	4.90	7.37	5.89	4.34
24	5.46	8.26	6.71	4.75
28	6.03	8.96	7.52	5.06
32	6.61	9.64	8.39	5.35
36	7.18	10.33	9.36	5.62
40	7.76	10.99	10.29	5.88

Table II
***H* VALUES FOR DAILY RATE OF RETURN DATA:**
[H ≡ (CORRELATION DIMENSION FOR THIS EMBEDDING DIMENSION MINUS THAT FOR THE NEXT LOWER EMBEDDING DIMENSION) DIVIDED BY 4]

m	Silver	Copper	Sugar	Coffee
4	—	—	—	—
8	0.27	0.42	0.34	0.24
12	0.21	0.35	0.26	0.19
16	0.18	0.29	0.22	0.15
20	0.16	0.25	0.21	0.13
24	0.14	0.22	0.21	0.10
28	0.14	0.18	0.20	0.08
32	0.15	0.17	0.22	0.07
36	0.14	0.17	0.24	0.07
40	0.15	0.17	0.23	0.07

structure detected is nonlinear rather than linear. H values for Table III estimates are presented in Table IV.

While these results strongly suggest the presence of nonlinear structure, it is prudent to check further the possibility that pure noise data could have generated these results in samples of these sizes. As discussed above, the shuffle test for the AR-transformed data for each commodity is performed using 20 shuffles at each of $m = 40$ and the m value for which H (taken out to three decimal places) is minimized (thus showing the strongest saturation). For each set of 20 shuffles, Table V gives the mean, the high, and the low values of the estimated correlation dimension, and compares them to that of the corresponding unshuffled series.

Table V shows that, out of a total of 160 shuffle correlation dimension estimates computed, not a single one is as low as the corresponding nonshuffle estimate. Thus one can reject, with high confidence, the possibility that the data reflect simply structureless noise masquerading as structured data.

Table III
CORRELATION DIMENSION ESTIMATES FOR AR RESIDUALS

	Correlation Dimension			
m	Silver (N = 5282)	Copper (N = 5302)	Sugar (N = 4441)	Coffee (N = 4070)
4	1.75	2.18	1.90	1.77
8	2.90	3.89	3.30	2.92
12	3.80	5.31	4.38	3.80
16	4.55	6.50	5.30	4.50
20	5.22	7.51	6.16	5.06
24	5.83	8.41	7.01	5.51
28	6.46	9.12	7.85	5.92
32	7.10	9.80	8.81	6.33
36	7.73	10.48	9.94	6.75
40	8.36	11.16	11.07	7.16

Table IV
***H* VALUES FOR AR RESIDUALS:**
[H ≡ (CORRELATION DIMENSION FOR THIS EMBEDDING DIMENSION MINUS THAT FOR THE NEXT LOWER EMBEDDING DIMENSION) DIVIDED BY 4]

m	Silver	Copper	Sugar	Coffee
4	—	—	—	—
8	0.29	0.43	0.35	0.29
12	0.23	0.36	0.27	0.22
16	0.19	0.30	0.23	0.18
20	0.17	0.25	0.22	0.14
24	0.15	0.23	0.21	0.11
28	0.16	0.18	0.21	0.10
32	0.16	0.17	0.24	0.10
36	0.16	0.17	0.28	0.11
40	0.16	0.17	0.28	0.10

Finally, to test for an ARCH explanation of the data, an ARCH(10) test is run on the above AR specifications, standardizing the ARCH residuals to remove the time-varying aspect of their variance, estimating the correlation dimensions of the standardized residuals, and conducting shuffle tests. The correlation dimension estimates, H values, and shuffle results appear in Tables VI, VII, and VIII, respectively (note that for three commodities the lowest H value occurs at the highest embedding dimension—40—so for those commodities only one shuffle test is needed).

A comparison of Table VI with Table III shows that the ARCH filtering process increases the estimated correlation dimensions, as would be expected of any filtering process with finite data. However, the increases are not very large. The H values are also increased, as shown by Table VII in comparison with Table IV, but they still reach minima much closer to the theoretical deterministic value of zero than to the theoretical noise value of one. The shuffle tests on the ARCH residuals uniformly confirm the earlier shuffle results. All 100 correlation dimension estimates for shuf-

Table V
SHUFFLE RESULTS, AR RESIDUALS: MEAN, HIGH, AND LOW VALUES OF CORRELATION DIMENSION ESTIMATES OVER 20 SHUFFLES

Silver		Sugar	
$m = 24$	Mean = 7.42	$m = 28$	Mean = 9.27
	Low = 5.97		Low = 8.08
	High = 10.43		High = 10.69
	Unshuffled = 5.83		Unshuffled = 7.85
$m = 40$	Mean = 12.42	$m = 40$	Mean = 13.32
	Low = 10.02		Low = 11.72
	High = 17.54		High = 15.72
	Unshuffled = 8.36		Unshuffled = 11.07
Copper		**Coffee**	
$m = 32$	Mean = 12.74	$m = 28$	Mean = 8.42
	Low = 11.75		Low = 7.46
	High = 14.75		High = 10.62
	Unshuffled = 9.80		Unshuffled = 5.92
$m = 40$	Mean = 16.01	$m = 40$	Mean = 12.20
	Low = 14.77		Low = 10.78
	High = 18.36		High = 15.83
	Unshuffled = 11.16		Unshuffled = 7.16

Table VI
CORRELATION DIMENSION ESTIMATES FOR STANDARDIZED ARCH RESIDUALS

	Correlation Dimension			
m	Silver (N = 5262)	Copper (N = 5282)	Sugar (N = 4421)	Coffee (N = 4050)
4	1.57	1.59	1.56	1.57
8	3.15	3.23	3.10	3.01
12	4.64	4.93	4.61	4.45
16	5.99	6.55	6.00	5.83
20	7.22	8.08	7.28	7.07
24	8.36	9.53	8.55	8.09
28	9.46	10.84	9.70	9.00
32	10.54	12.04	10.85	9.87
36	10.55	13.23	12.07	10.63
40	12.55	14.40	13.34	11.32

fled series of ARCH residuals are higher than the corresponding unshuffled correlation dimension estimate. This result is striking in light of the substantial filtering the data undergo, and it provides strong evidence for the proposition that there is some nonlinear process with a deterministic component underlying these data series.

Table VII

H VALUES FOR STANDARDIZED ARCH RESIDUALS

m	Silver	Copper	Sugar	Coffee
4	—	—	—	—
8	0.40	0.41	0.39	0.36
12	0.37	0.43	0.38	0.36
16	0.34	0.41	0.35	0.35
20	0.31	0.38	0.32	0.31
24	0.29	0.36	0.32	0.26
28	0.28	0.33	0.29	0.23
32	0.27	0.30	0.29	0.22
36	0.26	0.30	0.31	0.19
40	0.24	0.29	0.32	0.17

Table VIII

SHUFFLE RESULTS, STANDARDIZED ARCH RESIDUALS: MEAN, HIGH, AND LOW VALUES OF CORRELATION DIMENSION ESTIMATES OVER 20 SHUFFLES

Silver		Sugar	
$m = 40$	Mean = 16.04	$m = 28$	Mean = 10.79
	Low = 13.10		Low = 9.85
	High = 19.95		High = 13.84
	Unshuffled = 12.55		Unshuffled = 9.70
		$m = 40$	Mean = 15.28
			Low = 14.09
			High = 18.99
			Unshuffled = 13.34
Copper		**Coffee**	
$m = 40$	Mean = 16.32	$m = 40$	Mean = 13.97
	Low = 14.64		Low = 12.46
	High = 19.31		High = 17.05
	Unshuffled = 14.40		Unshuffled = 11.32

CONCLUSIONS

This study applies the correlation dimension technique, along with the associated AR and ARCH residual tests and shuffle test, to rate of return series for the futures prices of four commodities. The results for the rate of return data and for their autoregressive and ARCH residuals, both in regard to the level and saturation of the correlation dimension estimates, strongly suggest the presence of structure in the data. The fact that the results are relatively unchanged by the use of AR residuals implies that this structure is nonlinear in nature, and the fact that the results hold up under ARCH filtering shows that the apparent structure does not simply reflect heteroskedasticity. Further, the fact that the shuffle tests are passed with "flying colors" implies that the results are not a fluke which could have been achieved misleadingly, with unstructured data. It is worth noting that, since the data series had in excess of 4000 observations, the procedures employed are stronger than they would be with the much smaller data series commonly used with these procedures by economists.

A noise explanation of the data as well as a linear-structure-plus-noise explanation are rejected. Thus, the hypothesis of nonlinear structure is accepted. As Barnett and Hinich (1989) point out, this acceptance is necessary but not sufficient for the presence of chaos in the data. If chaos is viewed as the null hypothesis, this study fails to reject it. Evidence for the presence of chaos is provided, but, as Barnett and Hinich say "further research is needed before we can confirm or reject the discovery of chaos." In any event, if chaos is present, it is probably accompanied by noise, so even if the chaos is exactly identified, an incentive for risk-averse agents to engage in hedging behavior still exists.

The evidence of structure in futures prices raises further questions about the efficient markets hypothesis, since it creates the possibility that profitable, nonlinearity-based trading rules may exist. Clearly then, an important area for future research is to identify more precisely the nature of the nonlinearity in the data. Only then can it be established what, if any, profitable trading rules exist. Moreover, identification of the structure would establish whether prediction beyond some horizon is precluded, due to the sensitive dependence on initial conditions inherent in chaotic systems. Another important area for future research is to further develop equilibrium models of financial markets which can give rise to dynamic structure in rates of return. Previous work in this vein includes van der Ploeg (1986) and Lucas (1978). Perhaps this substrand of literature could further clarify the relation between market efficiency and nonlinear dynamics.

APPENDIX

Daily futures price data are gathered for four major commodities traded on exchanges in New York: (1) The Coffee "C" contract at the Cocoa, Sugar, and Coffee Exchange, Inc., in ¢/100 lbs.; (2) The Sugar No. 11 contract at the Cocoa, Sugar, and Coffee Exchange, Inc., in ¢/100 lbs.; (3) The Silver .999 Fine contract at the Commodity Exchange, Inc. in ¢/10 troy oz.; and (4) The Refined copper contract, at the Commodity Exchange, Inc., in ¢/100 lbs. The data are in the form of settlement prices and are from the Center for the Study of Futures Markets, Graduate Business School, Columbia University, New York.

The typical structure of commodity futures price series is that they are quoted for selected months of the year for which contracts are traded. The terms of these contracts are spaced from the near months to the more distant months. The accepted practice [Labys and Granger (1970); Taylor (1986)] is followed to compile the near future prices series. These series adopt prices from the contract nearest to the present or spot data; a jump is made to the prices of the successive contract at the beginning of the month in which the near contract comes to maturity. The series for silver and copper begin in January 1968; the series for sugar begins in January 1971; and the series for coffee begins October 1972. All series end in March 1989.

Bibliography

Barnett, W. A., and Chen, P. (1988): "The Aggregation-Theoretic Monetary Aggregates Are Chaotic and Have Strange Attractors," in *Dynamic Econometric Modeling,* Barnett, W., Berndt, E., and White, H. (eds.). Cambridge, UK: Cambridge University Press.

Barnett, W. A., and Hinich, M. J. (forthcoming): "Has Chaos Been Discovered with Economic Data?," in *Evolutionary Dynamics and Nonlinear Economics,* P. Chen and R. Day (eds.). Oxford: Oxford University Press.

Baumol, W., and Benhabib, J. (1989): "Chaos: Significance, Mechanism, and Economic Applications," *Journal of Economic Perspectives,* 3:77–105.

Baumol, W., and Quandt, R. (1985): "Chaos Models and Their Implications for Forecasting," *Eastern Economic Journal,* 11:3–15.

Blank, S. (1990, January): "'Chaos' in Futures Markets? A Nonlinear Dynamical Analysis," Columbia Business School Center for the Study of Futures Markets, Working Paper #204. Columbia University, New York.

Brock, W. (1986): "Distinguishing Random and Deterministic Systems: Abridged Version," *Journal of Economic Theory,* 40:168–195.

Brock, W., and Sayers, C. (1988): "Is the Business Cycle Characterized by Deterministic Chaos?," *Journal of Monetary Economics,* 22:71–90.

Brorsen, B.W., Von Bailey, D., and Richardson, J.W. (1984): "Investigation of Price Discovery and Efficiency for Cash and Futures Cotton Prices," *Western Journal of Agricultural Economics,* 9:170–176.

Cargill, T. F., and Rausser, G. C. (1972): "Time and Frequency Domain Representation of Futures Prices as a Stochastic Process," *Journal of the American Statistical Association,* 67:23–30.

Cargill, T. F., and Rausser, G. C. (1975): "Temporal Price Behavior in Commodity Futures Markets," *Journal of Finance,* 30:1043–1053.

Danthine, J. P. (1977): "Martingale, Market Efficiency and Commodity Prices," *European Economic Review,* 10:1–17.

DeCoster, G. P., and Mitchell, D.W. (1991a): "The Efficacy of the Correlation Dimension Technique in Detecting Determinism in Small Samples," *Journal of Statistical Computation and Simulation,* 39:221–229.

DeCoster, G. P., and Mitchell, D.W. (1991b): "Nonlinear Monetary Dynamics," *Journal of Business and Economic Statistics,* 9:455–461.

Fama, E. (1970): "Efficient Capital Markets: Review of Theory and Empirical Work," *Journal of Finance,* 25:383–417.

Frank, M., Gencay, R., and Stengos, T. (1988): "International Chaos?" *European Economic Review,* 32:1569–1584.

Frank, M., and Stengos, T. (1988a): "Chaotic Dynamics in Economic Time-Series," *Journal of Economic Surveys,* 2:103–133.

Frank, M., and Stengos, T. (1988b): "Some Evidence Concerning Macroeconomic Chaos," *Journal of Monetary Economics,* 22:423–438.

Frank, M., and Stengos, T. (1989): "Measuring the Strangeness of Gold and Silver Rates of Return," *Review of Economic Studies,* 56:553–567.

Gabisch, G. (1987): "Nonlinearities in Dynamic Economic Systems," *Atlantic Economic Journal,* 15:22–31.

Grassberger, P., and Procaccia, I. (1983): "Measuring the Strangeness of Strange Attractors," *Physica,* 9D:189–208.

Gross, M. (1988): "A Semi-Strong Test of the Efficiency of the Aluminum and Copper Markets at the LME," *Journal of Futures Markets,* 8:67–77.

Guimaraes, R. M., Kingsman, B. G., and Taylor, S. J. (1989): *A Reappraisal of the Efficiency of Financial Markets.* Berlin: Springer-Verlag.

Gupta, S., and Mayer, T. (1981): "A Test of the Efficiency of Futures Markets in Commodities," *Weltwirtschaftliches Archives,* 117:661–671.

Hinich, M. J., and Patterson, D. M. (1985): "Evidence of Nonlinearity in Daily Stock Returns," *Journal of Business and Economic Statistics,* 3:69–77.

Houthakker, H. (1961): "Systematic and Random Elements in Short-Term Price Movements," *American Economic Review* 51:164–172.

Hsieh, D. A. (1989): "Testing for Nonlinear Dependence in Daily Foreign Exchange Rates," *Journal of Business,* 62:339–368.

Jensen, M. C. (1978): "Some Anomalous Evidence Regarding Market Efficiency: An Editorial Introduction," *Journal of Financial Economics,* 6:95–101.

Kamara, A. (1982): "Issues in Futures Markets: A Survey," Working Paper CSFM-30, Center for the Study of Futures Markets, Columbia University, New York.

Kelsey, D. (1988): "The Economics of Chaos or the Chaos of Economics," *Oxford Economic Papers,* 40:1–31.

Labys, W. C., and Granger, C.W. J. (1970): *Speculation, Hedging and Commodity Price Forecasts.* Lexington, MA: Heath Lexington Books.

Leuthold, R. M. (1972): "Random Walk and Price Trends: The Live Cattle Futures Market," *Journal of Finance,* 27:879–889.

Lucas, R. E. (1978): "Asset Prices in an Exchange Economy," *Econometrica,* 46:1429–1445.

MacDonald, R., and Taylor, M. P. (1988): "Testing Rational Expectations and Efficiency in the London Metal Exchange," *Oxford Bulletin of Economics and Statistics,* 50:41–52.

Mandelbrot, B. (1963a): "New Methods in Statistical Economics," *Journal of Political Economy,* 41:421–443.

Mandelbrot, B. (1963b): "The Variation of Certain Speculative Prices," *Journal of Business,* 36:394–429.

Ploeg, F. van der (1986): "Rational Expectations, Risk and Chaos in Financial Markets," *Economic Journal* 96 (suppl.):151–162.

Ramsey, J. B., Sayers, C. L., and Rothman, P. (1988): "The Statistical Properties of Dimension Calculations Using Small Data Sets: Some Economic Applications," Research Report #88-10, C.V. Starr Center, New York University.

Ramsey, J. B., and Yuan, H.-J. (1987): "The Statistical Properties of Dimension Calculations Using Small Data Sets," Research Report #87-20, C.V. Starr Center, New York University.

Rocca, L. H. (1969): "Time Series Analysis of Commodity Futures Prices," Ph.D. dissertation, University of California, Berkeley.

Samuelson, P. (1965): "Proof that Properly Anticipated Prices Fluctuate Randomly," *Industrial Management Review,* 6:41–49.

Savit, R. (1988): "When Random is Not Random: An Introduction to Chaos in Market Prices," *Journal of Futures Markets,* 8:271–289.

Scheinkman, J. A., and Lebaron, B. (1989a): "Nonlinear Dynamics and Stock Returns," *Journal of Business,* 62:311–337.

Scheinkman, J. A., and LeBaron, B. (1989b): "Nonlinear Dynamics and GNP data," in *Economic Complexity,* Barnett, W., Geweke, J., and Shell, K. (eds.). Cambridge, UK: Cambridge University Press.

Sims, C. A. (1984): "Martingale-Like Behavior of Prices and Interest Rates," Discussion Paper No. 205, Department of Economics, University of Minnesota, Minneapolis.

Smidt, S. (1965/1977): "A Test of the Serial Independence of Price Changes in Soybean Futures," as reprinted in *Selected Writings on Futures Markets,* Peck, A. (ed.). Chicago: Chicago Board of Trade, pp. 257–277.

Stevenson, R. A., and Bear, R. M. (1970/1977): "Commodity Futures: Trends or Random Walk?", as reprinted in *Selected Writings on Futures Markets,* Peck, A. (ed.). Chicago Board of Trade, pp. 279–294.

Takens, F. (1980): "Detecting Strange Attractors in Turbulence," in *Dynamical Systems and Turbulence: Lecture Notes in Mathematics* #898, Rand, D. and Young, L. (eds.). Berlin: Springer-Verlag, pp. 366–382.

Taylor, S. J. (1980): "Conjectured Models for Trends in Financial Prices: Tests and Forecasts," *Journal of the Royal Statistical Society A,* 143:338–362.

Taylor, S. J., (1986): *Modeling Financial Time Series.* Chichester, UK: Wiley.

Working, H. (1958/1977): "A Theory of Anticipatory Prices," as reprinted in *Selected Writings of Holbrook Working.* Chicago: Chicago Board of Trade.