Estimating the Volatility of S&P 500 Futures Prices Using the Extreme-Value Method

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INTRODUCTION

Risk estimation and forecasting are critical issues in option valuation. The key input to virtually all option pricing formulas, including the classic Black–Scholes (1973) and Cox, Ross, and Rubinstein (1979) binomial models, is an estimate of the underlying asset's volatility over the remaining life of the option.

Traditionally, variances have been estimated using squared close-to-close daily returns. More recently, Parkinson (1980) and Garman and Klass (1980) have developed variance estimators which use the high and low prices observed during the day. Assuming trading is continuous and always monitored, and prices follow a lognormal distribution, these extreme-value estimators are significantly more efficient than the close-to-close estimator, because they incorporate the range or dispersion of prices observed over the entire day, not just a "snapshot" price at the end of the day. With a more efficient volatility estimate, traders can more easily identify situations in which options appear to be under or overpriced by the market.

Since trade is not continuous in real-world markets, the practical applicability of extreme-value estimators is questionable. Garman and Klass (1980) demonstrate that extreme-value estimators are downward biased when trading is discrete, because "true" highs and lows can remain unobserved. Another source of bias is discreteness in prices, or the "one-eighth" problem, as discussed in Marsh and Rosenfeld (1986). In the empirical literature, Edwards (1988) finds that Parkinson's estimator is significantly downward biased relative to the close-to-close estimator for the S&P 500 and Value Line cash indices. Wiggins (1991) reaches the same conclusion with respect to volatilities of individual stocks, with the downward bias most severe for stocks with relatively low trading volumes.

In contrast to the market for individual stocks, where discrete pricing and nontrading problems can be very serious, the market for near-maturity S&P 500 futures contracts closely approximates a continuous trading world. Trading volume is very high; the average daily volume of the contracts analyzed in the sample exceeds 43,000. High volume is associated with high trading continuity, which minimizes estimation errors from unobserved true extreme prices during the day. In addition,

Helpful comments from two anonymous reviewers are greatly appreciated.

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The Journal of Futures Markets, Vol. 12, No. 3, 265–273 (1992)
 CCC 0270-7314/92/030265-09$04.00

the minimum price increment of 0.05 is very small relative to the daily standard deviation of price changes, which minimizes biases from price discreteness.[1] Under these conditions, the extreme-value estimator could potentially outperform the close-to-close estimator.

This article presents an empirical analysis of the relative bias, efficiency, and forecasting properties of the close-to-close and extreme-value volatility estimators for the S&P 500 futures price series. The first section provides a description of the data set and volatility estimators, and the next section discusses the statistical procedures and presents the empirical tests. The results suggest that the Parkinson (1980) and Garman and Klass (1980) estimators are only slightly downward biased, and are significantly more efficient than the close-to-close estimator.

DATA DESCRIPTION

S&P 500 futures price data are from the Technical Tools Company. The raw data set contains complete daily opening, high, low, closing, cash price and volume records for all contracts over the April 1982 through December 1989 period. The data are screened for clear recording errors, such as an opening or low price exceeding the high price, and none are found.

The final data sample contains daily price records from the three months nearest to but not inclusive of each contract expiration month. For example, the daily data for the months of December through February are March contract prices, and the March through May data are June contract prices. This selection procedure ensures that each daily price record is an independent observation, and is drawn from an actively traded contract.[2]

Define in logarithms:

C = closing price on date t
C_{-1} = closing price on date $t-1$
O = opening price on date t
H = high price on date t
L = low price on date t
H' = high price on date t, inclusive of C_{-1}
L' = low price on date t, inclusive of C_{-1}

The variance estimators for date t are[3]:

$$\begin{aligned}
\sigma^2_{cct} &= (C - C_{-1})^2 \\
\sigma^2_{e1t} &= (O - C_{-1})^2 + 0.3607(H - L)^2 \\
\sigma^2_{e2t} &= 0.3607(H' - L')^2 \\
\sigma^2_{e3t} &= (O - C_{-1})^2 + 0.5(H - L)^2 - 0.3862(C - O)^2 \qquad (1)
\end{aligned}$$

[1]The S&P 500 contract has daily price limits which are very wide relative to the underlying volatility of the futures price, and have been effective only once (on October 13, 1989). For most of the sample (through October 18, 1988) there are no price limits. Limits on daily price changes bias all volatility estimators downward, but have their biggest impact on extreme-value estimators. If, for example, a futures price hits the upper limit and trading resumes at a price within the limits by the close of trading, the close-to-close estimator is unaffected, but the extreme-value estimator is biased.

[2]The average daily volume of the price series in the data set could be increased slightly by extending data collection from the nearest maturity contract through the first week of the expiration month.

[3]All of the estimators assume a mean daily log price relative of zero. Merton (1980) shows the variance estimation bias from this simplifying assumption, when using daily data, is minimal.

The first estimator, σ_{cc}^2, is the traditional close-to-close estimator. The next estimator, σ_{e1}^2, is computed by adding an estimate of the overnight variance using the traditional method to Parkinson's (1980) extreme-value formula for daytime variance. σ_{e2}^2 is a modified version of Parkinson's estimator, which includes the previous closing price as a possible high or low during the close-to-close period. This estimator will be downward biased relative to σ_{cc}^2, even if trading is continuous during exchange hours, because it does not incorporate true high or low prices occurring during nighttime hours. It has the advantages of avoiding the need to explicitly estimate a nighttime variance, and thus, the requirement of an opening price in the data set.

The final estimator, σ_{e3}^2, is computed by adding the overnight variance estimate to the Garman and Klass (1980) super-efficient daytime estimator. Garman and Klass demonstrate that σ_{e3}^2 is more than seven times more efficient than the close-to-close estimator in a continuous-trading world.

To maintain consistency with standard practice, the analysis which follows is performed using estimates of standard deviation or volatility, not variance, with volatilities expressed on an annualized basis. Volatilities are calculated over both calendar month and weekly (five trading day) intervals to see whether the properties of the estimators are sensitive to the measurement interval. For example, if month T contains N trading days, then month T annualized close-to-close volatility is computed as:

$$\sigma_{ccT} = \text{sqrt}[(253/N)^* \sum_t \sigma_{cct}^2] \tag{2}$$

The full sample contains 92 monthly and 374 weekly volatilities between May 1982 and December 1989, inclusive.

EMPIRICAL TESTS

Bias Tests

Summary statistics for monthly and weekly volatilities and formal statistical tests of estimator bias appear in Table I. The first two rows in each panel provide the sample mean and median of the annualized volatility estimates from each model, with the standard error of the mean in parenthesis. Formal tests of bias in the extreme-value estimators relative to the close-to-close estimator are provided in the last row of each panel. Differences between σ_{cc} and σ_{ei} are calculated for each monthly or weekly observation, and time-series means of these (independent) volatility differences are presented with standard errors of the means in parenthesis.

Panels (A) and (C) of Table I present monthly and weekly results for the full sample. The first extreme-value estimator, σ_{e1}, a combination of Parkinson's formula for daytime volatility and the traditional estimator for nighttime volatility, is effectively unbiased. Using monthly data, σ_{e1} is on average less than σ_{cc}, but the difference is not statistically significant. The second estimator, σ_{e2}, is significantly downward biased for both weekly and monthly intervals. This result is not surprising given that σ_{e2} essentially assumes nighttime volatility is zero. Garman and Klass's estimator, σ_{e3}, is significantly downward biased using monthly data but not using weekly data.[4]

[4]Notice that if the analysis had been performed in units of variance, not standard deviation, then the annualized sample mean of each estimator would be invariant to the measurement interval (weekly or monthly).

Table I
SUMMARY STATISTICS AND TESTS OF BIAS FOR WEEKLY AND MONTHLY CLOSE-TO-CLOSE AND EXTREME-VALUE VOLATILITIES

	σ_{cc}	σ_{e1}	σ_{e2}	σ_{e3}
(A) Monthly data, full sample				
Mean	17.56	17.17	16.31	16.80
	(1.54)	(1.62)	(1.32)	(1.57)
Median	13.54	14.25	13.54	13.91
Mean ($\sigma_{cc} - \sigma_{ei}$)		0.39	1.25	0.76
		(0.24)	(0.27)	(0.29)
(B) Monthly data, October 1987 excluded				
Mean	16.19	15.67	15.12	15.33
	(0.71)	(0.61)	(0.57)	(0.57)
Mean ($\sigma_{cc} - \sigma_{ei}$)		0.52	1.07	0.86
		(0.20)	(0.21)	(0.28)
(C) Weekly data, full sample				
Mean	16.62	16.65	15.84	16.31
	(0.85)	(0.86)	(0.71)	(0.84)
Median	13.53	13.69	13.44	13.47
Mean ($\sigma_{cc} - \sigma_{ei}$)		−0.03	0.77	0.31
		(0.35)	(0.23)	(0.38)
(D) Weekly data, October 1987 excluded				
Mean	15.60	15.53	14.95	15.22
	(0.45)	(0.35)	(0.34)	(0.34)
Mean ($\sigma_{cc} - \sigma_{ei}$)		0.07	0.65	0.38
		(0.20)	(0.19)	(0.26)

All volatilities are expressed on an annualized basis. The sample period is May 1982–December 1989, which includes 92 monthly and 374 weekly observations. Standard errors of the means are in parentheses.

Panels (B) and (D) present summary statistics and tests of bias using a sample excluding the market crash month of October 1987. The elimination of October 1987 results in a significant reduction in mean volatility, and the standard errors of the means are cut approximately in half. Using monthly data, all three extreme-value estimators are significantly downward biased, but using weekly data, only σ_{e2} is significantly downward biased. For σ_{e1}, the largest percentage bias across the four panels of Table I is only about 3%.

These results are encouraging for practical application of the extreme-value technique to the S&P 500 futures price series. The Parkinson (1980) estimator in particular contains little empirical bias relative to close-to-close volatility, suggesting the discrete-time and discrete-price trading problems discussed in Garman and Klass (1980) and Marsh and Rosenfeld (1986) are not particularly serious for an actively traded instrument with small price increments.

Efficiency Tests

This section evaluates the relative efficiency of close-to-close and extreme-value volatilities in estimating close-to-close volatility over the next week or month. For each week or month T, squared errors (SE) and absolute errors (AE) around week or month $T + 1$ close-to-close volatility are computed for each estimator, for example

$$\mathrm{SE}(\sigma_{ccT}) = (\sigma_{ccT} - \sigma_{ccT+1})^2$$
$$\mathrm{AE}(\sigma_{ccT}) = |\sigma_{ccT} - \sigma_{ccT+1}| \tag{3}$$

Relative mean squared error efficiencies of the extreme-value estimators versus the close-to-close estimator are calculated by taking the differences between $\mathrm{SE}(\sigma_{eiT})$ and $\mathrm{SE}(\sigma_{ccT})$ for each T, and then averaging these differences over the sample period.

Table II presents the results of the efficiency tests described above for the sample excluding the months of October and November 1987. The results are qualitatively the same over the full sample, though the standard errors are rather large in a few cases.

The first row of panel (A) presents mean squared errors for each of the four monthly volatility estimators. The mean of $\mathrm{SE}(\sigma_{cc})$ is approximately one-third higher than for the extreme-value estimators. The formal tests in the second row demonstrate the superiority of the extreme-value estimators, as the mean differences in squared errors are approximately two SEs greater than zero in each case. The mean absolute error results in the third and fourth rows are qualitatively similar yet more significant statistically. Based on the point estimates, both the modified version of Parkinson's estimator, σ_{e2}, and the Garman and Klass estimator, σ_{e3}, perform somewhat better than σ_{e1}, but the differences are not large.

Table II
TESTS OF RELATIVE EFFICIENCY OF WEEKLY AND MONTHLY CLOSE-TO-CLOSE AND EXTREME-VALUE VOLATILITY ESTIMATORS, OCTOBER AND NOVEMBER 1987 EXCLUDED

	σ_{cc}	σ_{e1}	σ_{e2}	σ_{e3}
(A) Monthly data				
Mean squared error (SE)	47.13	36.67	35.40	34.21
	(12.82)	(9.67)	(9.39)	(9.07)
Mean $[\mathrm{SE}(\sigma_{cc}) - \mathrm{SE}(\sigma_{ei})]$		10.46	11.72	12.91
		(4.95)	(5.85)	(6.76)
Mean absolute error (AE)	4.58	4.00	3.89	3.86
	(0.54)	(0.48)	(0.48)	(0.47)
Mean $[\mathrm{AE}(\sigma_{cc}) - \mathrm{AE}(\sigma_{ei})]$		0.58	0.69	0.72
		(0.19)	(0.22)	(0.26)
(B) Weekly data				
Mean squared error (SE)	73.04	56.25	56.45	56.01
	(10.53)	(9.81)	(9.97)	(9.92)
Mean $[\mathrm{SE}(\sigma_{cc}) - \mathrm{SE}(\sigma_{ei})]$		16.78	16.58	17.03
		(5.25)	(5.36)	(6.24)
Mean absolute error (AE)	6.24	5.33	5.33	5.29
	(0.31)	(0.28)	(0.28)	(0.28)
Mean $[\mathrm{AE}(\sigma_{cc}) - \mathrm{AE}(\sigma_{ei})]$		0.91	0.90	0.95
		(0.18)	(0.18)	(0.22)

Errors in the mean squared and mean absolute error computations are defined as the differences between the relevant estimator in period T and the close-to-close estimator in period $T + 1$. Standard errors are in parentheses.

In the weekly volatility efficiency tests in panel (B) of Table II, all of the above results are confirmed. The mean squared error of σ_{cc} is roughly 30% higher than the mean SEs of the other estimators, with the differences about three SEs above zero.

These results establish that all three versions of the extreme-value estimator provide a more efficient estimate of next week or month's S&P 500 futures price volatility than does the close-to-close estimator. If a single historical volatility estimate is to be used as a forecast of future volatility, each of the extreme-value estimators are superior to the close-to-close estimator.

Forecasting Tests

To complete the analysis of relative estimator performance, volatility forecasting tests are executed using both monthly and weekly volatility measurement intervals. Tables III and IV present the results of maximum likelihood estimation of autoregressive moving-average (ARMA) time series models for monthly and weekly estimated volatilities, respectively. These forecasting models represent a more sophisticated (and costly) approach to estimating volatility over the next week or month into the future than using a historical volatility, as was implied in the efficiency tests in the previous section.

The performance of each specification can be assessed using the standard error of the estimate (SSE) in the last column of each table. The monthly (weekly) regressions include one (two) moving-average terms to eliminate serial correlation in the

Table III
AUTOREGRESSIVE MOVING AVERAGE (ARMA) FORECASTING EQUATIONS FOR MONTHLY VOLATILITIES, OCTOBER AND NOVEMBER 1987 EXCLUDED

	MA(1)	SEE
(1) $\sigma_{ccT+1} = 1.92 + 0.87^{*}\sigma_{ccT}$	0.63	5.40
(1.08) (0.06)	(0.12)	
(2) $\sigma_{ccT+1} = 1.34 + 0.93^{*}\sigma_{e1T}$	0.52	5.18
(1.10) (0.07)	(0.11)	
(3) $\sigma_{ccT+1} = 0.73 + 1.01^{*}\sigma_{e2T}$	0.57	5.05
(1.00) (0.06)	(0.11)	
(4) $\sigma_{ccT+1} = 1.52 + 0.94^{*}\sigma_{e3T}$	0.45	5.16
(1.21) (0.08)	(0.11)	
(5) $\sigma_{ccT+1} = 1.34 + 0.88^{*}\sigma_{e1T} - 0.05^{*}\sigma_{ccT}$	0.54	5.17
(1.10) (0.35) (0.34)	(0.14)	
(6) $\sigma_{e1T+1} = 2.32 + 0.84^{*}\sigma_{e1T}$	0.47	4.05
(1.03) (0.06)	(0.13)	
(7) $\sigma_{e1T+1} = 5.35 + 0.62^{*}\sigma_{ccT}$	0.26	4.47
(1.63) (0.10)	(0.17)	
(8) $\sigma_{e1T+1} = 2.07 + 1.21^{*}\sigma_{e1T} - 0.35^{*}\sigma_{ccT}$	0.46	3.99
(1.01) (0.24) (0.23)	(0.13)	

The sample includes 89 monthly observations between May 1982 and December 1989 inclusive. All equations are estimated by maximum likelihood, with approximate standard errors in parentheses below the coefficient estimates. The MA(1) column presents moving average coefficient estimates and standard errors for each model. SEE is the standard error of the estimate, the maximum likelihood standard deviation of the regression residuals. All volatilities are expressed on an annualized basis.

Table IV
AUTOREGRESSIVE MOVING AVERAGE (ARMA) FORECASTING EQUATIONS FOR WEEKLY VOLATILITIES, OCTOBER AND NOVEMBER 1987 EXCLUDED

	MA(1)	MA(2)	SEE
(1) $\sigma_{ccT+1} = 0.74 + 0.95^{*}\sigma_{ccT}$	0.66	0.14	6.87
(0.32) (0.02)	(0.06)	(0.06)	
(2) $\sigma_{ccT+1} = 3.00 + 0.81^{*}\sigma_{e1T}$	0.10	0.15	7.11
(1.12) (0.07)	(0.07)	(0.06)	
(3) $\sigma_{ccT+1} = 1.40 + 0.95^{*}\sigma_{e2T}$	0.21	0.19	7.14
(0.94) (0.06)	(0.06)	(0.06)	
(4) $\sigma_{ccT+1} = 5.53 + 0.66^{*}\sigma_{e3T}$	−0.08	0.07	7.19
(1.39) (0.09)	(0.06)	(0.06)	
(5) $\sigma_{ccT+1} = 0.51 + 0.16^{*}\sigma_{e1T} + 0.80^{*}\sigma_{ccT}$	0.60	0.18	6.84
(0.33) (0.11) (0.10)	(0.07)	(0.06)	
(6) $\sigma_{e1T+1} = 0.79 + 0.94^{*}\sigma_{e1T}$	0.42	0.26	4.29
(0.32) (0.02)	(0.06)	(0.06)	
(7) $\sigma_{e1T+1} = 7.61 + 0.49^{*}\sigma_{ccT}$	−0.10	0.49	4.62
(0.86) (0.05)	(0.06)	(0.05)	
(8) $\sigma_{e1T+1} = 1.21 + 0.72^{*}\sigma_{e1T} + 0.19^{*}\sigma_{ccT}$	0.45	0.19	4.22
(0.35) (0.06) (0.05)	(0.06)	(0.06)	

The sample includes 365 weekly observations between May 1982 and December 1989, inclusive. All equations are estimated by maximum likelihood, with approximate standard errors in parenthesis below the coefficient estimates. The MA(1) column presents moving average coefficient estimates and standard errors for each model. SEE is the standard error of the estimate, the maximum likelihood standard deviation of the regression residuals. All volatilities are expressed on an annualized basis.

residuals; these moving-average errors in the time-series are at least partly caused by the inherent measurement error in the volatility estimates. The months of October and November of 1987 are excluded from the regressions; none of the specifications exhibit any significant forecasting ability when these observations are included.

The first four volatility equations in each table regress σ_{cc} against lagged values of each of the four competing estimators. In Table III, the specifications (2)–(4) using the extreme-value estimators are superior to the close-to-close model (1) based on their respective SEEs. The extreme-value estimator σ_{e2} appears to be slightly more efficient than the other two in predicting σ_{cc} using monthly data. The fifth regression runs σ_{cc} on lagged values of itself and σ_{e1}. The estimated coefficient on the lagged σ_{cc} is negative and statistically insignificant, and the SEE does not improve from using σ_{e1} alone in eq. (2).

Regressions (6) and (7) replicate (2) and (1), but with σ_{e1} as the dependent variable. The results again demonstrate that the extreme-value estimator is the superior predictor of monthly volatilities. Since σ_{e1} exhibits less sampling error than σ_{cc}, the SEE statistics are smaller in (6) and (7) than in (2) and (1), respectively. The results of (8) confirm those of (5); in a multiple regression including σ_{cc} and σ_{e1}, the optimal weight on the close-to-close estimator is less than or equal to zero using monthly data.

In the weekly measurement interval equations in Table IV, the close-to-close estimator performs much better relative to the extreme-value estimators. In equations (1)–(4), the lagged close-to-close estimator has a smaller SEE than any of the extreme-value estimators in predicting σ_{cc}. This is surprising and difficult to explain given the MSE results in Table II and the regressions using monthly data in Table III. Much of the explanatory power of (1) comes from the MA(1) term; when the equations are estimated without MA terms, the SEE of (1) is higher than the SEEs of (2)–(4). In eq. (5), adding σ_{el} to the close-to-close autoregression does little to improve the fit of the equation; the coefficient on σ_{el} is positive but statistically insignificant.

Eqs. (6) and (7) illustrate that σ_{el} is superior to σ_{cc} in predicting σ_{el} in weekly data, confirming the monthly results. Adding σ_{cc} to the σ_{el} autoregression in (8) yields a slight improvement in the fit; in contrast to Table III, the coefficient on σ_{cc} in (8) is positive and significant.

SUMMARY AND CONCLUSIONS

This article examines the empirical bias and efficiency of the Parkinson (1980) and Garman and Klass (1980) extreme-value volatility estimators for the S&P 500 futures price series. In a continuous-trading model, extreme-value estimators are much more efficient than the close-to-close estimator. But, in a discrete-time, discrete-price world, their properties can change dramatically, as noted by Garman and Klass (1980) and Marsh and Rosenfeld (1986). Discrete trading reduces the efficiency and creates downward bias in extreme-value estimators, making their empirical applicability suspect. The S&P 500 futures contract is heavily traded, has wide daily price limits, and has small minimum price increments. These characteristics suggest that extreme-value estimators should perform better here than in the market for individual stocks.

The empirical results demonstrate that both the Parkinson (1980) and Garman Klass (1980) estimators exhibit very little downward bias, and are much more efficient than the traditional close-to-close estimator. In a time-series ARMA forecasting context, the extreme-value estimator outperforms the close-to-close estimator using monthly data. Using weekly data, neither estimator is uniformly superior as a forecasting tool. In summary, if a single historical volatility estimate is to be used as predictor of future volatility, the extreme-value estimator is preferred; but, if a more sophisticated forecasting model is to be employed, the close-to-close estimator may still have some predictive value.

Several interesting research projects are suggested by the results of this article. Implied volatilities from S&P 500 futures options prices could be analyzed in relation to extreme-value to close-to-close volatilities. If implied volatilities appear more closely associated with close-to-close than with the more efficient extreme-value volatilities, appropriate futures option trading rules could be developed and tested. Given the success of Parkinson's estimator for the S&P futures, it would be interesting to examine its performance for other actively traded futures contracts with listed options, including Treasury bonds and Eurodollars.[5] Extreme-value volatility estimates from S&P 500 futures prices could be used in volatility forecasting models for the S&P 100 and S&P 500 cash indices, and thus, in the valuation of cash index options. Finally, the efficiency of extreme-value volatilities could be compared with the efficiency of estimators computed using intraday transactions data.

[5]Daily price limits are more often binding on these contracts than on the S&P 500, diminishing the usefulness of Parkinson's estimator. Also, the log-normal distribution assumption is less realistic for bond than for equity futures prices.

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