Effect of Institutional Realities on Dynamic Hedging Performance for a Grain Producer

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INTRODUCTION

Hedging using the futures market can serve as a way for grain producers to reduce price risk or uncertainty. Confronted with a highly variable yield the question of how much of expected production to hedge becomes especially important for grain producers. A common practice in the optimal hedging literature is to exclude from the model constraints faced in trading futures. These include contract fixity, commissions, and interest cost.

For the case where the hedge position is assumed to remain fixed (i.e., one-period model), there is evidence supporting the exclusion of commissions and interest costs on borrowed money [Nelson (1985); Alexander et al. (1986)]. Leuthold and Peterson (1987) provide evidence for a cattle feedlot suggesting that rounding continuous solutions to match fixed contract units may be acceptable. However, contract fixity may be an important constraint for the smaller producer (Nelson, 1985).

When the hedge position is allowed to change (i.e., dynamic hedging), the impact of institutional realities on hedging effectiveness may be quite different. Commissions may be an important cost in a dynamic hedging program since many futures transactions may occur. Interest expense may increase or decrease with dynamic hedging depending on the relative size of the average futures position during the hedging period and the corresponding margin requirements. Alexander et al. (1986) suggests that margin costs have little effect on optimal one-period hedge positions. It is not clear whether the same is true for dynamic hedge positions. Finally contract lumpiness may become more important when the hedge is revised since each adjustment is restricted.

The purpose of this article is to examine the effect of institutional realities on performance of optimal hedge positions for a grain producer. The effects of institutional realities on dynamic hedging is compared to the one-period hedge.

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DYNAMIC HEDGING MODEL

Karp (1987) provides a workable model which is used in this study to calculate optimal dynamic grain hedge positions. The underlying assumptions of the model are flexible enough to allow for a changing futures price variance and yield uncertainty. A significant seasonal component is found in grain futures price variance [Kenyon et al. (1987); Fackler (1986)] and yield variability is especially important for grain producers. Simultaneous inclusion of these phenomena is nonexistent in previous dynamic hedging models [Adler and Detemple (1988); Duffie and Jackson (1990); Karp (1988)].

The grain producer is assumed to receive revenue from two sources: (1) selling the crop at harvest; and (2) futures trading beginning around planting time. The present value of total revenue from period t to harvest is expressed as:

$$R_t = -\sum_{\tau=t}^{T} B^{\tau-t+1}\{F(\tau + 1) - F(\tau)\}h(\tau) + B^{T-t+1}(F(T + 1) + b(T + 1))Q(T + 1) \tag{1}$$

where:

$T + 1$ = harvest period,
B = discount rate per period (e.g., per month),
$F(t)$ = futures closing price at time t (\$/bu.),
$h(t)$ = quantity hedged at time t, where $h(t) > 0$ represents a short hedge (bu./acre),
$b(T + 1)$ = harvest basis (cash minus futures) (\$/bu.), and
$Q(T + 1)$ = yield (bu./acre).

Considering that future hedge positions will be chosen optimally also, the producer chooses the hedge in each period to maximize the expected utility of returns. An exponential utility function of the form $U(R_t) = -\exp(-kR_t)$ is assumed:

$$\max_{\{h(\tau)\}_{\tau=t \text{ to } T}} E_t\{-\exp[-k(R_t)]\} \tag{2}$$

where E_t is the expectation operator given information available at time t, and k is the constant "Arrow–Pratt" index of absolute risk aversion. Yield, basis, and futures price (i.e., the state variables) are each assumed to be normally distributed:

$$\begin{aligned} F(t + 1) &= aF(t) + e_1(t),\ F(t) \text{ given} \\ Q(t + 1) &= Q(t) + e_2(t),\ Q(t) \text{ given} \\ b(t + 1) &= b(t) + e_3(t),\ b(t) \text{ given} \end{aligned} \tag{3}$$

where $Q(t)$ is the yield forecast in period t ($t < T + 1$), $b(t)$ is the harvest basis *forecast* in period t ($t < T + 1$), $e_i(t)$ is normally distributed with mean zero and variance $\sigma_i(t)$, and $e_i(t)$ is independent of $e_i(t')$, $t' \neq t$, $i = 1,2,3$.[1] The futures price is expected to move up ($a > 1$) or fall ($a < 1$) by a constant percentage per period ($(a - 1) \cdot 100\%$), depending on the value of a. The yield and harvest basis forecasts in period t are equal to the expected value of the true values.

[1]The normality assumption is required for obtaining a solution, while the model can handle serially correlated errors. Evidence exists suggesting that futures prices do not follow a normal distribution [Hall et al. (1989); Stevenson and Bear (1970)]. However, Hudson and Leuthold (1986) suggest that futures prices may be becoming more normal. Also, Gordon (1985) provides evidence suggesting that futures prices may be normally distributed, but that the variance changes periodically.

To solve for the optimal dynamic hedge positions, the present value of returns [eq. (1)] is expressed as the sum of quadratic terms to conform to the *Linear Exponential Gaussian* (LEG) control problem format [Jacobson (1973); Karp (1987) (1988)]:

$$R_t = \tfrac{1}{2}\sum_{\tau=t}^{T} B^{\tau-t+1} y(\tau)' D(\tau) y(\tau) \tag{4}$$

where:

$$y(t) = [h(t), Q(t+1), F(t+1), F(t), b(t+1)]'$$

$$D(t) = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, t < T \begin{bmatrix} 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, t = T.$$

In matrix notation, $y(t)$ can be expressed as a linear combination of $y(t-1)$, the current hedge level, and the error terms:

$$y(t) = Ay(t-1) - (\beta)h(t) + \Gamma e(t) \tag{5}$$

where:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \text{“}a\text{” satisfying } F(t+1) = aF(t) + e_1(t)$$

$$\Gamma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \beta = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, e(t) = [e_1(t)\ e_2(t)\ e_3(t)]'$$

The hedging solution to eq. (2) is:

$$h^*(t) = G(t)y(t-1) = \alpha_1(t)Q(t) + \alpha_2(t)F(t) + \alpha_3(t)b(t) \tag{6}$$

where the $\alpha_i(t)$'s are parameters depending on a, covariances and variances of the $e_i(t)$'s in period t and future periods, and the discount rate B. Although forecasts of the yield and harvest basis do not appear in eq. (1), they enter the solution due to the distributional assumptions in eq. (3).[2] Formulas for calculating $G(t)$ are provided in Appendix A. Karp's (1987) formulas are generalized slightly to include the possibility of a singular state-variable covariance matrix, $\Sigma(t)$. The covariance matrix may be singular in a particular period if the yield and/or harvest basis forecast will not be updated. For example, in the early stages of growth, the producer's yield forecasts may not change until important weather variables become known.

Although not included in the model for calculating the optimal hedge position, commissions and margin costs are calculated and deducted from gross hedging re-

[2]Recall that the *expected value* of utility is maximized [eq. (2)].

turns in the empirical evaluation of hedging performance (discussed in a later section). Commissions are extremely difficult to include directly in the model. A commission is charged when a "new" position is established. In addition, commissions depend on the *change* in the hedge position. Margin costs may be included directly but are excluded to simplify presentation of the model.[3]

ESTIMATION OF OPTIMAL HEDGE POSITIONS

The Hedging Program

To examine the impact of institutional realities on dynamic hedging performance, an empirical analysis is conducted assuming the hedge is revised each month. Optimal hedge positions are estimated in each year from 1982–1987 for a hypothetical corn producer in Johnston county, North Carolina. Hedging begins in mid-March and ends at harvest, which is assumed to be the first Monday in October.

Data

Futures prices used are for the December corn contract on the Chicago Board of Trade. County level yield and harvest cash price data are used to generate yield and harvest basis forecasts since farm level data are not available.[4] The yield forecast is updated three times; at the end of June, in mid-July, and at harvest when the forecast is updated to the true value. A yield forecast is made using one of three equations depending on the time of year (Table I). The harvest basis is calculated using the cash price at Selma, located in Johnston county. The three-year average harvest basis serves as the forecast. The harvest basis is updated at harvest, when the true value becomes known.

Parameter Estimation

Forecasts of yield and harvest basis generated from 1973–1981 are then used, together with futures prices, to estimate variances and covariances of the forecast and futures price errors [i.e., the $e_i(t)$'s from eq. (3)]. Since the harvest basis and yield forecasts are updated simultaneously at harvest, an estimate of the covariance between the final forecast and the true value is required. It is calculated using the mean of the product of the forecast errors of the final forecasts. The covariance between the forecast update and the futures price is estimated as the mean of the product of the forecast change and the futures price error term.

[3]The interest expense on money borrowed to satisfy initial margin requirements could be included directly in the model as follows:

$$h^*(t)m(1 + r)^{T-t+1} - mh^*(t) = mh^*(t)\{(1 + r)^{T-t+1} - 1\},$$

where m is the per bushel initial margin requirement and r is the daily cost. In addition, funds may be borrowed to maintain the margin at the initial level and profits earn interest at the rate δ. Compounded interest costs and earnings are, respectively:

$$h^*(t)(F(t) - F(t + 1)\}(1 + r)^{T-t} - h^*(t)\{F(t) - F(t + 1)\} = h^*(t)\{F(t) - F(t + 1)\}\{(1 + r)^{T-t} - 1\} \quad \text{when } F(t) - F(t + 1) < 0$$

$$h^*(t)(F(t) - F(t + 1)\}(1 + \delta)^{T-t} - h^*(t)\{F(t) - F(t + 1)\} = h^*(t)\{F(t) - F(t + 1)\}\{(1 + \delta)^{T-t} - 1\}, \quad \text{when } F(t) - F(t + 1) > 0$$

These represent extensions of Nelson's (1985) formulas for the one-period case.

[4]Yield variability at the county level may be lower than that faced by an individual producer.

Table I
JOHNSTON COUNTY CORN YIELD REGRESSION EQUATIONS[a]

Explanatory Variable	Parameter Estimate	T-Ratio	*P*-Value
Used from March to June[b]			
INTERCEPT	16.223	9.115	0.0001
DUM	25.297	1.557	0.1315
TREND	3.345	9.122	0.0001
TSTAR	−2.579	−3.114	0.0045
DW[c] = 2.178; R^2 = 0.63			
Used from June to mid-July			
INTERCEPT	−5919.031	−4.757	0.0001
DUM	4044.320	2.464	0.0210
TREND	3.044	4.791	0.0001
TSTAR	−2.064	−2.471	0.0207
DRAIN	2.502	2.708	0.0120
DW = 2.102; R^2 = 0.71			
Used from mid-July to harvest			
INTERCEPT	−4512.366	−3.292	0.0031
DUM	3493.840	2.217	0.0363
TREND	2.326	3.325	0.0028
TSTAR	−1.782	−2.221	0.0360
DRAIN	2.348	2.679	0.0131
DTEMP	−2.152	−1.997	0.0573
DW = 2.105; R^2 = 0.75			

TREND is equal to 1952 in 1952, 1953 in 1953, etc.; DUM is trend intercept shifter equal to one after 1965 and zero otherwise; TSTAR is the trend slope shifter equal to TREND after 1965 and zero otherwise; DRAIN is the deviation of June rainfall from the average (inches), and DTEMP is the deviation of average temperature from the end of June to mid-July from the average.

Weekly rainfall and temperature data from Johnston county weather stations are contained in *Weather and Crops.*

[a]Regression equations are estimated using data from 1952–1981. Johnston county corn yield is the dependent variable.

[b]Evidence of heteroscedasticity is found using the Park-Glejser test (Pindyck and Rubinfeld). In correcting for heteroscedasticity, the trend variable is set to 1 for 1952, 2 for 1953, etc. Trend in the other two equations is equal to the year.

[c]DW is the Durbin–Watson statistic.

Futures prices are used also to estimate the coefficient on the lagged futures price. Following Fackler (1986), trigonometric functions are selected to capture the seasonal pattern in daily futures price variance. The futures price variance is assumed to be characterized by multiplicative heteroscedasticity (Judge et al. (1982); Harvey (1976)]:

$$\sigma^2(t) = \exp[S(t) + \alpha\overline{\mathrm{F}} + \mu]$$

$$S(t) = \sum_{i=1}^{9} \{\Theta(i)\cos(2\pi it/250) + \phi(i)\sin(2\pi it/250)\} \tag{7}$$

where $\sigma^2(t)$ is the futures price variance; $S(t)$ is a function capturing seasonality in futures price variance; μ, $\theta(i)$, $\phi(i)$, and α are parameters to be estimated;

$\pi = 3.141592654$; t is the tth business day of the calendar year; 250 is the number of business days in a year; and $\bar{F}$ is the average futures price in January and February. One advantage to the use of trigonometric functions is that a smooth transition of variance occurs over time.

Previous studies find a significant relation between grain price variability and the average futures price [see Kenyon et al. (1987) for a discussion of this literature]. The average futures price in January and February is included to account for differences in futures price variance across years. The January and February average is chosen since it can be used in mid-March, at the beginning of the hedging program, to estimate the futures price variance through October.[5]

The futures price equation [eq. (3)] is first estimated using Ordinary Least Squares (OLS). The log of the squared residuals are then regressed on the trigonometric functions and the average futures price in January and February.[6] Parameter estimates are presented in Table II. The exponential of the regression equation is used to estimate the futures price variance:

$$\hat{\sigma}^2(t) = \exp[\text{``}\textit{regression equation}\text{''}] = \exp[\hat{\mu} + \hat{S}(t) + \hat{\alpha}\bar{F}] \tag{8}$$

As expected, the estimate of the futures price variance is highest in the summer months, as important whether information becomes available (Figure 1).

The futures price equation is then re-estimated using OLS, after correcting for heteroscedasticity. The resulting coefficient estimate is provided in Table II.[7]

Optimal monthly hedge positions for three farm sizes and risk-aversion parameter $k = .007$ are presented in Table III. For risk-aversion parameters in this range, hedge positions tend to be large enough to get consistent hedging by even the smallest farm. Note that large adjustments in optimal hedge positions occur about the time when the yield forecasts are updated, i.e., June and July. However, the greatest adjustment occurs in the final hedge position when yield and harvest basis become known at harvest. These adjustments may be lost after rounding, as contract lumpiness reduces hedging responsiveness. Also, the rounding error is most important for the smallest farm, as suggested by the rounding error relative to the size of the hedge position.

EVALUATION OF HEDGING PERFORMANCE

The effect of institutional constraints on monthly hedging is compared to the effect on the one-period hedge for the years 1982–1987. The optimal one-period or fixed hedge position can be solved as a special case of the dynamic problem. This is accomplished by writing returns in the LEG control problem format and using the formulas contained in Appendix A. In this case, only one iteration is required, since one optimal hedge position is calculated. The variances and covariances of the state variables over the entire hedging program are contained in the covariance matrix, $\Sigma(t)$, instead of variances and covariances over a month.

Monthly optimal hedge positions and the one-period optimal hedge are calculated for three farm sizes and for three levels of absolute risk aversion. The range of

[5]The optimal hedge position is derived assuming that the hedge position in each future period is also chosen optimally. The optimal hedge in future periods requires an estimate of the futures price variance.

[6]Alternative estimation procedures exist for improving the efficiency of the parameter estimates (Harvey, 1976).

[7]See Judge et al. (1982) for a more detailed discussion of multiplicative heteroscedasticity and statistical properties of the estimators.

Table II
FUTURES PRICE VARIANCE REGRESSION EQUATION AND COEFFICIENT ON LAGGED FUTURES PRICE (1973–1981)

Explanatory Variable	Parameter Estimate	Asymptotic *P*-Value
Futures price variance equation:		
Intercept	−6177.800054[a]	0.0992
cos1	−11532.750037	0.0987
sin1	1261.507745	0.1646
cos2	−9391.809657	0.0962
sin2	2080.297847	0.1623
cos3	−6624.846760	0.0923
sin3	2245.449672	0.1589
cos4	−4000.642869	0.0875
sin4	1861.301460	0.1545
cos5	−2029.697315	0.0826
sin5	1226.434610	0.1494
cos6	−838.795742	0.0791
sin6	638.488801	0.1444
cos7	−267.812058	0.0800
sin7	252.342162	0.1406
cos8	−59.470944	0.0931
sin8	68.778896	0.1409
cos9	−6.925285	0.1426
sin9	9.865516	0.1547
$\bar{F}$	−0.212880	0.0427
$R^2 = 0.09$		
Futures price equation[b]:		
$F(t-1)$	1.000032	0.0001

[a]This estimate is adjusted by adding 1.2704 to obtain the intercept estimate (Judge et al., 1982).
[b]Corrected for heteroscedasticity using the futures price variance equation.

risk parameters is selected so that the optimal hedge levels lie between zero and expected production. For hedge positions outside this range, the producer is likely to face resistance by lenders. Only data available at the time the hedge is updated is used to calculate the optimal hedge. At the beginning of each year, all parameter estimates are updated. Selling the entire crop at harvest (i.e., no hedging) is included as the control strategy.

Optimal hedge positions are rounded to the nearest whole contract unit (i.e., 5000-bu. interval) and compared to solutions assuming no contract fixity for each farm size. Performance is evaluated by calculating the mean of actual returns and the root mean square (RMSE) of actual minus expected returns as a measure of risk or uncertainty of returns.

Commissions are calculated assuming a \$50/contract "round turn" commission charge. Interest costs are calculated assuming the producer pays the prime rate of interest on funds borrowed to meet initial margin requirements and offset losses in the futures market. Gains from the futures market earn interest at a rate equal to the three-month treasury-bill rate.

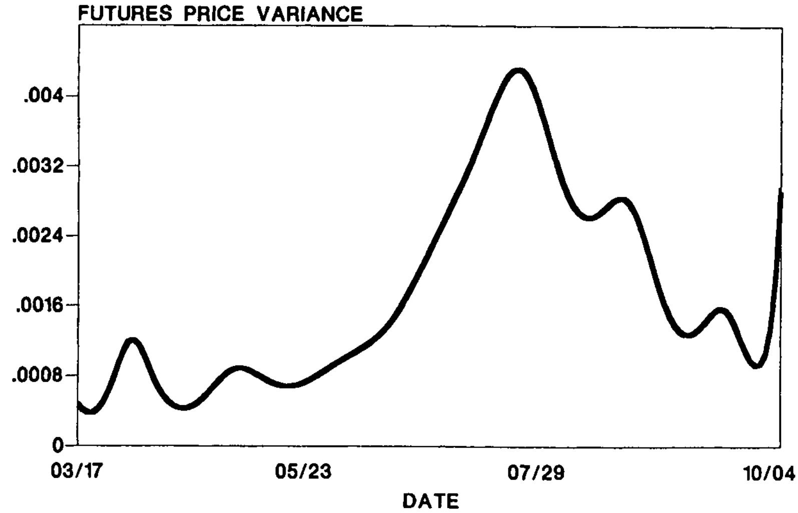

Figure 1
Futures price variance estimate.

PERFORMANCE RESULTS

Table IV contains a comparison of monthly and one-period hedging performances across risk parameters and farm sizes. Assuming no contract lumpiness, monthly hedging returns, net of interest costs, and commissions range from \$9.00/acre to \$5.00/acre higher than selling at harvest only, depending on the risk parameter. Commissions for the dynamic hedge range from 15¢/acre to 21¢/acre higher than the one-period hedge, and interest costs are slightly higher as well. Net returns from monthly hedge revisions are about \$1/acre higher than the fixed hedge. The RMSE of returns is higher also for the monthly revisions.[8]

As the risk parameter becomes larger under the "no lumpiness" scenario, the RMSE increases even though the producer places relatively more importance on a smaller variability of returns. This result suggests that higher moments than the mean and variance may be important for the range of risk parameters examined. Since preference rankings are invariant to monotonic transformations of the utility function, the expected value of utility, $E(-\exp[-kR])$, can be re-expressed as $E(1 - \exp[-kR])$. Based on the following equalities:

$$\begin{aligned} E(1 - \exp[-kR]) &= E(kR - k^2R^2/2! + k^3R^3/3! - \ldots) \\ &= kER - k^2E(R^2)/2! + k^3E(R^3)/3! - \ldots \end{aligned}$$

higher moments are weighted more heavily (i.e., higher moments become increasingly important relative to the first and second moments) for larger values of the risk parameter, k, since higher powers of k increase at a faster rate as k increases.

[8]Differences between means and RMSEs across marketing alternatives are mentioned here simply as a check on the model. Are results consistent with what one might expect and, if not, why aren't they? Due to the small number of observations (six years), statistical tests can not distinguish between means and RMSEs among marketing alternatives. However, since out-of-sample data is used to examine the hedging programs, it is felt that results are representative of actual results had the hedging model been used. In addition, consistent patterns of performance are observed for the unrounded hedge positions across risk parameters and frequency of revisions. This seems to indicate that differences in the mean and RMSE do not occur by chance.

Table III
ADJUSTMENTS IN OPTIMAL MONTHLY HEDGE POSITIONS AND ROUNDING ERRORS FOR HEDGE POSITIONS IN 1985[a] (k = 0.007)

Period	Optimal Hedge	Change in Optimal Hedge Position	Rounded Hedge Position	Rounding Error[b]
50 acres harvested (bu.)				
Mar.–Apr.	4450		5000	550
Apr.–May	4400	−50	5000	600
May–June	4300	−100	5000	700
June–July[c]	4000	−300	5000	1000
July–Aug.[c]	3600	−400	5000	1400
Aug.–Sept.	3700	100	5000	1300
Sept.–Oct.	4200	500	5000	800
Average rounding error (as percent of rounded hedge)				18%
150 acres harvested (bu.)				
Mar.–Apr.	13350		15000	1650
Apr.–May	13200	−150	15000	1800
May–June	12900	−300	15000	2100
June–July	12000	−900	10000	−2000
July–Aug.	10800	−1200	10000	−800
Aug.–Sept.	11100	300	10000	−1100
Sept.–Oct.	12600	1500	15000	2400
Average rounding error				13%
250 acres harvested (bu.)				
Mar. –Apr.	22250		25000	2750
Apr.–May	22000	−250	20000	−2000
May–June	21500	−500	20000	−1500
June–July	20000	−1500	20000	0
July–Aug.	18000	−2000	20000	2000
Aug.–Sept.	18500	500	20000	1500
Sept.–Oct.	21000	2500	20000	−1000
Average rounding error				7%

[a]1985 is typical of the six years examined.
[b]Rounded minus unrounded hedge position.
[c]Yield forecasts are updated within these periods.

This suggests that higher moments than the first and second may be important for the range of risk parameters examined. The effect of higher moments may also explain the higher RMSE from monthly hedge revisions compared to the fixed hedge. Since the effect of institutional realities on hedging performance is of prime interest here, conclusions reached are not affected.

The effect of contract lumpiness on dynamic hedging performance depends on both the farm size and risk attitudes. Rounding of optimal solutions actually results in higher mean returns and lower variability for the smallest farm with the smallest risk-aversion coefficient. For larger farms with higher risk-aversion

Table IV
MONTHLY VERSUS FIXED HEDGING PERFORMANCE WITH AND WITHOUT CONTRACT LUMPINESS[a] (1982–1987)

	Sell Only	Fixed Hedge			Monthly Hedge		
		$k = 0.025$	$k = 0.065$	$k = 0.105$	$k = 0.025$	$k = 0.065$	$k = 0.105$
No lumpiness ($/acre)							
Mean	164.39	174.22	171.71	168.78	175.39	172.84	170.06
RMSE	53.75	49.62	49.67	51.74	51.09	51.42	53.11
Commissions	0	0.63	0.41	0.19	0.78	0.56	0.4
Interest	0	0.77	0.5	0.23	0.79	0.51	0.27
NET	164.39	172.82	170.8	168.36	173.82	171.77	169.39
50 acres harvested ($/acre)							
Mean		180.86	164.39	164.39	179.93	169.53	166.53
RMSE		57.17	54.13	54.08	47.88	64.77	55.71
Commissions		1	0	0	1	1	1
Interest		0.67	0	0	1.02	0.26	0.33
NET		179.19	164.39	164.39	177.91	168.27	165.2

150 acres harvested ($/acre)						
Mean	175.37	169.88	169.89	175.05	172.15	170.79
RMSE	51.91	50.69	53.97	48.99	55.28	51.81
Commissions	0.67	0.33	0.33	0.67	0.67	0.33
Interest	0.83	0.41	0.2	0.76	0.5	0.23
NET	173.87	169.14	169.36	173.62	170.98	170.23
250 acres harvested ($/acre)						
Mean	174.27	170.98	168.79	175.55	172.04	168.95
RMSE	51.34	50.53	52.82	51.27	53.15	52.67
Commissions	0.6	0.4	0.2	0.8	0.6	0.4
Interest	0.75	0.5	0.19	0.78	0.54	0.27
NET	172.92	170.08	168.4	173.97	170.9	168.28

[a] Variables are defined in Appendix B.
[b] Optimal hedge positions are rounded to the nearest whole contract unit (5000-bu. increments).

parameters, mean net returns are not substantially affected by rounding of optimal hedge positions.

The relative performance of dynamic hedging and the fixed hedge is affected by contract lumpiness. After rounding, net returns from the monthly hedge improve relative to the fixed hedge for the small farm with higher values of the risk parameter. The optimal fixed hedge is rounded to zero for the smallest farm at higher levels of risk aversion. Net returns to the monthly hedge are no longer consistently above those from the one-period hedge for larger farms after rounding. The one-period hedge provides higher net returns than the monthly hedge for the largest, most risk-averse farm and the medium-sized, least risk-averse farm. Differences in the RMSE between one-period and monthly hedging are more substantial after rounding, especially for the smaller producer. In some cases, the RMSE is substantially higher compared to simply selling at harvest.

As expected, commissions are higher for the dynamic hedging program in most cases. Differences in commissions between the monthly hedge and the fixed hedge range from 0¢/acre to 83¢/acre, after rounding. In general, interest on margin requirements is slightly higher for monthly hedges due to greater short positions. Monthly hedge levels tend to be higher since the futures price is predicted to fall in most years (i.e., $\hat{a} < 1$). With less time between revisions (1 month) compared to the one-period hedge (7 months), there is greater confidence in the ability to predict gains from a short monthly hedge position. Therefore, a greater short commitment is undertaken.

CONCLUSIONS AND IMPLICATIONS

Revisions in the optimal hedge position, calculated assuming no commission charges, results in only a small increase in commission expenses. This suggests that dynamic hedging models, which ignore commissions, need not provide useless solutions for guiding grain producers in practical applications.

Interest costs on borrowed funds to meet initial margin requirements are affected only slightly by monthly hedging compared to the fixed hedge. As long as hedge positions remain between zero and expected production, interest on margin requirements is unlikely to be greatly affected by revision of the hedge position.

Rounding of continuous optimal hedging solutions causes the relative performance of dynamic hedging and the fixed hedge to become ambiguous for various risk parameters and farm sizes. Contract fixity has the greatest effect on hedging performance for the smaller producer in this study. Care must be taken when interpreting results generated from continuous dynamic hedging solutions when institutional realities are ignored. For smaller farms there is little that can be concluded from continuous solutions without directly incorporating contract lumpiness into the model.

The effect of rounding on the performance of continuous solutions for the larger farms is relatively small. The model employed in this study suggests that, for smaller contract size (e.g., 1000 bu.) or larger farm size, the effect of contract lumpiness on dynamic hedging performance is less important.

Finally, compared to the fixed hedge, dynamic hedging does not appear to be more affected by contract lumpiness. This suggests that conclusions drawn concerning the effect of contract lumpiness on the fixed hedge may apply to dynamic hedging as well.

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Appendix A

FORMULAS FOR CALCULATING OPTIMAL MULTIPERIOD HEDGE POSITIONS

Formulas

1. I = identity matrix.
2. $\Sigma(t) = E_t(e(t)e(t)') =$ Covariance matrix at time t.
3. $M(t) =$ Cholesky decomposition of $\Sigma(t)$ satisfying $M(t)M(t)' = \Sigma(t)$.
4. $\tilde{D}(t) = kBD(t)$.
5. $W(t) = \tilde{D}(t) + BW^*(t+1)$; $W(T) = \tilde{D}(T)$.
6. $\tilde{W}(t) = W(t) - W(t)\Gamma M(t)(I + M(t)'\Gamma'W(t))\Gamma M(t))^{-1}M(t)'\Gamma'W(t)$.
7. $W^*(t) = A'\{W(t) - W(t)\beta(\beta'\tilde{W}(t)\beta)^{-1}\beta'\tilde{W}(t)\}A$.
8. $G(t) = -(\beta'\tilde{W}(t)\ \beta)^{-1}\beta'\tilde{W}(t)A$.
9. $s(t) = s(t+1)|I + M(t)'\Gamma'W(t)\Gamma M(t)|^{-1/2}$; $s(T+1) = 1$.

Second-order condition for a maximum:

$$\beta'\tilde{W}(t)\beta < 0, \text{ for all } t$$

Requirement for calculating expected value of utility in each period:

$$I + M(t)'\Gamma'W(t)\Gamma M(t) \text{ positive definite, for all } t$$

Formulas for calculating expected revenue in the initial period:

$$V(t) = A + (\beta)G(t)$$

$$X(t) = .5V(t)'D(t)V(t) + V(t)'X(t+1)V(t);\ X(T) = .5V(T)'D(T)V(T)$$

$$c(t) = c(t+1) + \text{trace}\{\Gamma'(.5D(t) + X(t+1))\Gamma\Sigma(t)\};\ c(T) = \text{trace}(.5\Gamma'D(T)\Gamma\Sigma(T))$$

$$E_1R = y(0)'X(1)y(0) + c(1)$$

where R is returns and E_1 is the expectation operator given that $F(1)$, $Q(1)$, and $b(1)$ are each known [source: Karp (1985), eq. (8), p. 44].

Appendix B

DEFINITIONS FOR VARIABLES IN TABLE IV

$$\text{Mean} = \sum_{i=1}^{6} R_i/6,$$

where R_i is the actual total return in year i.

$$\text{RMSE} = \sqrt{\sum_{i=1}^{6} (R_i - \text{RAVG}_i)/6}$$

where $RAVG_i$ is the expected value of total returns at the beginning of the hedging program.

Commissions = Average commission expenditure over the six-year period ($50/contract).

Interest = Interest costs due to futures trading. An initial margin of $1000/contract is assumed. "Marking to market" method of accounting is used in calculating interest costs (income) from losses (gains) in the futures market. Interest rates are collected from *Economic Indicators.*

NET = Mean − Commissions − Interest.

k = Risk parameter in units of acres/$. Risk attitudes are assumed to be independent of farm size. This allows one to concentrate on the contract lumpiness issue.