Ex-Ante Hedging Strategy Selection Using Foreign-Exchange-Rate Forecasting Models

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INTRODUCTION

Commonly suggested hedging strategies in the academic and practitioner literature tend to focus on risk reduction, often ignoring expected return.[1] This concentration on reducing risk neglects two crucial factors in the ex-ante hedging strategy selection process: price expectation (i.e., forecast price change) and its relation to hedging objectives. Working (1962) stresses the critical role of price expectations for both selective and anticipatory hedging strategy selection. Goodman (1979) considers the potential benefit of forecast price change in evaluating the predictive accuracy of ten exchange-rate forecasting services. He concludes that the performances of technically oriented services may help in managing short-term exchange-rate risk exposure, but he does not provide a linkage between the forecast and the strategy selection process.

This article develops and evaluates ex-ante hedging decision models. Simple foreign-exchange-rate forecasts serve as proxies for investor expectations. Hedge ratios are then selected in accordance with the forecast and the hedging objective, i.e., whether the hedge is to protect against rising or falling currency prices. Hedge ratio magnitudes are selected according to the hedging objective, and by representative levels of hedging aggressiveness or the level of confidence a hedger places in a forecast. The results indicate that decision models similar to those examined in this study provide hedging performances that many hedgers may consider superior to more traditional, commonly recommended strategies.

[1]Howard and D'Antonio (1984) attempt to incorporate mean return into a hedging strategy in order to develop a risk-return measure of hedging effectiveness. This strategy, however, is appropriate only for investors who wish to maximize a return-to-standard deviation ratio. It also requires ex-ante estimation of five hedge ratio parameters if it is to be used in practice.

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THE HEDGING STRATEGY SELECTION PROBLEM

The most common hedging strategy in the academic literature is the variance-minimizing hedge suggested by Johnson (1960). Ederington (1979) and Franckle (1980) use it for ex-post evaluations of the T-bill futures market; Hill and Schneeweis (1982) and Dale (1981) use it for ex-post evaluations of currency futures. Lasser (1987) evaluates potential benefits of this strategy in an ex-ante setting by estimating the edge ratio in one period and applying it in a later period for T-bills and T-bonds. His results suggest that such ex-ante estimates are not significantly superior to the naive equal-and-opposite hedge. In a study of Canadian dollar futures, Marmer (1986) arrives at a similar conclusion.

In focusing on one or a few hedging strategies, the hedge ratio selection problem is often reduced to one of choosing among a few possible alternatives. Issues of how to go about ex-ante estimation of sometimes complex analytic hedge ratios is usually left to the reader. In most empirical studies, historical data serve as inputs to the estimation process. Few writers, however, provide viable methodologies for estimating future hedge ratio parameters. Further, most studies take the perspective of either long or short hedgers, but usually not both. In examining bounded sets of hedging strategies, however, Hammer (1988) draws a clear distinction between the objectives of short and long hedgers. He suggests that these objectives might best be met by selecting strategies skewed about the naive hedge, with long hedges on one side and short hedges on the other.

Figure 1 illustrates the strategy selection problem which this article addresses. Figure 1A shows a spot-futures opportunity locus drawn from the perspective of a short hedger who sells futures to protect a long position in a commodity against falling prices. This hedger expects spot prices to rise, providing positive returns to the unhedged position. Since the hedger anticipates favorable price movement, hedging in futures is undertaken strictly to reduce risk in the event expectations prove incorrect ex post. The variance-minimizing hedge and a hypothetical naive hedge are identified in the figure; both strategies reduce perceived risk, but also reduce expected return.[2] To the extent this hedger acts on price expectations, a hedge position somewhere between the variance-minimizing hedge and the unhedged position is desirable. If the hedger has a high degree of confidence in these expectations, then a small position in futures is desirable because it permits the hedger to participate in the expected price increase; the lower the confidence level, the greater will be the desired hedge ratio up to the minimum-variance or naive hedge.

Figure 1B shows the same opportunity locus from the perspective of a long hedger who buys futures to protect against rising spot prices, and who has the same price expectations as the short hedger. The opportunity locus is rotated about the horizontal axis because a long hedger having these expectations faces unfavorable expected price movement. To the extent that this hedger acts on price expectations, the minimum-variance and naive hedges are far superior to remaining unhedged, even though they provide negative expected returns. If confidence in these expectations is high, however, this long hedger may prefer a hedge ratio in excess of these positions, i.e., a position in futures that is in excess of the short spot position being hedged. This may require a hedge ratio greater than unity.

[2]The variance-minimizing and naive hedge positions are shown in close proximity in the figure because this occurs when spot and futures returns are highly correlated; statistically, these hedge ratios are frequently indistinguishable.

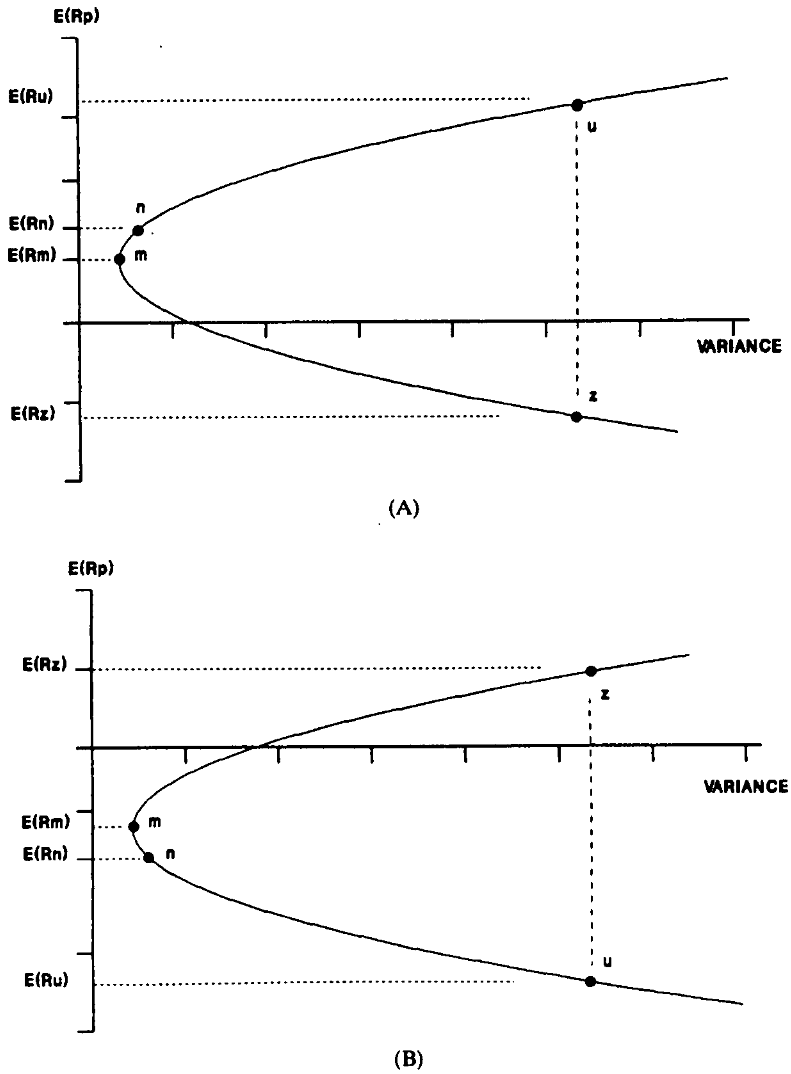

Figure 1

Illustrative spot-futures opportunity locus. *z* = same variance hedge as u; m = minimum-variance hedge; u = unhedged (spot) position; n = naive hedge. (A) A short hedger's perspective. (B) A long hedger's perspective.

Figure 1 shows that short and long hedgers who act on price expectations prefer, ex ante, hedge ratio selections that tend to fall on opposite sides of the naive and/or minimum-variance hedge. This is the strategy selection problem that the present study examines. Rather than estimating hedge ratio parameters, however, simple forecasts are used as proxies for investor expectations about future price movement. These proxies are then used to select long hedge ratios and short hedge ratios skewed to opposite sides of the naive hedge.

SIMPLE EXCHANGE-RATE FORECASTS

One simple forecast is to assume that prices are equally likely to rise or to fall. Where risk reduction is a primary concern, the naive hedge is often a suitable selection for this forecast. A second simple forecast is based on the assumption that spot prices follow a martingale process:

$$E(S_T) = S_t \tag{1}$$

i.e., the expected spot price at time T is today's price, S_t. Since no change in price is expected, the hedger may choose to take no position in futures. These two forecasts and corresponding strategies represent two extremes: a pure risk avoidance strategy, the naive hedge, and a high-risk strategy, remaining unhedged.[3] They provide contrasts to the performances of active hedging strategy decisions.

Giddy and Dufey (1975) evaluate a variety of exchange-rate forecasts. They conclude that models based on martingale and submartingale processes tend to outperform most others. A submartingale process assumes that prices change:

$$E(S_T) \lesseqgtr S_t \tag{2}$$

One approach for developing a forecast for this process is to assume that recent price movements will persist, on average, over the hedge period, i.e., set

$$\Delta S_s = S_t - S_{t-i} \tag{3}$$

where ΔS_S is a projected price change, i establishes the differencing interval, and the S subscript denotes this process.[4] To some extent, the differencing period used to compute eq. (3) is arbitrary. This point is considered in more detail in the empirical analysis section.

Cornell and Reinganum (1982) compare futures and forward prices to evaluate whether they are significantly different. They report only negligible differences. Thus, both are candidates for use as simple exchange-rate forecasts. Some suggest that futures and/or forward prices are unbiased estimates of future spot rates. Fama (1976), Jacobs (1982), and Wolff (1987), however, provide evidence of time-varying risk premiums in the pricing of forward foreign-exchange contracts. This suggests one approach for modeling the spot-futures/forward price relationship:

$$F_{T,t} = S_t + RP_{T,t} \tag{4}$$

where $F_{T,t}$ is the price of a futures or forward contract that matures at time T, S_t is today's spot price, and $RP_{T,t}$ is today's risk premium for a contract maturing at time T. Risk premiums may provide a "measure" of investor expectations. A positive premium may indicate that sellers of futures/forward contracts who are long the spot currency, expect spot prices to fall and demand a positive premium as compensation for holding the currency; having similar expectations, buyers who are short the spot currency, are willing to pay the premium because they expect to gain on their short spot position. A negative risk premium may indicate opposite expectations: sellers who are long the spot currency expect spot prices to rise and are discounting the futures/forward contract because they expect to gain on a spot price increase; buyers require a discount because they expect a loss to accrue to their short spot position. This logic is just the opposite of suggestions that futures/forward prices are unbiased estimates of future spot prices. The question of whether to use futures/forward prices as estimates of future spot prices, or to use

[3]The unhedged strategy has the highest risk strategy of any considered in this study; examination of Figure 1 confirms this observation.

[4]Giddy and Dufey (1975) use a different approach for developing forecasts using a submartingale process.

the risk premium as an estimate of future spot price change, is an empirical question. The results in this study show that the risk premium computed as

$$\Delta S_F = RP_{T,t} = F_{T,t} - S_t \tag{5}$$

where the F subscript identifies this projected price change, produces superior hedging results, and is the futures/forward forecast used in the decision models.

Both casual observation and rigorous analysis reveals that trends in foreign currency prices are often present in time series data. This suggests that moving average models may be useful in forecasting price movement. Although variations of moving average models are numerous, this study uses only the simplest: the average price, S_A, of a currency is computed over some period of time prior to hedge initiation and compared to the prevailing spot price, S_t, i.e.,

$$\Delta S_M = S_A - S_t \tag{6}$$

serves as a forecast in the hedging decision models, where the M subscript designates this projected price change. The time period used to compute S_A is considered in the empirical analysis section.

Cornell (1977) uses an autoregressive process to evaluate changes in foreign currency prices. Although his results show only marginal statistical significance, an autoregressive model may be useful in making ex-ante hedging decisions. The simple autoregressive process is used and specified by:

$$\Delta S_A = S_t - s_{t-i} = a_0 + a_1(S_{t-i} - S_{t-2i}) \tag{7}$$

as a forecast for the decision models, where the A subscript denotes this process. The differencing interval, i, and estimates for the coefficients for eq. (7), are discussed in the empirical analysis section.

DATA AND METHODOLOGY

This study uses spot, futures, and forward prices taken from *The Wall Street Journal* on the first trading day of each week between January 1981 and December 1988. Prices are for British pounds, Canadian dollars, Japanese yen, Swiss francs, and West German marks. Spot and forward prices are quotes by Bankers Trust Company as of 3 PM eastern standard time (EST). Futures prices are closing price quotations by the International Monetary Market, which stops trading in these contracts between 2:15 and 2:30 PM EST. While the timing of these quotations differ slightly, three factors tend to mitigate any nonsynchronous problem. First, overall trends in the direction of price change remain the same since time differences are small. Second, weekly sampling emphasizes price trends and attenuates noise that may be present in daily samples. Third, any minor differences due to the timing of price quotations appear small in light of the eight-year sample period.

Eqs. (3) and (5)–(7) provide forecasts for the direction of expected spot price change, ΔS_j. The hedging decision then requires one of three courses of action:

1. Select $|\text{SHR}| < 1.0$ and $|\text{LHR}| > 1.0$, if

$$(\Delta S_j / S_t) > \alpha \tag{8}$$

where α is a decision threshold, SHR is a short hedge ratio, and LHR is a long hedge ratio. In words, if spot prices are forecast to rise, select a SHR less than unity to allow the long spot position to participate in the price rise, and select a LHR greater than unity to better protect against the expected price rise.[5]

2. Select |SHR| > 1.0 and |LHR| < 1.0 if

$$(\Delta S_j/S_t) < -\alpha \tag{9}$$

If spot prices are forecast to fall, select a SHR greater than unity to protect the long spot position against expected spot price declines, and select a LHR less than unity to permit the short spot position to benefit from the expected price decline.

3. If the decision threshold is not broken, adopt the naive hedge as a neutral strategy for reducing risk over the hedge period.[6]

The first two decisions follow directly from a previons section on hedging strategy. The last reduces risk and suggests that prices are equally likely to move in either direction. Two decision thresholds are used for each model. The first sets $\alpha = 0\%$, which forces an active hedge decision at the beginning of each hedge period. The second is empirically determined by varying the threshold and rerunning the analysis until an active hedge decision occurs for approximately 30–70% of all hedges undertaken. This provides representative results for hedging decisions that fall between 100% active and 100% inactive. The impact and the motivation for using positive threshold values is shown in the empirical analysis.

SHRs and LHRs are set for three mangitudes, which represent increasing levels of hedging aggressiveness or increasing levels of confidence in a forecast:

1. If spot prices are forecast to increase, then set
   SHR = −0.9 and LHR = −1.1, for slightly aggressive/confident hedges;
   SHR = −0.7 and LHR = −1.3, for mildly aggressive/confident hedges; and
   SHR = −0.5 and LHR = −1.5, for highly aggressive/confident hedges.
2. If spot prices are forecast to decrease, then set
   SHR = −1.1 and LHR = −0.9, for slightly aggressive/confident hedges;
   SHR = −1.3 and LHR = −0.7, for mildly aggressive/confident hedges; and
   SHR = −1.5 and LHR = −0.5, for highly aggressive/confident hedges.

The magnitudes for each aggressive/confidence level assure risk reduction *vis-a-vis* an unhedged position, because spot and futures returns for the currencies in this study are highly correlated. The logic underlying this observation is shown in Figure 1. The spot-futures portfolio that gives the same variance as the unhedged position is labeled "z." It is shown in the Appendix that the hedge ratio which determines this position is twice the variance-minimizing hedge. Numerous studies show that variance-minimizing hedges for these currencies are "near" unity. Thus, the magnitudes of the ex-ante hedge ratios used in this study insure that variance is reduced,[7] i.e., correspond to hedge positions between u and z in Figure 1. In some

[5]Hedge ratios in excess of unity usually violate what the Commodity Futures Trading Commission calls "bona fide" hedges, and are frequently treated as speculative strategies. As Figure 1 shows, however, many hedge ratios that exceed unity provide risk reduction relative to the unhedged position. Further, the *Statement of Financial Accounting Standards No. 80* (August, 1984), which covers accounting procedures for most futures trading, stresses high correlation and risk reduction as hedge criteria. It does not appear to necessarily preclude hedge ratios that exceed unity by small amounts.

[6]The naive equal-and-opposite hedge is neutral in the sense that it reduces variance and requires no estimates by a hedger.

[7]That hedge ratios between −1.0 and −1.5 guarantee risk reduction in this study is confirmed in the empirical analysis.

ways, the naive hedge might be considered a pure risk-avoidance strategy, and hedges deviating from unity might be considered speculative. As Working (1962) long ago argued, however, "pure risk-avoidance hedging," is "unimportant and virtually nonexistent in modern business practice..." (1962, p. 442).

To assess the viability of the hedging decision models for managing short-term exchange-rate risk exposure, hedge lengths of four, eight, and 12 weeks are used. This permits computation of realized returns and variances within each hedge period. It also permits an examination of these characteristics over time. Finally, these hedge lengths correspond to hedge lengths which are common to exchange-rate risk-management activities.

EMPIRICAL ANALYSIS

Since four-, eight-, and 12-week hedging periods are evaluated in this study, corresponding differencing intervals are initially used to project price change for the submartingale forecast, eq. (3). Although the results are generally acceptable, they appear to deteriorate with increasing hedge length. For this reason, differencing intervals of one, two, and three weeks are then used for all hedge lengths. Overall, the results for the one-week differencing interval (computed at hedge initiation) appear to be as good as, and often better than, those for all other differencing intervals. As a consequence, only the results for a one-week differencing period are reported for the submartingale process.

A similar approach is used to evaluate price change projections for the moving average process, eq. (6). In this instance, results for different averaging periods are far more mixed than for the submartingale process. The use of a constant 12-week averaging period, computed just prior to hedge initiation, is generally representative of all averaging periods considered, including hedge-length matching averaging periods. These results are the only reported.

The differencing interval used for the autoregressive process, eq. (7), presents a unique problem. The number of samples used to estimate the regression coefficients, a_0 and a_1, must first be determined. To err on the conservative side, 30 samples of weekly observations just prior to hedge initiation are used. If hedge-length matching differencing intervals are used, 12-week differencing intervals require exchange-rate data going back to 1974 in order to begin the analysis in 1981. As a consequence, a one-week differencing interval is used for two reasons. First, the results for the submartingale and moving average processes suggest that hedge-length matching intervals appear unnecessary. Second, the forecasts per se are not the central focus of this study. They are proxies for investor price expectations; presumably, experienced and knowledgable hedgers form expectations that are, on average, at least as accurate as the simple forecasts used in the present study.

Table I provides an overview of the efficacy of the forecasts. Fourteen of 75 forecasts result in projecting price movement in the wrong direction. This causes the decision rules in the above section to assign hedge ratios skewed in the wrong direction about unity, i.e., it produces hedging results that make the investor worse off than the naive hedge. Table I reports the threshold levels for each incorrect forecast; these results hold for both short and long hedges. In all cases, the incorrect forecast is only for one threshold value, either 0% or some positive level. The forecasts for British pounds, Swiss francs, and West German marks are correct in all but two cases, where the moving average forecasts for eight-week hedges and positive thresholds fail. Canadian dollars and Japanese yen account for 12 of the incor-

Table I
FORECAST MODEL PERFORMANCE: INCORRECT FORECASTS[a,b]

	British Pounds	Canadian Dollars	Japanese Yen	Swiss Francs	West German Marks	Number of Incorrect Forecasts
Submartingale						
4-week hedges	—	—	—	—	—	0
8-week hedges	—	—	—	—	—	0
12-week hedges	—	—	—	—	—	0
Futures						
4-week hedges	—	—	—	—	—	0
8-week hedges	—	—	$\alpha = 0\%$	—	—	1
12-week hedges	—	—	$\alpha = 0\%$	—	—	1
Forward						
4-week hedges	—	$\alpha = \frac{1}{4}\%$	$\alpha = 0\%$	—	—	2
8-week hedges	—	—	$\alpha = 0\%$	—	—	1
12-week hedges	—	—	$\alpha = 0\%$	—	—	1
Moving average						
4-week hedges	—	$\alpha = 2\%$	—	—	—	1
8-week hedges	$\alpha = 2\%$	—	—	—	$\alpha = 2\%$	2
12-week hedges	—	$\alpha = 2\%$	—	—	—	1
Autoregressive						
4-week hedges	—	$\alpha = \frac{1}{4}\%$	—	—	—	1
8-week hedges	—	$\alpha = \frac{1}{4}\%$	$\alpha = 0\%$	—	—	2
12-week hedges	—	—	$\alpha = 0\%$	—	—	1
Number of incorrect forecasts	1	5	7	0	1	

[a]An "incorrect forecast" is a forecast that results in reducing average hedge return below the naive hedge return, which makes the hedger worse off in terms of return and standard deviation than the naive hedge.

[b]The α-value is the decision threshold level that produces an incorrect forecast for both long and short hedging strategies. α-Values are specified for all forecasts that are incorrect; all other forecasts improve return performance, but usually at the expense of higher standard deviation.

rect forecasts. These failures occur only for positive thresholds for Canadian dollars and 0% thresholds for Japanese yen. Notice that use of the risk-premium approach for futures and forward forecasts provides correct results in all cases except Japanese yen and, in one instance, Canadian dollars. The submartingale forecast is correct for all currencies and all hedge lengths. While forecasts for Canadian dollars and Japanese yen are somewhat disappointing, they should not overshadow the fact that these simple models are *correct,* on average, 81.3% of the time for all currency/hedge-length combinations.

Although all forecasts and hedging decisions are strictly ex ante, the evaluation of the mean-variance performances are average realized results for each currency/hedge-length group. Figures 2 and 3 help clarify this approach to evaluation. Figure 2A plots realized highly aggressive four-week short hedge returns, where $\alpha = 0\%$, for each of the 102 hedges executed over the sample period. The plot shows the results for British pounds using the submartingale forecast, generally the

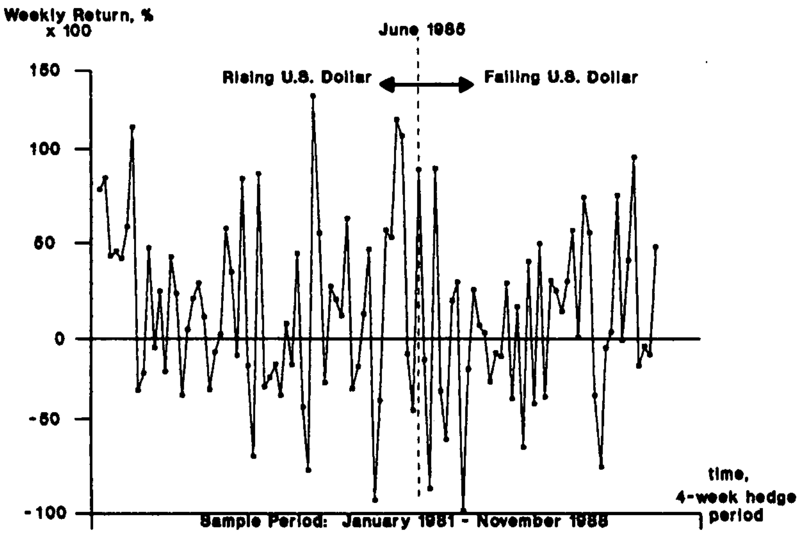

Figure 2A

Highly aggressive four-week short hedge return per hedge period: British pounds using the submartingale forecast. Means: naive return = −0.01, hedge return = 18.6, unhedged return = −7.8.

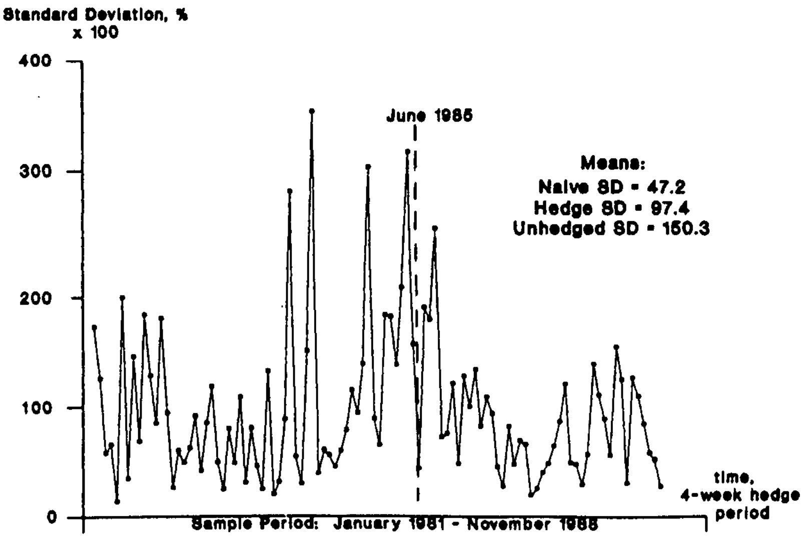

Figure 2B

Highly aggressive four-week hedge standard deviations: British pounds using the submartingale forecast. Note: Each standard deviation is computed for a four-week hedge using observed (ex-post) weekly returns over the hedge period. Standard deviations are reported for 102 four-week hedges.

best forecast for all currencies. As with all forecasts, the results are sometimes favorable, sometimes unfavorable. On average, however, this ex-ante hedging strategy provides significantly higher realized returns than either the naive hedge or the unhedged position, as reported in the graph. Figure 2B provides an indication of the period-by-period "cost" of the strategy. It plots realized standard deviations in each

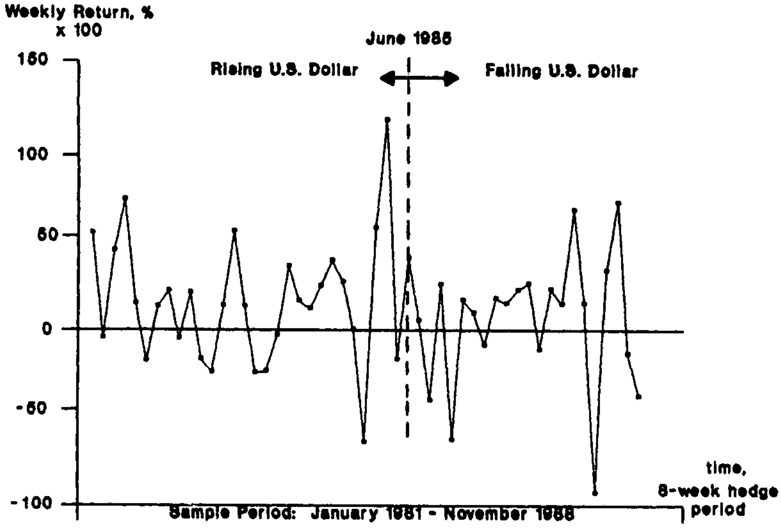

Figure 3A

Highly aggressive eight-week short hedge return per hedge period: British pounds using the submartingale forecast. Means: naive return = −1.1, hedge return = 16.6, unhedged return = −7.8.

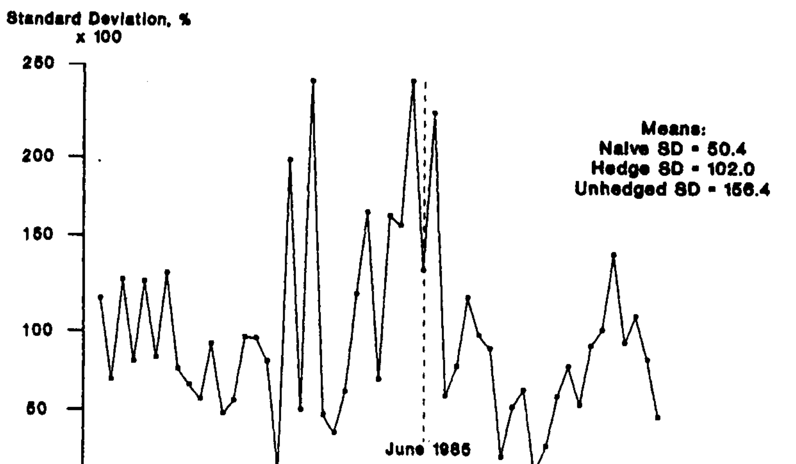

Figure 3B

Highly aggressive eight-week hedge standard deviations: British pounds using the submartingale forecast. Note: Each standard deviation is computed for a eight-week hedge using observed (ex-post) weekly returns. Standard deviations are reported for fifty-one eight-week hedges.

of the 102 hedge periods. It shows that variations differ greatly from one period to the next. Mean standard deviations for the 102 hedge periods for the naive hedge, the highly aggressive hedge, and the unhedged position are given in the Figure 2, and reflect what one might expect a priori: the naive hedge is least risky, the unhedged position is most risky, and the highly aggressive strategy falls approximately in the middle. Whether the increased riskiness of the aggressive strategy over the naive strategy is merited depends on an investor's aversion to risk. Figure 3 shows similar results for eight-week hedges. These figures are generally representative of hedging strategies across all currencies and hedge lengths.

Figures 2 and 3 also provide support for the hedging decision models from another perspective. From the beginning of the sample period to approximately June 1985, the value of the U.S. dollar rises, on average, vis-à-vis the other currencies. From approximately June 1985 to the end of the sample period, the value of the U.S. dollar falls, on average. This approximate transition date is identified in the figures. Notice that hedge returns and variances appear roughly comparable over both the rising trend and the falling trend. Thus, decision model performance is not much affected by market trends, and the results concerning the hedging decision models presented in this study are not sample (or time period) dependent.

The performances of the hedging decision models are evaluated in two ways for each currency/hedge-length combination or group. First, each combination that results in mean-variance stochastic dominance over all others for that group is selected as the best performing decision model. While this criteria identifies a large number of superior decision models, stochastic dominance is too strong a criteria to successfully apply to the 150 currency/hedge-length groups for short and long hedges evaluated in this study. A second criteria used to identify superior performances is the highest positive return-to-variance ratio for each group. Under a normality assumption, this ratio is directly proportional to Pratt's (1964) approximation for absolute risk aversion[8]; it may be interpreted as a measure of the return-variance tradeoff provided by a decision model. Where stochastic dominance occurs, the return-to-variance ratio is necessarily the highest in that currency/hedge-length group. Where the return-to-variance ratio is used for model evaluation, the hedging performance always dominates along one dimension (i.e., return or variance) and provides only marginally inferior performance along the other dimension.

The performances of the decision models are dependent on the accuracy and quality of investor price expectations. Since simple forecasts are serving as proxies for price expectations in this study, it is assumed that the performances of professional hedgers using the decision models will at least equal (if not surpass) the performances of the best forecasts. As a consequence, as well as to conserve space, Table II presents the results for the best performing long hedge ratio selections.[9] Mean-variance performances that stochastically dominate a currency/hedge-length group are underlined to facilitate interpretation. The standard deviations used for this evaluation are those of the hedge return series over the sample period, e.g., standard deviations such as those for the return series shown in Figures 2A and 3A, and not averages of the standard deviation series such as those shown in Figures 2B and 3B. Positive return-to-variance ratios that dominate a group are underlined also. These two performance criteria identify superior decision models for British pounds and Canadian dollars in every case. Notice that significant increases in return relative to the naive hedge are often attained for only modest increases in variance for these two currencies. The mildly and highly aggressive strategies frequently yield higher return than the unhedged strategy while simultaneously re-

[8]See Hammer (1988) for a discussion of this point.

[9]Complete results are available from the author on request.

Table II
RESULTS FOR THE BEST-PERFORMING EX-ANTE LONG HEDGE RATIO SELECTION MODELS[a,b]

Hedge Length/ Decision Model		Weakly Aggressive $E(R)\backslash\sigma$	Weakly Aggressive $\frac{E(R)}{\sigma^2}$	Mildly Aggressive $E(R)\backslash\sigma$	Mildly Aggressive $\frac{E(R)}{\sigma^2}$	Highly Aggressive $E(R)\backslash\sigma$	Highly Aggressive $\frac{E(R)}{\sigma^2}$
British pounds							
4-week hedges:	Naive:	0.001\0.191	0.03		Unhedged:	0.078\0.914	0.09
Submartingale	$\alpha = 0\%$	0.039\0.217	0.83	*0.114\0.337	1.00	*0.189\0.492	0.78
Submartingale	$\alpha = 1\%$	0.032\0.195	0.84	*0.094\0.258	1.41	*0.156\0.360	1.20
8-week hedges:	Naive:	0.011\0.101	1.08		Unhedged:	0.078\0.726	0.15
Submartingale	$\alpha = 0\%$	0.047\0.122	3.16	*0.117\0.227	2.27	*0.188\0.352	1.52
Submartingale	$\alpha = 1\%$	0.041\0.114	3.16	*0.099\0.177	3.16	*0.158\0.258	2.37
12-week hedges:	Naive:	0.012\0.086	1.62		Unhedged:	0.078\0.612	0.21
Futures	$\alpha = 0\%$	0.047\0.101	4.61	*0.118\0.182	3.56	*0.188\0.283	2.35
Submartingale	$\alpha = 1\%$	0.030\0.079	4.81	*0.066\0.122	4.43	*0.102\0.195	2.68
Canadian dollars							
4-week hedges:	Naive:	0.023\0.080	3.59		Unhedged:	0.007\0.318	0.07
Submartingale	$\alpha = 0\%$	0.033\0.081	5.03	*0.054\0.117	3.94	*0.074\0.172	2.50
Submartingale	$\alpha = \frac{1}{2}\%$	0.032\0.078	5.26	*0.049\0.098	5.10	*0.067\0.138	3.52
8-week hedges:	Naive:	0.022\0.041	13.1		Unhedged:	0.007\0.213	0.15
Autoregressive	$\alpha = 0\%$	0.031\0.045	15.3	*0.051\0.074	9.31	*0.070\0.112	5.58
Submartingale	$\alpha = \frac{1}{2}\%$	0.025\0.041	14.9	0.031\0.049	12.9	0.037\0.064	9.03
12-week hedges:	Naive:	0.022\0.044	11.4		Unhedged:	0.007\0.179	0.22
Autoregressive	$\alpha = 0\%$	0.028\0.044	14.5	0.039\0.065	9.23	0.050\0.098	5.21
Submartingale	$\alpha = \frac{1}{2}\%$	0.026\0.043	14.1	0.032\0.051	12.3	0.039\0.067	8.69
Japanese yen							
4-week hedges:	Naive:	−0.123\0.284	−1.53		Unhedged:	−0.113\0.882	−0.15
Submartingale	$\alpha = 0\%$	−0.091\0.302	−1.00	*−0.027\0.399	−0.17	*0.037\0.536	0.13
Submartingale	$\alpha = 1\%$	−0.096\0.310	−1.00	*−0.041\0.396	−0.26	*0.013\0.507	0.05

8-week hedges:	Naive:	−0.097\0.127	−6.01		Unhedged:	−0.113\0.657	−0.26
Submartingale	α = 0%	−0.073\0.135	−4.01	*−0.026\0.211	−0.58	*0.021\0.318	0.21
Submartingale	α = 1%	−0.077\0.136	−4.16	*−0.037\0.195	−0.97	*0.002\0.278	0.03
12-week hedges:	Naive:	−0.078\0.085	−10.8		Unhedged:	−0.113\0.524	−0.41
Submartingale	α = 0%	−0.059\0.101	−5.78	*−0.020\0.175	−0.65	*0.019\0.264	0.27
Submartingale	α = 1%	−0.059\0.098	−6.14	*−0.021\0.135	−0.90	*0.017\0.223	0.34
Swiss francs							
4-week hedges:	Naive:	−0.149\0.405	−0.91		Unhedged:	−0.037\1.045	−0.03
Submartingale	α = 0%	−0.100\0.416	−0.58	*−0.003\0.496	−0.01	*0.095\0.627	0.24
Submartingale	α = 1%	−0.112\0.405	−0.68	*−0.039\0.450	−0.19	*0.034\0.542	0.12
8-week hedges:	Naive:	−0.101\0.188	−13.0		Unhedged:	−0.037\0.779	−0.06
Submartingale	α = 0%	−0.064\0.114	−4.93	*0.009\0.231	0.17	*0.082\0.365	0.62
Submartingale	α = 1%	−0.081\0.098	−8.43	*−0.042\0.170	−1.45	*−0.003\0.261	−0.04
12-week hedges:	Naive:	−0.096\0.171	−3.28		Unhedged:	−0.037\0.606	−0.10
Submartingale	α = 0%	−0.073\0.177	−2.33	−0.027\0.235	−0.49	*0.020\0.326	0.19
Submartingale	α = 1%	−0.083\0.165	−3.05	−0.057\0.185	−1.67	−0.031\0.238	−0.55
West German marks							
4-week hedges:	Naive:	−0.085\0.151	−3.73		Unhedged:	−0.020\0.961	−0.02
Submartingale	α = 0%	−0.045\0.181	−1.37	*0.034\0.312	0.35	*0.113\0.471	0.51
Submartingale	α = 1%	−0.052\0.171	−1.78	*0.012\0.266	0.17	*0.076\0.388	0.51
8-week hedges:	Naive:	−0.076\0.058	−22.6		Unhedged:	−0.020\0.723	−0.04
Submartingale	α = 0%	−0.042\0.077	−7.08	*0.026\0.191	0.71	*0.093\0.317	0.93
Submartingale	α = 1%	−0.060\0.062	−15.6	*−0.030\0.129	−1.80	*0.001\0.213	0.02
12-week hedges:	Naive:	−0.071\0.064	−17.3		Unhedged:	−0.020\0.578	−0.06
Submartingale	α = 0%	−0.046\0.077	−7.76	*0.005\0.161	0.19	*0.056\0.260	0.83
Submartingale	α = 1%	−0.056\0.064	−13.7	*−0.025\0.122	−1.68	*0.007\0.199	0.18

[a] $E(R)$ is expected return and σ is standard deviation. All returns and standard deviations are weekly percentages. $E(R)\backslash\sigma$ combinations that stochastically dominate a hedge-length\aggressiveness group for all forecast models are underlined. The $E(R)\backslash\sigma^2$ ratio is proportional to Pratt's approximation for absolute risk aversion; please see the text for a discussion. These values are underlined where they dominate a hedge-length\aggressiveness group.

[b] Asterisks denote returns that differ from naive returns at the 0.05 level.

ducing variance. A moment's reflection reveals that this observation provides extraordinarily strong support for the decision models.

Long hedge ratio selections for Japanese yen, Swiss francs, and West German marks present a somewhat different but equally impressive picture. All three currencies have naive and unhedged returns that are negative in all cases. In spite of this, the decision models produce hedge returns that are often significantly higher and, in fact, returns that are often positive at higher levels of aggressiveness. Where returns are negative, the return-to-variance ratio cannot be used for performance evaluation. Where returns are positive, however, it clearly identifies superior return performance, but shows that the cost is higher variance. For West German marks, 14 of 18 decision model selections using the submartingale forecast stochastically dominate all others.

Table III presents the results for the best-performing short hedge ratio selections. The signs of the returns for the naive hedge and the unhedged positions reverse from those in Table II. They are negative for British pounds and Canadian dollars, and positive for the other three currencies. In spite of this, the decision models for British pounds provide positive hedge returns in all instances and permit identification of the best-performing models by the performance criteria. The results are not as strong for Canadian dollars, but are generally good relative to the naive hedge and the unhedged position. For Japanese yen, the decision models produce stochastically dominant results in 16 of 18 cases, all using the submartingale forecasts. Although not quite as strongly, the results for Swiss francs and West German marks appear impressive. Tables II and III show that the submartingale forecast tends to dominate all others; since this forecast relies only on the price change occurring in the week just prior to hedge initiation, it provides strong support for the previous contention that professional hedgers should perform at least as well. A comparison of Tables II and III reveals that the best-performing decision models do not always use the same forecast for long and short hedges.

Finally, Tables II and III show the general impact of using positive decision thresholds, i.e., $\alpha > 0\%$. With few exceptions, results for positive thresholds are lower return and lower risk than for $\alpha = 0\%$. The higher the decision threshold, in general, the lower the risk and return; at sufficiently high levels, of course, it always leads to the naive hedge. Threshold levels, therefore, are directly related to a hedger's aversion to risk. In practice, thresholds might be established on the basis of the quality (direction and magnitude) of price expectations. Where quantitative forecasts are used, historical data can establish the probability of correctly forecasting the direction of price change and the magnitude of price change; the greater the probability of correct forecasts, the lower the desired threshold. This is a complex issue, however, and is beyond the scope of this study. Where price expectations are subjectively formed, the hedger must provide subjective probabilities and set thresholds accordingly.

CONCLUDING REMARKS

This study provides a linkage between foreign currency price expectations and the hedging strategy selection process. In practice, price expectations are formed in many ways, from intuitive judgment to sophisticated quantitative models. Five simple forecasts are used in this study as proxies for price expectations. Hedging strategy selections are based on these forecasts and the specific objectives of short and long hedgers. An empirical evaluation of the hedging decision models uses weekly prices for five major foreign currencies over an eight-year sample period. The results show that the decision models provide viable hedging performances for hedgers who are not pure risk avoiders. Working (1962) suggests that pure risk avoidance hedging, in fact, is virtually nonexistent in modern business practice. The models often produce return performances that are superior to naive hedges for only small increases in variance, and return performances that are often superior to the unhedged position but at a lower variance.

The submartingale forecasts with the assumption of a short but persistent price change trend produce the best overall hedging performances for shorter hedge lengths. This suggests that professional hedgers might produce performances far superior to those found in this study. Since the evaluation covers an eight-year period which contains both rising and falling currency price trends, it provides strong support for the use of these and similar decision models in the ex-ante hedging decision process. In practice, quantitative models may be evaluated using historical data before they are used in the decision process. The high quality judgment of experienced hedgers, however, may prove even more beneficial. The results suggest that future efforts need to focus on developing and evaluating forecasts, and on formalizing the relation between ex-ante forecast accuracy and hedge ratio magnitudes. Similar forecasts may prove useful also for hedging decisions involving other commodities, e.g., interest rate futures. Such efforts may improve decision making in practical hedging environments.

Appendix

PROOFS CONCERNING THE HEDGE RATIO THAT YIELDS THE SAME VARIANCE AS THE UNHEDGED (SPOT-ONLY) POSITION

To identify the hedge ratio that produces the spot-futures portfolio having the same variance as the spot-only (or unhedged) position, consider the variance of a two-asset spot-futures portfolio:

$$\begin{aligned}\sigma_p^2 &= X_s^2\sigma_s^2 + X_f^2\sigma_f^2 + 2X_sX_f\sigma_s\sigma_f\rho \\ &= X_s^2(\sigma_s^2 + C_p^2\sigma_f^2 + 2C_p\sigma_s\sigma_f\rho)\end{aligned}$$

where the subscripts denote the spot and futures markets, respectively; the X_i's are the proportions of the portfolio represented by each market; the σ_i^2's are return variances in each market; ρ is the correlation between spot and futures returns; and $C_p = X_f/X_s$ is the hedge ratio for this two-asset portfolio. Setting X_s to plus or minus unity implies that 100% of the spot position is hedged; this is done for convenience. Next, the spot variance is set equal to the spot-futures portfolio variance and solved for the resulting hedge ratio:

$$\sigma_s^2 = \sigma_s^2 + C_p^2\sigma_f^2 + 2C_p\sigma_s\sigma_f\rho$$

$$C_p(C_p\sigma_f^2 + 2\sigma_s\sigma_f\rho) = 0$$

$$C_p = 0, \qquad \text{or}, \qquad C_p = -2\rho(\sigma_s/\sigma_f)$$

Since 0 is the unhedged position, the hedge ratio that produces the spot-futures portfolio with the same variance as the spot-only position must be

$$C_z = -2\rho(\sigma_s/\sigma_f)$$

Table III
RESULTS FOR THE BEST-PERFORMING EX-ANTE SHORT HEDGE RATIO SELECTION MODELS[a,b]

Hedge Length/ Decision Model		Weakly Aggressive $E(R)\backslash\sigma$	Weakly Aggressive $\frac{E(R)}{\sigma^2}$	Mildly Aggressive $E(R)\backslash\sigma$	Mildly Aggressive $\frac{E(R)}{\sigma^2}$	Highly Aggressive $E(R)\backslash\sigma$	Highly Aggressive $\frac{E(R)}{\sigma^2}$
British pounds							
4-week hedges:	Naive:	−0.001\0.191	−0.03		Unhedged:	−0.078\0.914	−0.09
Submartingale	$\alpha = 0\%$	0.036\0.205	0.86	*0.111\0.314	1.13	*0.186\0.466	0.86
Submartingale	$\alpha = 1\%$	0.030\0.209	0.69	*0.092\0.290	1.09	*0.153\0.398	0.97
8-week hedges:	Naive:	−0.011\0.101	−1.08		Unhedged:	−0.078\0.726	−0.15
Submartingale	$\alpha = 0\%$	0.024\0.121	1.64	*0.095\0.224	1.89	*0.166\0.350	1.36
Submartingale	$\alpha = 1\%$	0.018\0.109	1.52	*0.077\0.167	2.76	*0.135\0.247	2.21
12-week hedges:	Naive:	−0.012\0.086	−1.62		Unhedged:	−0.078\0.612	−0.21
Futures	$\alpha = 0\%$	0.024\0.103	2.26	*0.094\0.185	2.75	*0.165\0.285	2.03
Forward	$\alpha = \frac{1}{4}\%$	0.015\0.090	1.85	*0.067\0.152	2.90	*0.119\0.237	2.12
Canadian dollars							
4-week hedges:	Naive:	−0.023\0.080	−3.59		Unhedged:	−0.007\0.318	−0.07
Submartingale	$\alpha = 0\%$	−0.012\0.109	−1.01	*0.008\0.137	0.43	*0.029\0.195	0.76
Submartingale	$\alpha = \frac{1}{2}\%$	−0.014\0.090	−1.73	*0.004\0.126	0.25	*0.022\0.171	0.75
8-week hedges:	Naive:	−0.022\0.041	−13.1		Unhedged:	−0.007\0.213	−0.15
Autoregressive	$\alpha = 0\%$	−0.012\0.048	−5.21	*0.008\0.079	1.28	*0.027\0.118	1.94
Submartingale	$\alpha = \frac{1}{2}\%$	−0.019\0.043	−10.3	−0.012\0.054	−4.12	−0.006\0.070	−1.22
12-week hedges:	Naive:	−0.022\0.044	−11.4		Unhedged:	−0.007\0.179	−0.22
Autoregressive	$\alpha = 0\%$	−0.017\0.052	−6.29	−0.006\0.082	−0.89	0.005\0.117	0.37
Futures	$\alpha = \frac{1}{2}\%$	−0.018\0.040	−11.3	−0.010\0.043	−5.41	*−0.001\0.058	−0.30
Japanese yen							
4-week hedges:	Naive:	0.023\0.284	1.53		Unhedged:	0.113\0.882	0.15
Submartingale	$\alpha = 0\%$	0.155\0.291	1.83	*0.219\0.374	1.57	*0.283\0.506	1.11
Submartingale	$\alpha = 1\%$	0.150\0.273	2.01	*0.205\0.304	2.22	*0.259\0.386	1.74

8-week hedges:	Naive:	0.097\0.127	6.01		Unhedged:	0.113\0.657	0.26
Submartingale	$\alpha = 0\%$	0.120\0.147	5.55	*0.167\0.234	3.05	*0.214\0.343	1.82
Submartingale	$\alpha = 1\%$	0.116\0.137	6.18	*0.156\0.196	4.06	*0.195\0.279	2.51
12-week hedges:	Naive:	0.078\0.085	10.8		Unhedged:	0.113\0.524	0.41
Submartingale	$\alpha = 0\%$	0.098\0.095	10.9	*0.136\0.163	5.12	*0.175\0.252	2.76
Submartingale	$\alpha = 1\%$	0.097\0.089	12.2	*0.135\0.136	7.30	*0.173\0.203	4.20
Swiss francs							
4-week hedges:	Naive:	0.149\0.405	0.91		Unhedged:	0.037\1.045	0.03
Submartingale	$\alpha = 0\%$	0.197\0.418	1.13	*0.295\0.501	1.18	*0.392\0.634	0.98
Submartingale	$\alpha = 1\%$	0.185\0.422	1.04	*0.258\0.498	1.04	*0.331\0.605	0.90
8-week hedges:	Naive:	0.101\0.188	13.0		Unhedged:	0.037\0.779	0.06
Submartingale	$\alpha = 0\%$	0.137\0.111	11.1	*0.210\0.227	4.08	*0.284\0.362	2.17
Submartingale	$\alpha = 1\%$	0.120\0.104	11.1	*0.160\0.179	4.99	*0.199\0.271	2.71
12-week hedges:	Naive:	0.096\0.171	3.28		Unhedged:	0.037\0.606	0.10
Autoregressive	$\alpha = 0\%$	01173\0.174	3.86	0.161\0.230	3.04	*0.204\0.320	1.99
Autoregressive	$\alpha = \frac{1}{4}\%$	0.109\0.175	3.56	0.137\0.224	2.73	0.165\0.303	1.80
West German marks							
4-week hedges:	Naive:	0.085\0.151	3.73		Unhedged:	0.020\0.961	0.02
Submartingale	$\alpha = 0\%$	0.124\0.167	4.45	*0.203\0.287	2.47	*0.282\0.443	1.44
Submartingale	$\alpha = 1\%$	0.117\0.161	4.51	*0.181\0.246	2.99	*0.245\0.366	1.83
8-week hedges:	Naive:	0.076\0.058	22.6		Unhedged:	0.020\0.723	0.04
Submartingale	$\alpha = 0\%$	*0.109\0.096	11.8	*0.177\0.216	3.79	*0.244\0.343	2.07
Submartingale	$\alpha = 1\%$	0.091\0.082	13.5	*0.122\0.159	4.83	*0.152\0.245	2.53
12-week hedges:	Naive:	0.071\0.064	17.3		Unhedged:	0.020\0.578	0.06
Submartingale	$\alpha = 0\%$	0.097\0.089	12.2	*0.148\0.178	4.67	*0.199\0.279	2.56
Moving Average	$\alpha = 2\%$	0.077\0.071	15.3	0.087\0.134	4.85	0.097\0.214	2.12

[a] $E(R)$ is expected return and σ is standard deviation. All returns and standard deviations are weekly percentages. $E(R)\backslash\sigma$ combinations that stochastically dominate a hedge-length/aggressiveness group for all forecast models are underlined. The $E(R)/\sigma^2$ ratio is proportional to Pratt's approximation for absolute risk aversion (see the text). These values are underlined where they dominate a hedge-length/aggressiveness group.

[b] Asterisks denote returns that differ from naive returns at the 0.05 level.

which is the hedge ratio that identifies "z" in Figure 1. Finally, the variance-minimizing hedge ratio is well known and may be written by inspection as

$$C_m = -\rho(\sigma_s/\sigma_f)$$

Thus, $C_z = 2C_m$ which is obvious in Figure 1, since the opportunity locus is symmetrical about the variance-minimizing position.

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