An Empirical Evaluation of the Extended Mean-Gini Coefficient for Futures Hedging

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INTRODUCTION

For futures hedging, an extensive literature oriented toward risk-minimization exists. Originating with the contributions of Stein (1961) and Johnson (1960), this risk-minimization approach has dominated the academic literature on hedging physical commodities. It has been extended to hedging with financial futures by Edgerington (1979) and others.

As is well-known, the risk-minimization approach implicitly assumes either that returns are normally distributed or that investors have quadratic utility functions. A growing body of evidence suggests that the assumption of normally distributed returns is false and the assumption of quadratic utility is widely regarded as too restrictive. Stochastic dominance approaches to returns are proposed as remedies to the deficiencies of the mean-variance (M-V) approach presumed by risk-minimization models.[1]

The stochastic dominance approach is applied to hedging with derivative instruments by Cheung, Kwan, and Yip (CKY) (1990). CKY propose that Gini's mean difference be used as an alternative measure of risk, and they develop mean-Gini hedge ratios, which are consistent with a stochastic dominance approach. The attractive feature of using Gini's mean difference as a measure of risk is that mean Gini efficient sets are second-order stochastic dominant (SSD). The same cannot be said for the M-V efficient set. Because mean-Gini efficient sets are SSD, they do not require the assumption of normality or quadratic utility as the M-V model does.

CKY compare the results from hedging a given spot position with futures and options under a M-V framework and under their stochastic dominance framework. CKY find that using the M-V framework results in the selection of hedged positions that are sometimes dominated stochastically by another alternative. By contrast,

[1]Of course, M-V hedging does not require that hedgers adopt the risk-minimizing hedge position. M-V traders with some risk tolerance may well adopt hedge positions that are different from the risk-minimizing approach.

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their mean-Gini approach is specifically designed to exclude the adoption of a dominated position. In their analysis, CKY implicitly assume that hedgers using both techniques would have the same hedge ratio. Under this assumption, CKY compare the portfolio standard deviation with the portfolio Gini for a given hedged position. Depending on the hedge position, CKY show that the standard deviation and portfolio Gini give different preference rankings.

This article extends the analysis of CKY (1990) by proposing that the extended mean-Gini (EMG) coefficient be used to develop spot futures hedge ratios, rather than the mean-Gini coefficient. This study shows that the EMG approach is more general than the mean-Gini approach of CKY and it shows how the CKY model is a special case of the EMG model. The risk-minimization M-V hedge ratio is contrasted with the hedge ratio preferred by investors adopting the EMG approach.[2] For low level of risk aversion among EMG traders, it is shown that their hedge ratios are quite similar to the risk minimizing M-V hedge ratio. However, for higher levels of risk aversion, the hedge positions favored by EMG and risk-minimizing M-V hedgers differ substantially. This result differs substantially from the approach of CKY, who use the same hedge ratio for M-V and mean-Gini hedgers for the same degree of risk aversion.

THE EXTENDED MEAN-GINI MODEL

Yitzhaki (1983) defines the extended mean-Gini coefficient as:

$$\Gamma(V) = \int_a^b (1 - F(x))dx - \int_a^b (1 - F(x))^v dx \tag{1}$$

where:

$F(x)$ is the cumulative probability density function of x;

v is the risk-aversion parameter;

a and b are the lower and upper bounds of the distribution.

Yitzhaki (1983) demonstrates that the following two efficiency conditions are necessary for SSD efficiency:

$$\mu_f \geq \mu_g \tag{2}$$

and

$$\mu_f - \Gamma_f(v) \geq \mu_g - \Gamma_g(v) \tag{3}$$

where μ_f represents the expected return on prospect F. If prospect F satisfies conditions (2) and (3), then prospect F is EMG efficient and is contained within the SSD set.

The extended mean-Gini coefficient given in eq. (1) represents a family of coefficients of dispersion. Differences between these coefficients are characterized by a parameter V. This parameter represents a measure of risk aversion. The greater the value of V, the more risk averse an investor is. As demonstrated by Yitzhaki (1983), increasing the parameter V reduces the certainty equivalent. Investors characterized by such higher risk aversion are willing to pay a greater premium to offset their risk. Similar efficiency conditions can be defined for the extended mean-Gini coefficient, and prospects which are EMG efficient are also SSD efficient.

[2]The risk-minimizing hedge ratio is taken as a point of reference for M-V hedgers. It is not implied that all M-V hedgers would adopt the risk-minimizing hedge position.

Yitzhaki (1984) also shows that risk seekers have a value of the risk-aversion parameter, V, that lies between $0 \leq V < 1$. Risk neutral investors are identified by $V = 1$, and risk-averse investors are characterized by $V > 1$. As V tends to infinity, investors adopt a maxi-min strategy. In the mean-Gini case, as discussed by CKY, this parameter equals 2. Thus, the model explored by CKY is strictly a special case of the extended mean-Gini model; it is the special case of $V = 2$.

The practical implementation of the EMG coefficient is facilitated by the following form.[3]

$$\Gamma(V) = -V \operatorname{Cov}(R_p, (1 - F(R_p))^{V-1}) \tag{4}$$

where R_p is the return on the portfolio.

Calculation of the EMG coefficient is accomplished with the following technique: rank the observed portfolio returns, R_p, from the lowest to highest return. The complement of the cumulative probability density function may then be expressed as:

$$(1 - F(R_p)) = \frac{N - \operatorname{Rank}(R_p)}{N} \tag{5}$$

therefore:

$$(1 - F(R_p))^{V-1} = \frac{(N - \operatorname{Rank}(R_p))^{V-1}}{N} \tag{6}$$

From eq. (6), it is then possible to calculate $\Gamma(V)$ for any value of V.

To find the EMG hedge ratio, a portfolio of buying the underlying asset and selling futures contract is created. The return on this portfolio is given by:

$$R_{pt} = R_{st} + X_f R_{ft} \tag{7}$$

where:

R_{pt} is the return on the portfolio at time t,

R_{st} is the return on the asset time t,

R_{ft} is the return of the futures contract at time t,[4] and

X_f is the hedge ratio.

Substituting for R_p into eq. (4), and differentiating with respect to X_f, the global minimum EMG hedge ratio is obtained :

$$X_f = \frac{-\operatorname{Cov}(R_f, (1 - F(R_p))^{V-1}}{\dfrac{\partial \operatorname{Cov}(R_f, (1 - F(R_p))^{V-1})}{\partial X_f}} \tag{8}$$

As eq. (8) shows, the optimal hedge ratio in the EMG framework depends on the value of the risk-aversion parameter, V. When V is less than 1.0 for a particular individual, the trader is a risk seeker. For the risk-neutral trader, $V = 1$, and the hedge ratio in eq. (8) reduces to zero: the trader does not adjust his position based on the covariances in eq. (8).

[3]See Shalit and Yitzhaki (1984), p. 1467.

[4]The return on the futures contract is defined as the percentage change in the futures price. Strictly speaking, however, futures contracts have no return because they require no investment. Identifying the percentage price change on the futures as the futures return is fairly standard. See, for example, CKY (1990).

Consider now the approach of CKY as a special case of eq. (8). For their analyses, the mean-Gini model arises in the special case where $V = 2$. In that situation, eq. (8) becomes:

$$X_f = \frac{-\mathrm{Cov}(R_f,(1 - F(R_p)))}{\left(\dfrac{\partial\, \mathrm{Cov}(R_f,(1 - F(R_p))}{\partial X_f}\right)} \tag{9}$$

Because CKY held $V = 2$ for all their analyses, they did not study how X_f varies with V. This issue is considered below.

EMPIRICAL RESULTS

The empirical properties of the hedge ratio in eq. (8) are tested on five commodities: corn, gold, copper, German marks, and the S&P 500 stock index futures contracts. The data consist of daily cash and futures prices for each trading day in 1989. This gives a sample of approximately 250 observations per contract.[5]

The partial derivative of eq. (8) is difficult to evaluate. Therefore, for a particular value of the risk-aversion parameter, V, an iterative process is used to estimate X_f. The iteration begins with X_f set to some arbitrary value, and returns for the portfolio are generated assuming a position hedged with X_f futures. Based on these returns, the complement of the CDF raised to power V-1 is evaluated. This evaluation is performed using the technique elaborated above and shown in eqs. (5) and (6). With a value for the complement of the CDF, 1-$F(R_p)$, one can find the EMG coefficient in eq. (4). The iteration continues by varying values of X_f until the minimum value of the extended Gini coefficient for the particular value of V is found.

This iterative technique is applied for successive values of V and for the various commodities. In particular, the EMG coefficient is estimated for values from 2 to 50 incrementing by 1, and from 60 to 200 incrementing by 10. Thus, by beginning with a value of V greater than 1.0, only risk-averse investors are considered. Further, the initial value, $V = 2$, is comparable to the approach followed by CKY. Risk-aversion factors greater than 200 are considered also. However, these higher degrees of risk aversion do not lead to hedge ratios that differ from those below 200 and, therefore, the results for these higher values of V are not reported here.

Figure 1 shows the estimated hedge ratios for the five commodities. As the graph indicates, the hedge ratio changes for different levels of the risk-aversion parameter, V. Table I summarizes the hedge ratios for selected risk-aversion factors and shows the returns for the hedged portfolios. Figure 1 and Table I together highlight five important points. First, hedge ratios for low levels of risk aversion ($V = 2$ to $V = 5$) are similar to the risk-minimizing M-V hedge ratios.[6] As Table I shows, for

[5]Because data were not available for a few dates for some contracts, the number of observations differed slightly for the different commodities.

[6]The risk-minimizing M-V hedge position is found usually by regressing the percentage price change in the spot instrument S_t for day t on the percentage price change of the futures F_t for day t:

$$S_t = \alpha + \beta F_t + \varepsilon_t$$

The resulting β-coefficient is the hedge ratio. For an elaboration of this hedging model, see Johnson (1960) or Stein (1961). As this approach leads to a risk-minimizing hedge ratio within the M-V framework, it does not formally include a risk-aversion parameter. Risk aversion is absolute, because risk is being minimized without regard to expected return. Further, this approach does not include the price expectations of the hedger in the analysis.

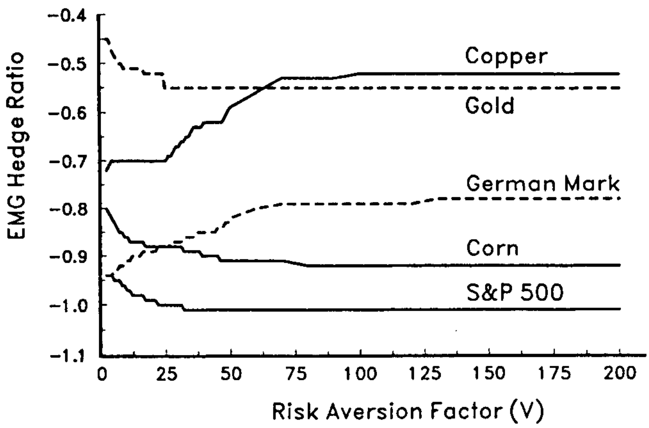

Figure 1
Hedge ratios for five commodities.

example, the risk-minimizing M-V hedge ratio for corn is $-.81$ and, with $V = 5$, the EMG hedge ratio is $-.83$. All five commodities exhibit this pattern. Second, Figure 1 and Table I both show that hedge ratios for higher levels of risk aversion are generally different from the risk-minimizing M-V hedge ratios. In Figure 1, each graph begins with a hedge ratio for $V = 2$, which is close to the risk-minimizing M-V hedge ratio. For higher values of V, the EMG hedge ratio differs significantly from the M-V hedge ratio. Notice also, that the EMG hedge ratio for higher levels of risk aversion can be either larger or smaller than the risk-minimizing M-V hedge ratio. Third, Figure 1 shows that EMG hedge ratios follow a step function. This is an important point, because it shows that traders with similar values of V can still have significantly different optimal futures positions. Fourth, while EMG hedge ratios for higher levels of V may differ widely from the M-V hedge ratio, they tend to become stable. As Figure 1 shows, for values of V above 75, the estimated hedge ratios remain quite steady. Although not shown in Figure 1, values of V above 200 are also nearly constant. Fifth, Table I shows that changes in the hedge ratio sometimes lead to significant changes in the return of the hedge portfolio. For example, in the corn contract, the EMG portfolios have a much higher return than the M-V portfolio.

The differences between the risk-minimizing M-V hedge ratio and the EMG ratios appear to be quite significant for most commodities. Comparing the M-V hedge ratio with the EMG ratio for $V = 100$, it is found that the hedge position differs by 13.58% for corn, 33.96% for copper, 17.72% for the mark, 10.87% for gold, and 7.45% for the S&P 500.[7] In some instances, these different hedge positions lead to very dramatic differences in returns, as Table I summarizes. Comparing the risk minimizing M-V hedge ratio with the EMG ratio for $V = 100$, these return differences range from the fairly trivial (German mark, 1.1%) to the quite significant (S&P 500, 17.6%), to the dramatically different (corn, 643%).

In M-V framework, it is expected that increasingly risk-averse traders hedge an increasing percentage of their positions by selling more futures. At the limit, the most risk-averse trader sells the number of futures contracts indicated by the risk minimizing M-V hedge ratio, and they are fully hedged. Under the EMG approach,

[7]These percentage differences are computed from the lower of the risk-minimizing M-V hedge ratio or the EMG ratio with $V = 100$.

Table I
COMPARISON OF HEDGE RATIOS ACROSS COMMODITIES

Commodity	Risk-Aversion Factor	Return	Percentage Change in Return Compared to MV Return	Hedge Ratio
Corn	M-V	−0.00000835	0.0	−0.81
	$V = 5$	0.00000238	128.0	−0.83
	$V = 25$	0.00002922	449.0	−0.88
	$V = 50$	0.00004533	643.0	−0.91
	$V = 75$	0.00004533	643.0	−0.91
	$V = 100$	0.00004533	643.0	−0.92
Copper	M-V	−0.00107800	0.0	−0.71
	$V = 5$	−0.00108100	−0.3	−0.70
	$V = 25$	−0.00108100	−0.3	−0.70
	$V = 50$	−0.00110700	−2.7	−0.59
	$V = 75$	0.00112180	−4.0	−0.53
	$V = 100$	0.00112180	−4.0	−0.53
German mark	M-V	0.00015590	0.0	−0.93
	$V = 5$	0.00015590	0.0	−0.93
	$V = 25$	0.00015590	0.0	−0.88
	$V = 50$	0.00015470	−0.8	−0.82
	$V = 75$	0.00015420	−1.1	−0.79
	$V = 100$	0.00015420	−1.1	−0.79
Gold	M-V	0.00017840	0.0	−0.46
	$V = 5$	0.00018660	4.6	−0.49
	$V = 25$	0.00019255	7.9	−0.51
	$V = 50$	0.00019255	7.9	−0.51
	$V = 75$	0.00019255	7.9	−0.51
	$V = 100$	0.00019255	7.9	−0.51
S&P 500	M-V	0.00021610	0.0	−0.94
	$V = 5$	0.00021610	0.0	−0.95
	$V = 25$	0.00017810	−17.6	−1.00
	$V = 50$	0.00017810	−17.6	−1.01
	$V = 75$	0.00017810	−17.6	−1.01
	$V = 100$	0.00017810	−17.6	−1.01

this intuition holds for some commodities, but not for others. For the examples of gold, corn, and the S&P 500, the percentage of the spot position hedged increases with the risk-aversion factor. However, for copper and the German mark, increasingly risk-averse traders hedge less of their position.

To understand this result more fully, consider the corn contract as an example. As investors become more risk averse, they sell more corn futures contracts. This reduction in risk is presumably offset with a lower level of return. Table I shows for corn, however, that the return increases with the percentage hedged and the degree of risk aversion. This apparently anomalous result arises because the average return for both spot corn and corn futures are negative over the estimation period. Therefore, by increasing the number of futures contracts sold, the trader actually achieves a greater return. If, on the other hand, both the average returns of the spot and the futures are positive, then selling more futures leads to a lower return, as is the case in the S&P 500 contract.

Figure 1 illustrates that as investors become more risk averse they sometimes sell more futures contracts than the risk-minimizing M-V hedge ratio implies—as in the case of corn, gold, and S&P 500. Sometimes, however, very-averse traders sell fewer contracts than the risk-minimizing M-V hedge ratio implies, as in the case of copper and German marks. This phenomena may be explained by the relationship between the spot and future prices used to estimate the hedge ratios. For corn, gold, and the S&P 500 contracts, the futures price usually exceeds the spot price. For copper, the spot price often exceeds the futures price. Gold and the S&P 500 almost always trade at full carry, while corn is approximately a cash-and-carry market.[8] Copper, by contrast, often trades below full carry, and even copper prices are characterized often by backwardation.[9] Generalized from these four commodities, it appears that EMG hedge ratios for highly risk-averse traders are likely to fall below risk-minimizing M-V hedge ratios for cash-and-carry commodities. Similarly, it seems that EMG hedge ratios for highly risk-averse traders for goods like copper are likely to be greater than risk-minimizing M-V hedge ratios. The results for the German mark, however, appear anomalous. The German mark is essentially a full carry commodity, but the EMG hedge ratio increases in Figure 1 for higher levels of risk aversion. Generally, for full carry commodities, investors sell more futures contracts as they become more risk averse. For a commodity with frequent backwardation, like copper, increasingly risk-averse traders do not necessarily increase the number of futures contracts sold.

So far in the analysis, consideration is given to hedge ratios estimated over the entire sample period of one year, or about 250 trading days per contract. Consider now how hedgers might adjust their hedge ratios as new information becomes available. It's assumed that a trader estimates the hedge ratio based on the most recent 50 days of trading activity and updates the hedge ratio each day. This gives a moving window of the 50 most recent trading days, and the trader relies upon this data to estimate the hedge ratio. The risk-minimizing M-V hedge ratio and EMG hedge ratios with risk-aversion parameters of $V = 5$, $V = 50$, and $V = 100$ are considered.

In general, the risk-minimizing M-V hedge ratios are quite stable across time. When they change, they do so gradually, consistent with the slight changes in data that arise each day as the oldest day is substituted and the most recent day is added. The dynamic hedge ratios for $V = 5$ follows essentially the same path as the M-V hedge ratios. Using a buy-and-hold strategy would appear to be adequate for weakly risk-averse investors as the hedge ratios do not change substantially over the life of the contract, but the same cannot be said for strongly risk-averse investors.

Compared with the risk-minimizing M-V hedge ratio and the EMG hedge ratio for slightly risk-averse traders ($V = 5$), the EMG ratios for highly risk-averse investors ($V = 50$ or $V = 100$) are quite volatile. As noted before, EMG hedge ratios follow a step function as the risk-aversion parameter increases. EMG hedge

[8]In saying that a market trades at full carry or that a commodity is a cash-and-carry commodity, it is meant that prices in the market closely conform to the relationship $F = S(1 + C)$. That is, the futures price (F) equals the spot price (S) grossed up by the costs of carrying the spot good to delivery against the futures contract (C). The principal elements of the cost-of-carry (C) are: interest expense, storage, and insurance.

[9]Backwardation means that the spot price strictly exceeds the futures price.

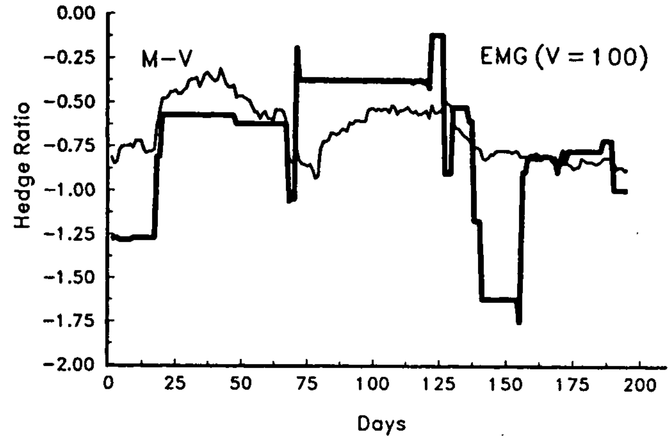

Figure 2
Dynamic hedge ratios for copper.

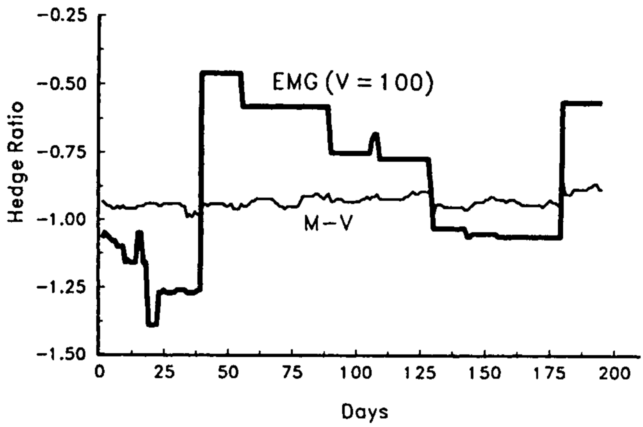

Figure 3
Dynamic hedge ratios for S&P 500.

ratios for highly risk-averse traders are also quite volatile over time, even for an investor with a constant risk-aversion parameter. Figures 2 and 3 illustrate the differences between risk-minimizing M-V and EMG hedge ratios across time for the copper and S&P 500 contracts, respectively.

Earlier, it was noted that copper is a commodity that is often in backwardation, so the commodity exhibits only a weak link between the futures price and the spot price plus the cost of carrying the spot commodity forward to the futures delivery. Not surprisingly, the risk-minimizing M-V hedge ratio is fairly volatile as the thin line in Figure 2 indicates. However, the EMG hedge ratio for $V = 100$, also shown in Figure 2, is much more volatile. The EMG hedge ratio generally follows the same pattern of increases and decreases as the M-V hedge ratio, but its amplitude is much greater. Further, the EMG hedge ratio follows a step function. This means that the replacement of just one observation in the estimation sample can lead to substantial changes in the optimal hedge ratio. This principle is further illustrated by Figure 3 for the S&P 500 contract. For the S&P 500, the risk-minimizing M-V hedge ratio is quite stable and has values near 0.9. However, the EMG hedge ratio for $V = 100$ varies quite dramatically, even when the M-V hedge ratio changes very little. As these graphs show, highly risk-averse investors who adopt a buy-and-hold strategy, may be significantly mishedged unless they adjust the hedge over time.

CONCLUSION

This study proposes the use of the extended mean-Gini coefficient to measure dispersion in portfolios hedged with futures. The theoretical advantage of the extended mean-Gini coefficient over variance is that extended mean-Gini efficient sets are second-order stochastic dominant, and apply to any probability density function, provided that the first absolute moment exists. By contrast, M-V efficient sets can be dominated stochastically. By using the extended mean-Gini coefficient to develop hedge ratios, it is shown how to determine hedge ratios suitable for different classes of risk-averse investors.

The empirical results of this article demonstrate that, for low levels of risk aversion ($V = 2$ to $V = 5$), investors adopt hedge ratios similar to risk-minimizing M-V hedge ratios. More risk-averse investors adopt hedge ratios that differ substantially from those M-V investors would select. Whether more risk-averse investors sell or buy more futures seems to depend upon the relationship between spot and futures prices. For commodities that trade in cash-and-carry markets, with the futures price exceeding the spot price by the cost-of-carry, more risk-averse investors tend to sell more futures contracts than the risk-minimizing M-V hedger. The opposite appears to hold if the market is not a cash-and-carry market. It is shown also that if one adopts a dynamic hedging strategy, EMG hedge ratios for moderately to strongly risk-averse investors are considerably more volatile than those favored by risk-minimizing M-V investors. As a consequence, strongly risk-averse investors that follow a buy-and-hold strategy run the risk of being significantly mishedged throughout the contract's life.

The questions left unanswered in this article constitute an agenda for future research: Do these results extend across other commodities and time frames? Why do the hedge ratios follow a step function, both with respect to changes in the risk-aversion parameter and across time? Though these and other questions remain unanswered, the extended mean-Gini approach appears to offer a promising alternative to the traditional risk-minimizing M-V approach to hedging with futures.

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