Minimum Variance Hedge Ratios for Stock Index Futures: Duration and Expiration Effects

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INTRODUCTION

Minimum variance hedge ratios and their accompanying hedging effectiveness measures have become common statistics in hedging articles since Ederington (1979) popularized their use.[1] Hill and Schneeweis (1981, 1982), Figlewski (1984, 1985) and others have shown empirically that many ex-post minimum variance hedge ratios are superior to the classic or naive hedge ratio in risk–return space. But other studies have tested the ex-ante usefulness of the minimum variance hedge ratio (hereafter, HR*) and the results are mixed. In out-of-sample tests, for example, both Marmer (1986) and Lasser (1987) find that hedging, according to HR*, does not yield significantly superior results compared to the naive hedging strategy. The stability of HR* is thus in question.

This study examines the stability of HR* for MMI and S&P 500 stock index futures contracts with respect to hedge duration and time to contract expiration. Hedge durations of one, two, and four weeks are compared, and these groups are further broken down by the number of weeks remaining until contract expiration. The HR*s are analyzed to see if they exhibit predictable trends, and statistical comparisons are made with the beta (or naive or classic) hedge ratio.

Previous research has addressed the duration effect. For example, Ederington (1979); Hill and Schneeweis (1982); Marmer (1986); and Chen, Sears, and Tzang (1987) find that HR*s increase with increasing hedge duration. However, the results for stock index futures is mixed. Figlewski (1984, 1985) finds that HR*s for S&P 500 stock index futures decreases for three- and four-week durations compared to daily and weekly durations. Previous research does not identify an expiration effect, nor does it consider duration and time to expiration together. Thus, this article adds to the growing body of empirical research relating to the risk-minimizing hedge ratio.

[1]See, for example, Franckle (1980); Cicchetti, Dale, and Vignola (1981); Maness (1981); Hill and Schneeweis (1981, 1982); Wilson (1983); Junkus and Lee (1985); Marmer (1986); Figlewski (1984, 1985); Peters (1986); Lasser (1987); Chen, Sears, and Tzang (1987); Lypny (1988); and Lindahl (1989).

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The next section provides some background on stock index futures hedging, defines the minimum variance and beta hedge ratios, and develops the theory of the expiration effect.

HEDGING WITH STOCK INDEX FUTURES

Hedging with stock index futures allows portfolio managers to reduce exposure to market risk without altering the composition of their portfolio. In equilibrium, full hedge coverage completely eliminates risk, and the combined cash/futures portfolio earns the risk-free rate of return. The equilibrium relationship is:

$$F = C + C(R_f - D) \tag{1}$$

where:

F = stock index futures price;
C = cash index price;
R_f = risk-free interest rate;
D = dividend yield on the cash index.

Eq. (1) shows that the futures price in equilibrium is equal to the cash price plus the cost of carrying the cash portfolio. Because futures can be bought with virtually zero margin,[2] whereas cash must be paid for stock, the interest rate multiplied by the cash price represents part of the cost of carrying the cash portfolio. However, since holders of the cash portfolio receive dividends, whereas owners of futures contracts do not, the dividend yield is subtracted from the interest rate to determine the final cost of carry. The yields are time-related, therefore, the yield differential becomes less important as contract expiration approaches and the futures price converges toward the cash index price. At expiration the futures price equals the cash price by definition.

The fact that futures prices converge toward cash prices as expiration approaches gives theoretical merit to analyzing the expiration effect. If futures prices are also less volatile as expiration approaches, the minimum variance hedge ratio can be expected to converge toward the beta hedge ratio. This is explained in greater detail after the ratios are defined.

Minimum Variance Hedge Ratio

The application of modern portfolio theory to hedging by Johnson (1960) and Stein (1961) has resulted in a decision rule commonly known as the minimum variance hedge ratio. They define the portfolio in terms of two assets—a futures position and a spot (or cash) position. Minimizing the variance formula for the two-asset portfolio with respect to the number of futures contracts held yields the optimal hedge ratio as measured by risk reduction. In equation form:

$$\min_{x_f}: \sigma^2(\Delta p_t) = x_s{}^2\sigma^2(\Delta s_t) + x_f{}^2\sigma^2(\Delta f_t) + 2x_s x_f \mathrm{cov}(\Delta s_t, \Delta f_t) \tag{2}$$

where:

Δs_t, Δf_t, Δp_t = the price change during period t of the spot, futures, and a portfolio of spot and futures, respectively;

[2]Under current rules, margin is very low or nonexistent since T-bills can often be put up as margin for institutional investors.

x_f = the proportion of the portfolio held in futures contracts;
x_s = the proportion of the portfolio held in the spot commodity which equals 1, by assumption;
σ^2 = variance of price changes.

The solution yields the minimum-risk hedge ratio, HR*:

$$x_f^* = \mathrm{HR}^* = \mathrm{cov}(\Delta s_t, \Delta f_t)/\sigma^2(\Delta f_t) \tag{3}$$

In words, HR* is the proportion of futures contracts needed to hedge an established cash position when the hedger's goal is to maximize risk reduction. Often called the portfolio model's optimal hedge ratio, HR* equals the covariance between spot and futures price changes divided by the variance of futures price changes. Equivalently, HR* can be defined as the coefficient of the independent variable in a regression of futures price changes (the independent variable) on spot price changes (the dependent variable). When hedging an existing long cash position, futures contracts are sold short; and, for an anticipatory hedge, futures contracts are bought. The degree of hedging effectiveness associated with this strategy is given by the coefficient of determination of the regression equation, and is commonly referred to as R^2.

The fact that HR* is based on an *established* cash position should be emphasized. This study assumes the cash position is predetermined and stable, i.e., when hedging a preexisting portfolio or a portfolio which will be acquired but whose size is previously determined. A retirement fund is a good example; the size of the current portfolio is known and the periodic increases can also be estimated closely. HR* does not apply to an index arbitrage situation, i.e., where the size of the cash and futures positions are yet to be determined.

Beta Hedge Ratio

Beta hedge ratio simply refers to the portfolio's beta. The classic 1:1 hedge ratio calls for a futures position that is equal in *magnitude* but opposite in sign to the cash position. At first glance, this might be interpreted as matching the cash and futures positions dollar for dollar. However, when the cash position is a stock portfolio, the number of futures contracts for full hedge coverage needs to be adjusted by the portfolio's beta—a statistic that describes the portfolio's tendency to rise or fall in value compared to the market.

Beta is the coefficient of the independent variable in a regression of market returns (the independent variable) on cash portfolio returns (the dependent variable). It is equal to the covariance between the portfolio's return and the market's return divided by the variance of the market's return. In equation form:

$$\mathrm{beta} = \mathrm{cov}(R_s, R_M)/\sigma^2(R_M) \tag{4}$$

where:

R_s = return on the spot portfolio;
R_M = return on the market, where the index underlying the futures contract is used as a proxy for the market.

A beta of 1 refers to a portfolio of average volatility. A beta of 1.18 means the portfolio's return will rise or fall 1.18 times as fast as the average market return, and

1.18 then becomes the appropriate hedge ratio. Portfolios with betas greater than 1 thus call for larger futures positions, and portfolios with betas less than 1 can be fully hedged with smaller futures positions.

Theory of an Expiration Effect

Note the similarity between the definitions of the risk-minimizing hedge ratio, [HR* in eq. (3)] and beta [eq. (4)]. Figlewski (1985) identifies the conditions when HR* is also the cash portfolio's beta. Basically, these conditions involve nonrandom cash dividends and an equilibrium relationship between cash and futures prices. While dividends tend to be stable over time, cash and futures prices can vary significantly from equilibrium. At contract expiration, however, an equilibrium relationship must exist because futures prices are equal to cash prices by definition. Thus, it follows that the minimum variance hedge ratio might approach the beta hedge ratio as equilibrium prices are approached at expiration. This logic assumes that futures prices become less volatile as expiration is approached. Samuelson (1965), however, theorizes that futures become more volatile as expiration is approached. The evidence, so far, is mixed.[3]

Figure 1 illustrates the typical relationship found by previous studies when an ex-post minimum variance hedge ratio is compared to the beta hedge ratio. The straight line between $h = 0$ and R_f represents the equilibrium relationship (if futures price changes are exactly what they should be in a perfect hedge). In equilibrium, HR* would equal the beta hedge ratio and they would both yield the risk-free return. Futures prices frequently vary from equilibrium, however, and

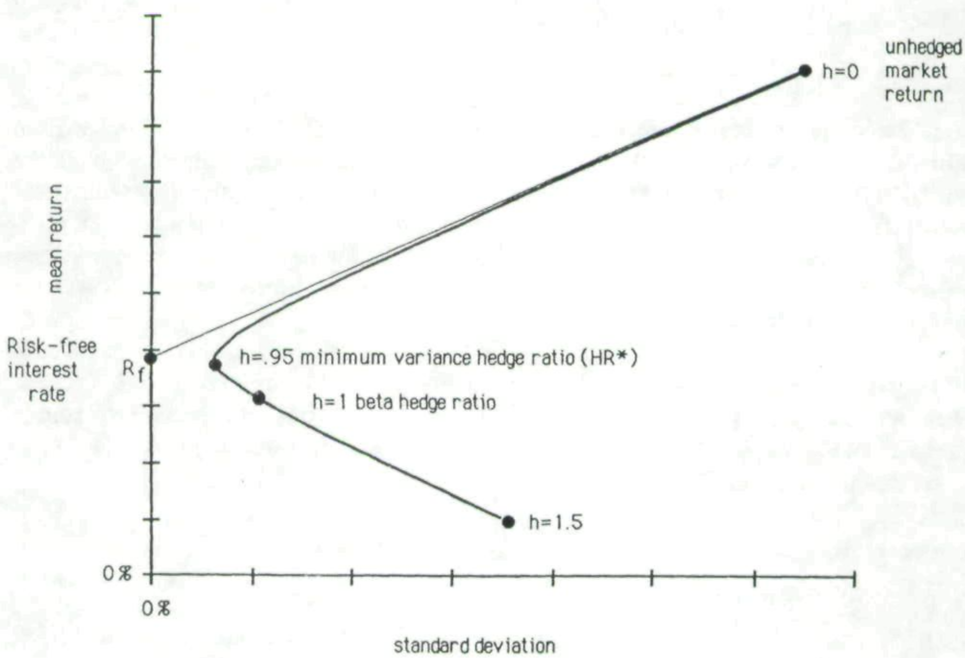

Figure 1
Typical risk–return profile for stock index futures hedges.

[3]See Castelino and Francis (1982).

they usually exhibit more volatility than cash prices. Thus, the minimum variance hedge ratio is usually less than the naive or beta hedge ratio. The curve represents the risk–return tradeoff profile as the hedge ratio is increased from 0 (the unhedged market return) to 1.5. When futures are priced closer to equilibrium, the closer the curved hedged risk–return profile is to the straight line, and the closer HR* is to the beta hedge ratio.

DATA AND METHODOLOGY

To test for the stability of the minimum variance hedge ratio, nonoverlapping one-, two-, and four-week hedge durations on the MMI and S&P 500 stock index futures contracts are defined for hedges lifted between 0 and 12 weeks before contract expiration. The respective cash indexes represent the cash portfolios, so the beta hedge ratios equal 1 for each portfolio. Minimum variance hedge ratios are calculated by regressing cash price changes against futures price changes.

Weeks are defined using Friday-to-Friday closing prices, and the futures price is represented by the nearest futures contract in the March–June–September–December series for both the MMI and S&P 500 contracts. S&P 500 futures trade only in this March quarterly cycle, while MMI futures trade in this cycle plus the first three consecutive months at any point in time.

An examination of the open interest and volume information on the MMI contracts shows that about 99% of the trading activity occurs during the last two months of contract life. A large surge in volume is evident when each of the two preceding contracts expire, and volume statistics prior to nine weeks before expiration are quite low. For this reason, MMI hedges are limited to those that are lifted within the last nine weeks of contract life, while the last 12 weeks are examined for the S&P 500 contract.

The study uses MMI data from August 1985–1989 and uses S&P 500 data from 1983–1989.[4] Cash and futures data on the MMI are from the Chicago Board of Trade through 1988. MMI data from 1989 is collected from *The Wall Street Journal.* Futures data on the S&P 500 index are from the Chicago Mercantile Exchange for 1983–1988, while cash S&P 500 data and 1989 futures data are from *The Wall Street Journal.*

To analyze the stability of HR*, with respect to hedge duration, ordinary least squares (OLS) simple regressions are run for the two respective time periods as a whole (August 1985–1989 for MMI and 1983–1989 for S&P 500) for nonoverlapping one-, two-, and four-week hedge durations that are lifted from zero to nine weeks of contract expiration for MMI and from zero to 12 weeks for S&P 500. This results in, for example, 364 observations for the one-week duration S&P 500 hedges (7 years of data * 4 quarterly expiration dates * 13 weeks per quarter).[5] The OLS simple regression can be expressed as follows:

$$\Delta s_t = \beta_0 + \beta_1 \Delta f_t + e_t \tag{5}$$

[4]The inception of the MMI contract was in August 1985. The S&P 500 contract began in 1982, but data points from that year were not included, since previous studies have reported unusual behavior during the first months that the contract traded. Deleting observations from 1983 to August 1985 were found to have no significant effect on the S&P 500 results.

[5]Similar computations result in 168 observations for the two-week S&P hedges (7 yrs * 4 qtrs * 6 per qtr); 84 observations for the four-week S&P hedges (7 yrs * 4 qtrs * 3 per qtr); 176 observations for one-week MMI hedges (4.4 yrs * 4 qtrs * 10 per qtr); 88 observations for two-week MMI hedges (4.4 yrs * 4 qtrs * 5 per qtr); and 35 observations for four-week MMI hedges (4.4 yrs * 4 qtrs * 2 per qtr).

where:

$\Delta s_t, \Delta f_t = s_t - s_{t-1}$ and $f_t - f_{t-1}$ for one-week hedges, $s_t - s_{t-2}$ and $f_t - f_{t-2}$ for two-week hedges, and $s_t - s_{t-4}$ and $f_t - f_{t-4}$ for four-week hedges;
β_0, β_1 = regression parameters where $HR^* = \beta_1$;
e_t = residual term.

To analyze the stability of HR* with respect to contract expiration date, separate OLS simple regressions [eq. (5)] are run for each of the subperiods. One-week duration hedges are separated into those that occur zero, one, two, ... etc. weeks before expiration; two-week duration hedges are subdivided into those that occur zero, two, four, ... etc. weeks before expiration; and four-week duration hedges are subdivided into those that occur zero, four, and eight weeks before expiration.[6] With four contract expiration dates per year, the 4.4 years of MMI data result in either 17 or 18 data points per subset, and the seven years of S&P 500 data result in 28 data points per subset.

To further analyze the expiration effect, HR*s are also estimated using multiple regression. The OLS multiple regression makes use of dummy variables to represent the different subsets of data (based on weeks to expiration), and is expressed as follows:

$$\Delta s_t = \beta_0 + \beta_1 \Delta f_t D_1 + \beta_2 \Delta f_t D_2 + \beta_3 \Delta f_t D_3 + \ldots + \beta_{n+1} \Delta f_t D_{n+1} + e_t \tag{6}$$

where:

$\beta_0, \beta_1, \beta_2, \ldots, \beta_{n+1}$ = regression parameters where $\beta_1, \beta_2, \ldots, \beta_{n+1}$ are HR* estimates for hedges with zero, one, two, ..., n weeks to expiration respectively;
D_1 = 1 for hedges with 0 weeks to expiration, 0 otherwise;
D_2 = 1 for hedges with 1 week to expiration, 0 otherwise;
$D_3 \ldots D_{n+1}$ = 1 for hedges with 2 ... n weeks to expiration, 0 otherwise;
n = 9 weeks for MMI hedges and 12 weeks for S&P 500 hedges;

All other variables are as defined in eq. (5). The multiple regression is statistically more powerful since only one constant is estimated, rather than one per subset as in the simple regressions.

A restricted least squares (RLS) multiple regression model is also used to analyze the data. In the RLS model, the coefficient for 0 weeks to expiration is restricted to equal 1, and this is referred to as the "base case." The coefficients of the dummy variables then represent differences from the base case. This restriction is justified both theoretically (since cash and futures prices must be at equilibrium at expiration) and empirically (since the results from the previous simple regressions show that hedges with zero weeks to expiration have HR*s that are not significantly different from 1). The RLS regression is expressed as:

$$\Delta s_t = \beta_1(\Delta s_t D_1 + \Delta f_t M_1) + \beta_2 \Delta f_t D_2 + \beta_3 \Delta f_t D_3 + \ldots + \beta_{n+1} \Delta f_t D_{n+1} + e_t \tag{7}$$

where:

β_1 = 1, by definition, for the base case (0 weeks to expiration);
$\beta_2, \ldots, \beta_{n+1}$ = regression coefficients representing the differences from the base case;
M_1 = 0 for hedges with 0 weeks to expiration, 1 otherwise;

[6]Since only the last ten weeks before expiration were examined for the MMI contract, the four-week duration subsets were divided into hedges that were lifted at zero weeks and at six weeks.

All other variables are as defined earlier in eqs. (5) and (6). When the regression coefficients representing the differences are added to the base case of 1, RLS estimates of HR* are obtained.

RESULTS

Tables I–VII summarize the duration and expiration effects for the MMI and S&P 500 stock index futures hedges. Table I provides information on the duration effect, Tables II–V provide information on the expiration effect, and the information in Tables IV and V is used to perform a linear trend analysis which is summarized in Tables VI and VII.[7]

The Duration Effect

Table I reports hedge ratio information for one-, two-, and four-week hedge durations for all hedges that are lifted within nine weeks of contract expiration for MMI futures and within 12 weeks for S&P 500 futures. The data indicates that:

1. Table I shows that the HR*s are significantly less than the beta hedge ratio, and that they increase as hedge duration increases from one to four weeks. The HR*s are 0.908, 0.95, and 0.971 for one-, two-, and four-week MMI hedges and 0.929, 0.965, and 0.97 for one-, two-, and four-week S&P 500 hedges. The four-week MMI HR* is significantly greater than the one-week HR* at the 95% confidence level, and the two-week MMI HR* is significantly greater than the one-week HR* at the 80% confidence level. The four-week S&P 500 HR* is significantly greater than the one-week HR* at the 99% confidence level, and the two-week S&P 500 HR* is significantly greater than the one-week HR* at the 95% confidence level. Earlier studies [Ederington (1979); Hill and Schneeweis (1982); Marmer (1986); and Chen, Sears, and Tzang (1987)] find similar increasing trends in HR*s; however, the evidence is mixed for stock index futures. Figlewski (1984, 1985) finds that HR*s for the S&P 500 stock index futures contract are 0.65 (daily), 0.85 (weekly), and 0.81 (three weeks) over seven months of data, and 0.67 (daily), 0.85 (weekly), and 0.83 (four weeks) over 16 months of data. Figlewski speculates that the drop in the HR*s of his three- and four-week durations are due to his small sample size. This is confirmed by the results reported in Table I. The fact that, on average, longer duration hedges are also lifted closer to contract expiration, influences the results in Table I; this is analyzed later.
2. The R^2 values in Table I also show an increase as hedge duration increases, as found in Ederington (1979); Hill, Liro, and Schneeweis (1983); Marmer (1986); and Chen, Sears, and Tzang (1987). Figlewski (1984, 1985) and Graham and Jennings (1987) find mixed results with stock index futures, and again, they speculate that the lower R^2 values for the three- and four-week durations are due to sampling error, as confirmed by this study.[8]

[7]Three outliers from the one-week MMI and one-week S&P 500 data sets that occurred during the October 1987 and October 1989 market crashes were deleted from the RLS multiple regressions and the resulting linear trend regressions that calculated the rates of convergence. The simple OLS regression results in Tables II and III are not adjusted for these outliers, and the multiple OLS regression results in Tables IV and V include both the adjusted and unadjusted figures.

[8]Even when patterns are found, however, summary statements must be made very carefully as the R^2 values are not directly comparable when the data sets of the dependent variable are different. In other words, when the length of the hedge duration and/or the proximity to contract expiration is changed, the data sets of the independent and dependent variables are also changed and R^2 values are no longer directly comparable. When this happens, larger R^2 values are not necessarily indicative of greater hedging effectiveness, since R^2 is a relative measure rather than an absolute measure of risk [see Lindahl (1989)].

Table I
MINIMUM VARIANCE VERSUS BETA HEDGE RATIOS: THE DURATION EFFECT

Stock Index Futures Contract	Hedge Duration (Weeks)	HR* Minimum Variance Hedge Ratio	Standard Errors	R^2 Coefficient of Determination	No. of Observations	Is HR* Significantly Less Than 1, the beta Hedge Ratio? (Confidence Level)
MMI	1	0.908[a]	0.014	0.959	176	Yes (99.9%)
	2	0.95	0.013	0.983	88	Yes (99.9%)
	4	0.971	0.015	0.992	35	Yes (90%)
S&P 500	1	0.929[b]	0.007	0.978	364	Yes (99.9%)
	2	0.965	0.01	0.984	168	Yes (99.9%)
	4	0.97	0.007	0.995	84	Yes (99.9%)

[a]Is significantly less than the two-week MMI hedge ratio at the 80% confidence level, and is significantly less than the four-week MMI hedge ratio at the 95% confidence level.

[b]Is significantly less than the two-week S&P 500 hedge ratio at the 95% confidence level, and is significantly less than the four-week S&P 500 hedge ratio at the 99% confidence level.

The Expiration Effect

Tables II and III report hedge ratio information for MMI and S&P 500 futures, respectively, when the data are segregated by number of weeks remaining until contract expiration. Separate simple OLS regressions are run for each of the subperiods. Data from the regressions reported in Table I are also included in Tables II and III for comparison purposes. Tables IV and V present the results from the OLS and RLS multiple regressions, which basically verify the trends shown in the simple OLS regression results of Tables II and III. The following conclusions are drawn from the data:

1. Tables II and III show that the HR* values increase toward 1, the beta hedge ratio, as hedges are lifted closer to contract expiration. The MMI results are quite consistent. For the one-, two-, and four-week hedge durations, all hedges that are lifted more than five weeks before contract expiration have HR* values that are significantly less than the beta hedge ratio. And all hedges that are lifted five weeks or closer to contract expiration have HR* values that are not significantly less than the beta hedge ratio. The 90% confidence level is used as the cutoff point for all "Yes and No" decisions. The S&P 500 results show the same trend. Only the results for week two and week eight of the one-week hedge durations are inconsistent with the pattern. All other results support the hypothesis that the minimum variance hedge ratio increases toward 1, the beta hedge ratio, as hedges are lifted closer to contract expiration.
2. The R^2 values in Tables II and III show no pattern of increasing as expiration nears, unlike previous studies. Chen, Sears, and Tzang (1987), for example, reported that the contracts closer to expiration have higher R^2 values, and thus are more effective in hedging energy price risk. Again, any summary statement involving R^2 values must be made very carefully, because R^2 is a relative measure.
3. The multiple regression results in Tables IV and V confirm the pattern exhibited by the simple regressions in Tables II and III. First, consider the OLS HR* columns for one- and two-week hedge durations for both MMI and S&P 500 futures. In general, the farther away the hedge is from expiration, the lower the minimum variance hedge ratio. No clear pattern exists for the four-week hedge durations, and the results are too sample sensitive to draw any conclusions.
4. The RLS results lend more insight to the significance of the numbers. The coefficient for the base case (zero weeks to expiration) is set equal to 1. The dummy variables are defined so that their coefficients represent the differences from the base case. As expected, larger negative differences are associated with the weeks farther from expiration. The corresponding t-statistics indicate the significance of the difference coefficients, and the largest t-statistics are usually found farthest from expiration. For example, consider the two-week hedge duration for MMI stock index futures. The t-statistics for the two-, four-, six-, and eight-week difference coefficients are -0.264, -0.658, -1.368, and -1.876, respectively. The six- and eight-week coefficients are significantly less than zero at the 82.5% and 93.6% confidence levels[9] respectively.

Linear Trend Analysis

The RLS "difference" data exhibited in Tables IV and V are further analyzed to determine a rate of convergence as the difference coefficients approach 0 and, as

[9]The confidence level is found by taking 1 minus the probability given in the $P > t$ column.

Table II
MINIMUM VARIANCE VERSUS BETA HEDGE RATIOS: THE EXPIRATION EFFECT MMI STOCK INDEX FUTURES

	Weeks to Expiration When Hedge Is Lifted	HR* Mimimun Variance Hedge Ratio	Standard Errors	R^2 Coefficient of Determination	No. of Observations	Is HR* Significantly Less Than 1, the Beta Hedge Ratio (Confidence Level)
1-week hedges	0	1.008	0.021	0.993	18	No
	1	1.039	0.018	0.995	18	No
	2	1.021	0.033	0.984	18	No
	3	0.968	0.08	0.902	18	No
	4	0.951	0.098	0.855	18	No
	5	0.978	0.024	0.99	18	No
	6	0.848	0.063	0.924	17	Yes (95%)
	7	0.846	0.068	0.912	17	Yes (95%)
	8	0.811	0.032	0.977	17	Yes (99.9%)
	9	0.926	0.04	0.973	17	Yes (90%)
	0–9	0.908	0.014	0.959	176	Yes (99.9%)
2-week hedges	0	1.029	0.013	0.997	18	No
	2	0.974	0.049	0.962	18	No
	4	0.97	0.045	0.966	18	No
	6	0.939	0.033	0.982	17	Yes (90%)
	8	0.932	0.018	0.995	17	Yes (99%)
	0–8	0.95	0.013	0.983	88	Yes (99.9%)
4-week hedges	0	0.958	0.035	0.979	18	No
	6	0.974	0.012	0.998	17	Yes (95%)
	0 and 6	0.971	0.015	0.992	35	Yes (90%)

the minimum variance estimates approach 1, the beta hedge ratio. To do this, the difference coefficients are added to the base case of 1, resulting in the RLS minimum variance hedge ratio (HR*) columns of Tables IV and V (last columns). These HR*s represent the dependent variable in the linear-trend analysis,[10] where the independent variable is represented by the number of weeks remaining until contract expiration. In equation form, the linear trend regression is expressed as:

$$HR_i^* = \beta_0 + \beta_1 X_i + e_i \tag{8}$$

where:

X_i = weeks 0, 1, 2, 3, . . . n to expiration for one-week durations, and weeks 0, 2, 4, . . . n to expiration for two-week durations;

n = 9 and 8 for MMI one- and two-week durations, respectively, and 12 and 10 for S&P 500 one- and two-week durations, respectively;

β_0, β_1 = regression parameters where β_1 indicates the rate of convergence per week of HR* toward the beta hedge ratio;

i = ith observation. The number of observations are 10, 5, 13, and 6 for one- and two-week MMI and one- and two-week S&P 500, respectively;

Other variables are as defined earlier.

Table VI presents the linear trend regression results for one- and two-week MMI hedges and one- and two-week S&P 500 hedges. Four-week hedge durations do not contain enough data points to perform the linear trend analysis. Figure 2 illustrates the linear trend regression result for the MMI two-week hedges. The rate of convergence equals the slope of the regression line which is −0.01 per week. In other words, MMI two-week hedges exhibit ex-post HR*s that increase, on average, by 0.02 for every two weeks closer to expiration. At expiration, the ex-post HR* is not significantly different from 1, the beta hedge ratio.

The results for the MMI one-week and S&P 500 one- and two-week durations are similar. The rate of convergence is −0.012 for MMI one week compared to −0.01 for MMI two-week, and −0.009 for the S&P 500 one-week compared to −0.008 for the S&P 500 two-week. All rates of convergence are significantly different from 0 at the 95% confidence level or greater, and all regression constants are not significantly different from 1, the beta hedge ratio. The rates of convergence for the two-week durations for each of the contracts are less than for the one-week durations, but not significantly so. And the HR* values for the two-week durations are larger than the corresponding HR* values for the one-week durations, but not significantly larger. The significant "duration effect" observed in Table I is really a joint duration–expiration result, since longer duration hedges are also lifted closer to contract expiration.

The average of the four rates convergence is −0.01, meaning that the minimum variance hedge ratio changes by about 1% per week during the last ten weeks of contract life. The results imply that hedging, according to the minimum variance strategy, is a dynamic process that should be monitored as hedge duration increases and as expiration is approached. To remain fully hedged, additional contracts need to be added to the original position. The rates of convergence are used to predict the minimum variance hedge ratios for the associated weeks to expiration and are given in Table VII.

[10]Linear trend regressions were also performed where the HR* values were weighted according to their standard errors, but no significant difference was found in the resulting rates of convergence.

Table III
MINIMUM VARIANCE VERSUS BETA HEDGE RATIOS: THE EXPIRATION EFFECT S&P 500 STOCK INDEX FUTURES

	Weeks to Expiration When Hedge Is Lifted	HR* Minimum Variance Hedge Ratio	Standard Errors	R^2 Coefficient of Determination	No. of Observations	Is HR* Significantly Less Than 1, the Beta Hedge Ratio? (Confidence Level)
1-week hedges	0	0.967	0.035	0.972	28	No
	1	0.997	0.027	0.984	28	No
	2	0.932	0.02	0.989	28	Yes (99%)
	3	1.004	0.026	0.985	28	No
	4	0.951	0.03	0.978	28	No
	5	0.942	0.039	0.964	28	No
	6	0.922	0.023	0.985	28	Yes (99%)
	7	0.902	0.042	0.953	28	Yes (95%)
	8	0.968	0.053	0.937	28	No
	9	0.927	0.014	0.994	28	Yes (99.9%)
	10	0.889	0.023	0.984	28	Yes (99.9%)

	11	0.928	0.036	0.969	28	Yes (90%)
	12	0.826	0.04	0.958	28	Yes (99.9%)
	0–12	0.929	0.007	0.978	364	Yes (99.9%)
2-week hedges	0	1.03	0.03	0.983	28	No
	2	0.979	0.031	0.979	28	No
	4	0.947	0.039	0.96	28	No
	6	0.921	0.024	0.984	28	Yes (99%)
	8	0.953	0.016	0.961	28	Yes (99%)
	10	0.921	0.021	0.988	28	Yes (99%)
	0–10	0.965	0.01	0.984	168	Yes (99.9%)
4-week hedges	0	0.997	0.02	0.992	28	No
	4	C.946	0.021	0.989	28	Yes (98%)
	8	0.969	0.009	0.998	28	Yes (99%)
	0–8	0.97	0.007	0.995	84	Yes (99.9%)

Table IV

MULTIPLE REGRESSION AND THE EXPIRATION EFFECT: MMI STOCK INDEX FUTURES[a]

		Regression Parameters for						
		Eq. (6), OLS		Eq. (7), RLS[b]				
Hedge Duration	Week before Expiration	Coefficient Estimates (HR*)	Standard Errors	Coefficient Estimates	Standard Errors	t-Statistic	$P > t$	HR*
1 week	0	1.011	0.052	1	0.051	19.633	0	1
	1	1.034	0.041	0.037	0.065	0.57	0.57	1.037
	2	1.006	0.04	0.011	0.065	0.175	0.862	1.011
	3	0.952	0.063	−0.051	0.08	−0.636	0.526	0.949
	4	0.952	0.066	−0.033	0.083	−0.405	0.686	0.967
	5	0.974	0.043	−0.024	0.066	−0.361	0.718	0.976
	6	0.935[a]	0.075	−0.057	0.09	−0.628	0.531	0.943
	7	0.942[a]	0.042	−0.053	0.066	−0.8	0.425	0.947
	8	0.909[a]	0.039	−0.088	0.064	−1.378	0.17	0.912
	9	0.923	0.023	−0.078	0.056	−1.397	0.164	0.922
	R^2	0.97		0.97				
	No. of obs.	173		173				

2 weeks	0	1.025	0.036	1	0.035	28.838	0	1
	2	0.981	0.036	−0.013	0.05	−0.264	0.793	0.987
	4	0.958	0.036	−0.033	0.05	−0.658	0.512	0.967
	6	0.913	0.041	−0.073	0.054	−1.368	0.175	0.927
	8	0.928	0.019	−0.074	0.039	−1.876	0.064	0.926
	R^2	0.985		0.984				
	No. of obs.	88		88				
4 weeks	0	0.964	0.027	1	0.014	72.41	0	1
	6	0.974	0.019	−0.024	0.017	−1.423	0.16	0.976
	R^2	0.992		0.998				
	No. of obs.	35		35				

[a]Data for OLS and RLS are adjusted for three outliers occurring during the October 1987 and October 1989 market crashes which affected weeks 6, 7, and 8 before expiration. Unadjusted OLS coefficients are 0.843, 0.844, and 0.810, respectively.

[b]The coefficient for week 0 is restricted to equal 1. The other coefficients are thus the "differences" from 1. The base case of 1 plus the differences equal the RLS minimum variance hedge ratios (HR*) listed in the last column.

Table V
MULTIPLE REGRESSION AND THE EXPIRATION EFFECT: S&P 500 STOCK INDEX FUTURES[a]

		Regression Parameters for						
		Eq. (6), OLS		Eq. (7), RLS[b]				
Hedge Duration	Week before Expiration	Coefficient Estimates (HR*)	Standard Errors	Coefficient Estimates	Standard Errors	t-Statistic	$P > t$	HR*
1 week	0	0.945	0.034	1	0.036	28.065	0	1
	1	0.989	0.039	0	0.053	0.004	0.997	1
	2	0.912	0.024	−0.088	0.043	−2.037	0.042	0.912
	3	0.998	0.024	−0.002	0.043	−0.048	0.962	0.998
	4	0.969	0.034	−0.008	0.049	−0.166	0.869	0.992
	5	0.932	0.028	−0.071	0.046	−1.544	0.123	0.929
	6	0.903	0.026	−0.093	0.044	−2.103	0.036	0.907
	7	0.962	0.03	−0.023	0.047	−0.502	0.616	0.977
	8	0.956[a]	0.031	−0.035	0.048	−0.726	0.468	0.965
	9	0.898[a]	0.038	−0.092	0.052	−1.767	0.078	0.908
	10	0.9	0.022	−0.099	0.042	−2.358	0.019	0.901
	11	0.936	0.025	−0.064	0.043	−1.476	0.141	0.936
	12	0.852	0.038	−0.156	0.052	−2.99	0.003	0.844

	R^2	0.976		0.975				
	No. of obs.	361		361				
2 weeks	0	0.992	0.033	1	0.034	29.719	0	1
	2	0.986	0.026	−0.014	0.043	−0.334	0.739	0.986
	4	0.905	0.033	−0.073	0.048	−1.522	0.13	0.927
	6	0.917	0.027	−0.069	0.044	−1.56	0.121	0.931
	8	0.9	0.031	−0.089	0.047	−1.899	0.059	0.911
	10	0.927	0.02	−0.07	0.039	−1.783	0.076	0.93
	R^2	0.979		0.977				
	No. of obs.	168		168				
4 weeks	0	995	0.019	1	0.019	52.355	0	1
	4	0.943	0.018	−0.03	0.027	−1.115	0.268	0.97
	8	0.973	0.009	−0.028	0.021	−1.295	0.199	0.972
	R^2	0.996		0.995				
	No. of obs.	84		84				

[a]Data for OLS and RLS are adjusted for three outliers occurring during the October 1987 and October 1989 market crashes which affected weeks eight and nine before expiration. Unadjusted OLS coefficients are 0.800 and 0.881 respectively.

[b]The coefficient for week 0 is restricted to equal 1. The other coefficients are thus the "differences from 1." The base case of 1 plus the differences equal the RLS minimum variance hedge ratios (HR*) listed in the last column.

Table VI
LINEAR TREND REGRESSION RESULTS[a]

Stock Index Futures Contract	Hedge Duration (Weeks)	Constant[b]	Standard Error (Constant)	Rate of Convergence Coefficient[c]	Standard Error (Coeff.)	R^2	No. of Observations
MMI	1	1.018	0.012	−0.012	0.002	0.767	10
	2	1.003	0.008	−0.01	0.002	0.938	5
S&P 500	1	0.996	0.019	−0.009	0.003	0.473	13
	2	0.988	0.016	−0.008	0.003	0.709	6

[a]The dependent variable is represented by the RLS coefficients from the last column in Tables IV and V and the independent variable is represented by the number of weeks to expiration.
[b]None of the constants are significantly different from 1.
[c]All regression coefficients are significant at the 95% level of confidence or greater.

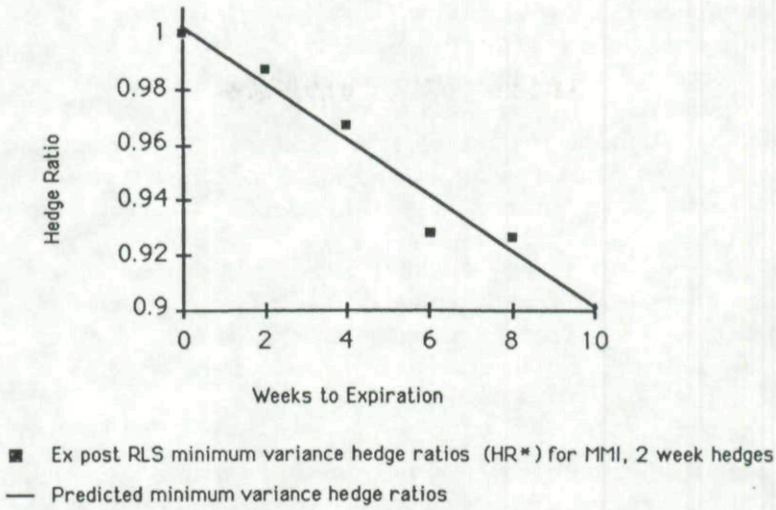

Figure 2
Linear trend regression MMI two-week hedges.

For example, consider a \$15 million stock portfolio that has a goal of matching the performance of the MMI index (beta equals 1). The date is October 9, and the manager of the fund is nervous because stocks are overvalued, and major market crashes have occurred in previous Octobers. The manager chooses to fully hedge the portfolio for two weeks to reduce all exposure to market risk. The MMI index is at 600, so one contract is worth $600 * 250 = \$150{,}000$. The nearest MMI futures

Table VII
PREDICTED MINIMUM VARIANCE HEDGE RATIOS

Contract Hedge Duration (Weeks to Expiration)	MMI (1 Week)	MMI (2 Weeks)	S&P 500 (1 Week)	S&P 500 (2 Weeks)
0[a]	1	1	1	1
1	0.988		0.991	
2	0.976	0.98	0.982	0.984
3	0.964		0.973	
4	0.952	0.96	0.964	0.968
5	0.94		0.955	
6	0.928	0.94	0.946	0.952
7	0.916		0.937	
8	0.904	0.92	0.928	0.936
9	0.892		0.919	
10			0.91	0.92
11			0.901	
12			0.892	
Rate of convergence[b]	−0.012	−0.01	−0.009	−0.008

[a]The base case (0 weeks) is set equal to 1, since the linear trend regression constants are not significantly different from 1.
[b]From the linear trend regression coefficients in Table VI.

contract expires in ten weeks, so the planned two-week hedge will be lifted when the contract is eight weeks from expiration. From Table VII, the appropriate minimum variance hedge ratio for a two-week MMI hedge that will be lifted at eight weeks is 0.92. The manager thus sells short \$15 million/150,000 * 92 = 92 contracts. Two weeks later, the hedge ratio should be increased to 0.94 to maintain full hedge coverage, and the manager should sell two additional contracts short. This biweekly increase continues until the hedge ratio is 1 (100 contracts) at expiration or until the hedge is lifted, whichever comes first.

Thus, hedging with futures should be viewed as a dynamic process, much like hedging with options. Just as the option delta is monitored, and contracts are added or subtracted based on changes in the option delta, so should contracts be added as futures hedges increase in duration and approach expiration.

CONCLUSIONS

The results of this study show that the minimum variance hedge ratios for MMI and S&P 500 stock index futures contracts increase significantly as hedge duration increases from one to four weeks. However, the duration effect is influenced by the fact that longer hedge durations are lifted closer to contract expiration. When the sample is subdivided by weeks to expiration, both simple OLS and multiple OLS and RLS regressions confirm that the minimum variance hedge ratio increases as hedges approach the contract expiration date.

Minimum variance hedge ratios are found to approach the beta hedge ratio at contract expiration and the expiration effect is further analyzed by estimating rates of convergence toward the beta hedge ratio. On average, the minimum variance hedge ratios for one- and two-week MMI and S&P 500 hedges increase by about 1% per week during the last ten weeks of contract life. The study concludes that when hedging an established cash position, hedging with futures should be viewed as a dynamic process—much like hedging with options. Just as the option delta is monitored, and contracts are added or subtracted based on changes in the option delta, so should contracts be added as futures hedges increase in duration and approach the contract expiration date.

Future research should test for the stability of the minimum variance hedge ratio in other commodities. Perhaps the trend toward the classic hedge ratio as contract expiration nears exists in other commodities as well. Also, perhaps this trend is a factor in why out-of-sample comparisons of the minimum variance hedge ratio with the classic strategy are inconclusive. The ex-ante usefulness of the portfolio model should be tested again, allowing for the rate of convergence toward the beta hedge ratio as expiration nears.

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