A GARCH Examination of the Relationship between Volume and Price Variability in Futures Markets

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INTRODUCTION

Despite a well-documented, positive relationship between trading volume and price changes of financial assets in equity markets, mixed results are reported for futures markets (see survey by Karpoff (1987)).[1] Most empirical investigations rely on convenient assumptions of a nearly continuous and (log)normally distributed price change process. However, there is substantial evidence that futures prices are not distributed normally or lognormally (e.g., Mann and Heifner (1976), Sterge (1989)). Hence, inferences based on classical statistical methods should be viewed with caution.

This article reexamines the distributional properties of futures price movements using Treasury-bond futures contracts. Then a GARCH model is used to reinvestigate the relationship between volume and price variability in Treasury-bond futures markets. The GARCH specification is found to be more appropriate than standard statistical models because it is consistent with a return distribution which is leptokurtic, and it also allows for a long-term memory in the variance of the conditional return distributions. As a result, the GARCH model is capable of mimicking observed statistical characteristics of many time series of return on financial assets.[2]

A reexamination of futures price volatility and the price variability–volume relationship has important implications for research into futures markets. First of

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[1]There is no consensus on how price change is defined. If absolute price change is used, a positive price–volume correlation is reported to exist in equity and futures markets. If price change *per se* (that is, without taking absolute value) is used, a positive price–volume correlation is reported for equity markets only. See Karpoff (1987) for a summary of empirical works.

[2]For example, Bollerslev (1987), Lamoureux and Lastrapes (1990), Baillie and Bollerslev (1989).

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all, the market crash of 1987 showed that volatilities of capital markets are subject to sudden movements which are, at best, only partially predictable. A better understanding of the movements of futures prices is important for asset pricing models and investors in futures markets. Second, the price variability–volume relationship has important implications for the design of new futures contracts (Cornell (1981)). A positive price variability–volume correlation suggests that a new futures contract can succeed only when there is "sufficient" price uncertainty with the underlying asset.

The results of this study indicate that the return process of Treasury-bond futures is best described by a GARCH (1, 1) specification. Current volatility of Treasury-bond futures prices can be explained by past volatility which tends to persist over time. When volume is included in the GARCH model and the simultaneity problem is accounted for, evidence is found of a positive price variability–volume correlation.[3] It suggests that volume helps to explain the volatility of Treasury-bond futures despite the persistence of past volatility. Contrary to the results reported by Lamoureux and Lastrapes (1990) for spot equity market, it is found that the GARCH effect persists in the Treasury-bond futures market even after volume is included in the model.

LITERATURE REVIEW

There is a saying on Wall Street that "It takes volume to make prices move." This street wisdom is confirmed by numerous empirical investigations. In a recent survey paper, Karpoff (1987) groups existing empirical investigations into: (1) those that examine the relationship between absolute price change and volume, and (2) those that examine the relationship between price change *per se* and volume. He finds that 18 of the 19 investigations in the first category report a positive correlation between absolute price change and volume; 12 of 16 studies in the second category find a positive relationship between price change *per se* and volume. In summary, he finds that there is a positive correlation between absolute price change and volume in both equity and futures markets, but there is a positive correlation between price change *per se* and volume for equity markets only.

Empirical investigations on the price–volume relationship in futures markets begin with the work of Clark (1973). He reports a positive relation between the *square* of price change and aggregated volume using daily data from the cotton futures market. Cornell (1981) examines 17 futures contracts and finds positive contemporaneous relations between changes in volume and changes in the variability of prices for each type of futures contract. Tauchen and Pitts (1983), using daily data of Treasury-bill futures, find that the variability of the price change is positively related to trading volume. Grammatikos and Saunders (1986) also report a positive correlation between volume and price-change variability for foreign currency futures.

One explanation for the positive volatility–volume correlation comes from research into the distribution of speculative prices. According to the Mixture of Distribution Hypothesis (MDH),[4] price volatility and volume traded should be positively correlated because of joint dependence on a common underlying direct-

[3]The simultaneity problem arises when volume is endogenous to the variance specification equation.

[4]See Clark (1973), Epps and Epps (1975), and Harris (1984).

ing variable, which could be interpreted as the rate of information flow to the market. In other words, both the price and trading volume change contemporaneously in response to new information.

Another explanation for the positive price–volume correlation is provided by the "sequential information" models of Copeland (1976), Jennings et al. (1981), and Smirlock and Starks (1984). In these models, information is disseminated to only one trader at a time in such a way that intermediate equilibriums occur prior to the final information equilibrium. Thus, sequential information models imply that there is a positive correlation between price volatility and trading volume in a sequential manner. In general, both the MDH and sequential information models provide theoretical support to the observed correlation between price changes and trading volume.

METHODOLOGY

The GARCH model used for this empirical analysis is developed from the autoregressive conditional heteroskedasticity (ARCH) model introduced by Engle (1982). The ARCH model allows the conditional variance to change over time as a function of past squared errors. The strength of the ARCH technique is that the conditional means and variances can be estimated jointly using traditional specified models for economic variables.

Engle's ARCH regression model is obtained by assuming that the mean of y_t (random variable) is given as $X_t\beta$ (independent variables), a linear combination of lagged endogenous and exogenous variables included in the information set Φ_{t-1} with β a vector of unknown parameters.

$$\begin{aligned} y_t|\Phi_{t-1} &\sim N(X_t\beta, h_t) \\ h_t &= \alpha_0 + \Sigma_i \alpha_i \epsilon^2_{t-i} \\ \epsilon_t &= y_t - X_t\beta \end{aligned} \tag{1}$$

Engle (1982) points out that even though the ordinary least squares produces unbiased estimators of eq. (1), an estimator using the ARCH structure, such as maximum likelihood, will be substantially more efficient.

Bollerslev (1986) extends the ARCH process to GARCH, which allows for a more flexible lag structure. Bollerslev shows that the extension of the ARCH process is very much like the extension of the standard time series process to a general ARMA model.

The GARCH (p, q) regression model is obtained by

$$\begin{aligned} \epsilon_t &= y_t - X_t\beta \\ \epsilon_t|\Phi_{t-1} &\sim N(0, h_t) \\ h_t &= \alpha_0 + \Sigma_{i=1}{}^q \alpha_i \epsilon^2_{t-i} + \Sigma_{i=1}{}^p \beta_i h_{t-i} \end{aligned} \tag{2}$$

For $p = 0$ the process reduces to the ARCH (q) process, and for $p = q = 0$ ϵ_t is just white noise. Bollerslev shows that the resulting GARCH (p, q) model is essentially a stationary ARCH (q) process. The orders of p and q can be identified by applying the traditional Box and Jenkins (1976) time series techniques to the autocorrelations and partial autocorrelations for the squared process of ϵ_t (see Bollerslev

(1987)). The following GARCH model is utilized to study the impact of volume on the Treasury-bond futures price volatility.

$$\begin{aligned} y_t &= \mu + \epsilon_t \\ \epsilon_t|\Phi_{t-1} &\sim N(0,h_t) \\ h_t &= \alpha_0 + \alpha_1\epsilon^2_{t-1} + \beta h_{t-1} + \delta V_t \end{aligned} \tag{3}$$

Where V is the volume, and y is the price change measured as $\ln(\text{close}_t/\text{close}_{t-1})$. Since the autocorrelation and partial autocorrelation for the squared residuals from eq. (3) cut off after lag one, GARCH (1,1) is selected as the appropriate model. Bollerslev (1987) also shows that GARCH (1,1) adequately fits many economic time series.

DATA AND EMPIRICAL RESULTS

Daily closing prices and volume data of Treasury-bond futures between January 1984 and August 1989 are from the Chicago Board of Trade (CBOT). Since most trading activities take place in the near-month contract, only near-month contract data are examined. As a result, this controls the maturity effect on futures prices (see Samuelson (1965)). A continuous sequence of 1434 observations of futures closing prices and volume data are gathered over the six-year period. The logarithm of the price relative is used to calculate price change. The use of logarithmic price changes prevents nonstationarity of the price level of the data from affecting futures price variability.

To assess the distributional properties of the daily bond price changes, various descriptive statistics are reported in Table I including: mean, variance, standard deviation, skewness, kurtosis, the Kolmogorov–Smirnov D statistics normality test, and a variety of order statistics. The null hypothesis of normality is rejected at the 1% level using K–S D statistics. Further evidence on the nature of deviation from normality may be gleaned from the sample skewness and kurtosis measures. While skewness is very close to zero, the kurtosis is very large.[5] This shows that much of the non-normality is due to leptokurtosis. Hence, a GARCH application is more appropriate than standard statistical models.

In Table II, the results of GARCH (1,1) analysis of price movements of Treasury-bond futures without volume are reported. Table II also reports the Box–Pierce portmanteau test statistics $Q(12)$ for an autoregressive or moving average process of order 12 in the residuals. This is one of the standard diagnostics used by Bollerslev (1987) and others. This test statistic is asymptotically equivalent to a Lagrange Multiplier and has asymptotic chi-squared distribution with 12 degrees of freedom under the null hypothesis of the residuals being uncorrelated (see Baillie and DeGennaro (1990)). This statistic provides the goodness-of-fit tests for the GARCH (1,1) model.

The results in Table II also indicate the presence of integrated GARCH (IGARCH). According to Engle and Bollerslev (1986), if $\alpha_1 + \beta = 1$ in the GARCH (1,1) process, then the model is known as IGARCH, which implies persistence of the conditional variance over all future horizons. The presence of near-integrated GARCH, or $\alpha_1 + \beta$ being close to but slightly less than unity, is documented by Bollerslev (1987) and Baillie and Bollerslev (1989).

[5]Kurtosis refers to excess kurtosis, so that a value of zero correspond to normality.

Table I
DISTRIBUTIONAL STATISTICS ON TREASURY BOND FUTURES PRICE CHANGE SERIES OF NEAR-MONTH CONTRACTS (1/3/84 TO 8/31/89)

Statistic	
Sample size	1433
Mean (μ)	0.000225
$t(\mu = 0)$	1.05063
Variance	0.0000657
Standard deviation	0.0081114
Skewness	0.0422398
Kurtosis	1.40305
D: Normal	0.050785*
Maximum	0.037844
Q3	0.004722
Median	0.000352
Q1	−0.004334
Minimum	−0.030261
Mode	0

D: Normal = Kolmogorov–Smirnov D test statistics,
Q3 = Third quartile, and Q1 = First quartile.
*Significance level = 1%.

Table II
MAXIMUM LIKELIHOOD ESTIMATES OF *GARCH* (1,1)

$$Y_t = \mu + \epsilon_t$$
$$\epsilon_t|\Phi_{t-1} \sim N(0, h_t)$$
$$h_t = \alpha_0 + \alpha_1\epsilon^2_{t-1} + \beta h_{t-1}$$

WHERE y_t IS TREASURY-BOND FUTURES PRICE CHANGE MEASURED AS LN(CLOSE$_t$/CLOSE$_{t-1}$)

Period	α_0(Thousands)	α_1	β	$\alpha_1 + \beta$	$Q(12)$
1984	0.0009	0.0192***	0.9791**	0.9983	8.79
	(0.22)	(1.75)	(75.89)		
1985	0.0018**	0.0024***	0.9646*	0.9970	4.22
	(1.92)	(1.82)	(54.67)		
1986	0.0051***	0.0053**	0.9806*	0.9800	4.48
	(1.75)	(2.03)	(122.17)		
1987	0.0069	0.0013*	0.9889*	0.9912	13.01
	(1.29)	(4.35)	(129.59)		
1988	0.0002	0.0001*	0.9992*	0.9993	18.26
	(0.16)	(2.27)	(445.58)		
1989	0.0026	0.0002*	0.9956*	0.9958	8.78
	(0.80)	(2.18)	(219.27)		
Overall	0.0013*	0.0040**	0.9940*	0.9980	11.04
	(2.47)	(2.32)	(649.25)		

Significance levels: * = 1%, ** = 5%, *** = 10%.
Asymptotic t-statistics appear in parentheses.
$Q(12)$ is the Box–Pierce portmanteau test statistics applied to the residuals.

THE RELATIONSHIP BETWEEN TRADING VOLUME AND PRICE VOLATILITY

Table III reports the coefficient estimates and asymptotic t-statistics of eq. (3). The parameters are estimated jointly, using numerical techniques to maximize the likelihood function. The GARCH analysis is performed on the entire sample period (1984–1990) and on each calendar year. The GARCH coefficients, α_1 and β, are statistically significant for all the subperiods and the overall period. These results differ from those of Lamoureux and Lastrapes (1990) in that the GARCH effect remains strongly significant for the overall period as well as in each calendar year when volume is included in the model. Moreover, the fact that the sums $(\alpha_1 + \beta)$ are fairly close to one in all situations indicates the persistence of past volatility in explaining current price volatility (see Engle and Bollerslev (1986)). These results also provide strong evidence that daily Treasury-bond futures price volatility can be characterized by a GARCH (1,1) specification. Since the estimates of the autoregressive parameters β are always greater than α_1, and the sum of these parameters is smaller than unity (except for subperiod 1984), both processes are likely to be stationary (Bollerslev (1987)). The GARCH results of Table III are consistent with previous findings for stock returns (see Akgiray (1989)), i.e., the time series of Treasury-bond futures prices exhibit significant levels of second-order dependence, and they cannot be modeled as white noise processes.

With respect to the positive price variability–volume relationship, statistically meaningful correlations are found in 1986 and 1988 only. There is no correlation between price variability and volume in 1984, 1985, 1987, 1989, and for the overall

Table III

MAXIMUM LIKELIHOOD ESTIMATES OF *GARCH* (1,1) WITH VOLUME

$$y_t = \mu + \epsilon_t$$

$$\epsilon_t | \Phi_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta h_{t-1} + \delta V_t$$

WHERE y IS TREASURY-BOND FUTURES PRICE CHANGE MEASURED AS LN(CLOSE$_t$/CLOSE$_{t-1}$)

Period	α_0(Thousands)	α_1	β	$\alpha_1 + \beta$	δ
1984	0.0002*	0.0529*	0.9543*	1.007	1.16×10^{-11}
	(2.11)	(2.31)	(5.58)		(0.15)
1985	0.0017***	0.0019*	0.9673*	0.9692	2.17×10^{-13}
	(1.68)	(2.19)	(52.71)		(0.32)
1986	0.0005	0.0076*	0.9705*	0.9781	7.98×10^{-12}**
	(0.56)	(6.07)	(85.97)		(1.90)
1987	0.0045	0.0011*	0.9918*	0.9929	1.77×10^{-13}
	(0.80)	(3.38)	(127.24)		(0.86)
1988	0.0003	0.0028*	0.9800*	0.9828	4.1×10^{-13}*
	(0.14)	(2.25)	(313.15)		(2.38)
1989	0.0026*	0.0021**	0.9956*	0.9977	2.1×10^{-12}
	(2.80)	(1.88)	(219.27)		(0.82)
Overall	0.0002*	0.0017*	0.9942*	0.9959	3.01×10^{-13}
	(2.829)	(11.129)	(927.66)		(1.583)

Significance levels: * = 1%, ** = 5%, *** = 10%.
Asymptotic t-statistics appear in parentheses.

sample period. However, one has to be cautious in interpreting the results due to the possible existence of a simultaneity problem discussed by Lamoureux and Lastrapes (1990), Bollerslev et al. (1990), and Harvey (1989). As Harvey (1989) points out, in many financial data, an equation may be part of a larger system of simultaneous equations. Volume in eq. (3) may be *endogenous* to this system; thus, the estimation procedures based on a likelihood function obtained by conditioning on these variables are inappropriate. If volume is correlated with the disturbances in the stochastic part of the model, the estimating procedures are likely to yield inconsistent estimators of the parameters in eq. (3). To overcome the simultaneity problem, Harvey (1989) points out that lagged values of endogenous variables should be used because they are classified, together with exogenous variables, as *predetermined*. Hence, treating volume as exogenous, eq. (3) is reestimated with lagged values of volume, and the results are reported in Table IV.

In Table IV, the GARCH analysis of eq. (3) is repeated with lagged volume. Similar to results in Table III, the GARCH effect is consistently significant in all calendar years and for the overall sample period. Again, the sums ($\alpha_1 + \beta$) are fairly close to one, which indicates that a larger part of Treasury-bond futures price volatility is explained by past volatility, which tends to persist over time. One major result shown in Table IV is that there is a positive and significant relationship between lagged volume and price variability for the overall period and most of the subperiods. It suggests the presence of a simultaneity problem and, therefore, lagged volume is a good instrument for contemporaneous volume in the system of simultaneous equations of eq. (3). Hence, results in Tables III and IV suggest that

Table IV
MAXIMUM LIKELIHOOD ESTIMATES OF *GARCH* (1,1) WITH LAGGED VOLUME

$$y_t = \mu + \epsilon_t$$
$$\epsilon_t | \Phi_{t-1} \sim N(0, h_t)$$
$$h_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta h_{t-1} + \delta V_{t-1}$$

WHERE y IS TREASURY-BOND FUTURES PRICE CHANGE MEASURED AS LN(CLOSE$_t$/CLOSE$_{t-1}$)

Period	α_0(Thousands)	α_1	β	$\alpha_1 + \beta$	δ
1984	0.0016	0.2561*	0.7911*	1.047	2.12×10^{-10}*
	(0.02)	(2.82)	(10.91)		(4.02)
1985	0.0015***	0.0016*	0.9675*	0.9691	1.27×10^{-12}***
	(1.68)	(2.47)	(58.94)		(1.78)
1986	0.0011	0.0050*	1.0076*	1.0126	6.09×10^{-11}*
	(0.48)	(2.17)	(48.98)		(6.61)
1987	0.0050	0.0010*	0.9919*	0.9929	1.017×10^{-14}
	(0.91)	(3.33)	(132.22)		(0.04)
1988	0.0009*	0.0023**	0.9770*	0.9793	3.45×10^{-14}*
	(2.42)	(1.93)	(283.42)		(6.17)
1989	0.0008*	0.0036*	0.9891*	0.9927	1.87×10^{-11}
	(2.55)	(6.63)	(47.08)		(1.60)
Overall	0.0021*	0.0018*	0.9938*	0.9956	3.99×10^{-12}*
	(2.82)	(12.28)	(998.58)		(2.35)

Significance levels: * = 1%, ** = 5%, *** = 10%.
Asymptotic t-statistics appear in parentheses.

volume is an endogenous variable in the system, and that there is a positive relationship between price variability and contemporaneous volume.

CONCLUSION

The empirical evidence presented in this article shows that conditional heteroskedastic processes that allow for autocorrelation in the first and second moments of returns distributions, over time, characterize Treasury-bond futures price movements satisfactorily. The analyses show that after accounting for persistency-in-volatility under a GARCH framework, trading volume contributes significant information to the return process of Treasury-bond futures. That is, similar to the findings for equity markets, the existence of a positive relationship is detected between price variability and volume in Treasury-bond futures markets. Also, the GARCH effect remains significant when volume is included in the variance equation.

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